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Pareto-Improving Congestion Pricing and Revenue Refunding with Heterogeneous Users

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Abstract

This study investigates Pareto-improving congestion pricing and revenue refunding schemes in general transportation networks, which make every road user better off as compared with the situation without congestion pricing. We consider user heterogeneity in value of time (VOT) by adopting a multiclass user model with fixed origin-destination (OD) demands. We first prove that an OD and class-based Pareto-improving refunding scheme exists if and only if the total system monetary travel disutility is reduced. In view of the practical difficulty in identifying individual user's VOT, we further investigate class-anonymous refunding schemes that give the same amount of refund to all user classes traveling between the same OD pair regardless of their VOTs. We establish a sufficient condition for the existence of such OD-specific but class-anonymous Pareto-improving refunding schemes, which needs information only on the average toll paid and average travel time for trips between each OD pair.

Keywords: traffic equilibrium, congestion pricing, Pareto-improving, revenue refunding

1. Introduction

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Although congestion pricing has a solid theoretical foundation and nowadays has very advanced technology support for its practical implementation, it has long been viewed as a political issue. In particular, congestion pricing proposals are frequently declined due to public opposition. For examples, the Edinburgh congestion charge plan was rejected by a referendum in 2005, the congestion pricing proposal for the West Midlands in England was rejected by councils in March 2008, and the New York congestion pricing proposal was declined in April 2008 due to public opposition.

There are several reasons for the public unacceptability of congestion pricing. First of all, for the public, congestion pricing is just like another tax collected by the "evil" government. As summarized by Cervero (1998), "middle-class motorists often complain they already pay too much in gasoline taxes and registration fees to drive their cars, and that to pay more during congested periods would add insult to injury. In the United States, few politicians are willing to champion the cause of congestion pricing in fear of reprisal from their constituents". Indeed, with congestion pricing implemented, users' surplus is transferred to the government in the form of the toll revenue. Thus users are unhappy even if the society as a whole (including the government and the users) gains. Furthermore, congestion pricing has the well-known social inequity problem, i.e. people with relatively low income are more likely to be priced out of driving. For this reason, congestion pricing is often considered as an elitist policy, which prices the poor off of roads so that the wealthy can move about unencumbered (Cervero, 1998). In addition, there might be the so-called spatial equity issue, namely congestion pricing may have different impact on people living and working in different places (Yang and Zhang, 2002). For these reasons, congestion pricing as a policy is often faced with public hostility, and its implementation is more like a political issue other than a theoretical or technological problem.

Revenue redistribution has long been considered as a possible way to solve the political issues of congestion pricing. By directly refunding the toll revenue to the users, in an even or uneven manner, all the problems mentioned above that lead to the public unacceptability of congestion pricing could be solved. That is, with proper revenue refunding schemes, congestion pricing no longer makes road users feel like being "exploited" by the government and the potential social and spatial inequity problems can be solved as well. Indeed, various forms of revenue distribution strategies have been proposed and discussed. Goodwin (1989) suggested a combination of revenue uses in order to offset several congestion pricing impacts. Small (1992) proposed a travel allowance for all commuters. Poole (1992) added that it might be possible to introduce off-peak discounts and peak-hour surcharges on a toll road. DeCorla-Souza (1994) proposed a cashing out strategy to induce shift of peak-period travelers to other modes, thus reducing the need for additional infrastructure. Parry and Bento (2001) recommended that income taxes be reduced to offset congestion pricing-related labor supply restriction. Kalmanje and Kockelman (2004) and Kockelman and Kalmanje (2005) proposed a novel strategy for practical implementation of revenue refunding, the credit-based congestion pricing (CBCP) strategy.

A number of theoretical and quantitative studies have been conducted on road pricing and revenue redistribution.[1] Bernstein (1993) examined the possibility of user-neutral congestion pricing with both positive and negative tolls (tolls and subsidies) in the Vickrey bottleneck congestion model (Vickley, 1969). Arnott et al. (1994) investigated the welfare effects of congestion tolls using the basic bottleneck model with heterogeneous but inelastic commuting demand. They considered the case when the toll revenues are rebated as an equal lump-sum payment to all drivers and analyzed when and how each group of drivers could be made better off with such a uniform rebate. Daganzo (1995) designed a Pareto-improving hybrid strategy between rationing and pricing in the bottleneck congestion model, where a fraction of drivers would be exempt from tolling each day. Nakamura and Kockelman (2002) applied this strategy to the San Francisco Bay Bridge corridor, and concluded that such a Pareto-improving strategy does not exist for that network and it would be difficult to find such a policy in general.

In a network context, Adler and Cetin (2001) discussed a direct distribution approach to

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[1] There is also a loosely related literature on Pareto-improving taxes in the general economic theory of taxation. The reader may refer to Guesnerie (1995) for a comprehensive treatment of this subject and to Geanakoplos and Polemarchakis (2008) for a recent theoretical study. These studies focus on how commodity taxes should be set to benefit all major groups rather than how revenue should be redistributed.

congestion pricing, in which the money collected from users on a more desirable route was directly transferred to users on a less desirable route using a two parallel route example with bottleneck congestion. For a single origin-destination (OD) pair connected by a number of parallel routes, Elliasson (2001) showed that a tolling system that reduces aggregate travel time and refunds the toll revenues equally to all users will make everyone better off than before the toll reform. Yang et al. (2004) developed an optimal integrated pricing model in a bi-modal transportation network with explicit consideration of subsidy to transit mode from road congestion pricing revenue. Liu et al. (2009) adopted a continuous value-of-time (VOT) distribution and examined the existence of Pareto-improving and revenue-neutral pricing scheme in a simple bi-modal network consisting a single road and a parallel transit line.

Recently, Lawphongpanich and Yin (2007, 2009) and Song et al. (2009) studied a class of Pareto-improving pricing schemes without revenue redistribution in networks.[2] They formulated the problem of finding Pareto-improving tolls as a mathematical program with complementarity constraints, and proposed a solution algorithm via manifold suboptimization. The existence of the Pareto-improving tolls without revenue redistribution in their study requires that the untolled equilibrium flow pattern be dominated by an alternative feasible flow pattern, under which some users are better off and no user is worse off than in the untolled equilibrium. The dominating flow distribution and the Pareto-improving tolls exist only for certain special networks that exhibit the generalized Braess paradox defined and characterized by Hagstrom and Abrams (2001).

