Communication

Parametric modelling of ECG signal

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1 Introduction

LIKE MANY other natural signals, the electrocardiograph (ECG) is also a non-stationary signal (GRENIER, 1983). The burst-like QRS feature contributes localised high-frequency components in the ECG signal, making it distinctly non-stationary (WALDO and CHITRAPA, 1991). Although this feature of the QRS wave has helped detection of the wave by filtering/linear prediction (FRIESEN *et al.*, 1990), it makes the modelling of the signal very difficult.

Most of the work in modelling a ECG is non-parametric in nature (GRAHAM, 1976; WOMBLE *et al.*, 1977; JALALEDDINE *et al.*, 1990). An attempt to represent a segment of the ECG by the impulse response of a pole-zero model was unsuccessful because of its prohibitively large order (MURTHY *et al.*, 1979). Later, modelling a small segment (about a period) of the ECG by damped sinusoids was found to be superior to the earlier attempt. The method, however, fails to exploit the global nature (e.g. pseudo-periodicity) of the ECG signal (NIRANJAN and MURTHY, 1993).

The time-dependent autoregressive (AR)/autoregressive moving average (ARMA) model is the representative of the general class of non-stationary signals (GRENIER, 1983). As such, the model can also be used for the ECG signal. However, the ECG has some distinctive features; its pseudo-periodicity, and different features of the constituent signals (P, QRS and T) representing actions of various parts of the heart (GUYTON, 1985) etc. It would be useful to know how the general time-dependent AR/ARMA model is restricted by the special features of the ECG-type signals.

We show that the amplitude modulated (AM) sinusoidal signal model, which is a special case of the time-dependent AR/ARMA model, can have the periodicity property, and the model can exhibit a burst-like feature very well, when the modulating signal is an exponential function. We propose that one or more AM sinusoidal signal(s) can be employed to model separately each feature of the ECG signal. The suitability of the developed model is then investigated for the ECG signal using an analysis-by-synthesis technique.

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2 Study of the model

2.1 *Model development*

Let a single order AR process be given by

$$\begin{aligned} x(n) &= ax(n-1) + w(n), \qquad n < n_0 \\ &= ad(n)x(n-1) + w(n), \qquad n \geq n_0 \end{aligned} \tag{1}$$

where $w(n)$ is the white noise input with zero mean and variance σ^2. Initially the coefficient a is set close to $1{\cdot}0 \exp(j\theta)$, and the variance σ^2 is adjusted to such a small value that $x(n)$ becomes a sinusoid before n_0 (KAY, 1988). At time $n = n_0$ the time-dependent factor $d(n)$ is introduced. To generate an amplitude modulated sinusoid, we choose

$$d(n) = \exp(b \sin \alpha n), \qquad n > n_0 \tag{2}$$

Accordingly, the skeleton structure (TONG, 1990) of eqn. 1 becomes

$$x(n) = ad(n)x(n-1), \qquad n > n_0 \tag{3}$$

With repeated substitution, at $n = n_0 + m$ we obtain

$$\begin{aligned} x(n_0 + m) &= a^m \prod_{i=1}^{m} d(i)x(n_0) \\ &= a^m D(m)x(n_0) \end{aligned} \tag{4}$$

where

$$\begin{aligned} D(m) &= \prod_{i=1}^{m} d(i) \\ &= \exp\left(\sum_{i=1}^{m} b \sin \alpha i\right) \\ &= \exp\left(\sum_{i=0}^{m} b \sin \alpha i\right) \\ &\simeq \exp\left(-\frac{b}{\alpha}(\cos \alpha m - 1)\right) \end{aligned} \tag{5}$$

In eqn. 5 the summation is approximated by the finite integral over the appropriate interval. The assumption is valid for small α. Therefore

$$x(n_0 + m) \simeq 1{\cdot}0 \exp(j\theta m) \exp\left(\frac{b}{\alpha}(1 - \cos \alpha m)\right) x(n_0) \tag{6}$$

