Partial Transmit Sequence and Selected Mapping Schemes to Reduce ICI in OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) is sensitive to the carrier frequency offset (CFO). We introduce the peak interference-to-carrier ratio (PICR) to measure the resulting intercarrier interference (ICI). This paper shows that PICR can be reduced by selected mapping (SLM) and partial transmit sequence (PTS) approaches. These schemes are analyzed theoretically and their performances are evaluated by simulation.**

***Index Terms*—Carrier frequency offset (CFO), intercarrier interference (ICI), orthogonal frequency division multiplexing (OFDM), peak interference-to-carrier ratio (PICR).**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is an attractive technique for mitigating the effects of multipath delay spread of a radio channel. Unfortunately, OFDM is sensitive to the carrier frequency offset (CFO), which is caused by misalignment in carrier frequencies and/or Doppler shift. The CFO violates the orthogonality of subcarriers and results in intercarrier interference (ICI) [1], [2]. In the open literature, several techniques have been proposed for reducing ICI [1], [2]. In this letter, we study the distribution for peak interference-to-carrier ratio (PICR) as a measure for ICI effects. Interestingly, PICR is analogous to the peak-to-average power ratio (PAR) issue for OFDM.

Controlling the PAR of an OFDM signal has gained much attention recently [3]–[5]. Generating several statistically independent OFDM frames for a data frame and selecting the one with the lowest PAR is a common approach [4], [5]. This approach improves the statistics of the PAR at the expense of additional complexity. Motivated by their success in reducing the PAR, we apply SLM [4] and PTS [4], [5] in this letter to reduce the PICR.

II. Peak Interference-to-Carrier Ratio (PICR)

A. OFDM Signaling

The complex baseband OFDM signal may be represented as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k e^{j2\pi k \Delta f t}, \qquad \text{for } 0 \le t \le T \quad (1)$$

where $j^2 = -1$, N is the total number of subcarriers and c_k is the modulated data symbol for the kth subcarrier. We shall write an ordered N-tuple $\mathbf{c} = (c_0, c_1, \ldots, c_{N-1})$. The frequency separation Δf between any two adjacent subcarriers is equal to $1/T$ where T is the OFDM symbol duration.

We assume that $s(t)$ is transmitted on an additive white Gaussian noise (AWGN) channel. The received signal sample for the kth subcarrier after discrete Fourier transform (DFT) demodulation can be written as

$$y_k = c_k S_0 + \sum_{l=0,\, l \ne k}^{N-1} S_{l-k} c_l + n_k, \qquad \text{for } k = 0, \ldots, N-1 \quad (2)$$

where n_k is a complex Gaussian noise sample. The second right term in (2) is the ICI term attributable to the CFO. The sequence S_k (the ICI coefficients) depends on the CFO and is given by

$$S_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp\left[j\pi\left(1-\frac{1}{N}\right)(k+\varepsilon)\right] \quad (3)$$

where ε is the normalized frequency offset defined as a ratio between the frequency offset (which remains constant over each symbol period) and the subcarrier spacing. For a zero frequency offset, S_k reduces to the unit impulse sequence. The ICI on the kth subcarrier can be expressed as (2)

$$I_k = \sum_{l=0,\, l \ne k}^{N-1} S_{l-k} c_l, \qquad \text{for } 0 \le k \le N-1. \quad (4)$$

Note that I_k is a function of both $\mathbf{c}$ and ε. Large I_k values cause high bit/symbol errors in subcarriers. In the sequel, we would be interested in reducing the peak magnitude of I_k.

B. PICR Problem

We define the PICR as

$$\mathrm{PICR}(\mathbf{c}, \varepsilon) = \max_{0 \le k \le N-1} \left\{ \frac{|I_k|^2}{|S_0 c_k|^2} \right\}. \quad (5)$$

Note that the PICR is a function of both $\mathbf{c}$ and ε. PICR is the maximum interference-to-signal ratio for any subcarrier. In other words, it specifies the worst-case ICI on any subcarrier.

To reduce ICI effects, (5) should be minimized and is zero for ICI-free channels. Interestingly, our definition (5) is similar to PAR issue in OFDM [3]–[5]. However, the PICR problem differs from the PAR issue in several ways.