This study investigates Pareto-improving congestion pricing cum revenue refunding (CPRR) schemes in general transportation networks that make *every* road user better off compared with the untolled case. The key point of the Pareto-improving CPRR scheme is that, compared with the "do-nothing" case, refunding (after tolling) reduces the net travel disutility of each individual user. Such a Pareto improvement can make congestion pricing as a policy

[2] A preliminary version of our article (Yang and Guo, 2005) was already made available a few years earlier than Lawphongpanich and Yin (2007, 2009) and Song et al. (2009). While both involving pricing and Pareto-improvement of network flow pattern, the research issues and the methodologies adopted by the two groups of authors, as mentioned above, are fundamentally different.

more acceptable to the public, because everyone is a winner under such a policy.[3] We consider multiclass users by VOT and fixed demand for each OD pair. This fixed demand model could apply when congestion pricing is introduced during the morning peak-demand period, in which the number of trips taken between each OD pair can be regarded as fixed because a large portion of the trips are work related and can not be easily forgone. The fixed demand assumption can be an even milder one for transportation networks with both mass transit and private car modes, because it only requires the combined demand fixed between each OD pair.[4] Congestion pricing in this fixed demand case is mainly aimed to rationalize users' route choices (or mode choices for two-mode networks). In this case, the low-income users are more likely to be forced to change routes (or modes) by congestion charge; it is thus essential to adopt a multiclass user model to capture the effects of user heterogeneity, because our goal is to make everyone better off.

We first show that an OD and class-based Pareto-improving refunding scheme exists if and only if the total system monetary travel disutility is reduced. In view of the practical difficulty in identifying individual user's VOT, we further investigate class-anonymous refunding schemes that give the same amount of refund to all user classes traveling between the same OD pair regardless of their VOTs.

Our OD-specific but class-anonymous Pareto-improving CPRR scheme takes advantage of users' utility-maximizing behavior. Specifically, each individual user chooses a route that minimize her travel disutility, taking into account her particular value of time; her generalized travel disutility at equilibrium (including both toll and travel time) will not be larger than that incurred by traveling the OD-average travel time while paying the OD-average toll. With this important observation, we are able to establish a sufficient condition for the existence of a

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[3] The CPRR scheme is studied in a general network. In the special case that the network exhibits the generalized Braess paradox (Hagstrom and Abrams, 2001) and Pareto-improving tolls exist (Lawphongpanich and Yin, 2007, 2009), the considered CPRR scheme of course exists in the special form of zero refunding.

[4] Because the whole study is based on a deterministic equilibrium model, for all the results to be valid on such a two-mode network, commuters' choice of the mass transit mode has to be modeled in a deterministic rather than a conventional (logit or probit based) stochastic manner.

class-anonymous Pareto-improving refunding scheme; namely, the tolling system reduces the average travel time of users between each OD pair on the networks. The actual design and implementation of the class-anonymous Pareto-improving CPRR schemes need only aggregate (or average) information of trips between each OD pair, namely the average toll paid and the average travel time experienced by users between each OD pair. This kind of aggregate (or average) OD-specific information is not difficult to estimate in practice. For example, we can simply estimate these numbers by running a network equilibrium assignment.

An OD based refunding scheme can be expected with foreseen widespread practical applications of the GPS kind of technology. Currently, drivers' origins and destinations for regular daily commuting are generally available based on their home and job locations. Also, drivers' destinations can be obtained from the current parking meters as well. Moreover, our OD based refunding schemes provide a way to calculate the origin (zone) based refunding scheme, i.e. taking the summation of the OD based Pareto-improving refund over all destinations for an origin will give (and justify) an (approximate but not necessarily Pareto-improving) amount that should be refunded to all users living in that origin zone.

The remainder of this paper is organized as follows. In next section, we introduce the multiclass user model and the corresponding multiclass system optimization (SO) and user equilibrium (UE) problems. In Section 3, the existence of a Pareto-improving refunding scheme is established in the sense that, if a congestion pricing scheme reduces the total system monetary travel disutility, redistributing the toll revenue to all users in an OD-specific and class-specific manner can make everyone better off. In Section 4, we study the existence and design of OD-specific but class-anonymous Pareto refunding schemes. Finally, some remarks and conclusions are provided in Section 5.

2. Preliminaries on Multiclass User Model

Let $G(N,A)$ denote a transportation network, with a set of nodes N and a set of links A, together with a set of OD pairs W. We consider separable link travel time function $t_a(v_a)$, $a \in A$, i.e. travel time on a link depends on the flow on that link only. The link travel time function $t_a(v_a)$, $a \in A$, is assumed to be monotonically increasing with v_a. We consider a discrete set of user classes corresponding to the groups of users with different socio-economic characteristics, such as income level. Let M denote the set of such user classes, and β_m, $\beta_m > 0$, be the VOT for users of class $m \in M$. Let d_w^m, $d_w^m > 0$, be the travel demand of user class $m \in M$ between OD pair $w \in W$, R_w the set of all simple paths connecting OD pair $w \in W$, and f_{rw}^m the flow of user class m on path $r \in R_w$. The flow v_a^m by user class $m \in M$ and the total aggregate flow v_a on link $a \in A$ can be expressed in terms of path flows as follows:

$$v_a^m = \sum_{w \in W} \sum_{r \in R_w} f_{rw}^m \delta_{ar}, \ a \in A, \ m \in M \tag{1}$$

$$v_a = \sum_{m \in M} v_a^m, \ \ a \in A \tag{2}$$

where $\delta_{ar} = 1$ if route r uses link a and 0 otherwise. For simplicity, we denote vectors as $\mathbf{f} = \left(f_{rw}^m, r \in R_w, w \in W, m \in M \right)$, $\mathbf{v} = (v_a, a \in A)$, and $\mathbf{v}^{\mathbf{M}} = (v_a^m, a \in A, m \in M)$.

In the presence of multiple user classes with different VOT, the system travel disutility can be measured either in time unit (time-based disutility or total system travel time) by

$$T = \sum_{a \in A} t_a(v_a) v_a = \sum_{a \in A} \sum_{m \in M} t_a(v_a) v_a^m \tag{3}$$

or in cost or monetary unit (cost-based disutility or total system cost) by

$$C = \sum_{a \in A} \sum_{m \in M} \beta_m t_a(v_a) v_a^m \tag{4}$$

Clearly, both the time-based and the cost-based system disutilities can be regarded as weighted sums of the travel times of all user classes in the network. The former has a uniform weighting factor equal to unity, while the latter has non-uniform weighting factors equal to the VOT of respective user classes. From an economic viewpoint, C is a more appropriate system disutility measure when users have different VOT. Nevertheless, in transportation context, T has long been accepted as a standard index of system performance. Thus both system time T and system cost C are useful criteria for network optimization (Guo and Yang, 2009).