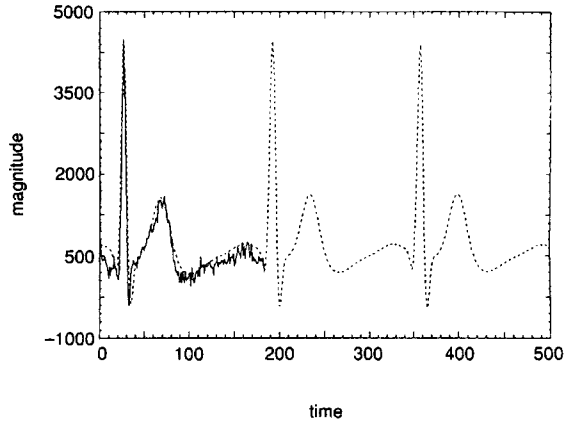

Fig. 1 *Original ECG signal, lead II configuration, 180 samples sampled at 250 Hz (solid line) and synthesised ECG signal (dashed line)*

where $a \simeq 1{\cdot}0 \exp(j\theta)$.

We define the sequence $y(m)$ as follows:

$$y(m) = \text{Real}\,[x(n_0 + m)] = A \exp\left(\frac{b}{\alpha}(1 - \cos \alpha m)\right) \cos(\theta m + \phi) \quad (7)$$

where $x(n_0) = A \exp(j\phi)$. It is apparent that $y(m)$ is an amplitude modulated (AM) sinusoid with carrier frequency θ and the modulating factor given by $\exp(-(b/\alpha)\cos \alpha m)$. Note that the term $\exp(b/\alpha)$ causes only a scaling of the signal. The presence of the exponential form helps in generating the burst-type periodic signals like ECG.

2.1.1 *Analysis-by-synthesis rules*: the signal parameters of the process model can be determined through the estimation of process parameters, as suggested by Grenier (GRENIER, 1983). In that case a suitable set of basis functions is used to express the time-varying process parameters. However, we adopt here an analysis-by-synthesis approach to find the parameters of the ECG signal directly. The approach provides some insight into the functioning of the model.

First, note that the carrier signal frequency θ must be an integer multiple of the modulating frequency α to have similar wave shape in every period of the signal $y(n)$. Under this condition, eqn. 7 can be rewritten as

$$y(m) = A \exp\left(\frac{b}{\alpha}(1 - \cos \alpha m)\right) \cos(k\alpha m + \phi) \quad (8)$$

where k is an integer.

