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Thin-Walled Structures 40 (2002) 691–702

THIN-WALLED STRUCTURES

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Parametric stress distribution in shell-of-revolution sludge digesters of parabolic ogival form

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Received 1 June 2001; received in revised form 20 October 2001; accepted 20 November 2001

Abstract

Egg-shaped sludge digesters have become popular in relatively recent times owing to their superior functional performance and lower maintenance costs in comparison with conventional cylindrical digesters. These innovative structures are usually constructed as thin shells of revolution in concrete, designed to withstand principally the hydrostatic pressure loading from the contained liquid. As regards the precise shape of the egg shell, a number of mathematical shell surfaces may be envisaged, and the stress distribution will very much depend on the chosen form. In this paper, it is desired to explore the possible adoption of the parabolic ogival shell as a sludge digester. The stress distribution in such a shell is expressed in terms of a single governing parameter ξ, greatly facilitating the investigation. For various values of ξ covering the most practical range for egg-shaped digester shells, recommendations are made regarding the positioning of supports. Taking into account maximisation of tank capacity, minimisation of peak stress resultants in the shell, and ease of prestressing, the best range of ξ for parabolic ogival digester shells is identified. The overall conclusion is that from a structural and functional point of view, the parabolic ogival profile is suitable for adoption in the design of egg-shaped concrete sludge-digester shells. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Egg-shaped sludge digesters; Shell structures; Containment structures; Shells of revolution; Ogival shells; Shell analysis; Membrane hypothesis; Bending theory of shells

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PII: S0263-8231(02)00020-4

1. Introduction

As is now generally well known, egg-shaped shell-of-revolution sludge digesters offer several distinct advantages over conventional digesters of wide cylindrical shape. The smooth curved profile of the egg-shaped digester permits better mixing of the sludge, while the greater volume-to-surface ratio of the egg shape reduces heat losses. The better circulation of sludge in the egg-shaped digester results in a reduced accumulation of deposits at the bottom of the digester, and consequent reductions in maintenance costs. Furthermore, the deposits that do settle to the bottom of the egg shell are easy to remove as they all collect in one relatively small area at the capular or pointed bottom of the digester, and the removal of the deposits may be carried out on a continuous basis. Similarly, the capular or tapered shape of the top of the egg-shaped digester allows the crust that forms on the surface of the sludge to be removed more conveniently than were the crust spread-out over the larger surface area of the conventional wide cylindrical digester. Evidently, the more complex geometry of the egg shell implies higher initial costs of construction, but these are offset in the long term by the lower maintenance costs. For all these reasons, a significant number of egg-shaped sludge digesters have been constructed in the recent past in countries such as the USA, Japan, Taiwan, Germany and Australia [1–3].

Despite this fairly widespread adoption of the egg shape for sludge containment, not much information is available in the literature on the detailed analysis and structural behaviour of egg-shaped sludge digesters, which partly explains why these large shell structures are not as common as they should be in other countries around the world. Taking into account both hydrostatic and seismic loading, Guggenberger [4] has considered the collapse design of egg-shaped steel digester tanks, with particular attention on the stability aspects of the structural problem. In an effort to increase the analytical data available to the designer of these structures, a study has just been completed on membrane and discontinuity effects in egg-shaped sludge digester shells comprising spherical ends and a middle circular ogival portion [5,6]. Another practical configuration currently under investigation is an assembly of two conical frusta (joined at their larger ends to form the equatorial junction of the digester) and two conical shells (forming the top and bottom closures of the digesters); closed-form results for discontinuity effects at the junctions of such shell assemblies have already been developed [7]. Noting the potential of the parabolic ogival shell as a form of egg-shaped sludge digester (the parabolic ogival shell has pointed ends and a bulging middle), the present study evaluates the structural feasibility of this shape on the basis of shell theory. Results of relevant stress resultants are presented in generalised parametric form, and design recommendations are made.