- ICI occurs at the receiver side, whereas high PAR values affect the transmitter.
- Exact computation of PAR requires oversampling, whereas max $|I_k|$ is obtained from N samples.
- As the transmitter does not know ε *a priori*, PICR can be computed only on the basis of a worst-case value, $\varepsilon_{wc} > 0$. Therefore, the performance of a PICR reduction scheme should hold for any $|\varepsilon| < \varepsilon_{wc}$.

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The PICR can be considered as a random variable. For some data frames the PICR can be high and for others the PICR can be low. The evaluation of complementary cumulative density function (CCDF) of PICR shows this trend. Naturally, one may apply PAR reduction techniques, which exploits the statistical distribution of an OFDM signal, to reduce PICR. As SLM and PTS schemes can reduce PAR, we investigate them to reduce PICR. In these schemes, a worst-case ε is assumed for the computation of PICR. Moreover, transmission of perfect side information (SI) is also assumed.

C. PICR and BER Relationship

It would be desirable to find the relationship between BER and PICR. As the exact BER is a complicated function of ICI, the precise relationship between PICR and BER has eluded us. We are however able to present a weaker relationship. In [6], the upper bound for the BER is derived in terms of maximum ICI ($I_{\max}$) for Binary Phase Shift Keying (BPSK). Using the approach of [6] and our definition of PICR, an upper bound for the BER of BPSK can be expressed as

$$\mathrm{BER} \leq \frac{1}{4}\left[\mathrm{erfc}\left\{\lambda(1-\sqrt{\mathrm{PICR}})\right\} + \mathrm{erfc}\left\{\lambda(1+\sqrt{\mathrm{PICR}})\right\}\right] \tag{6}$$

where $\lambda = (|S_0|/\sqrt{2}\sigma)$ and $\mathrm{erfc}(x) = 1-(2/\sqrt{\pi})\int_0^x e^{-t^2}\,dt$.

Evaluation of (6) shows that BER can be improved by reducing PICR.

III. Reducing PICR

A. Selected Mapping (SLM)

U statistically independent alternative transmit sequences $\mathbf{a}^{(u)}$ represent the same information. The sequence with lowest PICR is selected for transmission. To generate $\mathbf{a}^{(u)}$, we define U distinct fixed vectors $\mathbf{P}^{(u)} = \left[P_0^{(u)}, \ldots, P_{N-1}^{(u)}\right]$ with $P_v^{(u)} = e^{j\varphi_v^{(u)}}$, $\varphi_v^{(u)} \in [0, 2\pi]$, $0 \leq v < N$, $1 \leq u \leq U$. Then, each modulated symbol $\mathbf{c} = [c_0, c_1, \ldots, c_{N-1}]$ is multiplied carrierwise with the U vectors $\mathbf{P}^{(u)}$, resulting in a set of U different modulated symbols $\mathbf{c}^{(u)}$ with components

$$c_v^{(u)} = c_v P_v^{(u)}, \qquad \text{for } 0 \leq v < N \text{ and } 1 \leq u \leq U. \tag{7}$$

For simple implementation, we select $P_v^{(u)} \in [\pm 1, \pm j]$ for $0 \leq v < N$, $1 \leq u \leq U$. Using (4) and (7), the resulting ICI on the kth subcarrier can be expressed as

$$I_{k,\mathrm{SLM}} = \sum_{l=0,\ l\neq k}^{N-1} P_l^{(u)} c_l S_{l-k} \tag{8}$$

which is a function of the weighting sequence $\mathbf{P}^{(u)}$. Finally, the optimal PICR can be found as

$$\mathrm{PICR}_{\mathrm{optimal}} = \min_{\mathbf{P}^{(1)},\ldots,\mathbf{P}^{(U)}} \left[\frac{\max_{0\leq k\leq N-1} |I_{k,\mathrm{SLM}}|^2}{|S_0 c_k|^2}\right]. \tag{9}$$

B. Partial Transmit Sequences (PTS)

In PTS, the input data block is partitioned into disjoint subblocks or clusters which are combined to minimize the peaks. We partition the data frame $\mathbf{c}$ into M disjoint subblocks, represented by the vectors $\{\mathbf{c}_m,\ m = 1, 2, \ldots, M\}$, such that $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_M]$. It is assumed that each subblock consists of a contiguous set of subcarriers and the subblocks are of equal size. Then, each subblock is zero padded to make its length N and is multiplied by a weighting factor b_m $(m = 1, 2, \ldots, M)$. Thus, the ICI on the kth subcarrier can be expressed as

$$I_{k,\mathrm{PTS}} = \sum_{m=1}^{M} \sum_{l=0,\ l\neq k}^{N-1} b_m c_l^m S_{l-k} \tag{10}$$

where c_l^m is the data symbol in the newly formed mth subblock.

