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Research highlights:

- Flexible forming technologies are becoming increasingly significant with the rise of small batch and customised production in the market.
- Incremental Sheet Forming (ISF) is an emerging technology that promises high flexibility and formability but the low part accuracy limits its industry application.
- A model predictive control (MPC) algorithm was developed for two point incremental sheet forming (TPIF) with a partial die to improve part accuracy via two-directional toolpath correction.
- The control models are built based on the deformation nature of the TPIF process with a partial die.
- The TPIF process with MPC control leads to significant improvement in part accuracy compared with the traditional TPIF without toolpath control.

Part accuracy improvement in two point incremental forming with a partial die using a model predictive control algorithm

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Abstract

As a flexible forming technology, Incremental Sheet Forming (ISF) is a promising alternative to traditional sheet forming processes in small-batch or customised production but suffers from low part accuracy in terms of its application in the industry. The ISF toolpath has direct influences on the geometric accuracy of the formed part since the part is formed by a simple tool following the toolpath. Based on the basic structure of a simple Model Predictive Control (MPC) algorithm designed for Single Point Incremental Forming (SPIF) in our previous work [1] that only dealt with the toolpath correction in the vertical direction, an enhanced MPC algorithm has been developed specially for Two Point Incremental Forming (TPIF) with a partial die in this work. The enhanced control algorithm is able to correct the toolpath in both the vertical and horizontal directions. In the newly-added horizontal control module, intensive profile points in the evenly distributed radial directions of the horizontal section were used to estimate the horizontal error distribution along the horizontal sectional profile during the forming process. The toolpath correction was performed through properly adjusting the toolpath in two directions based on the optimised toolpath parameters at each step. A case study for forming a non-axisymmetric shape was conducted to experimentally validate the developed toolpath correction strategy. Experiment results indicate that the two-directional toolpath correction approach contributes to part accuracy improvement in TPIF compared with the typical TPIF process that is without toolpath correction.

Keywords: Two point incremental forming; Model predictive control; Geometric accuracy; Toolpath correction.

1. Introduction

ISF is a flexible sheet forming technology that offers a promising alternative to traditional sheet forming processes for the cost-effective production of small-batch or customised parts. Without using dedicated dies, it uses a simple tool with a hemispherical end to form the sheet parts. By travelling along a 3D CAM toolpath on the surface of the sheet, the tool end deforms the sheet incrementally with the plastic deformation localised near the tool end [2] during the forming process. The two main variations of ISF are SPIF and TPIF. The main difference is that TPIF uses simple dies besides the forming tool (Figure 1b and c) while SPIF (Figure 1a) only uses a single tool to form the parts. To be more specific, TPIF can be classified into two types, namely TPIF with a full die and TPIF with a partial die, as shown in Figure 1b and c. The simple dies are generally made of cheap materials like timber or resin [3] so that the cost in the fabrication and storage of dies is not huge. With the use of supporting dies, TPIF generally leads to better geometric accuracy of the formed parts than SPIF [4].

The major limitation of ISF is the poor geometric accuracy of the formed parts. The errors are usually caused by the sheet springback and sheet bending. The sheet springback accounts for the geometric inaccuracies in the most areas of the formed parts. In addition, the “pillow effect”, an unwanted curved surface, typically occurs on the flat base of the part formed in SPIF [5]. In order to improve the poor geometric accuracy, some attempts have been presented in the literature, including experimental investigation of process parameters [4, 6], hybrid ISF processes [7-9], the use of partially cut-out blanks [10], and a multi-stage strategy [3].

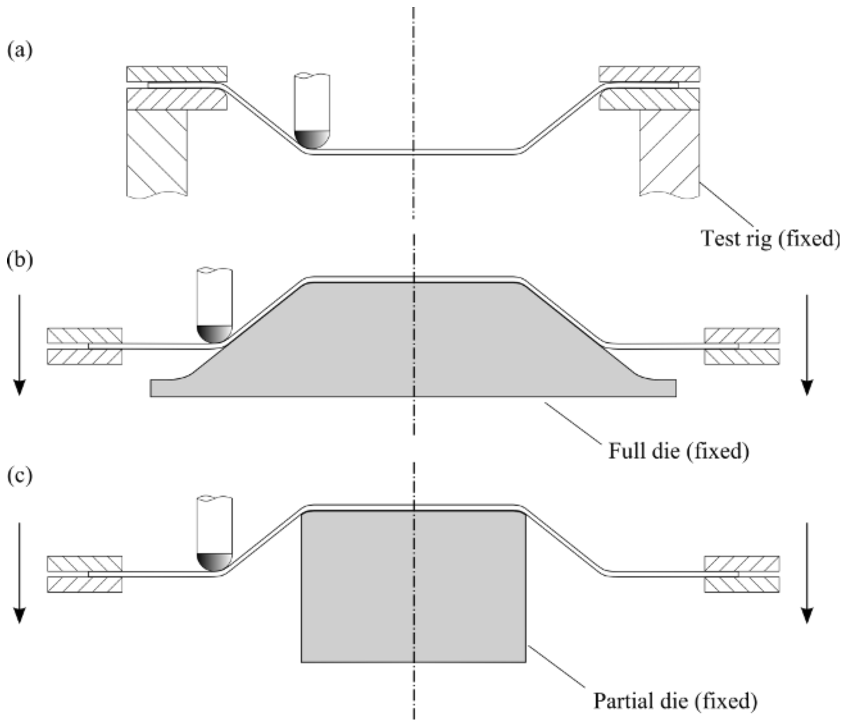

Figure 1 Typical ISF variations.

Recently, many studies concentrated on the toolpath correction/optimisation. The ISF toolpath can be corrected by using error compensation based on trial fabrications [11], a feature-based toolpath generation strategy [12], a Multivariate Adaptive Regression splines (MARS) correction strategy [13], iterative algorithms based on a transfer function [14], and an artificial cognitive system [15]. Moreover, some in-process toolpath correction approaches were performed in SPIF based on a control strategy using spatial impulse responses of the process [16] and a MPC strategy [17]. In particular, the MPC control strategy reported in [17] only dealt with the optimisation of the step depth, which is one of the two critical toolpath parameters in ISF. The predictive model in the control algorithm was obtained based on the formed shape in SPIF without toolpath control. In our previous work [1], a different MPC control strategy was developed to improve geometric accuracy via in-process toolpath correction. The MPC control algorithm used an analytical predictive model to vertically correct the toolpath by optimising the step depth during the forming process. The results showed that the geometric errors were improved in the base areas of the formed part, but the errors in the part wall areas were still relatively large since the proposed control algorithm only dealt with toolpath correction in the vertical direction. Additionally, obvious "pillow effect" was observed at the flat bases of the parts formed in the SPIF processes. The "pillow effect" is one of typical geometric inaccuracies in SPIF, however, there is no notable "pillow effect" in TPIF with the use of dies to support the flat base [18].

Currently, there are not many studies that have been reported on in-process toolpath correction in TPIF. In TPIF with a single full die, it is difficult to correct the geometric errors by freely adjusting the toolpath because the tool movement is greatly limited by the full die with a definite shape as shown in Figure 1b. A possible way for toolpath correction in TPIF with a full die is to use different full dies with modified shapes at the intermediate stages of the forming process. It would be time-consuming to fabricate multiple full dies for forming a part. On the contrary, TPIF with a partial die is more flexible and the toolpath can be freely adjusted in a large range for geometric accuracy improvement.

