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Particle breakage during shearing of a carbonate sand

M. R. COOP*, K. K. SORENSEN†, T. BODAS FREITAS* and G. GEORGOUTSOS‡

A series of ring shear tests was conducted to investigate the development of particle breakage with shear strain for a carbonate sand. It was found that at very large displacements the soil reached a stable grading, but that the final grading was dependent on both the applied normal stress and the initial grading. The particle breakage caused a volumetric compression, which again ceased only when the stable grading had been attained, emphasising that critical states as observed at much smaller strains in triaxial tests are not rigorously defined. Despite the severe degradation of the soil particles the mobilised angle of shearing resistance was found not to change significantly.

KEYWORDS: calcareous soils; sands; shear strength

Nous avons mené une série d'essais de cisaillement annulaire afin d'enquêter sur le développement des cassures de particules avec une déformation de cisaillement pour un sable carbonate. Nous avons trouvé qu'avec de très importants déplacements, le sol parvenait à une granulométrie stable mais que la granulométrie finale dépendait de la contrainte normale appliquée et de la granulométrie initiale. La cassure des particules provoque une compression volumétrique qui, encore une fois, ne cesse que lorsque la stabilité granulométrique est atteinte, soulignant le fait que les états critiques tels que ceux qui sont observés avec des déformations bien plus petites dans les essais triaxiaux ne sont pas rigoureusement définis. Nous avons trouvé que malgré la dégradation sévère des particules de sol, l'angle mobilisé de résistance au cisaillement ne changeait pas de manière significative.

INTRODUCTION

By means of triaxial testing over an extended range of pressures, Coop & Lee (1993) concluded that for a variety of sands of different mineralogies there was a unique relationship between the amount of particle breakage that occurred on shearing to a critical state and the value of the mean normal effective stress. The identification of a unique critical state line was then used as the basis for a critical state framework for sands at both large strains and small (Jovicic & Coop, 1997) and for the analysis of the behaviour of geotechnical structures in sands, such as driven piles (Klotz & Coop, 2001). The approach had much in common with the state parameter framework of Been & Jefferies (1985).

The implied assumption in the framework was that at the critical state a sand would reach a stable grading at which the particle contact stresses would not be sufficient to cause further breakage. An alternative assumption, based on the work by Chandler (1985) for material with deformable grains and implemented by Baharom & Stallebrass (1998) for soils with breakable grains, is that a critical state reached in the triaxial apparatus represents a balance between volumetric compression arising from particle breakage and volumetric dilation from particle rearrangement.

Luzzani & Coop (2002) identified that there was some doubt, particularly at higher stress levels, as to whether samples did reach a true constant-volume state in triaxial tests and therefore whether the particle breakage had completely ceased. To investigate this further they carried out ring shear tests on two sands, one with a quartz and one a carbonate mineralogy. In both cases particle breakage was found to continue to strains very much higher than those reached in the triaxial apparatus, so confirming the hypothesis that any constant-volume state observed in a triaxial test must be the result of counteracting components of volumetric strain and not because a stable grading has been reached. However, the displacements or strains that were achievable in their tests were insufficient to see whether the soil would eventually reach a stable grading or not. For tests on the carbonate sand at one stress level a constant ratio was found between volumetric strain and the amount of particle breakage as quantified with relative breakage, B_r, defined by Hardin (1985), suggesting that the volumetric strain would cease only when particle breakage stopped and a stable grading was reached.

Through improved testing techniques it has proved possible to reach higher displacements in the ring shear apparatus, allowing an investigation to be made of the evolution of grading at even larger strains, so identifying whether or not a stable grading is ever reached. As for most of the tests presented by Luzzani & Coop, a carbonate sand has been used for this study, to maximise the breakage and so minimise the strains that would be required to reach a constant grading, should it exist. However, the patterns of behaviour are likely to be applicable to sands of other mineralogies, although it is still not possible to reach the even higher strains that would be required to confirm the conclusions presented here for a quartz sand. Luzzani & Coop also restricted their investigation to one initial grading and loose samples at higher stress levels, so that the carbonate sand was starting from an initial state that was on its normal compression line, or limiting compression curve (Pestana & Whittle, 1995). Here a variety of gradings, densities and stress levels has been investigated.

MATERIAL TESTED AND PROCEDURES

The soil tested was Dog's Bay sand, a biogenic carbonate sand that was tested extensively in the triaxial apparatus by Coop (1990) and Coop & Lee (1993). The particles are predominantly foraminifera and mollusc shells and shell fragments, and so are angular with frequent intra-particle voids. They are also delicate, and break easily under load. As has been observed by others and is confirmed here, particle breakage is greater for uniformly graded than for well-graded sands, and so to maximise the breakage most of

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Discussion on this paper closes 1 October 2004, for further details see p. ii.

