Parity Forwarding For Multiple-Relay Networks

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***Abstract*—This paper proposes a relaying strategy for networks with multiple relays where each relay forwards parities of decoded codewords. This parity-forwarding scheme can be thought of as a generalization of Cover and El Gamal's well-known decode-and-forward strategy for the classic three-terminal relay channel to networks with multiple relays. As compared to previous multiple-relay decode-and-forward strategies, the parity-forwarding scheme is more flexible and can achieve a higher rate. The proposed strategy can be easily applied to networks with complex topologies. We show that relay networks can be degraded in more than one way, and parity-forwarding is capacity achieving for a new form of degraded relay networks.**

I. Introduction

A fundamental understanding of the role of relays in a network is crucial in the design of efficient network protocols for future wireless systems. Although the capacity of the simplest, yet the most fundamental, three-terminal relay channel [1] is still unknown, recent surge of interest in relay networks has resulted in not only new achievable rates for multiple-relay networks [2], [3], but also novel network protocols derived from the information theoretical insights [4], which improve previous protocols [5].

This paper generalizes Cover and El Gamal's decode-and-forward strategy [6, Theorem 1] for a single-relay network to a network with multiple relays. This generalization is simple yet flexible when applied to large networks with complex topologies. In particular, we show that a relay network can be degraded in more than one way and parity-forwarding protocols can be used to achieve the capacity of a new class of degraded networks with two relays.

The three-terminal relay channel was originally introduced in [1]. Key results for the single-relay channel were derived by Cover and El Gamal in their classic work [6], where two fundamental relay strategies, decode-and-forward [6, Theorem 1] and compress-and-forward [6, Theorem 6], are devised. Several coding strategies for networks with multiple-relays have since been proposed [4], [7], [3], [8], [9], [10], [2], [11]. In [2], an achievable rate is derived using successive interference cancelation at each relay. In [7], an improved rate is derived using a decode-and-forward scheme based on Carleial's coding strategy for the generalized multiple-access channel with feedback (named 'regular encoding' in [3]) which achieves the capacity of a certain class of degraded multiple-relay networks. In the latter scheme, each source message is repeatedly sent by the source and all the relays that have decoded the message so far. The destination decodes a message only after all relays have decoded the message and have participated in the cooperative transmission. One drawback of this scheme is that the achievable rate is bounded by the condition that all relays must decode the source message successfully. This can be restrictive in cases where the source-relay channel is poor. This paper proposes a new parity-forwarding scheme for the relay network in which each relay has the ability to facilitate transmission between any two communicating nodes in the network (and not just communication by the source.) This additional flexibility results in a higher overall transmission rates for a general relay network as compared to [7].

This paper focuses on decode-and-forward-like strategies in a relay network and does not explore the possibility of compress-and-forward. Compress-and-forward for relay networks has been considered in [3], where the relay nodes are divided into two groups: the decode-and-forward nodes and the compress-and-forward nodes. In addition, [3] also considers the possibility that a compress-and-forward node may partially decode the other relay's message to enhance its estimation of the source message. We note that compress-and-forward can be layered on top of the parity-forwarding scheme proposed in this paper to further enhance the overall transmission rate, although this is not explored in detail here.

We name our proposed scheme "parity-forwarding" because binning for the relay channel may be interpreted as parity generation. In Cover and El Gamal's decode-and-forward strategy [6, Theorem 1], the relay transmits a bin index of the source message to facilitate the decoding of the source message at the destination. In a linear coding context, binning can be realized by identifying parity bits (or syndrome) as the bin index [12]. This interpretation also facilitates practical code constructions for the relay channel as shown in our previous work [13].

The parity-forwarding strategy proposed in this paper bears a resemblance to network coding [14]. In both schemes, the intermediate nodes forward parities to efficiently help a pair of terminals communicate.

In this paper, directed acyclic graphs are used to visualize relay protocols. Each parity-forwarding scheme can be associated with a graph, which can also be interpreted as a *routing* scheme. To obtain the best overall transmission rate, all routing possibilities should be considered.