2. Geometric aspects of the parabolic ogival shell of revolution

The parabolic ogival shell of revolution is formed by rotating through 360° a parabola that is symmetrical about the horizontal x-axis, about the vertical y-axis (which therefore is the axis of revolution of the shell), giving a meridional cross-section as depicted in Fig. 1. The shell is therefore symmetrical not only about the vertical *y*-axis of rotation, but also about the horizontal 'equatorial' plane containing the *x*-axis. Let the overall height of the shell be *H*, and the equatorial diameter be *D*, as shown in Fig. 1.

With the origin *O* taken at the intersection of the axis of revolution and the equatorial plane, the equation of the generating meridian of the shell of revolution is

$$\frac{D}{2}-x = ky^2 \tag{1}$$

where k is a constant.

When $x = 0$, $y = \pm H/2$. From this condition, it follows that

$$k = \frac{2D}{H^2} \tag{2}$$

From Eq. (1), and making use of result (2), we may write

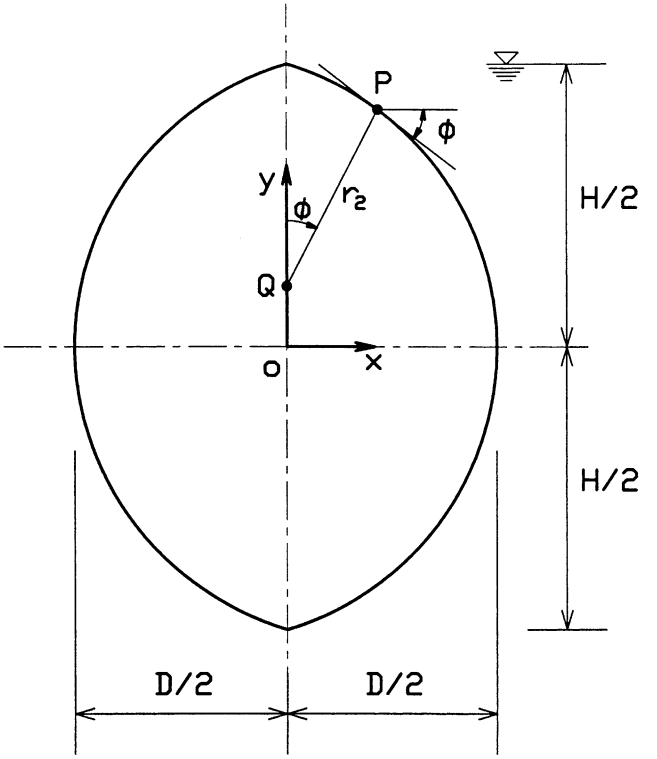

Fig. 1. Geometrical parameters of the parabolic ogival shell of revolution.

$$y = \pm\frac{1}{\sqrt{k}}\left(\frac{D}{2}-x\right)^{1/2} = \pm\frac{H}{\sqrt{2D}}\left(\frac{D}{2}-x\right)^{1/2} \tag{3}$$

If we define the angular coordinate ϕ as the angle measured from the upward direction of the axis of revolution of the shell to the normal to the shell midsurface at the point P in question (refer to Fig. 1), it is evident that

$$\tan\phi = -\frac{\mathrm{d}y}{\mathrm{d}x} = \pm\frac{H}{2\sqrt{2D}}\left(\frac{D}{2}-x\right)^{-1/2} \tag{4}$$

At the upper pole ($x = 0$, $y = +H/2$), if $\phi = \phi_o$, then

$$\phi_o = \tan^{-1}\left(\frac{+H}{2D}\right) \tag{5a}$$

Similarly, at the lower pole ($x = 0$, $y = -H/2$), if $\phi = \phi_o'$, then

$$\phi_o' = \tan^{-1}\left(\frac{-H}{2D}\right) = \pi-\phi_o \tag{5b}$$

From Eq. (4), we have

$$\tan^2\phi = \frac{H^2}{8D}\left(\frac{D}{2}-x\right)^{-1} \tag{6}$$

leading to the result

$$x = \frac{4D^2\sin^2\phi - H^2\cos^2\phi}{8D\sin^2\phi} \tag{7}$$

When $\phi = 90°$, Eq. (7) yields $x = D/2$, as expected.