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the tests were conducted on soil from the sieve interval 300–425 μm. To investigate the effects of particle size a test (RS7) was also carried out on a second uniform grading defined between the 212 μm and 300 μm sieve sizes. Another test (RS8) investigated the influence of the uniformity of the initial grading, for which a well-graded sample was created with 25% of the particles from each of the four sieve intervals 63–150 μm, 150–212 μm, 212–300 μm and 300–425 μm. In each case the initial particle size was defined carefully by wet-sieving the soil into its constituent particle sizes before selecting the grading to be used or reconstituting a sample in the chosen proportions.

Ring shear tests were conducted in the Bishop type of apparatus (Bishop *et al.*, 1971), which allows the shearing to occur at the mid-height of the sample by means of split confining rings. This system also allows the friction lost on the side walls to be quantified by measuring the resistance to raising the upper rings, so creating a gap between the two sets of rings. During the ‘gap open’ stages, the shear stress applied to the sample may also be measured, whereas when the gap is closed the shear stress is unknown owing to the friction between the two pairs of rings. However, these gap-open stages were reduced to a minimum as the displacement that could be reached during a test was determined by the amount of soil lost through the gap between the rings, which accelerated for the gap-open stages. All tests were stopped before the sample became so thin that the fins attached to the top platen would enter the central part of the sample, which is that principally used for the particle size analysis as described below. It is the loss of soil through the gap, which occurs even when the gap is closed, that prevents a similar investigation being carried out for a quartz sand, as the displacements required to reach a constant grading would be very much larger.

A summary of the tests undertaken is given in Table 1. Tests denoted LC are those that were previously presented by Luzzani & Coop (2002). The net vertical effective stress (σ'_v) quoted for each test includes the deduction made for the side friction. The nominal vertical stresses applied were 1 MPa (LC1-4 & RS1-8), 400 kPa (RS9-14) or 100 kPa (RS15-19), and it is the side friction that accounts for the variation in σ'_v. Whereas for the tests that were stopped at smaller displacements the side friction was fairly constant, that for the tests taken to very large displacements varied significantly, largely because of the changing sample thickness, and so a range is given for the net vertical stress.

The uniformly graded samples were created by water pluviation, but moist tamping had to be used for the well-graded sample to avoid segregation, after which the water bath was flooded to saturate the samples. Denser samples were created in five layers using gentle hand-tamping of each layer after pluviation. Shearing was conducted at a rate of about 1·9 mm/min for consistency with previous work.

The data are presented in this paper in terms of shear and vertical strains rather than displacements, so that:

$$\gamma = \frac{\delta h}{H_0} \tag{1}$$

$$\varepsilon_v = \frac{\delta v}{H_0} \tag{2}$$

where δh and δv are the horizontal and vertical displacements and H_0 is the initial sample height. Although the particle size data do indicate some non-uniformity of strains, albeit not excessive, presenting the data in terms of strains has the advantage that qualitative comparisons are more easily made with data from other apparatus. One objective of this research is also to emphasise the very large strains that are required to reach a stable grading, and this is better achieved by plotting strains rather than displacements. However, it should be stressed that the values of strain are purely notional, and that both vertical and shear strains are likely to be similarly affected by strain localisation.

The vertical strains have been corrected for the soil lost through the gap between the rings during shearing, and are based on the weight of soil recovered from outside the rings after the test. As discussed above, the rings were kept closed for most of the test to minimise the loss. Although the soil loss accelerated during the gap-open stages, it is not known by how much, and as for these tests the gap-open stages

Table 1. Details of the tests conducted

Test	Notes	Initial grading: μm	e prior to shearing	Net σ'_v during shearing: kPa	Final γ: %
LC1		300–425	1·58	≈ 805	207
LC2		300–425	1·61	≈ 805	730
LC3		300–425	1·57	≈ 805	104
LC4		300–425	1·56	≈ 805	251
LCSB1	Shear box test	300–425	1·46	930	52
RS1		300–425	1·51	≈ 670	171
RS2		300–425	1·46	≈ 670	2 860
RS3		300–425	1·55	650–660	11 100
RS4		300–425	1·52	≈ 670	1 430
RS5		300–425	1·46	740–860	11 030
RS6		300–425	1·43	750–820	2 780
RS7	Finer grading	212–300	1·41	750–850	2 910
RS8	Well graded	63–425	0·96	725–825	11 710
RS9		300–425	1·50	250–280	10 920
RS10		300–425	1·47	248–346	3 350
RS11		300–425	1·46	283–375	13 280
RS12		300–425	1·45	296–368	1 180
RS13		300–425	1·59	288–386	26 650
RS14		300–425	1·60	290–343	285
RS15		300–425	1·50	60–77	147 000
RS16		300–425	1·79	62–70	9 040
RS17		300–425	1·72	66–80	31 700
RS18		300–425	1·68	78–94	23 900
RS19		300–425	1·69	68–97	37 500

LC, test conducted by Luzzani & Coop (2002).

were few, most of the soil loss actually occurred during gap closed stages. The simple assumption has therefore been made that the correction to the vertical strain is proportional to the displacement, regardless of whether the gap was open or closed, although this does mean that for the gap-open stages there is a jump in vertical strain, which represents the acceleration of soil loss. A detailed comparison between uncorrected and corrected data was made by Luzzani & Coop (2002). For the test at the higher stress level that reached the largest strain (RS8) the correction represented a reduction of the volumetric strain by 5·4% on a total measured value of about 26%, but the corrections for most of the other tests were much smaller.