Throughout the paper, we use the random variable X to denote the symbol transmitted by the source and Y as the symbol received by the destination. Random variables X_k and Y_k denote the transmitted and received symbols at the k'th relay. The source message in block i is denoted by w_i; s_i^k represents the transmitted message by the k'th relay.

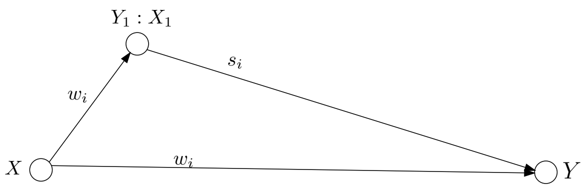

Fig. 1. Single-relay network.

II. Parity-forwarding for Single-Relay Network

Parity forwarding refers to a strategy in which a relay sends parities of the decoded codeword to facilitate transmission by other nodes. In this section, we recast Cover and El Gamal's decode-and-forward scheme as a parity forwarding strategy and briefly review the fundamental capacity result for a degraded single-relay channel. A single-relay network is shown in Fig. 1. The decode-and-forward strategy [6, Theorem 1] works by allowing the relay to decode the source message then forward a bin index of the source message to facilitate the ultimate decoding at the destination. As mentioned earlier, in a linear coding context, binning can be realized via *parity generation* [12]. Parity bits may be interpreted as a bin index because the set of codewords satisfying a particular parity forms a coset, and the set of all cosets form a partition of the entire codebook. Thus, decode-and-forward is equivalent to parity forwarding. In this paper, we use the following parity-generation function to describe the binning process:

Definition 1 (The Binning Function): Consider a set of integers, $Q = \{1, 2, \dots, 2^{nR_Q}\}$. Let $B = \{S_1, S_2, \dots, S_{2^{nR_B}}\}$ denote a random uniform partitioning of Q into 2^{nR_B} subsets $S_1, S_2, \dots, S_{2^{nR_B}}$ of size $2^{n(R_Q - R_B)}$ each. The binning function $P_{R_B,B}(w) : Q \to \{1, 2, \dots, 2^{nR_B}\}$ is defined by $P_{R_B,B}(w) = q$ if $w \in S_q$.

Using the binning function, the strategy of [6, Theorem 1] can be described as follows. The source uses a doubly indexed codebook $\mathcal{X}(w|s)$, $w \in \{1, \cdots, 2^{nR}\}$, $s \in \{1, \cdots, 2^{nR_1}\}$, generated uniformly according to $p(x|x_1)$. The relay uses a codebook $\mathcal{X}_1(s)$, $s \in \{1, \cdots, 2^{nR_1}\}$, generated uniformly according to $p(x_1)$. Let B_1 be a 2^{nR_1} uniform partition of $\{1, \cdots, 2^{nR}\}$. In each block i, the source transmits a new message $w_i \in \{1, \cdots, 2^{nR}\}$ by selecting the codeword $\mathbf{x}(w_i|s_i)$ where $s_i = P_{R_1,B_1}(w_{i-1})$, while the relay simultaneously decodes w_i and cooperatively transmits $\mathbf{x_1}(s_i)$. For the relay to successfully decode w_i, we need:

$$R < I(X; Y_1|X_1). \tag{1}$$

At the destination, s_i is decoded first, which is feasible if

$$R_1 < I(X_1; Y). \tag{2}$$

Once $s_i = P_{R_1,B_1}(w_{i-1})$ is known, w_{i-1} is restricted to the bin indexed by s_i (which is of the size $2^{n(R-R_1)}$.) Hence, the destination can now decode w_{i-1} provided that

$$R - R_1 < I(X; Y|X_1). \tag{3}$$

Equations (1)-(3) give us the degraded relay channel capacity.

III. Parity-forwarding With Two Relays

In a relay network with two relays, parity forwarding is more flexible and can achieve a higher rate as compared to other decode-and-forward schemes known in literature [7], [2], [3]. In this section, we devise three parity-forwarding protocols each suitable for a different set of channel characteristics. In contrast to a single-relay network, several independent transmission links exist in a two-relay network. Depending on the strengths of different links, each relay may choose which links to help the transmission for. This flexibility results in different relaying protocols for different two-relay networks.