This paper presents a MPC control algorithm for TPIF with a partial die to improve geometric accuracy via in-process toolpath correction in the horizontal and vertical directions. Based on our previous work for SPIF toolpath correction only in the vertical direction [1], an enhanced MPC algorithm has been developed specially for TPIF with a partial die to correct the toolpath through optimising the step depth and the horizontal step increment. In particular, a horizontal control module is added in the enhanced MPC algorithm for toolpath correction in the horizontal direction. In the horizontal control module, intensive profile points in the evenly distributed radial directions of the horizontal section were used to estimate the horizontal error distribution along the horizontal sectional profile during the forming process, which provide an achievable method to estimate the horizontal error distribution in forming general shapes. Two analytical models, based on the deformation nature in TPIF with a partial, were used for shape state predictions in two directions so that in-process toolpath control and correction can be directly conducted. During the forming process, shape measurement is performed at each time-step to provide shape feedback in the control algorithm. In the two (vertical and horizontal) separate MPC modules, optimised values of step depth and horizontal step increment are used to correct the toolpath in the vertical and horizontal directions, respectively. A non-axisymmetric shape was used to experimentally validate the developed control strategy. The experimental results were analysed by comparing the typical TPIF process and the TPIF process with toolpath correction in terms of horizontal sectional profiles, the error colour map, and the percentage distribution of geometric deviations. This work offers an achievable approach on in-process toolpath control/correction in TPIF with improved geometric accuracy on the final parts.

2. MPC control strategy for TPIF with a partial die

MPC is an advanced control technology and it is able to use linear models to deal with the control of constrained non-linear systems in various industry processes [17, 19]. This control technology is used to drive the system state following the target trajectory by minimising the predicted states and the target states in a finite horizon. Suppose a nonlinear system is modelled as,

$$y(k+1) = f\,(y(k), \Delta u(k)). \tag{1}$$

The optimisation problem solved by MPC at each time instant is expressed in the following equation,

$$\min \ J(k) = \left\| \hat{Y}(k) - W(k) \right\|^2 + \lambda \left\| \Delta U(k) \right\|^2, \tag{2}$$

$$\text{subject to} \quad \hat{y}(k+1 \mid k) = Ay(k) + B\Delta u(k)$$

$$\Delta u_{\min} \le \Delta u \le \Delta u_{\max}$$

where $J(k)$ is the cost function, $\hat{Y}(k) = \left[\hat{y}(k+1|k), \hat{y}(k+2|k), \ldots, \hat{y}(k+N_p|k) \right]^T$ contains the predicted system outputs and $\hat{y}(k+1|k) = Ay(k) + B\Delta u(k)$ is the linear simplification of the system, $W = \left[w(k+1), w(k+2), \ldots, w(k+N_p) \right]^T$ is the target trajectory, $\Delta U = \left[\Delta u(k+1), \Delta u(k+2), \ldots, \Delta u(k+N_p) \right]^T$ collects the system inputs over the prediction horizon (N_p).

The process model is dynamic because the prediction model at each sampling instant is updated based on the currently measured state and the estimated future inputs. Control actions over the finite horizon are obtained by solving an optimisation problem at the current sampling instant but only the first move of the control actions will be applied in the future.

The in-process toolpath correction strategy for TPIF with a partial die is developed based on a two-directional MPC control algorithm. The basic concept of toolpath correction in TPIF is illustrated in Figure 2. There are geometric errors on the formed part using the initial toolpath. By correcting the toolpath based on shape feedback in the MPC controlled process, the geometric errors can be reduced on the final parts. More specifically, the toolpath is corrected in two perpendicular directions during the forming process. This is achieved by properly modifying the values of two critical toolpath parameters, namely the horizontal step increment (Δu_r) and the step depth (Δu_z) in Figure 4, based on the optimisation in the MPC algorithm.

Figure 3 illustrates the structure of feedback control of TPIF using the two-directional MPC control algorithm. The forming process consists of a number of forming steps and each forming step corresponds to the forming a single contour. The control algorithm takes each forming step as a time-step. At time-step k, the measured shape states of currently formed shape are imported to the predictive models for shape state predictions of future several steps in the two directions. To drive the predicted shape states as close as to target shape states, the MPC optimisers will optimise Δu_z and Δu_r of the next several steps, however only the optimised Δu_z and Δu_r of the following single step ($k+1$) are used to correct the toolpath. In this way, the control of toolpath parameters will be repeated step by step till the end of the whole forming process.

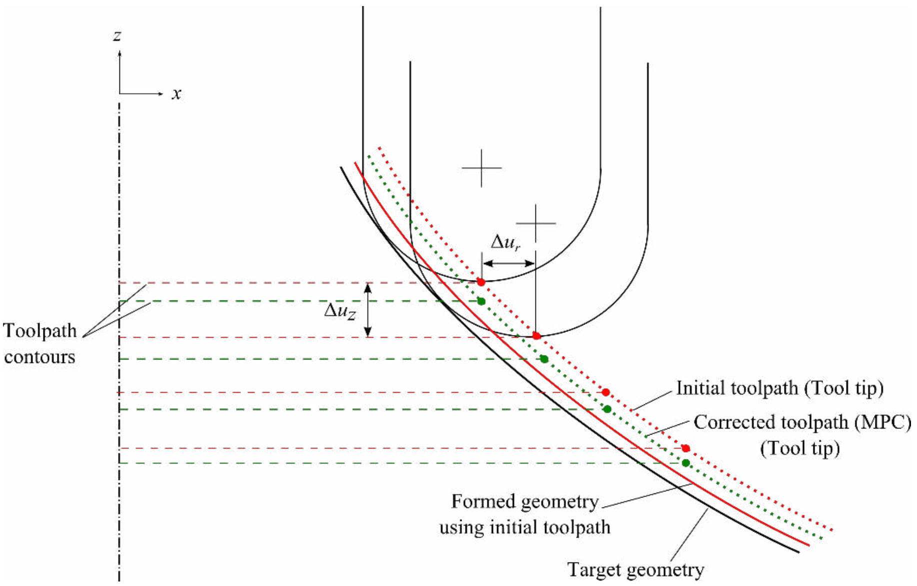

Figure 2 Toolpath correction by using a MPC control algorithm (section view).

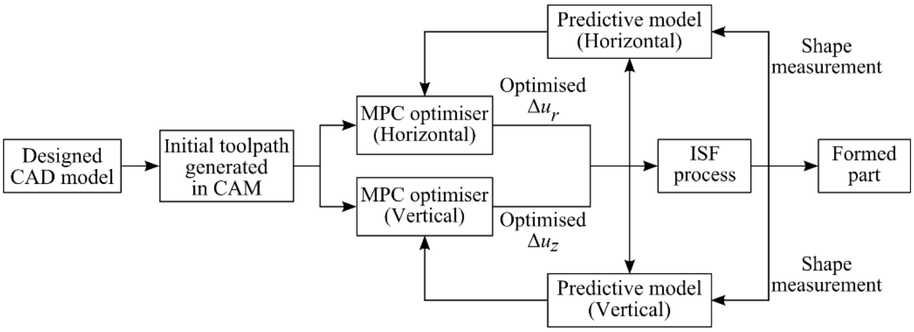

Figure 3 MPC control of two toolpath parameters in ISF.

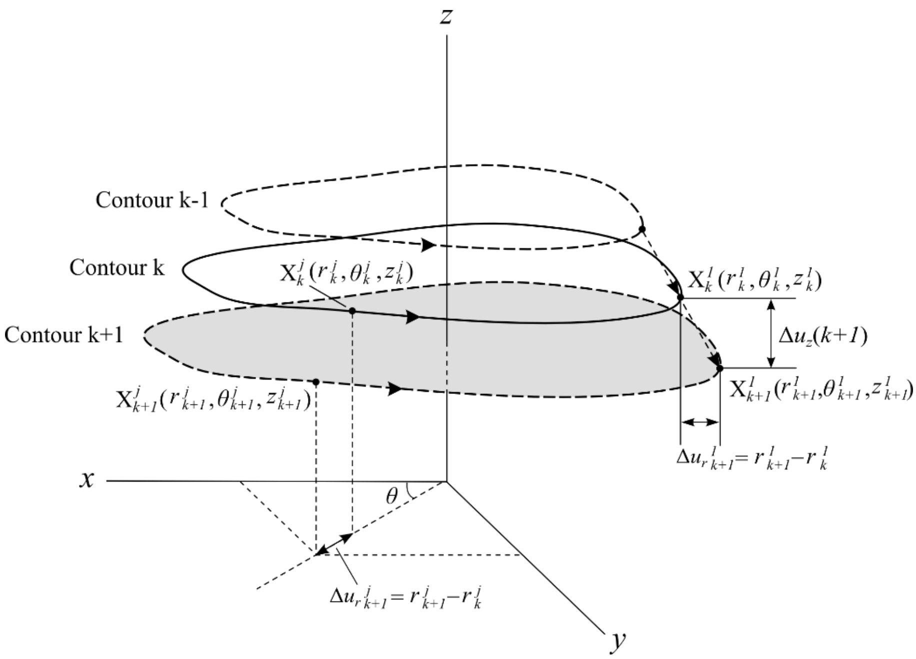

Figure 4 Two critical parameters of the contour toolpath.