For simplicity, let Ω be the feasible set of path flows defined as

$$\Omega = \left\{ \mathbf{f} : \sum_{r \in R_w} f_{rw}^m = d_w^m;\ f_{rw}^m \geq 0; w \in W,\ m \in M \right\} \tag{5}$$

With the definition of T and C, the time-based SO problem is formulated as

$$\min_{\mathbf{f} \in \Omega}\ T(\mathbf{f}) = \sum_{a \in A} t_a(v_a) v_a \tag{6}$$

and the cost-based SO problem is formulated as

$$\min_{\mathbf{f} \in \Omega}\ C(\mathbf{f}) = \sum_{a \in A} \sum_{m \in M} \beta_m v_a^m t_a(v_a) \tag{7}$$

The system optimal flows given by the above SO problems are generally different from the equilibrium flows in the absence of toll pricing, because each user tries to minimize her own travel time. Thus congestion pricing has to be introduced to decentralize an SO or other second-best target flow pattern into (or support it as) a multi-class UE flow pattern. That is, a tolling system can alter the generalized travel disutility of links and paths faced by each class of users, and thereby induce new multi-class UE flow patterns, which are exactly the target (SO or other second-best) flow patterns (Yang and Huang, 2004; Guo and Yang 2009).[5] Note that toll differentiation across user classes is unrealistic and difficult to implement in reality, because users differ from one another in VOT only, which is *observationally indistinguishable* (Arnott and Kraus, 1998; Yang and Huang, 2004). Therefore, in the present study context, we only consider congestion pricing schemes with anonymous link tolls, which means that the same amount of toll is levied on each link for all user classes.

[5] While the time or cost based SO and the first-best pricing are of primary theoretical interest, our subsequent analysis of CPRR schemes is developed for any second-best pricing schemes in general networks.

Let u_a denote the toll charged on link $a \in A$, and $\mathbf{u} = (u_a, a \in A)$ be the vector of all link toll charges. With a toll scheme $\mathbf{u}$ implemented, it is well known (e.g. Yang and Huang, 2004) that the multi-class UE problem can be formulated as the following equivalent minimization problem:

$$\min_{\mathbf{f} \in \Omega} \sum_{a \in A} \int_0^{v_a} t_a(\omega)\,\mathrm{d}\omega + \sum_{a \in A} \sum_{m \in M} \frac{1}{\beta_m} v_a^m u_a \tag{8}$$

Observe that objective function (8) is strictly convex in aggregate link flow $\mathbf{v}$ for monotonically increasing link travel time function $t_a(v_a)$, but linear in class-specific link flow $\mathbf{v}^{\mathbf{M}}$. Thus, under a given tolling system $\mathbf{u}$, the equilibrium link flow by user class $\mathbf{v}^{\mathbf{M}}$ is generally not unique (neither is the UE path flow $\mathbf{f}$), while the aggregate UE link flow $\mathbf{v}$ is unique. Since $\mathbf{v}$ is unique, the system time T at equilibrium given by (3) is unique. The uniqueness of the equilibrium system cost C is not so obvious and will be shown later. Hereinafter, we frequently make a comparison between the tolled situation ($\mathbf{u} \neq 0$) and the untolled situation ($\mathbf{u} = 0$) for the traffic equilibrium model (8). To distinguish the two situations, we let $\tilde{\mathbf{f}}$, $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{v}}^{\mathbf{M}}$ be the flows in the absence of tolls ($\mathbf{u} = 0$), and $\bar{\mathbf{f}}$, $\bar{\mathbf{v}}$ and $\bar{\mathbf{v}}^{\mathbf{M}}$ be the flows under a toll pattern $\mathbf{u} \neq 0$.

For the untolled situation, the following UE conditions (in time unit) hold

$$\sum_{a \in A} t_a(\tilde{v}_a)\delta_{ar} = \tilde{\mu}_w^{m,\mathrm{t}}, \text{ if } \tilde{f}_{rw}^m > 0,\ r \in R_w,\ w \in W,\ m \in M \tag{9}$$

$$\sum_{a \in A} t_a(\tilde{v}_a)\delta_{ar} \geq \tilde{\mu}_w^{m,\mathrm{t}}, \text{ if } \tilde{f}_{rw}^m = 0,\ r \in R_w,\ w \in W,\ m \in M \tag{10}$$

where $\tilde{\mu}_w^{m,\mathrm{t}}$ is the minimum travel time between OD pair $w \in W$ for user class $m \in M$ in the absence of toll charge. It is noted here the equilibrium travel time is identical for all user classes between the same OD pair without toll charge; the equivalent UE conditions in cost unit can be obtained by multiplying both sides of (9) and (10) by β_m, $m \in M$). Multiplying both sides of (9) and (10) by equilibrium path flow $\tilde{f}_{rw}^m$ and summing over all $r \in R_w,\ w \in W,\ m \in M$, we have

$$\sum_{a\in A}\sum_{m\in M}\sum_{r\in R_w} t_a\left(\tilde{v}_a\right)\tilde{f}_{rw}^m\delta_{ar}=\sum_{w\in W}\sum_{m\in M}\sum_{r\in R_w}\tilde{\mu}_w^{m,\mathrm{t}}\tilde{f}_{rw}^m \tag{11}$$

In view of $\sum_{r\in R_w}\tilde{f}_{rw}^m=d_w^m$, $\sum_{w\in W}\sum_{r\in R_w}\tilde{f}_{rw}^m\delta_{ar}=\tilde{v}_a^m$ and $\sum_{m\in M}\tilde{v}_a^m=\tilde{v}_a$, eqn. (11) becomes

$$\tilde{T}=\sum_{a\in A}t_a\left(\tilde{v}_a\right)\tilde{v}_a=\sum_{w\in W}\sum_{m\in M}\tilde{\mu}_w^{m,\mathrm{t}}d_w^m \tag{12}$$

Similarly, by rewriting the untolled UE conditions in monetary unit or multiplying both sides of (9) and (10) by positive β_m, we can obtain

$$\tilde{C}=\sum_{a\in A}\sum_{m\in M}\beta_m t_a\left(\tilde{v}_a\right)\tilde{v}_a^m=\sum_{w\in W}\sum_{m\in M}\tilde{\mu}_w^{m,\mathrm{c}}d_w^m \tag{13}$$

where $\tilde{\mu}_w^{m,\mathrm{c}}=\beta_m\tilde{\mu}_w^{m,\mathrm{t}}$ is the minimum travel cost (time converted into money) between OD pair $w\in W$ for user class $m\in M$ in the absence of toll charge.