In Section III-A, we consider the case in which the second relay helps the transmission between the first relay and the destination only without decoding the source message. This results in a higher overall achievable rate than previous schemes. (In fact, it is capacity achieving for a new form of degraded relay networks in which the link between the source and the second relay is weak.) In Section III-B.1, we consider the case in which the links from both the source and the first relay to the second relay are strong. In this case, the second relay helps the transmissions of the messages from both the source and the first relay to the destination. In Section III-B.2, we describe a protocol suitable for the case where the second relay has a strong channel from the source but a weak channel from the first relay. In both of the two latter cases, parity-forwarding encompasses rates achievable by previous schemes.

A. The One-Way Relay Protocol:

Consider a two-relay network shown in Fig. 2 where the second relay has a poor channel from the source but a strong channel from the first relay. Instead of requiring all relays to decode the message from the source [7], [2], [3], a parity-forwarding scheme in which the second relay helps the first relay only can achieve a higher rate. This channel configuration can be abstractly described by a special sense of degradedness in which the source can communicate no more information to the second relay than to the destination. The main result of this section is a new protocol which is capacity achieving for this class of relay networks.

In this protocol, the first relay decodes w_i, the message from the source, and forwards a random bin index (or its parities) to the destination. The second relay, due to its poor channel to the source, does not try to decode w_i. Instead, it helps the transmission of parities from the first relay to the destination by forming *extra* parities on the parities sent by the first relay. The directed graph in Fig. 2 describes such a protocol. Since the second relay does not decode w_i, the direct link between the source and the second relay is eliminated. The upper triangle serves to illustrate that the second relay is only relaying the message over the link between the first relay and the destination (which is why this protocol is called the *one-way relay protocol* in this paper.)

Mathematically, let $w_i \in \{1, \cdots, 2^{nR}\}$ denote the new message in block i. Let $s_i^1 = P_{R_1,B_1}(w_{i-1})$ and $s_i^2 = P_{R_2,B_2}(s_{i-1}^1)$ where B_1 and B_2 are fixed independent uniform random partitions of size 2^{nR_1} and 2^{nR_2} for $\{1, \cdots, 2^{nR}\}$ and $\{1, \cdots, 2^{nR_1}\}$, respectively.

The complete transmission scheme occurs in three successive blocks as follows. Assume that in block i, the source and the first relay know s_i^1 and s_i^2. (It will be cleared later that this is a valid assumption.) In each block i, the source encodes a new message w_i using a codebook $\mathcal{X}(w|s_i^1, s_i^2)$ of rate R generated according to $p(x|x_1, x_2)$. The first relay fully decodes w_i based on Y_1, which is feasible if

$$R < I(X; Y_1|X_1, X_2). \quad (4)$$

Upon decoding w_{i-1} in block $i-1$, the first relay forms $s_i = P_{R_1,B_1}(w_{i-1})$ and forwards it in block i to both the second relay and the destination using a codebook $\mathcal{X}_1(s^1|s_i^2)$ of rate R_1 generated according to the probability distribution $p(x_1|x_2)$. The second relay fully decodes s_i^1 in block i based on Y_2, which is possible if R_1 satisfies

$$R_1 < I(X_1; Y_2|X_2). \quad (5)$$

In block i, the second relay sends $s_i^2 = P_{R_2,B_2}(s_{i-1}^1)$ to the destination using a codebook $\mathcal{X}_2(s^2), s^2 \in \{1, 2, \cdots, 2^{nR_2}\}$, of rate R_2 generated according to $p(x_2)$. The destination can successfully decode s_i^2 in block i if

$$R_2 < I(X_2; Y). \quad (6)$$

Upon decoding s_i^2, the destination now attempts to decode s_{i-1}^1, (which is encoded using a codebook of rate R_1 and generated according to $p(x_1|x_2)$.) There are $2^{n(R_1-R_2)}$ possible s_{i-1}^1 messages inside the bin indexed by s_i^2. Therefore, the decoding of s_{i-1}^1 will be successful if R_1 and R_2 satisfy