There are no specialised modules for generating ISF toolpath in commercial CAM softwares. The most commonly-used toolpath type in ISF is the parallel contour toolpath generated in the milling module of CAM softwares [20]. It consists of a certain number of contours that are parallel to each other, as shown in Figure 4. In this control model, each contour is represented by a certain number (*m*) of contour points defined in the cylindrical coordinate system. Therefore, the toolpath can be horizontally corrected in enough number of radial directions when forming general shapes, such as non-axisymmetric shapes. Δu_r collects the radial distances between on two successive contours in terms of the predefined contour points (Figure 4). Δu_r can be represented as,

$$\Delta u_r(k+1) = \left[\Delta u_r^1(k+1), \Delta u_r^2(k+1), \ldots, \Delta u_r^j(k+1), \ldots, \Delta u_r^m(k+1)\right], \tag{3}$$

$$\text{and } \Delta u_r^j(k+1) = r_{k+1}^j - r_k^j, j = 1,2,...,m$$

where m is the number of contour points at each step, $\Delta u_r^j(k+1)$ is the radial distance between corresponding profile points from two neighbouring contours.

Step depth, Δu_z , is the vertical depth that the tool travels down between two consecutive steps. In particular, the number of contours depends on the Δu_z value and the total depth of the shape design. Therefore, the number of steps is also determined by Δu_z and the shape depth since the forming of a single contour is taken as a time-step. At each step, the z coordinates of all contour points on a single contour are the same since these points are on the same z-level plane. This can be expressed as,

$$\Delta u_z(k+1) = z_{k+1} - z_k . \tag{4}$$

Between two neighboring contours of the toolpath, Δu_z and Δu_r account for the tool movements in the vertical direction and the horizontal direction, respectively. Therefore, two separate MPC control modules are designed to optimise the two parameters separately.

2.1 MPC control algorithm in TPIF

A simple MPC control algorithm that only optimises Δu_z in SPIF was demonstrated in our previous study [1] that it was able to properly correct the toolpath in the vertical direction. Toolpath correction in the vertical direction leads to geometric accuracy improvement in the base area of the formed parts whilst the part accuracy in the wall areas requires further improvement. This is consistent with the work reported in [17]. In this work, an enhanced MPC algorithm has been developed specially for TPIF with a partial die to correct the toolpath in the vertical direction as well as the horizontal direction. To be more specific, a new horizontal MPC module is added into the control algorithm to correct the toolpath in both the horizontal and vertical directions to reduce dimensional deviations of the formed part. The predictive models in the vertical and horizontal control modules are analytical models that are built based on the deformation nature of TPIF with a partial die in terms of the two directions.

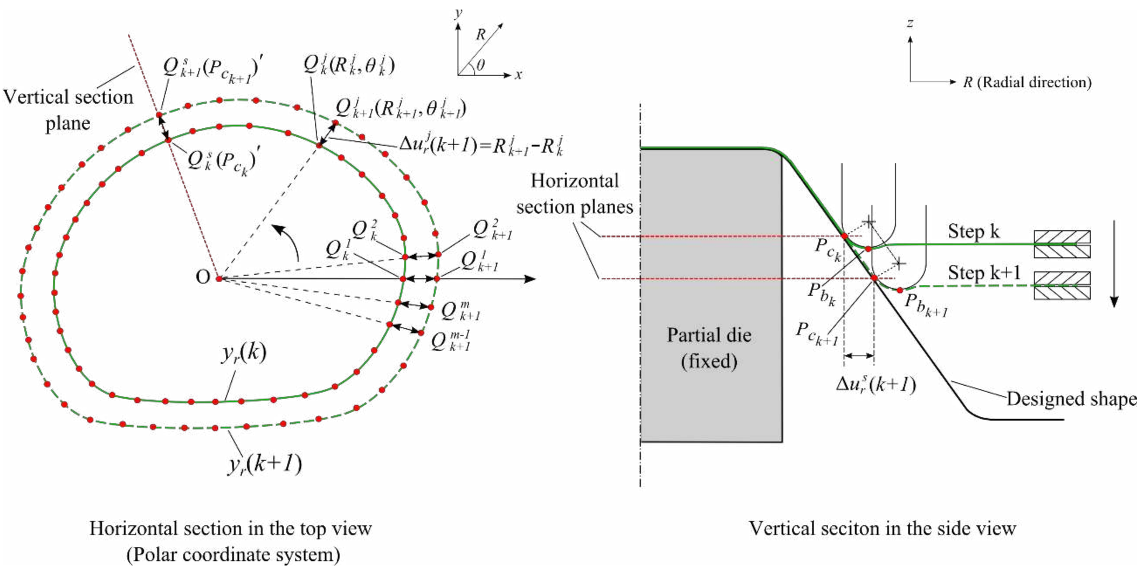

Figure 5 Cross-sectional profiles of formed shapes during TPIF.

Figure 5 shows the cross-sectional profiles of the formed shape in two directions during the forming process of TPIF. At each step, the formed shape is scanned and sectioned horizontally and vertically to get the cross-sectional profiles in the horizontal and vertical directions, respectively. The locations of the contact point P_{c_k} and the bottom point P_{b_k} are geometrically identified based on the local feature of the currently formed shape after scanning the formed shape. The cross-sectional profiles in the two directions are taken as the shape states for optimisation in the control modules. Each vertical sectional profile is obtained in a vertical section plane through a predefined radial direction $\theta_k^s = \theta_k^j (j = 1, 2, \cdots, m)$. On the vertical sectional profiles, contact points P_{c_k} and $P_{c_{k+1}}$ in the side view are the points Q_k^s and Q_{k+1}^s in the top view, respectively. The top view illustrates the horizontal sections, which corresponds to the horizontal section planes in the side view, at two neighbouring steps.

Since the deformation models for building the predictive models in the vertical and horizontal modules are different and they are obtained by sectioning the part in the vertical and horizontal directions, respectively, the two control modules are described and explained separately for clear understanding.

2.1.1 Horizontal control module

The newly-added horizontal MPC module is aimed to optimise Δu_r for toolpath correction in the horizontal direction. In the horizontal model, the horizontal sectional profile at each step is represented by intensive profile points in different radial directions, as seen in the top view of Figure 5. This method can provide proper accuracy for the estimation of horizontal error distribution in forming general shapes. As a matter of the resolution for representing a curve, more profile points can give more accurate estimation of the formed sectional profiles, especially. To guarantee enough accuracy in representing the formed profiles and the horizontal error distribution, the horizontal sectional profile at each step is sampled in 360 uniformly distributed radial directions in this study, as shown in the top view of Figure 5. That is, the number of profile points is set as *m*=360. At time-step *k*, $Q_k^j(R_k^j, \theta_k^j)$ is the *j*th profile point of horizontal profile $y_r(k)$ and its radial coordinate (R_k^j), is taken as the profile state of this point. Consequently, the horizontal profile state of step *k* is defined in the following equation,

$$y_r(k) = \left[y_r^1(k), y_r^2(k), \ldots, y_r^j(k), \ldots, y_r^m(k) \right]. \tag{5}$$