At any given point P of the shell midsurface, the two principal radii of curvature are denoted by r_1 and r_2. The first (r_1) is the actual radius of curvature of the parabolic meridian at the point P, while the second (r_2) is equal to the distance between P and Q, where Q is the point of intersection of the axis of revolution of the shell, and the normal to the shell midsurface at point P (refer to Fig. 1). Thus, from Fig. 1, we may write

$$r_2 = \frac{x}{\sin\phi} = \frac{4D^2\sin^2\phi-H^2\cos^2\phi}{8D\sin^3\phi} \tag{8}$$

The other radius of curvature is given by the usual relationship

$$r_1 = \frac{\left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{3/2}}{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} \tag{9}$$

Now, from Eq. (4),

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mp\frac{H}{4\sqrt{2D}}\left(\frac{D}{2}-x\right)^{-3/2} \tag{10}$$

Eqs. (4), (7) and (10) enable the evaluation of r_1 from Eq. (9), yielding

$$r_1 = \frac{[4D(D-2x) + H^2]^{3/2}}{4DH} = \frac{H^2}{4D\sin^3\phi} \tag{11}$$

(The negative symbol has been dropped, since r_1 is positive throughout.)

3. Loading components and shell stress resultants

Assuming the shell is completely filled with liquid of weight γ per unit volume, the depth of liquid at the vertical coordinate y is given by

$$d = \frac{H}{2}-y \tag{12a}$$

From Eqs. (3) and (4),

$$y = \pm\frac{H}{\sqrt{2D}}\left(\frac{D}{2}-x\right)^{1/2} = \left(\pm\frac{H}{\sqrt{2D}}\right)\left(\pm\frac{H}{2\sqrt{2D}\tan\phi}\right) = \frac{H^2}{4D\tan\phi} \tag{12b}$$

Eq. (12a) becomes

$$d = \frac{H}{2}-\frac{H^2}{4D\tan\phi} = \frac{2HD\sin\phi-H^2\cos\phi}{4D\sin\phi} \tag{12c}$$

Thus, the loading component p_r normal to the shell midsurface is given by

$$p_r = \gamma d = \gamma\left(\frac{2HD\sin\phi-H^2\cos\phi}{4D\sin\phi}\right) \tag{13}$$

The loading component p_ϕ in the direction of the tangent to the shell meridian is, of course, zero, since hydrostatic pressure acts purely perpendicular to the shell midsurface (Eq. (13)).

As is well known, for shells of revolution subjected to distributed loadings that vary smoothly, continuously and ‘not too rapidly’ [8] over the surface of the shell, the membrane or ‘momentless’ hypothesis [9] accurately predicts the state of stress in the interior of the shell, provided the shell geometry (thickness of shell, slope of the meridian, principal radii of curvature) also exhibits the same smoothness properties. Both the loading and shell geometry of present considerations conform to these requirements, so that the membrane solution should be adequate throughout, except in the lowest zones surrounding the bottom pole, over which the shell is assumed to be supported.

Since hydrostatic loading is axisymmetric, the only interior shell stress resultants of relevance in the present problem are N_ϕ (in the meridional direction) and N_θ (in the hoop direction); these are forces per unit length of the respective edge of a shell element, considered positive when tensile. General expressions for these are as follows (see, for instance, Flügge [10], Zingoni [11] or Gould [12]):

$$N_\phi = \frac{1}{r_2 \sin^2\phi}\left[\int r_1 r_2 (p_r \cos\phi - p_\phi \sin\phi)\sin\phi \, d\phi + C\right] \tag{14a}$$

$$N_\theta = r_2\left(p_r - \frac{N_\phi}{r_1}\right) \tag{14b}$$

where C is a constant of integration.