$$R_1 - R_2 < I(X_1; Y|X_2). \quad (7)$$

In the last step, with the help of s_{i-1}^1, the destination then attempts to decode w_{i-2}, (which is encoded using a codebook of rate R and generated according to $p(x|x_1, x_2)$.) There are $2^{n(R-R_1)}$ w_{i-2} messages in the bin indexed by s_{i-1}^1. Therefore, the successful decoding criterion is given by:

$$R - R_1 < I(X; Y|X_1, X_2). \quad (8)$$

The group of inequalities {(6), (7), (8)}, {(5), (8)}, and (4) result in the following achievable rate which equals the capacity of a new form of degraded two-relay networks:

$$\begin{aligned} R &< I(X; Y|X_1, X_2) + I(X_1; Y|X_2) + I(X_2; Y) \\ &= I(X, X_1, X_2; Y) \quad (9a) \\ R &< I(X; Y|X_1, X_2) + I(X_1; Y_2|X_2) \quad (9b) \\ R &< I(X; Y_1|X_1, X_2). \quad (9c) \end{aligned}$$

Definition 2: A doubly degraded two-relay network is defined by $p(y, y_1, y_2|x, x_1, x_2)$, where $X - (X_1, X_2, Y_1) - (Y_2, Y)$, $X_1 - (X_2, Y_2) - Y$ and $X - (X_1, X_2, Y) - Y_2$ form Markov chains.

Theorem 1: The capacity of a doubly degraded two-relay network is given by (9) maximized over $p(x, x_1, x_2)$.

Proof: It is straightforward to see that the upper bound derived from the generalized maximum min-cut theorem in [6] matches (9) for this channel. ■

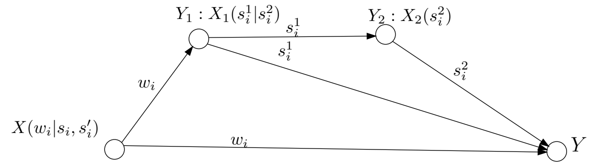

Fig. 2. The directed graph representing the one-way relay protocol. The second relay provides extra parities for the message of the first relay.

B. Two-Way Relay Protocols:

In this section, we show that parity forwarding is sufficiently general to encompass previous results obtained by the regular encoding decode-and-forward strategy [7], [3] for a general two-relay network (shown in Fig. 3) where the link between the source and the second relay may also be strong.

In order to achieve the rate obtained by regular encoding [7] for a general network with two relays, two complementary parity-forwarding protocols are needed, depending on the relative strengths of the channel between the first relay and the second relay (i.e. $I(X_1; Y_2|X_2)$) and the channel between the first relay and the destination (i.e. $I(X_1; Y|X_2)$). Section III-B.1 describes a relaying protocol for the case that $I(X_1; Y_2|X_2) \geq I(X_1; Y|X_2)$. Section III-B.2 focuses on the case that $I(X_1; Y_2|X_2) \leq I(X_1; Y|X_2)$. These two-way relay protocols guarantee an achievable rate equal to that of regular encoding for a general network with two relays.

1) Two-Relay Protocol A : Fig. 3 schematically describes the two-way relay protocol A. The relay channel is assumed to satisfy $I(X_1; Y_2|X_2) \geq I(X_1; Y|X_2)$. The main difference between this protocol and the one-way relay protocol is that $s^2 \in \{1, \cdots, 2^{nR_2}\}$ now encodes two messages, $u \in \{1, \cdots, 2^{nR_u}\}$ and $v \in \{1, \cdots, 2^{nR_v}\}$. The message v helps the destination decode s^1 while u represents extra parities for w, and $R_2 = R_u + R_v$. Otherwise, random codebook construction for the source, the first relay and the second relay are exactly the same as the one-way relay protocol.

In block i, we have $s_i^1 = P_{R_1,B_1}(w_{i-1})$ and $s_i^2 = (u_i, v_i)$ for $u_i = P_{R_u,B_u}(w_{i-2})$ and $v_i = P_{R_v,B_v}(s_{i-1}^1)$ where B_1, B_u, and B_v are fixed and independent uniform random partitions of sizes 2^{nR_1}, 2^{nR_u} and 2^{nR_v}, respectively.