From the definition of Δu_r (Figure 4), namely the horizontal distance that the tool moves between two consecutive steps, the horizontal distance between Q_k^j and Q_{k+1}^j can be taken to be $\Delta u_r^j(k+1)$, as shown in the side view of Figure 5. In TPIF, the part is formed incrementally step by step with the sizes of Δu_r and Δu_z typically being small. Consequently, the amounts of springback in two neighbouring steps are very close to each other and can be taken to be equal to each other. Based on this, a linear model is built for MPC control to predict the horizontal profile states during the forming process. Since the linear model is dynamic and the feedback of the measured shape state is updated at each step, the prediction errors brought by the linear model can be partially compensated. Therefore, the linearisation can be taken as reasonable for in-process control in ISF. After the shape measurement at step *k*, the state of point Q_{k+1}^j can be predicted in the next equation.

$$\hat{y}_r^j(k+1) = y_r^j(k) + \Delta u_r^j(k+1) \tag{6}$$

Taking into account the profile points in all radial directions, the profile state of next step can be estimated as,

$$\hat{y}_r(k+1|k) = y_r(k) + \Delta u_r(k+1), \tag{7}$$

where $\hat{y}_r(k+1) = \left[\hat{y}_r^1(k+1), \hat{y}_r^2(k+1), \ldots, \hat{y}_r^j(k+1), \ldots, \hat{y}_r^m(k+1)\right]$ is the predicted profile state of next step, $\Delta u_r(k+1) = \left[\Delta u_r^1(k+1), \Delta u_r^2(k+1), \ldots, \Delta u_r^j(k+1), \ldots, \Delta u_r^m(k+1)\right]$ is the horizontal step increment of next step.

In the future several steps, the predicted profile states can also be obtained in the following equation,

$$\begin{aligned}
&\hat{y}_r(k+1|k) = y_r(k) + \Delta u_r(k+1) \\
&\hat{y}_r(k+2|k) = \hat{y}_r(k+1|k) + \Delta u_r(k+2) \\
&\qquad\qquad\quad = y_r(k) + \Delta u_r(k+1) + \Delta u_r(k+2) \\
&\qquad\vdots \\
&\hat{y}_r(k+N_p|k) = y_r(k) + \Delta u_r(k+1) + \Delta u_r(k+2) + \ldots + \Delta u_r(k+N_p)
\end{aligned} \quad , \tag{8}$$

where N_p is the prediction horizon in MPC control and is set as 6 in this work after tuning.

By collecting Equation (8) together, the matrix-vector form of this equation can be obtained,

$$\hat{Y}_r(k) = Y_r(k) + L_r \Delta U_r(k), \tag{9}$$

where $Y_r(k) = [y_r(k), y_r(k), y_r(k), \ldots, y_r(k)]^T$ is the measured profile states at current step, $\hat{Y}_r(k) = \left[\hat{y}_r(k+1|k), \hat{y}_r(k+2|k), \ldots, \hat{y}_r(k+N_p|k)\right]^T$ is the collection of the predictions of future horizontal profile states, $\Delta U_r(k) = \left[\Delta u_r(k+1), \Delta u_r(k+2), \ldots, \Delta u_r(k+N_p)\right]^T$ stands for the horizontal inputs of next N_p steps; L_h is the coefficient matrix that indicates how the horizontal input influences the horizontal profile state in the future N_p steps and it is expressed in the next equation,

$$L_r = \begin{bmatrix} I_{m\times m} & 0 & 0 & 0 \\ I_{m\times m} & I_{m\times m} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ I_{m\times m} & I_{m\times m} & \cdots & I_{m\times m} \end{bmatrix}. \tag{10}$$

At time-step *k*, the control problem in the horizontal module can be summarised as an optimisation problem to optimise the horizontal toolpath inputs through minimising the difference between the predicted profile states and target profile states in the prediction horizon:

$$\min\ J_r = \left\|\hat{Y}_r - W_r\right\|^2 + \lambda_r \left\|\Delta U_r - \Delta U_{r0}\right\|^2, \tag{11}$$

$$\text{subject to } \hat{y}_r\left(k+j \mid k\right) = y_r(k) + \sum_{i=1}^{j} \Delta u_r(k+i),\ j = 1,2,\ldots,N_p$$

$$\left|\Delta u_r(k+j) - \Delta u_{r\,0}(k+j)\right| \le e,\ j = 1,2,\ldots,N_p$$

where J_r stands for the cost function, W_r is the target profile states in the horizontal direction, ΔU_r contains the horizontal inputs of several future steps, ΔU_{r0} is the collection of the initial horizontal inputs in the initial toolpath. Besides the minimisation of the difference between $\hat{Y}_r\left(k\right)$ and W_r, another item is added in the cost function to limit the control inputs in a predefined range around the initial inputs with e is set as 0.5, λ_r is the weighting coefficient and is adopted as 0.7 in this work after tuning;

2.1.2 Vertical control module

To optimise Δu_z for toolpath correction in the vertical direction, the vertical MPC module is developed based on a different deformation model that only uses one specified point to represent the vertical shape state in the vertical module while the control structure is similar to the horizontal control module.

In the side view of Figure 5, the vertical cross-sectional profile at each step is a curve which also can be represented by a number of points. At step k, P_{c_k} is the contact point where the flat local wall is tangent to the hemispherical end of the tool while P_{b_k} is the point where the bottom point of the ball end touches the metal blank. Due to the springback, the formed depth of the shape is generally smaller than the target depth. This control module is to drive the metal blank to the target depth by using the optimised Δu_z. That is, the vertical control module aims to drive the bottom point on the cross-sectional profile to the target position. Considering the vertical sectional profiles in all the radial directions ($\theta_k = \theta_k^1, \theta_k^2 \cdots, \theta_k^j, \cdots, \theta_k^m$) in the top view of Figure 5, the z coordinates of the bottom points of these vertical sectional profiles are taken as the shape state, $y_z(k)$, in the vertical direction.

$$y_z(k) = \left[y_z^1(k), y_z^2(k), \ldots, y_z^j(k), \ldots, y_z^m(k) \right] \tag{12}$$

Based on the incremental nature of TPIF, the shape is formed incrementally step by step and the size of Δu_z typically is small. Therefore, the amounts of springback in two neighbouring steps are very close to each other and can be considered to be equal to each other for building a linear model for MPC control. Under this simplification, the vertical distance between P_{b_k} and $P_{b_{k+1}}$ in the radial direction $\theta=\theta_k^s$ will be the step depth of step *k+1*, as expressed in the following equation,

$$\Delta u_z^s(k+1) = \hat{y}_z^s(k+1|k) - y_z^s(k). \tag{13}$$

In the contour toolpath, the contours are parallel to each other. The contour points of each contour are on the same z-level plane, which is consistent with the way they are defined in the initial contour toolpath. Therefore, the $\Delta u_z(k+1)$ values in all the radial directions are the same. That is, $\Delta u_z(k+1) = \left[\Delta u_z^1(k+1), \Delta u_z^2(k+1), \ldots, \Delta u_z^j(k+1), \ldots, \Delta u_z^m(k+1)\right]$ can be expressed as,

$$\Delta u_z(k+1) = \Delta u_z^s(k+1) I_{1\times m} . \tag{14}$$

Based on this, the optimised $\Delta u_z(k+1)$ can be obtained in the optimisation of just one element, namely $\Delta u_z^s(k+1)$, in a selected radial direction. In this work, the radial direction through the middle of one of the curved fillets ($\theta_k^s = 45°$) was used for optimising $\Delta u_z(k+1)$ in the vertical control module.

