Partial Derivatives of an Extended Regular Expression

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Motivations

- Generalizing the partial derivatives' method by Antimirov (96),
- Computing an NFA from an extended regular expression.

Regular Expressions

Definition

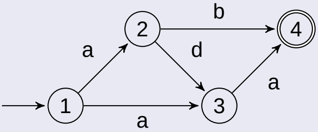
A regular expression E over an alphabet Σ is inductively defined by :

$$\begin{array}{ll} E = \emptyset, & E = F + G, \\ E = \varepsilon, & E = F \cdot G, \\ E = a, & E = F^*, \end{array}$$

with a a symbol of Σ and F and G two regular expressions.

Regular Expressions

Example



$$L = \{aa, ab, ada\}$$
$$E = aa + ab + ada$$
$$F = a(a + b + da)$$
$$G = (a + ad)a + ab$$

Regular Expressions and Automata

For every regular expression E, an automaton A recognizing $L(E)$ can be computed with respect to one of the following methods:

- Inductive computation of automata :Step-by-step,
- Positions : Glushkov, Follows,
- Derivatives : Brzozowski, Antimirov, Champarnaud and Ziadi.

Quotient of a Language/Derivative of an Expression

Definition

The quotient of a langage L over an alphabet Σ by a word w of Σ^* is the language $w^{-1}(L)$ defined by:

$$w^{-1}(L) = \{w' \in \Sigma^* \mid ww' \in L\}.$$

- Extending the idea of quotient over regular expressions: The derivative $\frac{d}{d_w}(E)$ of an expression E by a word w.

Lemma

Let E be a regular expression and w be a word.

$$L(\frac{d}{d_w}(E)) = w^{-1}(L(E))$$

Brzozowski' derivatives

Definition

Let E be a regular expression over Σ and a a symbol of Σ.
The derivative $\frac{d}{d_a}(E)$ is inductively defined as follows:

$$\frac{d}{d_a}(a) = \varepsilon, \qquad \frac{d}{d_a}(b) = \frac{d}{d_a}(\emptyset) = \frac{d}{d_a}(\varepsilon) = \emptyset,$$

$$\frac{d}{d_a}(F + G) = \frac{d}{d_a}(F) + \frac{d}{d_a}(G), \qquad \frac{d}{d_a}(F^*) = \frac{d}{d_a}(F) \cdot F^*,$$

$$\frac{d}{d_a}(F \cdot G) = \begin{cases} \frac{d}{d_a}(F) \cdot G + \frac{d}{d_a}(G) & \text{if } \varepsilon \in L(F), \\ \frac{d}{d_a}(F) \cdot G & \text{otherwise.} \end{cases}$$

Brzozowski's Automaton

Definition

Brzozowski' s automaton for E is defined by $(\Sigma, Q, I, F, \delta)$:

- $Q = \mathcal{D}_E \cup \{E\}$, the set of dissimilar derivatives,
- $I = \{E\}$,
- $F = \{E' \in Q \mid \varepsilon \in L(E')\}$,
- for every symbol a of Σ, for every derivatives F, G of Q:
$$\delta(F, a) = G \Leftrightarrow \frac{d}{d_a}(F) = G.$$

Brzozowski's Automaton: An Example

Example

Let $E = (a+b)^* a (a+b)^2$.