Using expressions (8), (11) and (13) for r_2, r_1 and p_r, respectively, and noting that $p_\phi = 0$, we evaluate the integral in Eq. (14a) which, after some simplifications, leads to the result

$$\begin{aligned} N_\phi = {} & \frac{\gamma H^3}{16D^2}\left(\frac{\sin\phi}{4D^2\sin^2\phi - H^2\cos^2\phi}\right)\Bigg[-\frac{D(4D^2+H^2)}{\sin^2\phi} + \frac{DH^2}{2\sin^4\phi} - H(4D^2 \\ & + H^2)\left(\frac{\cos\phi}{\sin\phi}\right) + \frac{H}{3}(4D^2+2H^2)\left(\frac{\cos\phi}{\sin^3\phi}\right)(1+2\sin^2\phi) - \frac{H^3}{15}\left(\frac{\cos\phi}{\sin^5\phi}\right)(3 \\ & + 4\sin^2\phi + 8\sin^4\phi) + C\Bigg] \end{aligned} \tag{15}$$

At the apex ($\phi = \phi_o$), $N_\phi = 0$ so that

$$\begin{aligned} C = {} & \frac{D(4D^2+H^2)}{\sin^2\phi_o} - \frac{DH^2}{2\sin^4\phi_o} + H(4D^2+H^2)\left(\frac{\cos\phi_o}{\sin\phi_o}\right) - \frac{H}{3}(4D^2 \\ & + 2H^2)\left(\frac{\cos\phi_o}{\sin^3\phi_o}\right)(1+2\sin^2\phi_o) + \frac{H^3}{15}\left(\frac{\cos\phi_o}{\sin^5\phi_o}\right)(3+4\sin^2\phi_o+8\sin^4\phi_o) \end{aligned} \tag{16}$$

With N_ϕ now known, the hoop stress resultant follows from Eq. (14b) which, after eliminating r_1, r_2 and p_r, may be rewritten as

$$N_\theta = (4D^2\sin^2\phi - H^2\cos^2\phi)\left[\frac{\gamma(2HD\sin\phi - H^2\cos\phi)}{32D^2\sin^4\phi} - \frac{N_\phi}{2H^2}\right] \tag{17}$$

The actual stresses in the meridional and hoop directions are calculated in the usual manner:

$$\sigma_\phi = \frac{N_\phi}{t}\,;\ \sigma_\theta = \frac{N_\theta}{t} \tag{18}$$

4. Volume capacity of tank

Considering elemental horizontal discs of radius x and thickness dy, the volume of the tank is twice the integral summation of such discs between the equatorial plane and the apex, that is

$$V = 2\int_0^{H/2} \pi x^2 \mathrm{d}y \tag{19}$$

From Eqs. (1) and (2),

$$x = \frac{D}{2} - ky^2 = \frac{D}{2} - \frac{2D}{H^2}y^2 \tag{20}$$

Therefore

$$V = 2\pi \int_0^{H/2} \left(\frac{D^2}{4} - 2\frac{D^2}{H^2}y^2 + 4\frac{D^2}{H^4}y^4\right)\mathrm{d}y = 2\pi\left[\frac{D^2}{4}y - \frac{2D^2}{3H^2}y^3 + \frac{4D^2}{5H^4}y^5\right]_0^{H/2} \tag{21}$$

$$= \frac{2\pi}{15}D^2H$$

5. Parametric results

Making use of Eq. (5a) to eliminate $\sin\phi_o$ and $\cos\phi_o$ from expression (16), we obtain, after simplifications

$$C = \frac{D}{30H^2}(112D^4 + 120D^2H^2 + 15H^4) \tag{22}$$

Defining a non-dimensional parameter of the ogival shell

$$\xi = \frac{H}{D} \tag{23}$$

(that is, ξ is the height-to-diameter ratio of the tank) allows Eq. (22) to be recast in the form

$$C = \frac{D^3}{30\xi^2}(112 + 120\xi^2 + 15\xi^4) \tag{24}$$

The constant C may be eliminated from expression (15) on the basis of relation (24); when further use is made of relation (23) to eliminate D from expressions (15) and (17), the following non-dimensional form of the results for the stress resultants is finally obtained:

$$\frac{N_\phi}{\gamma H^2} = \frac{\xi}{16}\left(\frac{\sin\phi}{4\sin^2\phi - \xi^2\cos^2\phi}\right)\left[-\left(\frac{4 + \xi^2}{\sin^2\phi}\right) + \left(\frac{\xi^2}{2\sin^4\phi}\right) - \xi(4\right.$$