Consequently, only one element of $y_z(k) = \left[y_z^1(k), y_z^2(k), \ldots, y_z^j(k), \ldots, y_z^m(k)\right]$, namely $y_z^s(k)$, is investigated in the vertical control module. The vertical shape state at each step can be simplified as,

$$y_z(k) = y_z^s(k) . \tag{15}$$

$y_z^s(k)$ can be obtained in the cross-sectional profile in the selected radial direction ($\theta_k^s = 45°$) after measuring the formed shape at step *k*. The vertical state of next step can be predicted as,

$$\begin{aligned}\hat{y}_z(k+1|k) &= y_z(k) + \Delta u_z^s(k+1) \\ &= y_z^s(k) + \Delta u_z^s(k+1)\end{aligned}, \tag{16}$$

where $\hat{y}_z(k+1)$ is the predicted profile state of next step. Similarly, the vertical shape states over the prediction horizon can be obtained by,

$$\begin{aligned}\hat{y}_z(k+1|k) &= y_z^s(k) + \Delta u_z^s(k+1) \\ \hat{y}_z(k+2|k) &= \hat{y}_z(k+1|k) + \Delta u_z^s(k+2) \\ &= y_z^s(k) + \Delta u_z^s(k+1) + \Delta u_z^s(k+2) \\ &\vdots \\ \hat{y}_z(k+N_p|k) &= y_z^s(k) + \Delta u_z^s(k+1) + \Delta u_z^s(k+2) + \Delta u_z^s(k+N_p)\end{aligned} \quad . \tag{17}$$

Equation (17) can be gathered into the matrix-vector form,

$$\hat{Y}_z(k) = Y_z(k) + L_z \Delta U_z(k), \tag{18}$$

where $\hat{Y}_z(k) = \left[\hat{y}_z(k+1|k), \hat{y}_z(k+2|k), \ldots, \hat{y}_z(k+N_p|k)\right]^T$ collects the state predictions of future N_p steps, $Y_z(k) = [y_z^s(k), y_z^s(k), \ldots, y_z^s(k)]^T$ is the measured shape state at time-step *k*, $\Delta U_z(k) = \left[\Delta u_z^s(k+1), \Delta u_z^s(k+2), \ldots, \Delta u_z^s(k+N_p)\right]^T$ is the vertical toolpath inputs, L_z is a matrix obtained by collecting Equation (17) together and is expressed as,

$$L_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ 1 & 1 & \cdots & 1 \end{bmatrix}. \tag{19}$$

To the drive the formed depth as close as possible to the target depth at each step, the control problem in the vertical control module at time-step *k* can also be summarised as an optimisation problem in the following equation. The second item is used to limit the size of vertical toolpath inputs that are negative at all steps. λ_z is set as 0.2, which is the same as in [1].

$$\min\ J_z = \left\| \hat{Y}_z - W_z \right\|^2 + \lambda_z \Delta U_z^{\ T} \Delta U_z \tag{20}$$

$$\text{subject to } \Delta U_z(k) = \left[\Delta u_z^s(k+1), \Delta u_z^s(k+2), \ldots, \Delta u_z^s(k+N_p) \right]^T, k = 1, 2, \ldots, N$$

$$\hat{y}_z(k+j \mid k) = y_z^s(k) + \sum_{i=1}^{j} \Delta u_z^s(k+i), j = 1, 2, \ldots, N_p$$

$$\Delta u_{z\ \min} \le \Delta u_z(k+j) \le \Delta u_{z\ \max}, j = 1, 2, \ldots, N_p$$

where J_z is the cost function, W_z is the target vertical states. The min and max values of Δu_z are 0.5mm and 2 mm, respectively.

In summary of Section 2.1, two separate MPC control modules have been developed to get the optimal Δu_r and Δu_z for in-process toolpath correction at each step. To solve the optimisation problems in the two control modules, the cost functions can be transformed into typical Quadratic Programming (QP) problems and would be solved using the method in our previous study [1]. After solving the optimisation problems, ΔU_z^* and ΔU_r^*, the optimised toolpath inputs over the next N_p steps, will be obtained. Only the first optimal move, namely the optimised toolpath inputs in the next one step, will be applied to form the contour in the next single step. Control actions will be conducted using the two-directional MPC control algorithm at each subsequent step until the shape is finally formed.

2.2 Application in TPIF with a partial die

The developed control strategy was applied to TPIF using a partial die to form parts. The implementation of the control system in the lab is shown in Figure 6. After the forming of a certain step was finished, currently formed shape was measured by a 3D digitiser. The shape states of current shape used for feedback control were generated by sectioning the scanned geometry. The shape states then were imported into the MPC control module where toolpath parameters were optimised in the well-defined optimisers. As a result, the metal blank was deformed by the tool following a corrected toolpath in the next step. As the tool formed down step by step, the toolpath was continuously corrected based on the shape feedback to get improved geometric accuracy in the final part.

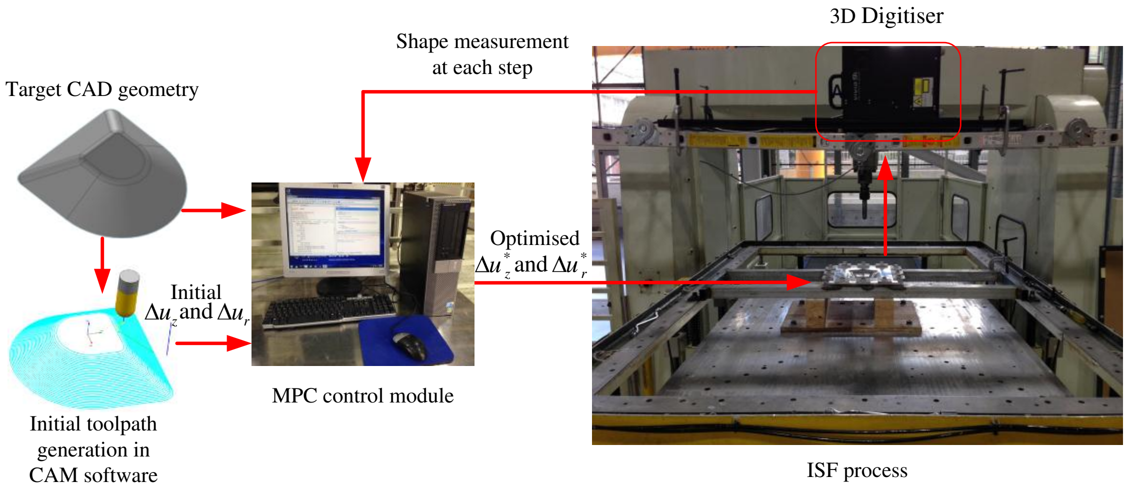

Figure 6 Structure of the closed-loop control system for ISF.

3. Experimental validation

The developed control strategy for TPIF with a partial die was experimentally validated in forming a non-axisymmetric shape. The control system for TPIF is built based on an ISF machine from AMINO® Corporation (Figure 6). During the forming process with control, the shape measurement of the formed parts for feedback is completed using a 3D Digitiser (VIVID 9i) placed on the top of the forming platform. It takes about 2.5 seconds for each scan and the scanning accuracy is in the range of ±0.05 mm, which is of sufficient accuracy for shape measurement in ISF. Then, the geometric data of scanned formed parts is collected in GEOMAGIC Qualify and is imported to the control algorithm programmed in Python. When the forming process is completed, the 3D comparison between the scanned formed part and the designed CAD model is performed using GEOMAGIC Qualify. The experiment results from uncontrolled and controlled TPIF processes were compared and analysed in terms of the geometric accuracy.

3.1 Case studies

The test shape used for the experiments was a non-axisymmetric shape, which contains both flat and curved walls (Figure 7). The wall angle was 40° and the total depth was 35mm. There are a number of z-level contours in the initial toolpath, as shown in Figure 7. The initial step depth was set as 1mm. Consequently, the number of steps in the forming process was calculated as 35. The metal sheet used for tests was made of aluminium (AA 7075-O) and the raw thickness was 1.6 mm. The unformed blank size was 300 mm × 300 mm. A ball-ended tool with a 20 mm diameter was used in the experiments. The feed rate in the forming process was 4000 mm/min. Based on the test shape, a partial die (Figure 8) made of timber was fabricated for the TPIF forming process in the test. Lubricating oil was used for lubrication during the forming process.