Under a toll $\mathbf{u}\neq 0$, the UE conditions (in monetary unit) are

$$\sum_{a\in A}\beta_m t_a\left(\bar{v}_a\right)\delta_{ar}+\sum_{a\in A}u_a\delta_{ar}=\bar{\mu}_w^{m,\mathrm{c}},\ \text{ if }\ \bar{f}_{rw}^m>0,\ r\in R_w,\ w\in W,\ m\in M \tag{14}$$

$$\sum_{a\in A}\beta_m t_a\left(\bar{v}_a\right)\delta_{ar}+\sum_{a\in A}u_a\delta_{ar}\geq\bar{\mu}_w^{m,\mathrm{c}},\ \text{ if }\ \bar{f}_{rw}^m=0,\ r\in R_w,\ w\in W,\ m\in M \tag{15}$$

where $\bar{\mu}_w^{m,\mathrm{c}}$ is the minimum travel cost (inclusive of equivalent cost of travel time) between OD pair $w\in W$ for user class $m\in M$ (the equivalent UE conditions in time unit are obtained by dividing both sides of (14) and (15) by β_m, $m\in M$). It follows readily from the above UE conditions that

$$\sum_{a\in A}\sum_{m\in M}\beta_m t_a\left(\bar{v}_a\right)\bar{v}_a^m+\Pi=\sum_{w\in W}\sum_{m\in M}\bar{\mu}_w^{m,\mathrm{c}}d_w^m$$

or equivalently

$$\bar{C}=\sum_{a\in A}\sum_{m\in M}\beta_m t_a\left(\bar{v}_a\right)\bar{v}_a^m=\sum_{w\in W}\sum_{m\in M}\bar{\mu}_w^{m,\mathrm{c}}d_w^m-\Pi \tag{16}$$

where $\Pi=\sum_{a\in A}u_a\bar{v}_a$ is the total toll revenue generated from all classes of users in the network. Since $\bar{\mu}_w^{m,\mathrm{c}}$ and Π are unique at equilibrium, eqn. (16) means that the cost-based system disutility $\bar{C}$ is unique.

3. Existence of Pareto-Improving Refunding Schemes

Now we move on to investigate revenue redistribution. We first consider a class and OD based refunding scheme $\Phi = \left\{ \Phi_w^m,\ w \in W,\ m \in M \right\}$, where Φ_w^m is the amount of refund to be *equally* refunded to the d_w^m users of class $m \in M$ between OD pair $w \in W$. In spite of its practical difficulty for implementation, the class-specific refunding scheme is introduced here to serve as a theoretical benchmark, more practical refunding schemes will be considered later.

Definition 1. *A refunding scheme* $\Phi = \left\{ \Phi_w^m,\ w \in W,\ m \in M \right\}$ *is said to be Pareto-improving if it holds that*

$$\sum_{w \in W} \sum_{m \in M} \Phi_w^m = \Pi \tag{17}$$

$$\bar{\mu}_w^{m,c} - \phi_w^m < \tilde{\mu}_w^{m,c}, \text{ for all } w \in W,\ m \in M \tag{18}$$

where $\phi_w^m = \Phi_w^m / d_w^m$ *is the amount of refund to each user in class* $m \in M$ *between OD pair* $w \in W$.

Equation (17) simply means that the total toll revenue is refunded to all users. The left-hand side of (18) is the net travel disutility of a user after tolling and refunding, and the right-hand side is the travel disutility in the absence of toll charge, thus (18) means that *every* user is made better off. Note that in this definition, the disutility unit used (time or money) is immaterial.

Lemma 1. *If a pricing scheme reduces the total system cost, namely* $\bar{C} < \tilde{C}$*, then there exists a Pareto-improving refunding scheme.*

Proof: Consider a refunding scheme $\Phi = \left\{ \Phi_w^m,\ w \in W,\ m \in M \right\}$ given by

$$\Phi_w^m = \left(\bar{\mu}_w^{m,c} d_w^m - \tilde{\mu}_w^{m,c} d_w^m\right) + \alpha_w^m \left(\sum_{w\in W} \sum_{m\in M} \tilde{\mu}_w^{m,c} d_w^m - \sum_{w\in W} \sum_{m\in M} \bar{\mu}_w^{m,c} d_w^m + \Pi \right),$$

$$w \in W, \ m \in M \quad (19)$$

where α_w^m is a positive number satisfying

$$\alpha_w^m > 0, \ w \in W, \ m \in M, \text{ and } \sum_{m\in M} \sum_{w\in W} \alpha_w^m = 1$$

Then Φ satisfies (17) as follows:

$$\sum_{m\in M} \sum_{w\in W} \Phi_w^m = \sum_{m\in M} \sum_{w\in W} \left(\bar{\mu}_w^{m,c} d_w^m - \tilde{\mu}_w^{m,c} d_w^m\right) + \left(\sum_{m\in M} \sum_{w\in W} \alpha_w^m \right) \left(\sum_{w\in W} \sum_{m\in M} \tilde{\mu}_w^{m,c} d_w^m - \sum_{w\in W} \sum_{m\in M} \bar{\mu}_w^{m,c} d_w^m + \Pi \right) = \Pi$$

We proceed to prove that Φ satisfies (18) as well. From (19), we have

$$\bar{\mu}_w^{m,c} d_w^m - \Phi_w^m = \tilde{\mu}_w^{m,c} d_w^m - \alpha_w^m \left(\sum_{w\in W} \sum_{m\in M} \tilde{\mu}_w^{m,c} d_w^m - \sum_{w\in W} \sum_{m\in M} \bar{\mu}_w^{m,c} d_w^m + \Pi \right),$$

$$w \in W, \ m \in M \quad (20)$$

From (13) and (16), $\bar{C} < \tilde{C}$ is equivalent to

$$\sum_{w\in W} \sum_{m\in M} \bar{\mu}_w^{m,c} d_w^m - \Pi < \sum_{w\in W} \sum_{m\in M} \tilde{\mu}_w^{m,c} d_w^m \quad (21)$$

From (21), the second term of the right-hand side of (20) is positive, and then it follows readily from (20) that

$$\bar{\mu}_w^{m,c} d_w^m - \Phi_w^m < \tilde{\mu}_w^{m,c} d_w^m, \ w \in W, \ m \in M \quad (22)$$

Note that (22) is simply equivalent to (18). This completes the proof. ♦

Lemma 1 states that $\bar{C} < \tilde{C}$ is a sufficient condition for the existence of a Pareto-improving refunding scheme. In fact, it is also a necessary condition, which can be readily proved by summing (22) over all $w \in W$ and $m \in M$ and thus obtaining (21). Thus we have the following theorem.