$$\frac{d}{d_a}(E) = E + (a+b)^2$$

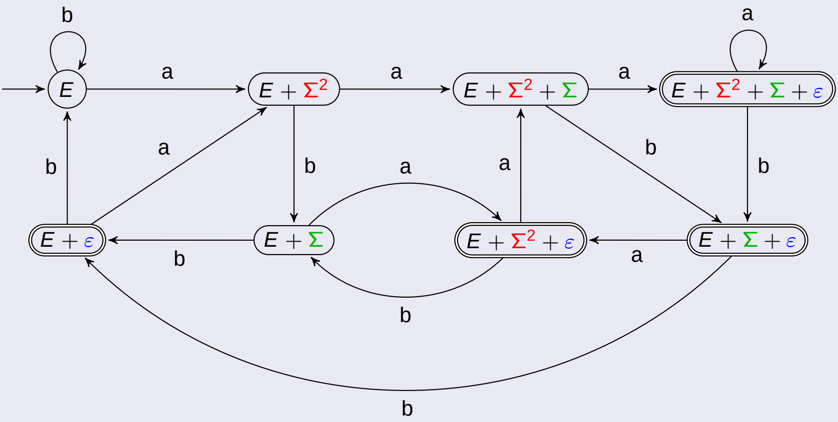
$$\frac{d}{d_b}(E) = E$$

$$\frac{d}{d_a}((a+b)^2) = \frac{d}{d_b}((a+b)^2) = (a+b)$$

$$\frac{d}{d_a}((a+b)) = \frac{d}{d_b}((a+b)) = \varepsilon$$

Brzozowski's Automaton: An Example

Example



Antimirov's Derivatives

Brzozowski's automaton is deterministic:
Then for $E_n = (a + b)^* a(a + b)^n$, the size is $2^{(n+1)}$.

Antimirov gives an alternative to derivatives:
Partial derivatives.

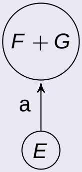
If the result of a derivative is a sum,
the derivative can be *split* into a set of expressions.

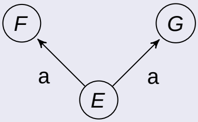
Antimirov's Derivatives

Example

Let $E = a \cdot F + a \cdot G$

The derivative of E by a is the expression $\frac{d}{d_a}(E) = F + G$.





Antimirov's Derivatives

Definition

Let E be an expression over Σ and a be a symbol of Σ.
The partial derivative $\frac{\partial}{\partial_a}(E)$ is the set inductively computed as follows:

$$\frac{\partial}{\partial_a}(a) = \{\varepsilon\}, \qquad \frac{\partial}{\partial_a}(b) = \frac{\partial}{\partial_a}(\emptyset) = \frac{\partial}{\partial_a}(\varepsilon) = \emptyset$$

$$\frac{\partial}{\partial_a}(F+G) = \frac{\partial}{\partial_a}(F) \cup \frac{\partial}{\partial_a}(G), \quad \frac{\partial}{\partial_a}(F^*) = \frac{\partial}{\partial_a}(F) \cdot F^*,$$

$$\frac{\partial}{\partial_a}(F \cdot G) = \begin{cases} \frac{\partial}{\partial_a}(F) \cdot G \cup \frac{\partial}{\partial_a}(G) & \text{if } \varepsilon \in L(F), \\ \frac{\partial}{\partial_a}(F) \cdot G & \text{otherwise} \end{cases}$$

with for $\mathcal{F}$ a set of expressions and G an expression:

$$\mathcal{F} \cdot G = \{F \cdot G \mid F \in \mathcal{F}\}.$$

Antimirov's Automaton

Definition

Antimirov's automaton of E is $(\Sigma, Q, I, F, \delta)$:

- $Q = \mathcal{D}'_E \cup \{E\}$,
- $I = \{E\}$,
- $F = \{E' \in Q \mid \varepsilon \in L(E')\}$,
- for every symbol a of Σ, for every derivated term F, G of Q:
$$G \in \delta(F, a) \Leftrightarrow G \in \frac{\partial}{\partial_a}(F).$$

Antimirov's Automaton: An Example

Example

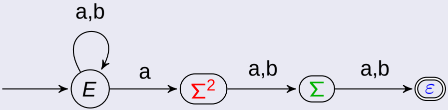
Let $E = (a+b)^* a(a+b)^2$.

$$
\begin{aligned}
\frac{\partial}{\partial_a}(E) &= \{E, (a+b)^2\} \\
\frac{\partial}{\partial_b}(E) &= \{E\} \\
\frac{\partial}{\partial_a}((a+b)^2) = \frac{\partial}{\partial_b}((a+b)^2) &= \{(a+b)\} \\
\frac{\partial}{\partial_a}((a+b)) = \frac{\partial}{\partial_b}((a+b)) &= \{\varepsilon\}
\end{aligned}
$$