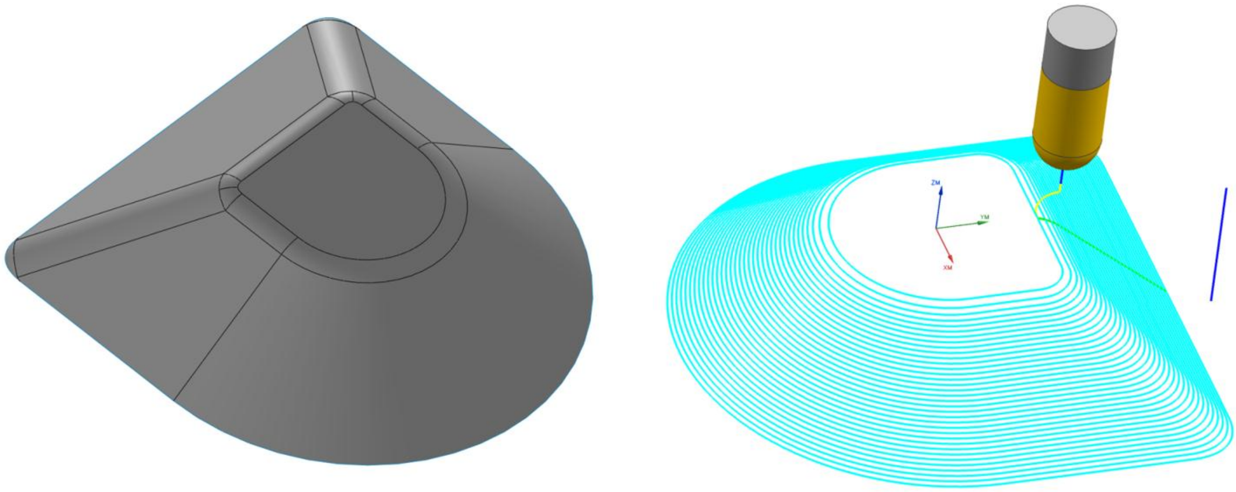

Figure 7 Test shape and the initial toolpath from CAM software.

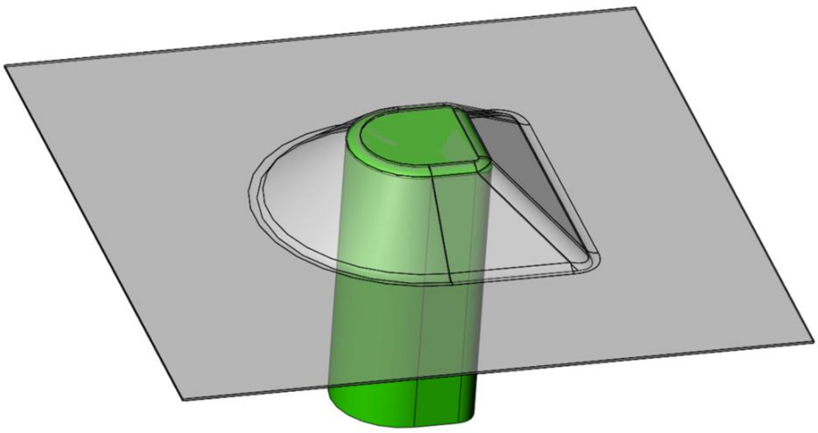

Figure 8 The partial die used in the TPIF process.

3.2 Results and discussion

In this section, the results of toolpath correction in TPIF are firstly illustrated by the comparison of initial toolpath and the corrected toolpath. Figure 9 shows the contours of the corrected toolpath and initial toolpath in terms of three sample steps (10th, 20th, and 30th). The initial depth of the contours at the three steps are z=-10, -20, -30mm, respectively, since the initial step depth was set as 1mm. From the comparison in the isometric view and the top view, it can be observed that the toolpath is corrected in the horizontal and vertical directions. This is achieved based on the optimised Δu_z and Δu_r at each step during the forming process.

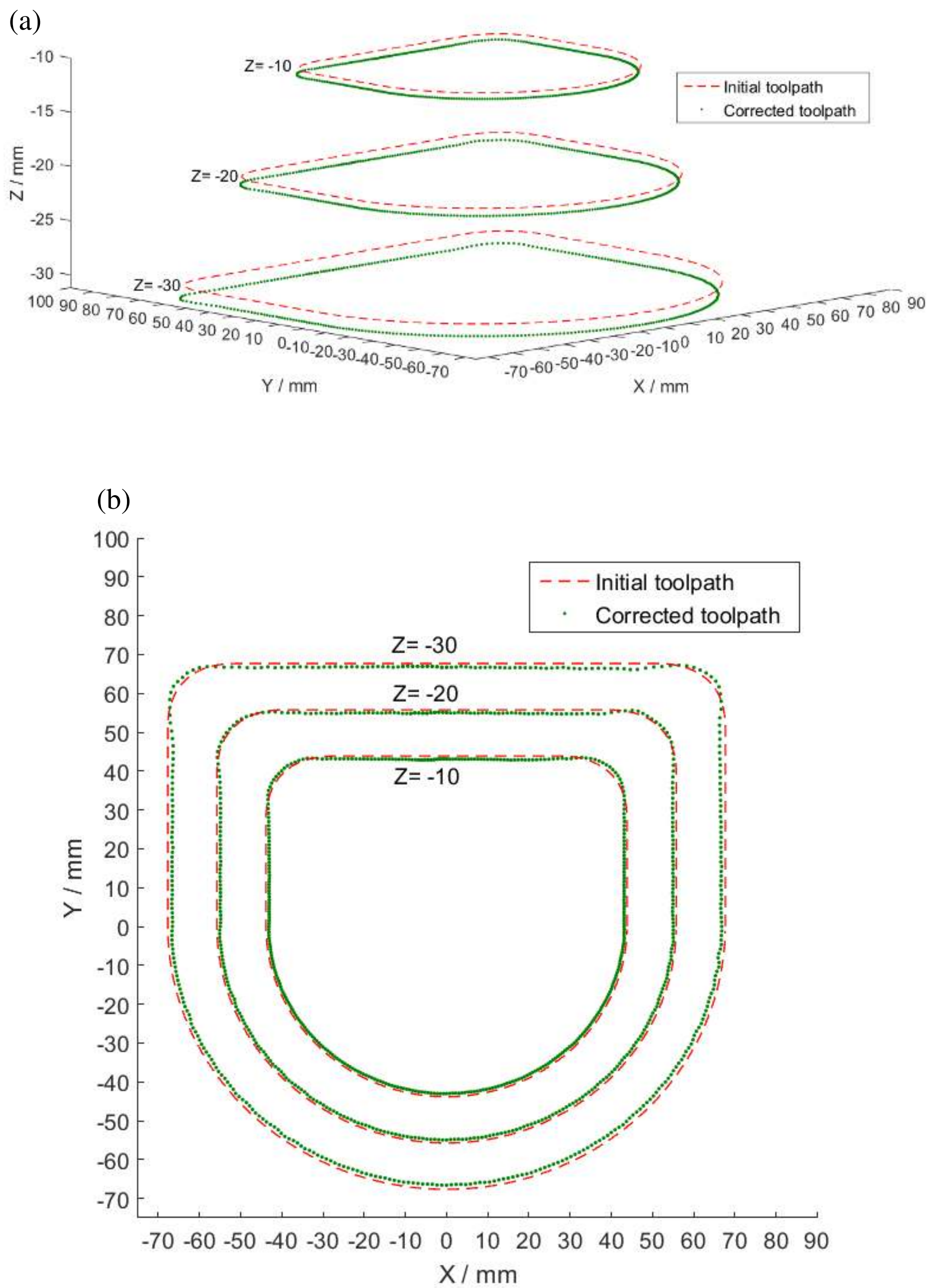

Figure 9 Contours of the corrected toolpath and the initial toolpath at three sample steps: (a) Isometric view; (b) Top view;

To analyse the geometric accuracy of uncontrolled and controlled TPIF processes, formed parts in the unclamped condition are compared using horizontal sectional profiles (Figure 10) and error distribution colour maps from the top view (Figure 11). The sectional profiles in Figure 10 were obtained in the horizontal sections at three z levels (z=-10, -20, -30mm).