Theorem 1. *There exists a Pareto-improving refunding scheme if and only if* $\bar{C} < \tilde{C}$*, i.e. the congestion pricing scheme reduces the total system cost.*

Theorem 1 relates individual user's disutility reduction (represented by (18)) to system disutility reduction ($\bar{C} < \tilde{C}$), and implies that, when refunding is adopted, it is the cost-based system disutility C, rather than the time-based system disutility T, that has direct impact on the individual user's travel disutility.

Theorem 1 can be further refined by considering a Pareto-uniformly-improving refunding scheme that gives perfect equity, i.e., all users are equally better off irrespective of their VOT and OD pairs. Suppose one's better-off degree is measured by the percentage reduction of travel cost after tolling and refunding, then a Pareto-uniformly-improving revenue refunding scheme requires that

$$\frac{\bar{\mu}_w^{m,c} - \phi_w^m}{\tilde{\mu}_w^{m,c}} = e,\ w \in W,\ m \in M \tag{23}$$

where e is the uniform percentage reduction of travel cost for users of all classes and OD pairs in the whole network. From (23), we have

$$\phi_w^m = \bar{\mu}_w^{m,c} - e\tilde{\mu}_w^{m,c},\ w \in W,\ m \in M \tag{24}$$

Multiplying both sides of (24) by d_w^m and summing over all $w \in W,\ m \in M$, we have

$$\sum_{w\in W}\sum_{m\in M} \phi_w^m d_w^m = \sum_{w\in W}\sum_{m\in M} \bar{\mu}_w^{m,c} d_w^m - e\sum_{w\in W}\sum_{m\in M} \tilde{\mu}_w^{m,c} d_w^m \tag{25}$$

In view of $\Phi_w^m = \phi_w^m d_w^m$ and (17), (25) simply gives

$$e = \frac{\sum_{w\in W}\sum_{m\in M} \bar{\mu}_w^{m,c} d_w^m - \Pi}{\sum_{w\in W}\sum_{m\in M} \tilde{\mu}_w^{m,c} d_w^m} \tag{26}$$

From (13) and (16), (26) is equivalent to

$$e = \frac{\sum_{a\in A}\sum_{m\in M} \beta_m t_a(\bar{v}_a)\bar{v}_a^m}{\sum_{a\in A}\sum_{m\in M} \beta_m t_a(\tilde{v}_a)\tilde{v}_a^m} = \frac{\bar{C}}{\tilde{C}} \tag{27}$$

Once e is calculated from (26) or (27), the Pareto-uniformly-improving refunding scheme is readily given by (24).

Equation (27) simply states that the uniform reduction ratio e of individual disutility is equal to the reduction ratio of the cost-based system disutility after introducing the pricing scheme. Therefore, (23) and (27) imply that minimizing the cost-based system disutility C

will maximize the average better-off degree of all users for the Pareto-uniformly-improving CPRR scheme considered here.

4. Anonymous Pareto-improving Refunding Schemes

Although the aggregate VOT distribution among the population can be estimated, the VOT of each individual user is in general unobservable. This makes it unrealistic to implement refunding scheme ϕ_w^m by user class. Therefore, we introduce anonymous OD-based refunding schemes, which give the same amount of refund to users of all classes between the same OD pair. An anonymous OD-based refunding scheme is denoted by $\Phi = \{\Phi_w,\ w \in W\}$, where Φ_w is the total amount of refund to be *equally* refunded to users of all classes between OD pair $w \in W$.

Definition 2. *An anonymous refunding scheme* $\Phi = \{\Phi_w,\ w \in W\}$ *is said to be Pareto-improving if it holds that*

$$\sum_{w \in W} \Phi_w = \Pi \tag{28}$$

$$\bar{\mu}_w^{m,c} - \phi_w < \tilde{\mu}_w^{m,c}, \text{ for all } w \in W,\ m \in M \tag{29}$$

where $\phi_w = \Phi_w / d_w$ *is the amount of refund to each user between OD pair* $w \in W$ *regardless of individual VOT , and* $d_w = \sum_m d_w^m$ *is the total travel demand between OD pair* $w \in W$.

Definition 2 is the counterpart of Definition 1 when refunding schemes are anonymous. Similar to Definition 1, eqn. (28) simply means that the total toll revenue is refunded to all users, and (29) means that each individual user is made better off after tolling and refunding.

To investigate the existence of anonymous Pareto-improving revenue refunding schemes, we begin with an important lemma.

Lemma 2. *For any congestion pricing scheme, the following inequality holds*

$$\bar{\mu}_w^{m,c} - \frac{\Pi_w}{d_w} \le \beta_m \frac{\sum_{a\in A} t_a(\bar{v}_a)\bar{v}_{a,w}}{d_w}, \ w\in W, \ m\in M \tag{30}$$

where $\Pi_w = \sum_{a\in A} u_a \bar{v}_{a,w}$ *is the total toll revenue generated from all users traveling between OD pair* $w\in W$, $\bar{v}_{a,w} = \sum_{r\in R_w}\sum_{m\in M} \bar{f}_{rw}^m \delta_{ar}$ *is the link flow by all users traveling between OD pair* $w\in W$.