Antimirov's Automaton: An Example

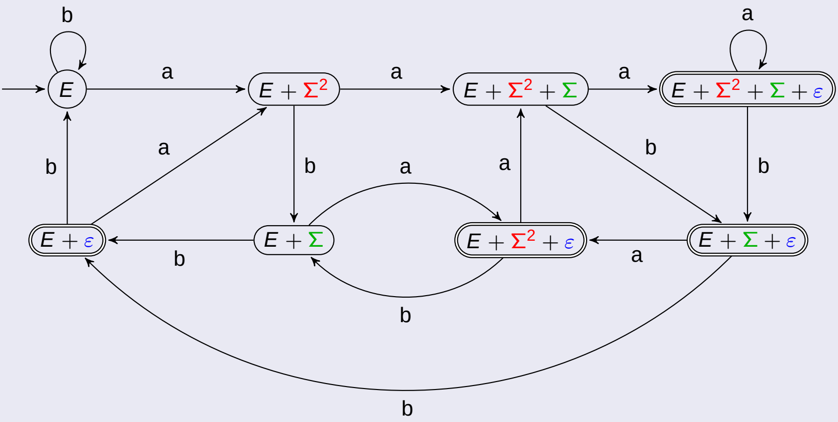
Example

Let $E = (a+b)^*a(a+b)^2$.



Antimirov's automaton: an example

Example



Extended Expressions

Definition

$$\neg L = \{w \in \Sigma^* \mid w \notin L\},$$
$$L \cap L' = \{w \in \Sigma^* \mid w \in L \land w \in L'\}.$$

Definition

An extended expression E over an alphabet Σ is:

- a simple regular expression,
- $E + F$, $E \cdot F$ and E^* where E and F are two extended expressions,
- the complement $\neg F$ of an extended expression,
- the intersection $F \cap G$ of two extended expressions,

With: $L(\neg F) = \neg L(F)$,
$L(F \cap G) = L(F) \cap L(G)$.

Extended Expressions: Derivatives

Lemma

Let L and L' be two languages over Σ, and w a word of Σ^*.

$$w^{-1}(L \cap L') = w^{-1}(L) \cap w^{-1}(L'),$$
$$w^{-1}(\neg L) = \neg(w^{-1}(L)).$$

Definition

Let F and G be two extended expressions over Σ and a a symbol of Σ.

$$\frac{d}{d_a}(F \cap G) = \frac{d}{d_a}(F) \cap \frac{d}{d_a}(G),$$
$$\frac{d}{d_a}(\neg F) = \neg \frac{d}{d_a}(F).$$

Extended Expressions: An Example

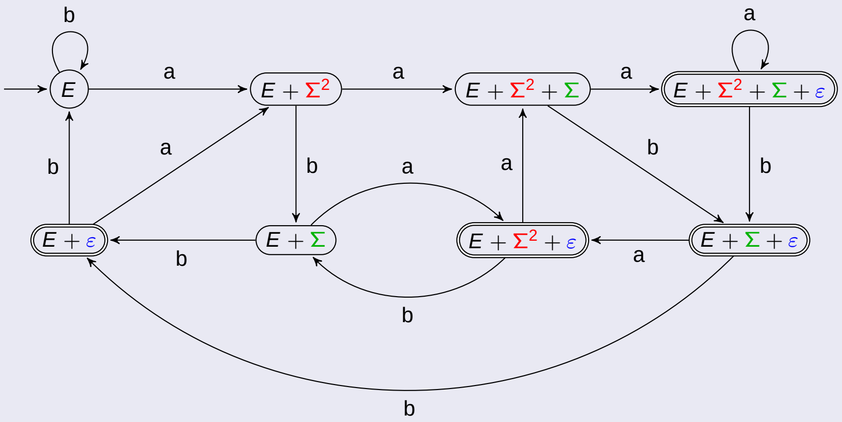
Example

Let $E = (\neg(\neg a \cap \neg b))^* a(a+b)^2$.

$$\begin{aligned}
\frac{d}{d_a}(E) &= E + (a+b)^2 \\
\frac{d}{d_b}(E) &= E \\
\frac{d}{d_a}((a+b)^2) &= \frac{d}{d_a}((a+b)^2) = (a+b) \\
\frac{d}{d_a}((a+b)) &= \frac{d}{d_a}((a+b)) = \varepsilon
\end{aligned}$$

Extended Expressions: An Example

Example



Extended Expressions: Partial Derivatives

Antimirov's derivatives does not take into account extended operators:

Antimirov: "*It would be useful to find an appropriate definition of partial derivatives of extended regular expressions (with intersection, complementation, and other operations). Then, in particular, our NFA construction would directly extend to this class of regular expressions*."

Our goal is to extend partial derivatives to extended operators.