We can write (11) as

$$I_{k,\mathrm{PTS}} = \sum_{m=1}^{M} b_m I_k^m \tag{11}$$

where I_k^m is the interference on kth subcarrier of block m. Thus, the total ICI is the weighted sum of ICI from each subblock and the total ICI and PICR can be reduced by optimizing the phase sequence $\mathbf{b} = [b_1, b_2, \ldots, b_M]$.

A drawback in the PTS approach is the complexity of the optimization of phase factors. To reduce of this complexity, we only consider binary phase factors (i.e., $b_m = \pm 1$). Without loss of generality, we can set $b_1 = 1$ and observe that there are $(M-1)$ binary variables to be optimized. Finally, the optimal PICR can be found as

$$\mathrm{PICR}_{\mathrm{optimal}} = \min_{b_1,\ldots,b_M} \left[\frac{\max_{0\leq k\leq N-1} |I_{k,\mathrm{PTS}}|^2}{|S_0 c_k|^2}\right]. \tag{12}$$

IV. Simulation Results

The simulation results for SLM and PTS were obtained for an OFDM system with $N = 64$. The subcarriers are modulated with quadrature phase-shift keying (QPSK) and an AWGN channel is assumed throughout this study.

A. SLM Approach

Fig. 1 shows the CCDF of PICR per OFDM block as a function of U for $\varepsilon = 0.1$. In SLM approach with $U = 8$, only 1 out 10^4 of all OFDM blocks exceeds the PICR of −8.5 dB whereas in normal OFDM, 1 out 10^4 of all OFDM blocks exceeds the PICR of −5.5 dB. That amounts to a 3.0 dB reduction in PICR. Moreover, PICR reduction increases with increasing U. However, the computational complexity also increases with U. Thus, the performance can be traded off against complexity.

Fig. 2 shows the BER performance of QPSK modulated SLM OFDM System in an AWGN channel. Exact knowledge of SI is assumed at the receiver. For $U = 16$, an SNR gain of 3 dB is obtained at 10^{-4} BER over normal OFDM. In fact, SLM OFDM removes the error floor caused by ICI. Moreover, the SNR gain is large when M is large.

B. PTS Approach

Fig. 3 shows the CCDF of PICR per OFDM for $\varepsilon = 0.1$ and $M = 4$, 8 and 16. In the PTS approach, the PICR exceeds −7 dB for only 1 out 10^4 of all OFDM blocks whereas that of normal OFDM is for only 1 out 100. There is 3 dB reduction in PICR over normal OFDM with $M = 8$.

Fig. 3 also shows how the performance varies with M. When M is large, PICR reduction is large. However, the computa-

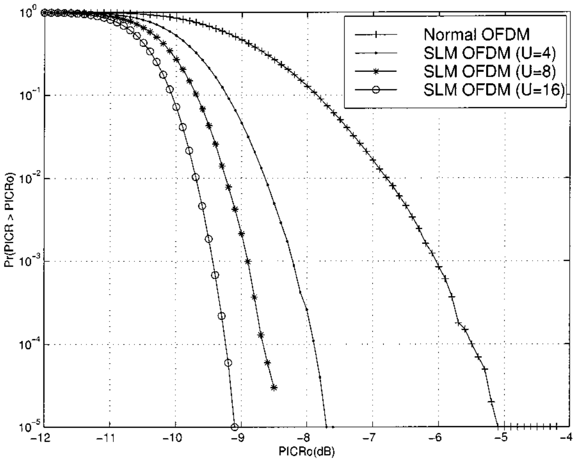

Fig. 1. CCDF of PICR of SLM OFDM system with $\varepsilon = 0.1$.