$$+ \xi^2)\left(\frac{\cos\phi}{\sin\phi}\right) + \frac{\xi}{3}(4 + 2\xi^2)\left(\frac{\cos\phi}{\sin^3\phi}\right)(1 + 2\sin^2\phi) - \frac{\xi^3}{15}\left(\frac{\cos\phi}{\sin^5\phi}\right)(3 \tag{25a}$$

$$+ 4\sin^2\phi + 8\sin^4\phi) + \frac{1}{30\xi^2}(112 + 120\xi^2 + 15\xi^4)\Bigg]$$

$$\frac{N_\theta}{\gamma H^2} = \frac{1}{32\xi^2}(4\sin^2\phi - \xi^2\cos^2\phi)\Bigg[\left(\frac{2\xi\sin\phi - \xi^2\cos\phi}{\sin^4\phi}\right)$$
$$-\left(\frac{\xi\sin\phi}{4\sin^2\phi - \xi^2\cos^2\phi}\right)\times\Bigg[-\left(\frac{4+\xi^2}{\sin^2\phi}\right) + \left(\frac{\xi^2}{2\sin^4\phi}\right) - \xi(4+\xi^2)\left(\frac{\cos\phi}{\sin\phi}\right) \tag{25b}$$
$$+ \frac{\xi}{3}(4 + 2\xi^2)\left(\frac{\cos\phi}{\sin^3\phi}\right)(1 + 2\sin^2\phi) - \frac{\xi^3}{15}\left(\frac{\cos\phi}{\sin^5\phi}\right)(3 + 4\sin^2\phi + 8\sin^4\phi)$$
$$+ \frac{1}{30\xi^2}(112 + 120\xi^2 + 15\xi^4)\Bigg]\Bigg]$$

From these final expressions, it is evident that for tanks with the same shape (that is, tanks of the same height-to-diameter ratio ξ), stress resultants in the shell are directly proportional to H^2 (or to D^2, since $D \propto H$). For instance, doubling the height H or diameter D of the tank, while maintaining the parameter ξ constant, will quadruple the stress resultants N_ϕ and N_θ in the shell. This is how the *scale* of the structure will affect its design.

Non-dimensional stress variations $N_\phi/\gamma H^2$ and $N_\theta/\gamma H^2$ have been plotted versus the meridional angle ϕ, for various values of ξ ranging from 1.0 to 3.0, which covers the most practical proportions for egg-shaped sludge digesters. The results are shown in Fig. 2. Note, from Eqs. (5), that the shell lies wholly in the interval $\phi_o \leq \phi \leq \phi_o'$, that is, $\tan^{-1}(\xi/2) \leq \phi \leq \pi - \tan^{-1}(\xi/2)$, and hence the computation of stress variations is only relevant within this interval.

Having identified ξ as the single parameter that governs the relative variation of stress resultants over the surface of the shell, we may as well express the volume capacity of the tank (Eq. (21)) in terms of this parameter, as follows:

$$V = \left(\frac{2\pi\xi}{15}\right)D^3 = \left(\frac{2\pi}{15\xi^2}\right)H^3 \tag{26}$$

Table 1 summarises the ranges of ϕ (ϕ_o to ϕ_o') and non-dimensional tank capacities (V/H^3) for the ξ values appearing in Fig. 2.

6. Discussion of results

For all values of the parameter ξ, hoop stress resultants N_θ remain positive (tensile) throughout the parabolic digester, rising from zero at the apex ($\phi = \phi_o$), to some peak value below the equatorial level, before beginning to drop in magnitude with further increase in ϕ. In terms of the non-dimensional stresses, $N_\theta/\gamma H^2$, the magnitude and location of this peak value are approximately {0.46; 143°}, {0.24; 129°}, {0.16; 118°}, {0.125; 110°} and {0.100; 107°} for ξ = 1.0, 1.5, 2.0, 2.5 and 3.0, respectively (Fig. 2b).