Brzozowski and Antimirov Derivative Combination

Definition

Let E be an extended expression and a a symbol of the alphabet Σ. The trivial derivative of E by a is the set $\partial_a(E)$ inductively computed by:

$$\partial_a(a) = \{\varepsilon\}, \qquad \partial_a(b) = \partial_a(\emptyset) = \partial_a(b) = \emptyset,$$

$$\partial_a(F+G) = \partial_a(F) \cup \partial_a(G), \quad \partial(F^*) = \partial_a(F) \cdot F^*,$$

$$\partial_a(F \cdot G) = \begin{cases} \partial_a(F) \cdot G \cup \partial_a(G) & \text{if } \varepsilon \in L(F), \\ \partial_a(F) \cdot G & \text{otherwise,} \end{cases}$$

$$\partial_a(\neg F) = \{\tfrac{d}{d_a}(\neg F)\}, \quad \partial_a(F \cap G) = \{\tfrac{d}{d_a}(F \cap G)\}.$$

Extended Expressions: An Example

Example

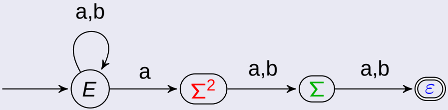
Let $E = (\neg(\neg a \cap \neg b))^* a(a+b)^2$.

$$
\begin{aligned}
\partial_a(E) &= \{E, (a+b)^2\} \\
\partial_b(E) &= \{E\} \\
\partial_a((a+b)^2) = \partial_a((a+b)^2) &= \{(a+b)\} \\
\partial_a((a+b)) = \partial_a((a+b)) &= \{\varepsilon\}
\end{aligned}
$$

Extended Expressions: An Example

Example

Let $E = (\neg(\neg a \cap \neg b))^* a(a+b)^2$.



Extended Expressions: An Example

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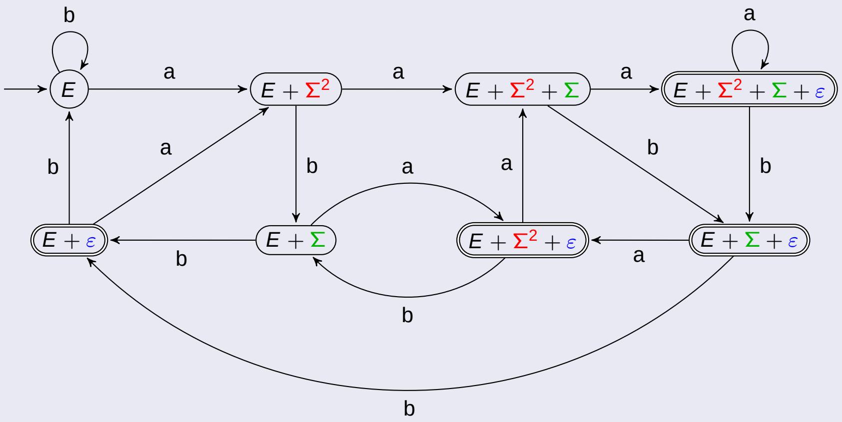

Extension of the Idea of Antimirov

$$E = ((\neg(\neg a \cap \neg b))^* a(a+b)^2) \cap \neg(a^* b a^*)$$

$$\longrightarrow$$

computing sets of sets
to deal with the intersection operator

Sets of sets of expressions: DNF

Definition

The language of a set of expressions $\mathcal{E}$ is $L(\mathcal{E})$:

$$L(\mathcal{E}) = \bigcap_{E \in \mathcal{E}} L(E).$$

Definition

The language of a SSE $\mathbb{E}$ is $L(\mathbb{E})$:

$$L(\mathbb{E}) = \bigcup_{\mathcal{E} \in \mathbb{E}} L(\mathcal{E}).$$

Sets of Sets of Expressions: An Example

$$\circleddash\{\{E_1, E_2\}, \{E_3\}\} \equiv \neg((E_1 \cap E_2) + E_3)$$
$$\equiv (\neg E_1 + \neg E_2) \cap \neg E_3$$
$$\equiv (\neg E_1 \cap \neg E_3) + (\neg E_2 \cap \neg E_3)$$
$$\equiv \{\{\neg E_1, \neg E_3\}, \{\neg E_2, \neg E_3\}\}$$

$$\{\{E_1\}, \{E_2\}\} \circledcap \{\{F_1\}, \{F_2\}\} \equiv (E_1 + E_2) \cap (F_1 + F_2)$$
$$\equiv (E_1 \cap F_1) + (E_1 \cap F_2) + (E_2 \cap F_1) + (E_2 \cap F_2)$$
$$\equiv \{\{E_1, F_1\}, \{E_1, F_2\}, \{E_2, F_1\}, \{E_2, F_2\}\}$$

$$\{\{E_1, E_2\}, \{E_3\}\} \odot F \equiv ((E_1 \cap E_2) + E_3) \cdot F)$$
$$\equiv (E_1 \cap E_2) \cdot F + (E_3 \cdot F)$$
$$\equiv \{\{(E_1 \cap E_2) \cdot F\}, \{E_3 \cdot F\}\}$$

Sets of Sets of Expressions

Definition

Let $\mathbb{E}$ and $\mathbb{F}$ be two sets of sets of expressions, and G be an expression.