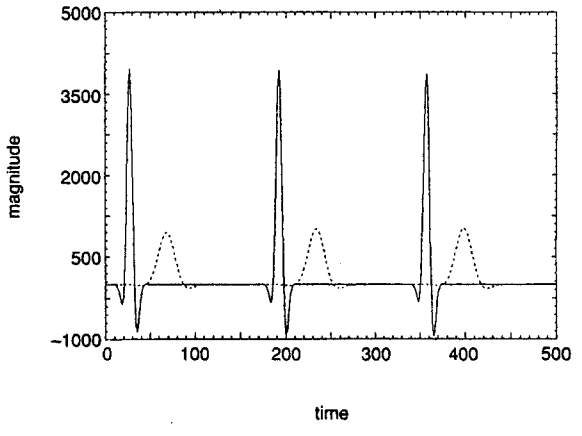

Fig. 2 *Synthesised QRS wave (solid line) and synthesised T wave (dashed line)*

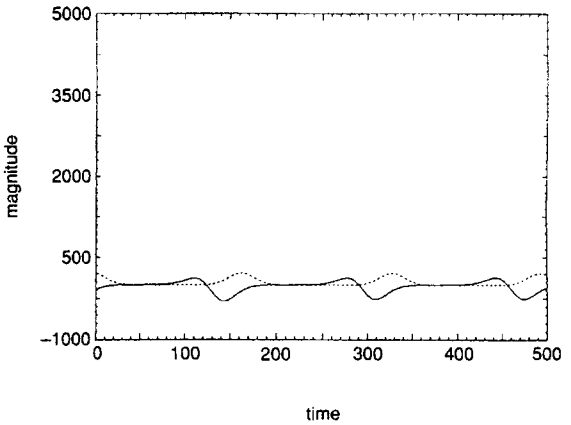

Fig. 3 *Synthesised components of P wave, P1 (solid line) and P2 (dashed line)*

To generate a complicated signal pattern, more than one AM sinusoid is needed. The most prominent feature of the signal is first fitted by one AM wave and, once fitted, this part of the original signal is removed. We continue this process on the residual signal until all the features of the signal under investigation have been exhausted. For the purpose of fitting an AM sinusoid, the following four steps are employed.

(i) We start with some initial guess of A, b and k. At the starting $\phi = 0$, and $\alpha = 2\pi/T$, where T is the period of the signal under investigation.

(ii) The values of the parameters b and k are to be altered to obtain the proper shape of the wave. The phase ϕ is also changed to decide the sign of the peak and the skewness of the hump. Therefore, the parameters b, k and ϕ are changed iteratively until the shape of the synthesised signal comes close to the targeted feature of the ECG signal, e.g. QRS complex.

(iii) Then the signal is scaled by a factor c and the origin is shifted to m_0, so that the target feature of the original signal coincides with the synthesised one. If the fitting is not satisfactory at this stage, we need to go back to the preceding and retry. The synthesised signal $z(m)$ is expressed as

$$z(m) = c\, y(m + m_0) \quad (9)$$

(iv) Once the synthesised signal is ready, we subtract it from the signal under investigation. In this way, we obtain the residual signal. If the residual signal still has a prominent feature, we need to go to the first step to start with a new AM sinusoid. When all features of the signal are exhausted, we can stop the process.

2.2 *ECG signal fitting*

The lead II ECG signal of a normal male patient, collected for one period and sampled at 250 Hz, is shown in Fig. 1. The T and QRS waves are synthesised by two separate AM sinusoids. For the P wave, two more AM sinusoids are needed, and the corresponding waves are named P1 and P2. A DC signal is also needed to match the DC bias of the original ECG signal. The successive waves are matched in the following order: T, QRS, P1 and then P2. These waves are shown in Figs. 2 and 3. While fitting the T wave, the DC bias of the signal is adjusted.

To synthesise the ECG signal components, we start with $|a| = 0{\cdot}9999$, $\sigma_w = 10^{-28}$ and $n_0 = 15000$, as defined in eqn. 1. The large n_0 value is needed for the transient to die down and for the signal to reach a steady state. However, once we have the knowledge of $x(n_0)$ through inspection, n_0 can be set to zero and the recurrence equation can be utilised with a

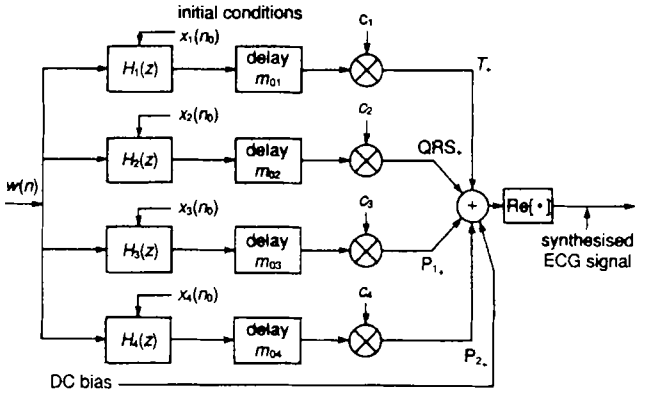

Fig. 4 *Block diagram of ECG signal synthesiser by first-order complex AR models; $H_i(z) = 1/(1 - a_i d_i(n) z^{-1})$, $x_i(n_0) = A_i \exp(j\phi_i)$; x_+ = pre-envelope of x*