In block i, the first relay decodes w_i which is encoded by the codebook $\mathcal{X}(w|s_i^1, s_i^2)$ of size 2^{nR} and generated according to $p(x|x_1, x_2)$. The decoding is successful if (4) is satisfied. Upon decoding w_i, the first relay forms $s_{i+1}^1 = P_{R_1,B_1}(w_i)$ for the next block.

The second relay decodes s_i^1 in block i, which requires (5) to hold. Having decoded s_i^1, the second relay now decodes w_{i-1}. Benefiting from s_i^1 as the bin index, the total number of valid w_{i-1} messages now reduces to $2^{n(R-R_1)}$. Since $\mathcal{X}(w|s_i^1, s_i^2)$ is a codebook of rate R generated according to $p(x|x_1, x_2)$, successful decoding is possible if

$$R - R_1 < I(X; Y_2|X_1, X_2). \quad (10)$$

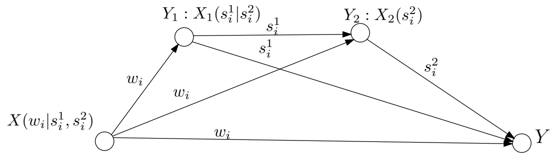

Fig. 3. The directed graph representing the two-way relay protocols.

Knowing w_{i-1} and s_i^1, the second relay may now form $s_{i+1}^2 = \big(P_{R_u,B_u}(w_{i-1}), P_{R_v,B_v}(s_i^1)\big)$ for the next block.

In block i, the destination first decodes s_i^2 (and extracts u_i and v_i) which is encoded by the codebook $\mathcal{X}_2(s^2)$ of size $2^{n(R_u+R_v)}$. This requires that

$$R_2 = R_u + R_v < I(X_2; Y). \tag{11}$$

The destination now uses all available information to decode w_{i-2}. It does so in several steps. First, since a bin index of rate R_v for s_{i-1}^1 is provided by v_i, the total number of possible choices for s_{i-1}^1 reduces to $2^{n(R_1-R_v)}$ within a bin indexed by v_i. The destination may further narrow down the choice for s_{i-1}^1 by looking inside this bin and forming a list, Ψ, of all s_{i-1}^1's for which $(\mathbf{x_1}(s_{i-1}^1|s_{i-1}^2), \mathbf{x_2}(s_{i-1}^2), \mathbf{y}^{i-1})$ are jointly typical. Since the codebook $\mathcal{X}_1(s^1|s_{i-1}^2)$ is generated according to $p(x_1|x_2)$, Ψ contains $2^{n\tilde{R}_1}$ s_{i-1}^1's where

$$R_1 - \tilde{R}_1 - R_v < I(X_1; Y|X_2). \tag{12}$$

Note that the destination may not uniquely decode s_{i-1}^1. Nevertheless, the list Ψ confines w_{i-2} to a bin Ω of size $2^{n(R-R_1+\tilde{R}_1)}$. The bin Ω is formed as the union of all $2^{n\tilde{R}_1}$ bins of size $2^{n(R-R_1)}$ indexed by elements of Ψ, i.e $\Omega = \big\{w|\exists s^1 \in \Psi : s^1 = P_{R_1,B_1}(w)\big\}$.

Finally, the destination decodes w_{i-2} knowing that it belongs to two independent random bins: Ω of size $2^{n(R-R_1+\tilde{R}_1)}$ and the bin indexed by u_i of size $2^{n(R-R_u)}$. Intersecting these two independent random bins restricts the number of valid choices for w_{i-2} to $2^{n(R-R_1+\tilde{R}_1-R_u)}$. Since the codebook for encoding w_{i-2} is generated according to $p(x|x_1, x_2)$, the decoding of w_{i-2} would be successful if

$$R - R_1 + \tilde{R}_1 - R_u < I(X; Y|X_1, X_2). \tag{13}$$

Combining {(13), (12), (11)}, {(10), (5)} and (4) gives the following achievable rate

$$\begin{aligned} R &< I(X; Y|X_1, X_2) + I(X_1; Y|X_2) + I(X_2; Y) \\ &= I(X, X_1, X_2; Y) && \text{(14a)} \\ R &< I(X; Y_2|X_1, X_2) + I(X_1; Y_2|X_2) \\ &= I(X, X_1; Y_2|X_2) && \text{(14b)} \\ R &< I(X; Y_1|X_1, X_2). && \text{(14c)} \end{aligned}$$

The above rate maximized over $p(x, x_1, x_2)$ is achievable for a general two-relay network. In particular, it is also the capacity if the relay network is *serially degraded* in the sense that $X - (Y_1, X_1, X_2) - (Y_2, Y)$ and $(X, X_1) - (Y_2, X_2) - Y$ form Markov chains [7, Definition 2.3 and Theorem 3.2].