- $\mathbb{E} \circledcap \mathbb{F} = \bigcup_{\mathcal{E} \in \mathbb{E}, \mathcal{F} \in \mathbb{F}} \{\mathcal{E} \cup \mathcal{F}\}$
- $\circleddash(\mathbb{E}) = \circledcap_{\mathcal{E} \in \mathbb{E}} (\bigcup_{E \in \mathcal{E}} \neg E)$
- $\mathbb{E} \odot G = \bigcup_{\mathcal{E} \in \mathbb{E}} \{(\bigcap_{E \in \mathcal{E}} E) \cdot G\}$.

Lemma

- $L(\mathbb{E} \circledcap \mathbb{F}) = L(\mathbb{E}) \cap L(\mathbb{F})$
- $L(\circleddash(\mathbb{E})) = \neg(L(\mathbb{E}))$
- $L(\mathbb{E} \odot G) = L(\mathbb{E}) \cdot L(G)$.

Derivative Formulas

Definition

Let E be an extended expression and a be a symbol of Σ.
The extended derivative of E by a is the SSE $\partial'_a(E)$ inductively defined by:

$$\partial'_a(a) = \{\{\varepsilon\}\}, \qquad \partial'_a(b) = \partial'_a(\varepsilon) = \partial'_a(\emptyset) = \emptyset,$$

$$\partial'_a(F+G) = \partial'_a(F) \cup \partial'_a(G), \qquad \partial'_a(F \cap G) = \partial'_a(F) \circledcap \partial'_a(G),$$

$$\partial'_a(F^*) = \partial'_a(F) \odot F^*, \qquad \partial'_a(\neg F) = \circleddash(\partial'_a(F)),$$

$$\partial'_a(F \cdot G) = \begin{cases} \partial'_a(F) \odot G \cup \partial'_a(G) & \text{if } \varepsilon \in L(F), \\ \partial'_a(F) \odot G & \text{otherwise.} \end{cases}$$

Extended Expressions: An Example

Example

Let $E = ((\neg(\neg a \cap \neg b))^* a(a+b)^2) \cap \neg(a^* b a^*)$.
Let $F = ((\neg(\neg a \cap \neg b))^* a(a+b)^2)$ and $G = \neg(a^* b a^*)$.

$$\partial'_a(E) = \partial'_a(F) \circledcap \partial'_a(G)$$

$$\partial'_a(F) = \partial'_a((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 \cup \partial'_a(a) \odot (a+b)^2$$

$$\partial'_a(F) = \partial'_a((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 \cup \{\{(a+b)^2\}\}$$

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$$\partial'_a(F) = \{\{(\neg(\neg a \cap \neg b))^* \cdot a(a+b)^2\}\} \cup \{\{(a+b)^2\}\}$$

Extended Expressions: An Example

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$$\partial'_a(E) = \partial'_a(F) \mathbin{\bigcirc\hspace{-0.78em}\cap} \partial'_a(G)$$

$$\partial'_a(F) = \partial'_a((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 \cup \partial'_a(a) \odot (a+b)^2$$

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$$\partial'_a(E) = \partial'_a(F) \circledcap \partial'_a(G)$$

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Extended Expressions: An Example

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$$\partial'_a(E) = \partial'_a(F) \circledcap \partial'_a(G)$$

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Extended Expressions: An Example

Example

Let $E = ((\neg(\neg a \cap \neg b))^* a(a+b)^2) \cap \neg(a^* b a^*)$.
Let $F = ((\neg(\neg a \cap \neg b))^* a(a+b)^2)$ and $G = \neg(a^* b a^*)$.