Proof: From UE conditions (14)-(15), we have

$$\bar{\mu}_w^{m,c} \le \sum_{a\in A} u_a\delta_{ar} + \beta_m \sum_{a\in A} t_a(\bar{v}_a)\delta_{ar}, \ r\in R_w, \ w\in W, \ m\in M \tag{31}$$

Multiplying both sides of (31) by equilibrium path flow $\bar{f}_{rw} = \sum_{m\in M} \bar{f}_{rw}^m$ aggregated over all user classes and summing over all $r\in R_w$, we have

$$\bar{\mu}_w^{m,c} \sum_{r\in R_w} \bar{f}_{rw} \le \sum_{a\in A} u_a \sum_{r\in R_w} \bar{f}_{rw}\delta_{ar} + \beta_m \sum_{a\in A} t_a(\bar{v}_a) \sum_{r\in R_w} \bar{f}_{rw}\delta_{ar}, \ w\in W, \ m\in M \tag{32}$$

In view of $\sum_{r\in R_w} \bar{f}_{rw} = d_w$ and $\sum_{r\in R_w} \bar{f}_{rw}\delta_{ar} = \bar{v}_{a,w}$, (32) becomes

$$\bar{\mu}_w^{m,c} d_w \le \Pi_w + \beta_m \sum_{a\in A} t_a(\bar{v}_a)\bar{v}_{a,w}, \ w\in W, \ m\in M \tag{33}$$

We thus obtain (30) immediately from (33). This completes the proof. ♦

Note that in (30) of Lemma 2, Π_w/d_w is the average toll paid by users of OD pair $w\in W$, and $\sum_{a\in A} t_a(\bar{v}_a)\bar{v}_{a,w}/d_w$ is the average travel time of users between OD pair $w\in W$. Thus Lemma 2 means that, under any congestion pricing scheme, the generalized travel disutility for each individual user is not larger than that incurred by traveling the OD-average travel time while paying the OD-average toll. This result is due to the product differentiation effect between toll and travel time: with congestion pricing implemented, higher-VOT users will choose routes with lower travel times and higher tolls, while lower-VOT users will choose routes with higher travel times and lower tolls, and as a result, everyone is “happier” than traveling the average time and paying the average toll. This is the basic idea behind our design of class-anonymous Pareto-improving refunding schemes based on OD-average information only.

With Lemma 2, we first suppose that the tolling system reduces the total travel time of users between each OD pair as given below.

$$\sum_{a \in A} t_a\left(\bar{v}_a\right)\bar{v}_{a,w} < \sum_{a \in A} t_a\left(\tilde{v}_a\right)\tilde{v}_{a,w},\ \forall w \in W \tag{34}$$

Combining (30) and (34), we obtain

$$\bar{\mu}_w^{m,\mathrm{c}} - \frac{\Pi_w}{d_w} < \beta_m \frac{\sum_{a \in A} t_a\left(\tilde{v}_a\right)\tilde{v}_{a,w}}{d_w},\ w \in W,\ m \in M \tag{35}$$

Since $\tilde{\mu}_w^{m,\mathrm{t}}$ is identical for all user classes between OD pair $w \in W$ in the absence of toll charge, it follows from UE conditions (9)-(10) that

$$\sum_{a \in A} t_a\left(\tilde{v}_a\right)\tilde{v}_{a,w} = \tilde{\mu}_w^{m,\mathrm{t}} d_w,\ w \in W,\ m \in M \tag{36}$$

Then (35) becomes

$$\bar{\mu}_w^{m,\mathrm{c}} - \frac{\Pi_w}{d_w} < \beta_m \tilde{\mu}_w^{m,\mathrm{t}} = \tilde{\mu}_w^{m,\mathrm{c}},\ w \in W,\ m \in M \tag{37}$$

Consider a class-anonymous refunding scheme $\Phi_w = \Pi_w,\ w \in W$, this refunding scheme readily meets (28), and from (37), it also satisfies (29). Thus it is an anonymous Pareto-improving refunding scheme. In addition, this OD-specific Pareto-improving refunding scheme uses the revenue collected from users traveling between the same OD pair only and refunds the revenue equally to all users of the same OD pair. Therefore, it does not require cross-subsidy among users traveling between different OD pairs. To sum up, we have the following theorem.

Theorem 2. *If a congestion pricing scheme reduces the total travel time of users between each OD pair in the network, then there exists a Pareto-improving class-anonymous refunding scheme without requiring cross-subsidization among users of different OD pairs.*

Theorem 2 is a direct result of Lemma 2: according to Lemma 2, the tolled travel disutility of each individual user is not larger than that incurred by traveling the OD-average travel time while paying the OD-average toll, therefore, if each user obtains a refund equal to the OD-average toll, and the OD-average travel time is reduced, then the net disutility of each user is reduced as compared with the untolled situation.

Applying Theorem 2 to a single-OD network, we have the following corollary.

Corollary 1. *For a transportation network with a single OD pair, if a congestion pricing scheme reduces the total system travel time, then refunding the total toll revenue equally to all users regardless of their VOT will make everyone better off.*

Recall that Theorem 1 in last section shows that a necessary and sufficient condition for the existence of a Pareto improving discriminatory refunding scheme by user class is the total system cost reduction; here Theorem 2 shows that the total travel time reduction in each OD pair guarantees existence of a class-anonymous Pareto-improving revenue refunding scheme, which also implies existence of the Pareto improving discriminatory refunding scheme. Therefore, we can conclude that reduction in travel time between each OD pair implies reduction in total system cost on a multiclass traffic equilibrium network. Naturally, for a single-OD network, a pricing scheme that reduces the system time also reduces the system cost. Nevertheless, for general multiple-OD network, a reduction in total system time by toll charge does not necessarily imply reduction in total system cost and vice versa.

Theorem 2 needs a relatively strong condition that the total travel time of each OD pair is reduced, which may not be the case in reality. We now explore the possibility of an anonymous Pareto-improving revenue refunding scheme when a pricing scheme reduces the total system travel time, but allows for increases in total travel times of certain OD pairs. In this case, cross-subsidization among different OD pairs is expected. Specifically, positive and negative subsidies should be allocated to OD pairs with total travel time increase and decrease, respectively.

To proceed, we now introduce additional notation. Let $T_w = \sum_{a \in A} t_a(v_a) v_{a,w}$ be the total travel time of users between OD pair $w \in W$ and $\Delta T_w = \overline{T}_w - \tilde{T}_w,\ w \in W$ be the corresponding change after introducing the pricing scheme; similarly, let $\Delta T = \sum_{w \in W} \Delta T_w = \overline{T} - \tilde{T}$ be the change in the total system travel time. By our assumption, $\Delta T < 0$; $\Delta T_w,\ w \in W$ can be either negative or positive.

