

PARAMETRIC STOCHASTIC CONVEXITY AND CONCAVITY OF STOCHASTIC PROCESSES*

MOSHE SHAKED¹ AND J. GEORGE SHANTHIKUMAR²

¹*Department of Mathematics, University of Arizona, Tucson, AZ 85721, U.S.A.*

²*School of Business Administration, University of California, Berkeley, CA 94720, U.S.A.*

(Received October 19, 1988; revised June 27, 1989)

Abstract. A collection of random variables $\{X(\theta), \theta \in \Theta\}$ is said to be parametrically stochastically increasing and convex (concave) in $\theta \in \Theta$ if $X(\theta)$ is stochastically increasing in θ , and if for any increasing convex (concave) function ϕ , $E\phi(X(\theta))$ is increasing and convex (concave) in $\theta \in \Theta$ whenever these expectations exist. In this paper a notion of directional convexity (concavity) is introduced and its stochastic analog is studied. Using the notion of stochastic directional convexity (concavity), a sufficient condition, on the transition matrix of a discrete time Markov process $\{X_n(\theta), n = 0, 1, 2, \dots\}$, which implies the stochastic monotonicity and convexity of $\{X_n(\theta), \theta \in \Theta\}$, for any n , is found. Through uniformization these kinds of results extend to the continuous time case. Some illustrative applications in queueing theory, reliability theory and branching processes are given.

Key words and phrases: Sample path convexity and concavity, Markov processes, directional convexity and concavity, single stage queues, super-modular and submodular functions, L -superadditive functions, reliability theory, branching processes, shock models, total positivity.