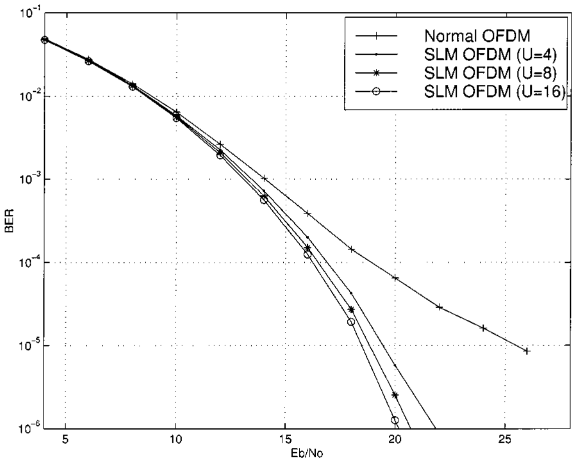

Fig. 2. BER performance of SLM OFDM system in AWGN channel with $\varepsilon = 0.1$.

tional complexity depends on M. Thus, it is a trade off between the performance and complexity. Moreover, optimized phase sequence requires 2^{M-1} computations of PICR. Note that an N-point IFFT and an N-point FFT are required at the transmitter to compute PICR. Thus, computation of optimized phase sequence is difficult. Instead, several selections of $\mathbf{b}$ can be generated randomly until PICR is reduced. Even 100 trials achieve a performance level that is nearly optimal for $M = 8$.

Fig. 4 shows the CCDF of PICR per OFDM block as a function of ε for both PTS and SLM. Fig. 4 reveals that assumption of ε_{wc} is effective for another ϵ provided $|\varepsilon| < \varepsilon_{wc}$. That is, any mismatch between the actual CFO and the worst-case CFO can be handled under this condition.

V. Conclusion

In this letter, new solutions to the ICI problem in OFDM systems have been presented. The definition of PICR is analogous to that of PAR. Consequently, PAR reduction schemes can be applied to reduce PICR. We investigated the SLM and PTS methods to reduce PICR. They improves the PICR statistics of an OFDM signal at the expense of additional complexity, but with little loss in efficiency. For an OFDM system with $N = 64$ and $\varepsilon = 0.1$, SLM with $U = 3$ and PTS with $M = 8$ reduce PICR by 3 dB. Moreover, both schemes work independent of ε, provided $|\varepsilon| < \varepsilon_{wc}$.

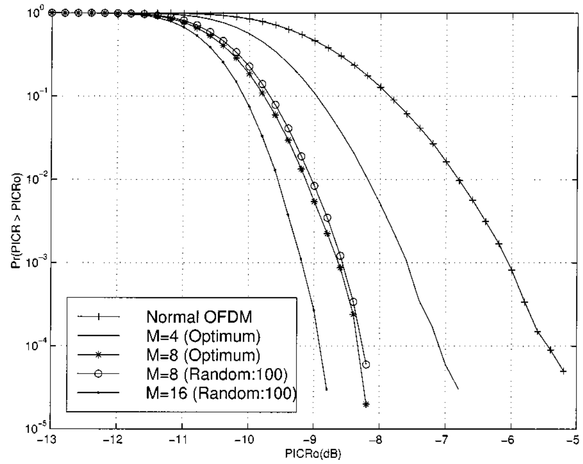

Fig. 3. CCDF of PICR of PTS OFDM system with $\varepsilon = 0.1$.

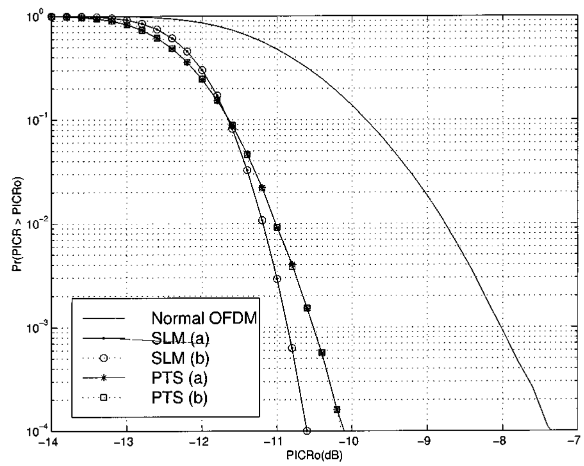

Fig. 4. CCDF of PICR of an OFDM system with (a) $(\varepsilon = 0.08, \varepsilon_{wc} = 0.08)$ and (b) $(\varepsilon = 0.08, \varepsilon_{wc} = 0.1)$.

Finally, we stress that our proposed scheme has been applied to an AWGN or a frequency nonselective fading channel. If the the fading channel is dispersive, the transmitter requires knowledge of the channel in order to perform the PICR optimization. This issue is currently under investigation.

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