In the TPIF process without toolpath control, the formed part has low geometric accuracy in the wall areas and the errors reach as large as 3 mm near the outside (bottom) edges, as shown in Figure 10 and Figure 11a. This is mainly caused by the sheet springback. However, from the top view (Figure 11), the inside base area (top) supported by the partial die is of high accuracy ($\pm$0.3 mm) in both controlled and uncontrolled TPIF processes. Also, there is no notable "pillow effect", which occurs in the flat base of the formed part in SPIF, observed in the base areas in the TPIF processes.

Compared with the part formed in uncontrolled TPIF, the part formed with MPC control has improved accuracy in most areas, as shown in Figure 11b. This can also be demonstrated by the comparison of the sectional profiles (Figure 10). In particular, there is significant improvement (from $\pm$3 mm to $\pm$0.3 mm) on the geometric accuracy in terms of the wall areas. Nevertheless, the accuracy in the certain areas of the corner fillets is still out of the desirable range and is slightly worse than the result from TPIF without toolpath control. In the areas of corner fillets, the local curvature of the shape changes rapidly in a relatively small range since the fillet radius equals the tool radius in the shape design. This could cause sudden changes of the strain when the tool deforms this area. As a result, the springback in the corner fillet areas varies rapidly so that the springback would be more complex and more difficult to capture. The springback of these areas was not well compensated in the current control model, which is the primary limitation of the current work.

Figure 12 shows the comparison of deviation distributions (percentage) between TPIF processes with and without control. More specifically, the formed shape was scanned into a large number of scatter points. This figure illustrates how dimensional deviations of all the points are distributed in different deviation ranges in the form of percentage distribution. The percentage of deviations ranging from +0.6 mm to +3.0 mm is greatly shortened from 78% to 8.5% with the use of MPC control algorithm. The percentage of deviations in the range $\pm$0.3 mm significantly increases from 19.5% to 70%. Compared with uncontrolled TPIF, the deviations of the points are more intensively distributed near the desirable range in terms of the TPIF with toolpath control.

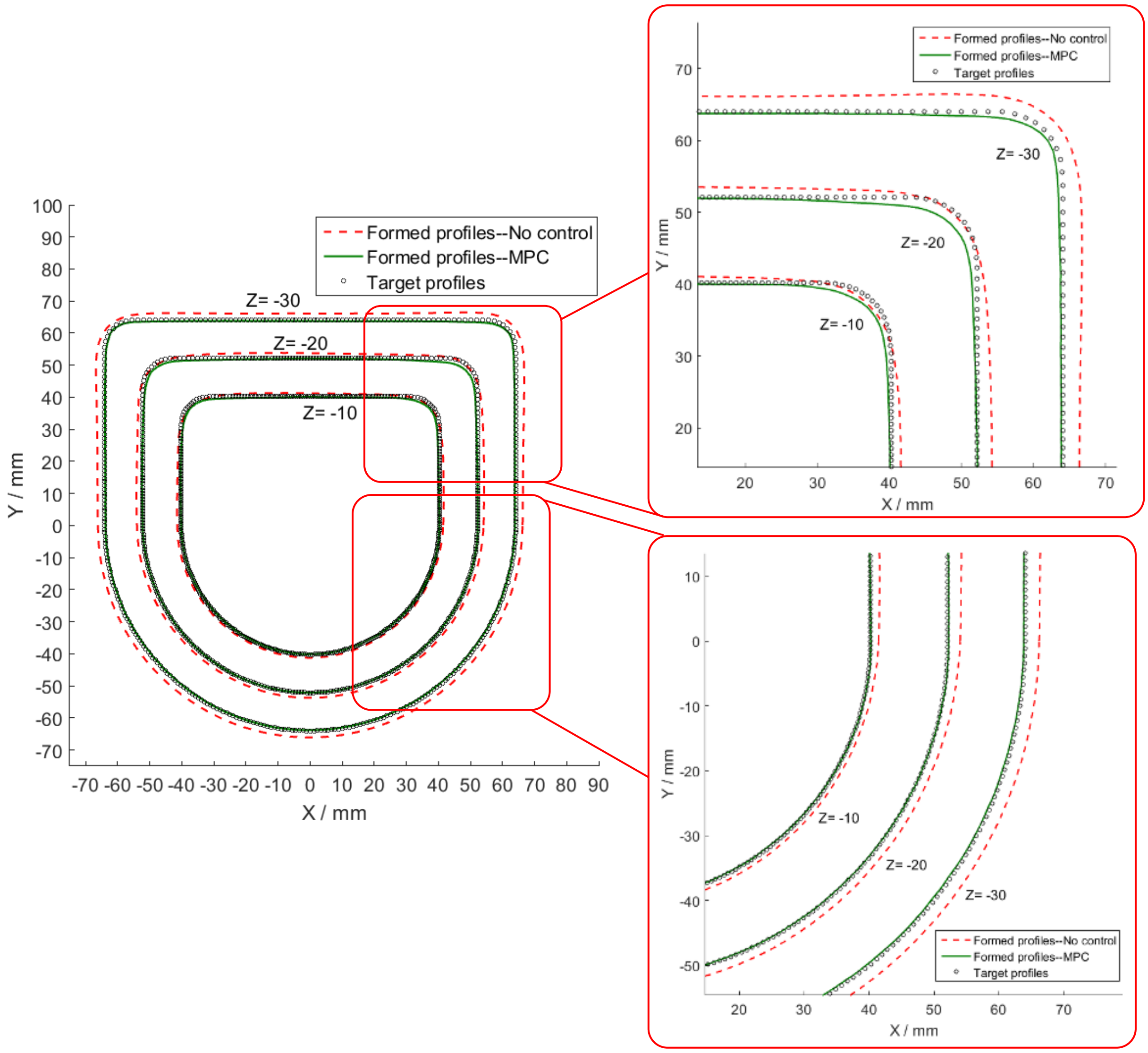

Figure 10 Comparison of sectional profiles from three horizontal sections.

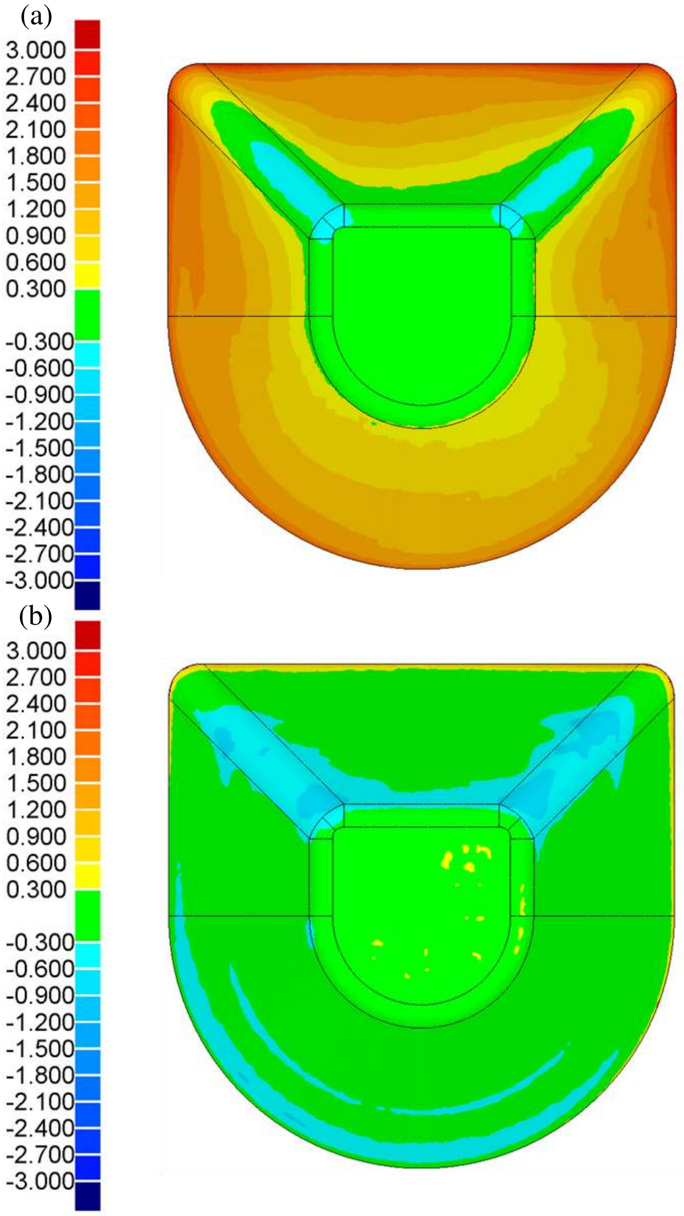

Figure 11 Geometric accuracy colour maps: (a) No control; (b) MPC control.