Note that the rate (14) is achievable using this protocol only if conditions (4)-(5) and (10)-(13) are consistent. In particular, condition (5) requires $R_1 < I(X_1; Y_2|X_2)$; condition (12) requires $R_1 - \tilde{R}_1 - R_v < I(X_1; Y|X_2)$. Therefore, the two-way protocol A is applicable only when $I(X_1; Y_2|X_2) \geq I(X_1; Y|X_2)$, (which is always true in a serially degraded relay network.)

2) Two-Way Relay Protocol B : We now describe a slightly different scheme, named the two-way relay protocol B, to achieve the rate given in (14) for the case $I(X_1; Y_2|X_2) \leq I(X_1; Y|X_2)$. Unlike the two-way protocol A, in this scheme, the second relay performs partial decoding of the first relay's message.

In this protocol, the operations of the source and the first relay are exactly the same as the case A. As before, s_i^1 and w_i are encoded using $\mathcal{X}(w|s_i^1, s_i^2)$ and $\mathcal{X}_1(s^1|s_i^2)$. The first relay decodes w_i, which is possible if (4) is satisfied.

The second relay forms a list Φ of all likely s_i^1 messages for which $(\mathbf{x_1}(s_i^1|s_i^2), \mathbf{x_2}(s_i^2), \mathbf{y_2}^i)$ are jointly typical. The list Φ contains $2^{nR_1'}$ candidates for s_i^1 provided that

$$R_1 - R_1' < I(X_1; Y_2|X_2). \tag{15}$$

Each element of Φ corresponds to a bin of w messages of size $2^{n(R-R_1)}$. Therefore, Φ restricts w_{i-1} to be inside a bin of size $2^{n(R-R_1+R_1')}$. Hence, the second relay can successfully decode w_{i-1} in block i given that

$$R - R_1 + R_1' < I(X; Y_2|X_1, X_2). \tag{16}$$

In block i, the second relay transmits $s_i^2 = P_{R_2,B_3}(w_{i-2})$ to the destination where B_3 is an independent random partitioning of the w space (of size 2^{nR}) into 2^{nR_2} bins of size $2^{n(R-R_2)}$.

The destination first decodes s_i^2 in block i, provided that

$$R_2 < I(X_2; Y). \tag{17}$$

Next, the destination decodes s_{i-1}^1 after canceling the interference from the second relay which is feasible if

$$R_1 < I(X_1; Y|X_2). \tag{18}$$

Finally, the destination decodes w_{i-2} using s_i^2 and s_{i-1}^1 as bin indices. Intersection of the two bins indexed by s_i^2 and s_{i-1}^1 forces w_{i-2} to be inside a bin of size $2^{n(R-R_1-R_2)}$. Consequently, w_{i-2} can be successfully decoded if

$$R - R_1 - R_2 < I(X; Y|X_1, X_2). \tag{19}$$

Combining {(19), (18), (17)}, {(16), (15)} and (4) results in the rate (14). Note that the condition $I(X_1; Y_2|X_2) \leq I(X_1; Y|X_2)$ is necessary in order for (15) and (18) to be consistent.

Note that for a general two-relay network, the rate (14) is an improvement over the rate (9) if $I(X; Y_2|X_1, X_2) > I(X; Y|X_1, X_2)$. That is, the second relay can improve the overall data rate from the source to the destination if the link between the source and the second relay is stronger than the link between the source and the destination.

IV. Parity-forwarding in Larger Networks

Parity-forwarding is flexible enough to apply to networks with complex topologies, since it allows for a relay node to choose which links to facilitate the decoding for. In this section, we outline a relaying protocol in an example network which shows how the one-way and the two-way relay protocols can be generalized and combined in a larger network.