Consider an anonymous refunding scheme $\Phi^* = \{\Phi_w^*,\ w \in W\}$ given by

$$\Phi_w^* = \Pi_w + \Delta\Pi_w,\ w \in W \tag{38}$$

where $\Delta\Pi_w$ denotes the amount of revenue transfer (positive or negative) to OD pair $w \in W$, and is determined by

$$\Delta\Pi_w = \begin{cases} \overline{\beta}\Delta T_w, & \text{if } \Delta T_w > 0 \\ \underline{\beta}\Delta T_w, & \text{if } \Delta T_w < 0 \\ 0, & \text{if } \Delta T_w = 0 \end{cases} \tag{39}$$

where $\overline{\beta} = \max_{m \in M} \{\beta_m\}$ and $\underline{\beta} = \min_{m \in M} \{\beta_m\}$.

Lemma 3. *The anonymous refunding scheme Φ^* given by (38) and (39) satisfies the following inequality*

$$\overline{\mu}_w^{m,c} - \phi_w^* \le \tilde{\mu}_w^{m,c},\ w \in W,\ m \in M \tag{40}$$

where $\phi_w^* = \Phi_w^* / d_w$, $w \in W$, and (40) can take equality for only three cases

$$\begin{aligned} &a)\ \Delta T_w = 0 \\ &b)\ \Delta T_w > 0,\ \textit{and}\ \beta_m = \overline{\beta} \\ &c)\ \Delta T_w < 0,\ \textit{and}\ \beta_m = \underline{\beta} \end{aligned} \tag{41}$$

Proof: From (30) of Lemma 2, we have

$$\overline{\mu}_w^{m,c} - \frac{\Pi_w}{d_w} \le \beta_m \frac{\overline{T}_w}{d_w},\ w \in W,\ m \in M \tag{42}$$

Then it follows that

$$\bar{\mu}_w^{m,c} - \phi_w^* = \bar{\mu}_{w,m}^{c} - \frac{\Pi_w}{d_w} - \frac{\Delta\Pi_w}{d_w}$$

$$\le \beta_m \frac{\bar{T}_w}{d_w} - \frac{\Delta\Pi_w}{d_w} \tag{43}$$

$$\le \beta_m \frac{\bar{T}_w}{d_w} - \beta_m \frac{\Delta T_w}{d_w} \tag{44}$$

$$= \beta_m \frac{\tilde{T}_w}{d_w}$$

$$= \tilde{\mu}_w^{m,c},\ w \in W,\ \ m \in M \tag{45}$$

where (43) follows from (42), (44) from (39) (the definition of $\Delta\Pi_w$), and (45) from $\tilde{T}_w = \sum_{a\in A} t_a\left(\tilde{v}_a\right)\tilde{v}_a = \tilde{\mu}_w^{m,t} d_w$ or from (36). From the definition of $\Delta\Pi_w$ given by (39), (44) takes equality only for the three cases given by (41). The proof is completed. ♦

Lemma 3 states that the anonymous refunding scheme Φ^* makes everyone "weakly" better off. The term "weakly" is used because inequality (40) can take equality for three groups of users given by (41), which means that the net travel disutility of these users after tolling and refunding can be (at most) equal to that of the untolled case. Lemma 3 holds because the anonymous refunding scheme Φ^* given by (38) and (39) is carefully designed to fulfill the "weakly" Pareto-improving conditions. Specifically and intuitively, for an OD pair with travel time increase ($\Delta T_w > 0$), we need a positive amount of subsidy to compensate the travel time increase of this OD pair ($\Delta\Pi_w > 0$). Since we can not observe individual VOT and we need everyone to be better off, we have to consider the most "vulnerable" user class. In this case, because we are using revenue to compensate the loss in time, the most "vulnerable" user class is the one with the highest VOT. Therefore, we have to let $\Delta\Pi_w = \bar{\beta}\Delta T_w$, which regards all travelers between the OD pair as having the highest VOT. In the same spirit, when we take money away from an OD pair with travel time reduction ($\Delta T_w < 0$), we have to regard all travelers as having the lowest VOT and thus let $\Delta\Pi_w = \underline{\beta}\Delta T_w$.

Because the anonymous refunding scheme Φ^* barely guarantees the non-increase of every individual user's travel disutility, it determines a *critical* amount of refund to users of each OD pair: if the amount of refund to users of OD pair $w \in W$ is less than Φ_w^*, then some user classes of this OD pair may suffer disutility increase; conversely, if the amount of refund is larger than Φ_w^*, then every user of this OD pair will have disutility reduction. Therefore, if $\sum_{w \in W} \Phi_w^* = \Pi$ or $\sum_{w \in W} \Delta\Pi_w = 0$ holds, then, while Φ^* itself is a self-financing and "weakly" Pareto-improving refunding scheme, the existence of a "strictly" Pareto-improving refunding scheme is not guaranteed; if $\sum_{w \in W} \Phi_w^* < \Pi$ holds, i.e. the critical refunding scheme Φ^* does not use up the total revenue, then refunding the extra amount of revenue to all users will result in an anonymous Pareto-improving revenue refunding scheme. We formally state these observations as a theorem with a rigorous proof. For easy presentation, we define the following two subsets of OD pairs

$$W^+ = \left\{ w \,\middle|\, \Delta T_w > 0,\ w \in W \right\}; \quad W^- = \left\{ w \,\middle|\, \Delta T_w < 0,\ w \in W \right\} \tag{46}$$

Theorem 3. *For a pricing scheme that reduces the total system travel time, namely* $\bar{T} < \tilde{T}$, *there exists a Pareto-improving class-anonymous refunding scheme if it holds that*

$$\frac{\sum_{w \in W^+} \Delta T_w}{\tilde{T} - \bar{T}} < \frac{\underline{\beta}}{\bar{\beta} - \underline{\beta}} \tag{47}$$

Proof: We shall first prove that inequality (47) is equivalent to $\sum_{w \in W} \Phi_w^* < \Pi$. From (38), $\sum_{w \in W} \Phi_w^* < \Pi$ is equivalent to $\sum_{w \in W} \Delta\Pi_w < 0$. From (39), we have

$$\begin{aligned} \sum_{w \in W} \Delta\Pi_w &= \bar{\beta} \sum_{w \in W^+} \Delta T_w + \underline{\beta} \sum_{w \in W^-} \Delta T_w \\ &= \left(\bar{\beta} - \underline{\beta}\right) \sum_{w \in W^+} \Delta T_w + \underline{\beta} \sum_{w \in W} \Delta T_w \\ &= \left(\bar{\beta} - \underline{\beta}\right) \sum_{w \in W^+} \Delta T_w + \underline{\beta} \Delta T \end{aligned}$$