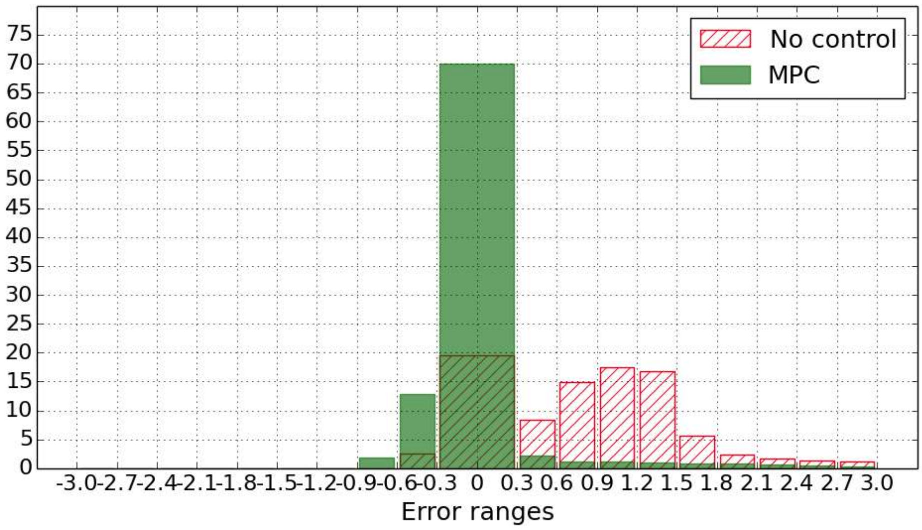

Figure 12 Deviation distributions (percentage) of different error ranges in the colour maps.

4. Conclusions and Future work

This paper reports an in-process toolpath correction strategy specially developed for TPIF with a partial die using MPC control for its ability to use linear models to achieve good control of constrained nonlinear systems in various industries [17, 19]. The MPC control algorithm was developed based on the deformation nature of TPIF with partial die, and a simple MPC algorithm for SPIF in our previous work [1]. The control algorithm presented in this paper is able to deal with toolpath correction in the horizontal and vertical directions through optimising two toolpath parameters (Δu_r and Δu_z) in two separate control modules. This toolpath correction strategy was experimentally tested to form a non-axisymmetric shape. Compared with the typical TPIF process that has no toolpath correction, fairly good improvement in geometric accuracy was achieved with the use of the toolpath correction strategy in TPIF with a partial die while the geometric accuracy in the partial fillet areas requires further improvement. This work provides a helpful approach to achieve in-process toolpath control/correction in TPIF.

One of the primary limitations of this work is that current control approach is not able to perfectly compensate the springback in the partial corner fillet areas of the test shape. The springback in the fillet areas with high curvature could be more complex because the local curvature changes rapidly in a small range and rapid changes of the strain could occur when the tool deforms the corner fillets. In the current control model, the Δu_z values of the contour points in different radial directions on each contour are taken to be constant, which follows the way of defining Δu_z in the typical contour toolpath. To effectively correct the errors in the areas with high curvature, such as corner fillets, the z positions of the contour points on each contour might need to be adjusted differently in different radial directions.

In the future, this limitation might be solved by using more complex predictive models and the further development of current MPC control algorithm through using varying Δu_z values in different radial directions at each step as well as coupling two toolpath parameters in the control algorithm. What's more, the geometric errors in the region that has already been formed might be further corrected by the integration of the MPC control algorithm with a multi-stage toolpath.

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References

[1] Lu H, Kearney M, Li Y, Liu S, Daniel WT, Meehan P. Model predictive control of incremental sheet forming for geometric accuracy improvement. The International Journal of Advanced Manufacturing Technology. 2015:1-14.
[2] Li Y, Daniel WJT, Meehan PA. Deformation analysis in single-point incremental forming through finite element simulation. The International Journal of Advanced Manufacturing Technology. 2016:1-13.
[3] Bambach M, Taleb Araghi B, Hirt G. Strategies to improve the geometric accuracy in asymmetric single point incremental forming. Production Engineering. 2009;3:145-56.
[4] Attanasio A, Ceretti E, Giardini C, Mazzoni L. Asymmetric two points incremental forming: Improving surface quality and geometric accuracy by tool path optimization. Journal of Materials Processing Technology. 2008;197:59-67.
[5] Fiorentino A, Feriti GC, Giardini C, Ceretti E. Part precision improvement in incremental sheet forming of not axisymmetric parts using an artificial cognitive system. Journal of Manufacturing Systems. 2015;35:215-22.
[6] Li Y, Lu H, Daniel WT, Meehan P. Investigation and optimization of deformation energy and geometric accuracy in the incremental sheet forming process using response surface methodology. The International Journal of Advanced Manufacturing Technology. 2015;79:2041-55.
[7] Araghi BT, Manco GL, Bambach M, Hirt G. Investigation into a new hybrid forming process: Incremental sheet forming combined with stretch forming. Cirp Ann-Manuf Techn. 2009;58:225-8.
[8] Taleb Araghi B, Göttmann A, Bambach M, Hirt G, Bergweiler G, Diettrich J, et al. Review on the development of a hybrid incremental sheet forming system for small batch sizes and individualized production. Production Engineering. 2011;5:393-404.
[9] Ambrogio G, Napoli L, Filice L. A novel approach based on multiple back-drawing incremental forming to reduce geometry deviation. International Journal of Material Forming. 2009;2:9-12.
[10] Allwood JM, Braun D, Music O. The effect of partially cut-out blanks on geometric accuracy in incremental sheet forming. Journal of Materials Processing Technology. 2010;210:1501-10.
[11] Hirt G, Ames J, Bambach M, Kopp R, Kopp R. Forming strategies and Process Modelling for CNC Incremental Sheet Forming. CIRP Annals - Manufacturing Technology. 2004;53:203-6.
[12] Lu B, Chen J, Ou H, Cao J. Feature-based tool path generation approach for incremental sheet forming process. Journal of Materials Processing Technology. 2013;213:1221-33.
[13] Behera AK, Verbert J, Lauwers B, Duflou JR. Tool path compensation strategies for single point incremental sheet forming using multivariate adaptive regression splines. Computer-Aided Design. 2013;45:575-90.
[14] Fu ZM, Mo JH, Han F, Gong P. Tool path correction algorithm for single-point incremental forming of sheet metal. Int J Adv Manuf Tech. 2013;64:1239-48.
[15] Fiorentino A, Giardini C, Ceretti E. Application of artificial cognitive system to incremental sheet forming machine tools for part precision improvement. Precision Engineering. 2015;39:167-72.
[16] Allwood JM, Music O, Raithathna A, Duncan SR. Closed-loop feedback control of product properties in flexible metal forming processes with mobile tools. CIRP Annals - Manufacturing Technology. 2009;58:287-90.
[17] Wang H, Duncan S. Constrained model predictive control of an incremental sheet forming process. Control Applications (CCA), 2011 IEEE International Conference on2011. p. 1288-93.

[18] Reddy NV, Lingam R, Cao J. Incremental Metal Forming Processes in Manufacturing. In: Nee AYC, editor. Handbook of Manufacturing Engineering and Technology: Springer London, 2014. p. 411-52.
[19] Camacho EF, Bordons C, Camacho EF, Bordons C. Model predictive control: Springer London, 2004.
[20] Malhotra R, Reddy NV, Cao JA. Automatic 3D Spiral Toolpath Generation for Single Point Incremental Forming. J Manuf Sci E-T Asme. 2010;132.