$$\partial'_a(E) = \partial'_a(F) \circledcap \partial'_a(G)$$

$$\begin{aligned}
\partial'_a(F) &= \partial'_a((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 && \cup \partial'_a(a) \odot (a+b)^2 \\
\partial'_a(F) &= \partial'_a((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 && \cup \{\{(a+b)^2\}\} \\
\partial'_a(F) &= \partial'_a(\neg(\neg a \cap \neg b)) \odot ((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 && \cup \{\{(a+b)^2\}\} \\
\partial'_a(F) &= \circleddash(\partial'_a(\neg a \cap \neg b)) \odot ((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 && \cup \{\{(a+b)^2\}\} \\
\partial'_a(F) &= \{\{\varepsilon\}\} \odot ((\neg(\neg a \cap \neg b))^*) \odot a(a+b)^2 && \cup \{\{(a+b)^2\}\} \\
\partial'_a(F) &= \{\{(\neg(\neg a \cap \neg b))^* \cdot a(a+b)^2\}\} && \cup \{\{(a+b)^2\}\}
\end{aligned}$$

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Let $F = ((\neg(\neg a \cap \neg b))^* a(a+b)^2)$ and $G = \neg(a^* b a^*)$.

$$\partial'_b(F) = \{\{F\}\}$$

$$\partial'_a(G) = \{\{G\}\}$$

$$\partial'_b(G) = \{\{\neg(a^*)\}\}$$

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Let $E = ((\neg(\neg a \cap \neg b))^* a(a+b)^2) \cap \neg(a^* b a^*)$.
Let $F = ((\neg(\neg a \cap \neg b))^* a(a+b)^2)$ and $G = \neg(a^* b a^*)$.

$$
\begin{aligned}
\partial'_a(F \cap G) &= \{\{F, G\}, \{\Sigma^2, G\}\} \\
\partial'_b(F \cap G) &= \{\{F, \neg(a)^*\}\} \\
\partial'_a(F \cap \neg(a)^*) &= \{\{F, \neg(a)^*\}, \{\Sigma^2, \neg(a)^*\}\} \\
\partial'_b(F \cap \neg(a)^*) &= \{\{F\}\} \\
\partial'_a(\Sigma^2 \cap \neg(a)^*) &= \{\{\Sigma \cap \neg(a)^*\}\} \\
\partial'_b(\Sigma^2 \cap \neg(a)^*) &= \{\{\Sigma\}\} \\
\ldots
\end{aligned}
$$

Antimirov Extended Automaton

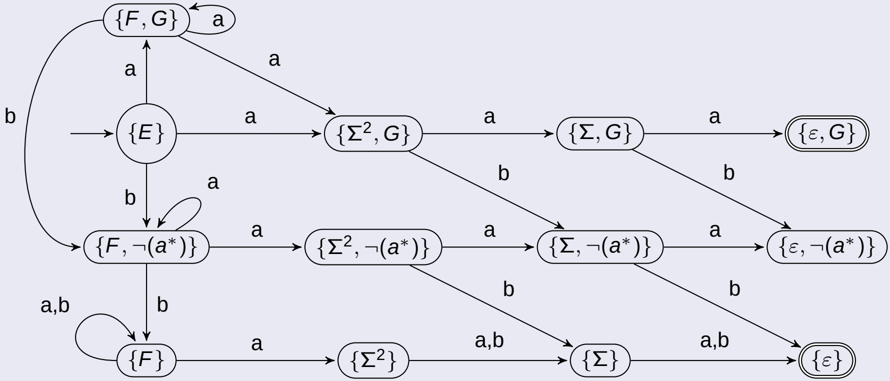
Definition

Antimirov extended automaton of E is $(\Sigma, Q, I, F, \delta)$:

- $Q = \mathcal{D}''_E \cup \{\{E\}\}$,
- $I = \{\{E\}\}$,
- $F = \{\mathcal{E} \in Q \mid \varepsilon \in L(\mathcal{E})\}$,
- for every symbol a of Σ, for every derivated term $\mathcal{E}_1, \mathcal{E}_2$ of Q:
$$\mathcal{E}_2 \in \delta(\mathcal{E}_1, a) \Leftrightarrow \mathcal{E}_2 \in \frac{\partial}{\partial_a}(\mathcal{E}_1).$$

Extended Expressions: An Example

Example



Conclusion

- We have defined $\circledcap$, $\odot$, $\circleddash$ over SSE,
- We can easily extend formulas over every boolean operators.
- We have provided a new algorithm computing an NFA from an extended RE.