Consider the network depicted in Fig. 4. Different relaying protocols suitable for different link specifications can be designed using the parity generation function for this network. In this example, it is assumed that the channel between the second relay and the third relay is stronger than the channel between the second relay and the destination.

In this network, the second relay has a poor channel to the source and only helps the first relay (i.e., the second relay is a one-way relay). The third relay helps the destination to decode the message from both the second relay and the source (i.e., the third relay is a two-way relay).

Referring to Fig. 4, this protocol is described using parity generation functions as follows: In block i, the source knows $(w_i, s_i^1, s_i^2, h_i, l_i)$, while the first relay knows (s_i^1, s_i^2, h_i, l_i), the second relay knows (s_i^2, l_i), and the third relay knows (h_i, l_i). We have $s_i^1 = P_{R_1,B_1}(w_{i-1})$ and $s_i^2 = P_{R_2,B_2}(s_{i-1}^1)$ (i.e., the second relay acts as an one-way relay.) Similar to the two-way relay protocol A, s^3 encodes two messages representing parities for both w and s^2, i.e., $s_i^3 = (h_i, l_i) = \big(P_{R_h,B_h}(w_{i-3}), P_{R_l,B_l}(s_{i-1}^2)\big)$ and $R_3 = R_h + R_l$. Here, B_1, B_2, B_h, and B_l denote independent uniform random partitions of size 2^{nR_1}, 2^{nR_2}, 2^{nR_h} and 2^{nR_l} of the message spaces of w, s^1, w and s^2, respectively.

The decoding process at each node takes advantage of the dependencies between the messages through the binning functions. For example, the third relay benefits from s_i^2 to decode s_{i-1}^1. Then, it uses s_{i-1}^1 as the index of a bin of the size $2^{n(R-R_1)}$ of candidate w-messages in order to decode w_{i-2}. The destination decodes the message of the third relay and incorporates l_i to partially decode s_{i-1}^2. The source message w_{i-3} is related to the bin index s_{i-1}^2 through s_{i-2}^1. The bin index s_{i-1}^2 confines w to be inside a bin of size $2^{n(R-R_2)}$, because it is possible to find an equivalent direct parity generator function for successive generation of parities.

Using this protocol, it is possible to prove that the following rate is achievable for this network.

Theorem 2: An achievable rate for the three-relay network example shown in Fig. 4 is given by (20) maximized over $p(u)p(x_3|u)p(x_2|u)p(x, x_1|x_2, x_3, u)$.

$$R < I(X; Y_1|X_1, X_2, X_3, U) \quad (20a)$$
$$R < I(X, X_1; Y_3|X_2, X_3, U) + I(X_2; Y_3|U) \quad (20b)$$
$$R < I(X; Y_3|X_1, X_2, X_3, U) + I(X_1; Y_2|X_2, X_3, U) \quad (20c)$$
$$R < I(X; Y|X_1, X_2, X_3, U) + I(X_2; Y|U) + I(X_3, U; Y) \quad (20d)$$

This rate is achievable with the outlined protocol if

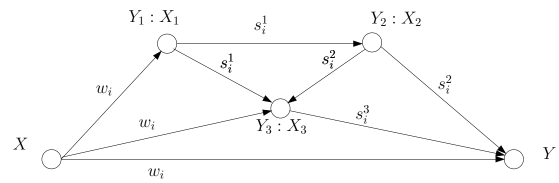

Fig. 4. A three-relay network.

$I(X_2; Y_3|U) \geq I(X_2; Y|U)$. Another protocol is needed for the case $I(X_2; Y_3|U) < I(X_2; Y|U)$ in which the second relay performs the two-way relay protocol B. A detailed proof is presented in the extended version of this paper.

V. Conclusion

A generalization of decode-and-forward relaying strategy is developed for multiple-relay networks in which each relay forwards a random bin index (or equivalently parity bits) of its decoded messages. A new class of degraded networks is identified. It is shown that this strategy achieves the capacity of a new class of degraded two-relay networks while encompasses previous results obtained by regular encoding [7]. The proposed parity-forwarding scheme is flexible when applied to larger networks with various topologies.

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