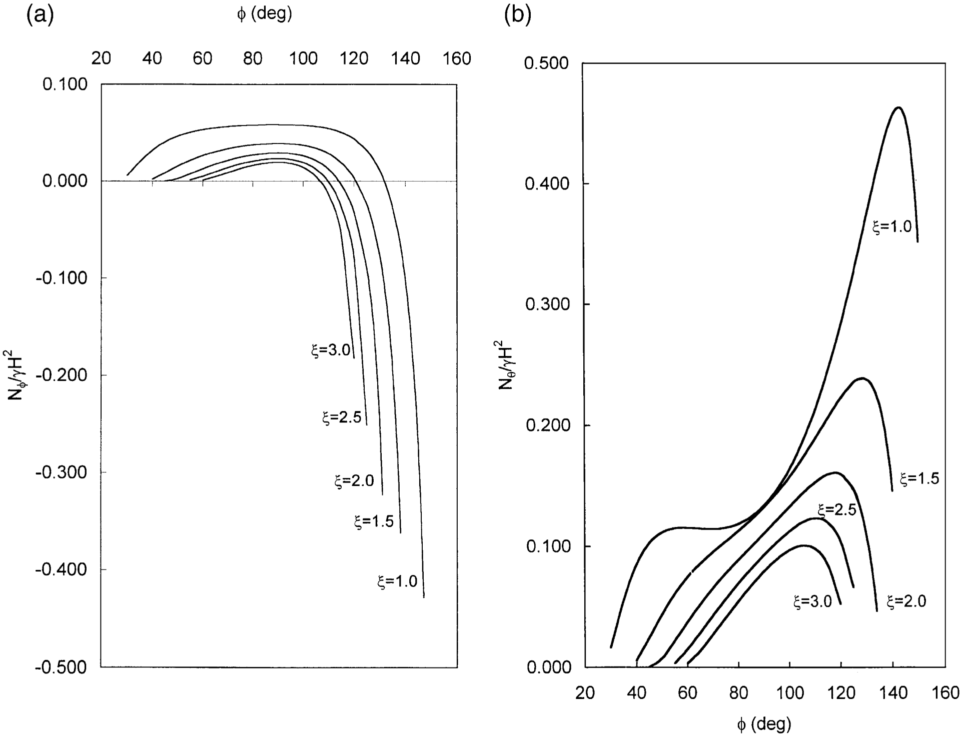

Fig. 2. Non-dimensional stress variations for the liquid-filled parabolic ogival shell: (a) meridional stresses; (b) hoop stresses.

Table 1
Ranges of ϕ and non-dimensional tank volumes for various ξ

ξ	Range of ϕ (°)	V/H^3
1.0	26.6–153.4	0.41888
1.5	36.9–143.1	0.18617
2.0	45.0–135.0	0.10472
2.5	51.3–128.7	0.06702
3.0	56.3–123.7	0.04654

Thus, for a given height H of the digester, the peak value of the hoop stress resultant N_θ increases rapidly as ξ is reduced from 3.0, through 2.5, 2.0 and 1.5, to 1.0 (that is, as the diameter D of the tank is increased). However, by reference to Table 1, the capacity gain as ξ is reduced is more rapid than the increase in the peak hoop stress, implying that structural efficiency increases with reduction in ξ. Here, structural efficiency η is being defined as the ratio of non-dimensional tank volume to non-dimensional peak hoop-stress resultant, that is

$$\eta = \frac{V}{H^3} \times \frac{\gamma H^2}{(N_\theta)_{\text{peak}}} = \frac{V\gamma}{H(N_\theta)_{\text{peak}}} \tag{27}$$

For $\xi = 1.0$, 1.5, 2.0, 2.5 and 3.0, the parameter η works out at approximately 0.91, 0.78, 0.65, 0.54 and 0.47, respectively.

A peculiarity of the hoop-stress variation for $\xi = 1.0$ is the insensitivity of the hoop stress resultant to changes in ϕ over the region $\phi = 54°$ to $\phi = 76°$, a behaviour which might be expected of the region of the parabolic meridian in the neighbourhood of the equator ($\phi = 90°$, around which ϕ is changing rapidly with relatively little change in the vertical coordinate y), but not away from the equator ($\phi = 54°–76°$). Also, for a tank of given height H, the equatorial value of the hoop stress resultant appears to be insensitive to ξ in the range $\xi = 1.0$ to $\xi = 1.5$ (note the touching curves).