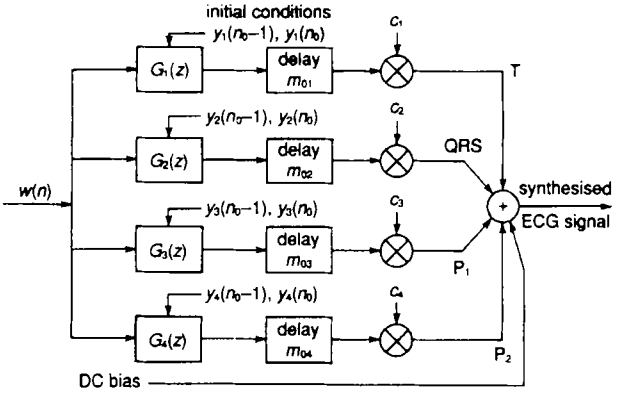

Fig. 5 *Block diagram of ECG signal synthesiser by second-order real AR models; $G_i(z) = 1/((1 - a_i d_i(n) z^{-1})(1 - a_i^* d_i(n) z^{-1}))$ $y_i(n_0 - 1) = A_i \exp((b_i/\alpha_i)(1 - \cos \alpha_i)) \cos(\phi_i - k_i \alpha_i)$, $y_i(n_0) = A_i \cos \phi_i$*

proper initial condition $x(n_0)$. The DC bias is found to be 500. The values of the other parameters for the constituent waves T, QRS, P1 and P2 are given in Table 1.

The synthesised ECG signal is shown in Fig. 1, and it closely resembles the original signal over the period of consideration. The signal-to-noise ratio (SNR) is calculated to be 9·06 dB, when the SNR is defined as

$$\mathrm{SNR} = 10.0 \log \frac{\sum_{i=1}^{n} (x_{org}(i))^2}{\sum_{i=1}^{n} (x_{org}(i) - x_{syn}(i))^2}$$

where $x_{org}(i)$ and $x_{syn}(i)$ are the original and synthesised ECG signals at the ith instant, and n is the period of the ECG signal (165 samples). The correlation coefficient ρ between the synthesised and original ECG is calculated to be 0·933. The correlation coefficient is defined by

$$\rho = \frac{\sum_{i=1}^{n} (x_{org}(n) - m_{org})(x_{syn}(n) - m_{syn})}{\sqrt{\sum_{i=1}^{n} (x_{org}(n) - m_{org})^2 \sum_{i=1}^{n} (x_{syn}(n) - m_{syn})^2}}$$

where n, $x_{org}(i)$ and $x_{syn}(i)$ are as defined earlier, and m_{org} and m_{syn} are the mean of original and synthetic ECG signals, respectively. The generating process can be represented as shown in Fig. 4. Equivalently, Fig. 5 provides the complete representation of the signal. Note that the functional block, Real [·] does not appear in Fig. 5.

3 Conclusions

The method presented here provides a way to model parametrically the ECG signal, and also its constituent waves. The method is a synthesis technique, and it guarantees the preservation of waveform. Moreover, the use of a small number of parameters (only 25 parameters are required) promises applications of the model for superior data compression, but this could only be asserted after further research.

For the analysis of the signal, direct evaluation of model parameters leads to a set of highly nonlinear equations. As an alternative, we may estimate the equivalent process parameters by the method cited by Grenier (GRENIER, 1983). The equivalence between the two approaches is an important concept, and we have formulated it in this work. Furthermore, the analysis-by-synthesis approach as demonstrated here is simple to implement. The approach clearly provides a good insight into modelling of the ECG signal (GUYTON, 1986).

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Table 1 Model parameters and their values for different waves

parameter	T wave	QRS wave	P_1 wave	P_2 wave
α, rad	0·03808	0·03808	0·03808	0·03808
b	0·19	1·1	0·1	0·1
k	2	7	1	1
ϕ, rad	0.25π	1.15π	1.6π	π
m_0	5	54	155	85
c	0·78557	4.778×10^{-21}	40·0	15·0
A	7.258×10^{-2}	6.917×10^{-2}	2.505×10^{-2}	2.505×10^{-2}

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