Thus $\sum_{w\in W}\Delta\Pi_w < 0$ is equivalent to $\left(\bar{\beta}-\underline{\beta}\right)\sum_{w\in W^+}\Delta T_w + \underline{\beta}\Delta T < 0$, which in turn is equivalent to inequality (47) in view of $\Delta T = \bar{T} - \tilde{T} < 0$. Therefore, inequality (47) is equivalent to $\sum_{w\in W}\Phi_w^* < \Pi$. If condition (47) holds, i.e. $\sum_{w\in W}\Phi_w^* < \Pi$ holds, then there exists an anonymous refunding scheme $\Phi = \{\Phi_w,\ w\in W\}$ such that $\sum_{w\in W}\Phi_w = \Pi$ and $\Phi_w > \Phi_w^*,\ w\in W$. It follows from $\phi_w > \phi_w^*,\ w\in W$ and (40) of Lemma 3 that

$$\bar{\mu}_w^{m,c} - \phi_w < \bar{\mu}_w^{m,c} - \phi_w^* \le \tilde{\mu}_w^{m,c},\ w\in W,\ m\in M \quad (48)$$

Thus $\Phi = \{\Phi_w,\ w\in W\}$ is a Pareto-improving refunding scheme that satisfies both (28) and (29). This completes the proof. ♦

Theorem 3 states that condition (47) is a sufficient condition for the existence of an anonymous Pareto-improving revenue refunding scheme if the pricing scheme reduces the total system travel time. Observe that both sides of inequality (47) have explicit intuitive meanings. The left-hand side of (47) represents the ratio of the total increase in OD travel times for the subset W^+ of the time-increased OD pairs to the total system travel time reduction for the whole network; it can be regarded as a measure of the non-uniformity of travel time changes in various OD pairs brought about by the pricing scheme. The right-hand side of (47) is the ratio of the lowest VOT to the difference between the highest and the lowest ones, which is a reasonable measure of the disparity of users' VOT distribution or the degree of user heterogeneity (infinity means homogeneous users). Hence, Theorem 3 implies that, to ensure the existence of anonymous Pareto-improving revenue refunding schemes, the discrepancy in travel time changes (increase and decrease) among the various OD pairs in the network resulting from the pricing scheme can not be too large, as dictated by the degree of user heterogeneity.

The above way of interpreting Theorem 3 places more emphasis on the travel time changes of different OD pairs. From another perspective, Theorem 3 can be interpreted in terms of user heterogeneity[6]: if the heterogeneity in users' VOT is small enough (the right-hand side of (47) large enough), then there exists an anonymous Pareto-improving revenue refunding scheme. This way of interpretation allows us to link the heterogeneous user case under study with the traditional homogeneous user case. That is, if individuals have the same VOT, by redistributing toll revenue across OD pairs, it is possible to make everyone strictly better off. Then, by continuity, if individuals are sufficiently similar, it is also possible to make everyone better off by redistributing toll revenue across OD pairs.

Recall that the goal of congestion pricing is to support a target (SO or other second-best) flow pattern as a multiclass UE flow pattern, thus sometimes it is more essential to relate Theorem 3 directly to the target flow pattern instead of the pricing scheme. From this point of view, condition (47) has no "direct" linkage with specific pricing schemes: the left-hand side of (47) depends on flow pattern only and the right-hand side is predetermined by the VOT distribution. Therefore, the *existence* of an anonymous Pareto-improving revenue refunding scheme is essentially determined by the target flow pattern. That is, given a target flow pattern that satisfies condition (47), any pricing scheme that can support the given flow pattern as UE can give rise to anonymous Pareto-improving revenue refunding schemes.

Checking condition (47) and calculating the critical refunding scheme Φ^* require only aggregate information (ΔT_w and Π_w) and general VOT distribution information ($\bar{\beta}$ and $\underline{\beta}$), while the identification of individual user's VOT is not involved. Thus we can easily use condition (47) and Φ^* to design anonymous Pareto-improving revenue refunding schemes. Also, condition (47) applies to certain special cases and yields meaningful results. If the total travel time of each OD pair is reduced as in the case of Theorem 2, i.e. $\Delta T_w < 0$ for all $w \in W$, then condition (47) must hold. Hence Theorem 2 (and more specifically, Corollary 1) states an anonymous Pareto-improving revenue refunding scheme without cross-subsidization

[6] Similar observation was made in the basic bottleneck congestion model with heterogeneous commuters by Arnott et al. (1994). They found that, whether or not a uniform rebate can make everyone better off depends on the relative number of commuters of each group and their differences in the shadow value of queuing time and the unit cost of schedule delay.

as a special case of Theorem 3. If all network users share the same VOT, namely $\bar{\beta} = \underline{\beta}$, then the right-hand side of (47) approaches infinity and condition (47) holds naturally. This gives the following corollary.

Corollary 2. *For a transportation network with homogeneous users, if a congestion pricing scheme reduces the total system travel time, then there exists a Pareto-improving revenue refunding scheme.*

Note that Corollary 2 can also be obtained from Theorem 1. Namely, system time and system cost are two identical measures when users are homogeneous. Corollary 2 is specifically presented here to show an application of Theorem 3.

5. Conclusions

In this paper we studied Pareto-improving CPRR schemes for the fixed demand case with multiclass users. We proved that, an OD and class-based Pareto-improving refunding scheme exists if and only if the total system travel cost is reduced. In this context our results highlight the importance of system cost other than system time, while the latter has long been accepted as a standard index of transportation system performance.

We then studied the existence and design of more plausible class-anonymous Pareto-improving revenue refunding schemes. Our basic idea relies on the observation that, at network equilibrium, each individual user will choose a route based on her own VOT such that her generalized travel disutility (including both toll and travel time) will not be larger than that incurred by traveling the OD-average travel time while paying the OD-average toll. We also found that, to ensure the existence of a class-anonymous Pareto refunding scheme, the discrepancy in travel time changes (increase and decrease) among the various OD pairs resulting from the pricing scheme should not be too large, as dictated by the degree of user heterogeneity.

The fixed demand assumption adopted in this paper is amenable to some cases, such as the peak-demand period with most work-related trips which can not be easily forgone, but may be too restrictive for other cases, like the daytime downtown congestion pricing schemes (as in London), where a large portion of trips are consumption or entertainment related. In this case travel demand is clearly elastic. We shall continue our investigation on the Pareto-improving CPRR schemes for the difficult and challenging elastic demand case.

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