Turning now to the meridional stress variations (Fig. 2a), it is noted that the stress resultants N_ϕ rise from zero at the apex ($\phi = \phi_o$), to a peak tensile value around the equator ($\phi = 90°$), before beginning to decrease and becoming negative (compressive) in the regions of the tank below the equatorial level. The peak tensile meridional values obtained at the equatorial level are all considerably lower than the peak tensile hoop values that were noted earlier, being approximately (in non-dimensional terms) 0.058, 0.039, 0.029, 0.023 and 0.019 for $\xi = 1.0$, 1.5, 2.0, 2.5 and 3.0, respectively.

The case $\xi = 1.0$ exhibits the greatest insensitivity of the meridional stress-resultant variation with meridional angle ϕ in the neighbourhood of the equatorial plane ($\phi = 90°$), partly because in this region, the meridional angle ϕ changes more rapidly with respect to the vertical coordinate y the smaller ξ becomes: when $\xi = 1.0$, ϕ changes significantly over the neighbourhood of the equator, while the depth coordinate y and the meridional stress resultant N_ϕ (which is roughly inversely proportional to $\sin\phi$ in the neighbourhood of the equatorial plane) change relatively little.

The crossover from tension to compression occurs at $\phi \approx 132°$, 120°, 114°, 110° and 106° for $\xi = 1.0$, 1.5, 2.0, 2.5 and 3.0, respectively. Beyond these changeover values of ϕ, compression rapidly increases with ϕ (that is, with depth of liquid), so that the supports of the tank must be positioned not too far below the changeover values of ϕ in order to cut off the excessive meridional compression that would otherwise occur in the shell were the shell to continue unsupported over its lower regions. This would reduce the likelihood of meridional buckling of the shell in the lower regions. The advantage of locating the supports exactly at the tension-to-compression changeover values of ϕ would be not only the total elimination of zones of compression (and hence of zones of possible local instability), but also the cut-off of the higher hoop-tension peaks (corresponding to $\xi = 1.0$, 1.5 and 2.0) noted earlier.

Assuming, then, that compression in the tank can be eliminated or minimised by careful choice of support location as indicated above, the design of the concrete shell may be based on the noted hoop and meridional tensile actions. For instance, to cater for the peak hoop tensile non-dimensional stress resultant of 0.46 noted for the case $\xi = 1.0$, one should design the shell to withstand a tensile force of 0.46 γH^2

(per metre width) in the hoop direction. For a very large tank with $H = 50$m, this force amounts to $0.46(9.81)(2500) = 11\ 281.5$kN/m which, over a shell thickness of say 0.5 m, would result in a tensile stress of 22.6 N/mm^2 in the material of the shell. Clearly, there would be a need to provide hoop steel reinforcement and/or prestressing to withstand this stress while minimising cracking, and this can easily be achieved following the usual design procedures.

7. Summary and conclusions

The stress distribution in liquid-filled parabolic ogival tanks has been expressed in terms of a single governing parameter ξ (the ratio of tank height to tank diameter), greatly facilitating a study of the distribution. It has been shown that the stress resultants in the shell are directly proportional to H^2 (or D^2) for tanks of the same shape (that is, tanks of the same height-to-diameter ratio ξ).

For various values of ξ covering the practical range for egg-shaped sludge-digester shells, recommendations have been proposed regarding the positioning of supports. The location of supports that eliminates or minimises meridional compression in the lower parts of the tank, as well as cutting off the peaks of the tensile hoop-stress variations in the same regions, has been identified for each case of ξ.

Although the structural efficiency η of the tank as defined by Eq. (27) (that is, ratio of non-dimensional tank capacity to non-dimensional peak hoop-stress resultant) is highest at the lower end of the range of ξ (that is, as ξ approaches 1.0), the range $1.5 \leq \xi \leq 2.0$ is recommended for practical egg-shaped digesters of parabolic ogival profile, since the slope of the shell is sufficiently steep at the poles ($37° \leq \phi_o \leq 45°$) to allow effective prestressing.

From a structural and functional point of view, the parabolic ogival profile is suitable for adoption in the design of egg-shaped sludge digester shells.

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