Following each test the soil was carefully retrieved from the apparatus in three layers. The central layer, Zone 2, was that within ±2·5 mm of the split in the rings; Zone 1 was that soil above and Zone 3 the soil below. Luzzani & Coop found that the particle breakage in Zones 1 and 3 was always similar, but that there was significantly higher breakage in Zone 2, confirming some non-uniformity of strains. For some of the tests taken to the largest strains at the highest stress levels a well-defined band of shearing could be identified after the test, with a cohesive central zone of soil that separated easily from the remainder of the sample, in which case the soil in this band was taken as Zone 2. An example is shown in Fig. 1. However, it was found that the thickness of the shear band was in any case generally around 5 mm.

Following the tests the final particle size distribution was determined by wet-sieving by hand, which was complemented by sedimentation tests using the hydrometer method in some cases. For some tests the particle size distribution of only Zone 2 was determined, whereas in other cases all three zones were examined. The amount of particle breakage was quantified by means of Hardin's (1985) relative breakage, B_r, the definition of which is given in Fig. 2. This is partly for consistency with previous work. However, there are many means of quantifying breakage that have been chosen by different authors. In particular, the total surface area of the particles, as used for example by Miura & Yamanouchi (1977) and Miura & O-Hara (1979), has many advantages, but the general assumption made in calculating the surface area is that the particles are spherical. Dog's Bay sand was chosen for this study as it highlights particle breakage, and—as will be seen later—many of the conclusions reached might not have been possible for sands with stronger, solid and spherical particles. However, one disadvantage is that the delicate, open and angular particles that promote breakage also mean that an assumption of a spherical shape would be unjustifiable. For simplicity, and again for consistency with previous work, rather than defining B_r from the baseline of the US standard 74 μm sieve, the British Standard 63 μm sieve size has been used, but this makes only a small difference to the values calculated.

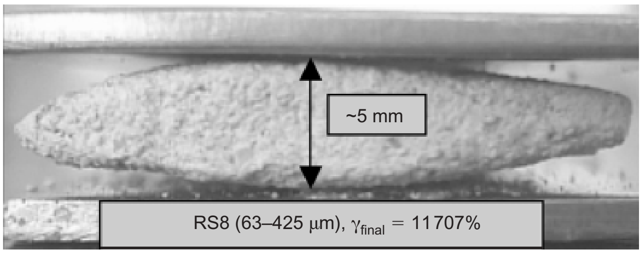
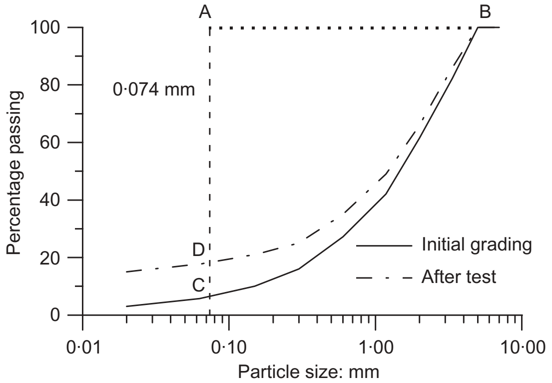


Fig. 1. Cross-section of a shear band retrieved from test RS8



Total breakage, B_t = area BCDB
Breakage potential, B_p = area BCAB
Relative breakage, $B_r = B_t/B_p$

Fig. 2. Definition of relative breakage, B_r (Hardin, 1985)

VOLUME CHANGE AND EVOLUTION OF GRADING

Figure 3 shows the volumetric strains measured for a selection of the tests. The jumps in the data, for example at around 500% shear strain for test RS8, result from the gap-open stages, for which there was an acceleration of soil loss through the gap that is evident even after the data have been corrected, because the correction has assumed that the loss is proportional to the displacement irrespective of whether the gap is open or not, which, as discussed above, is inaccurate. Tests often ended with a gap-open stage to check the final stresses, and so jumps in the data of this kind are sometimes evident at the end of the data. Nevertheless, the volumetric strain data do indicate that for most of these tests, which were carried out at vertical stress levels in the range 650–860 kPa, a constant volumetric strain is reached at a shear strain of around 2000%. It had been intended that each of these tests (RS3, RS5 and RS7) should have had similar vertical stresses. For the tests by Luzzani & Coop

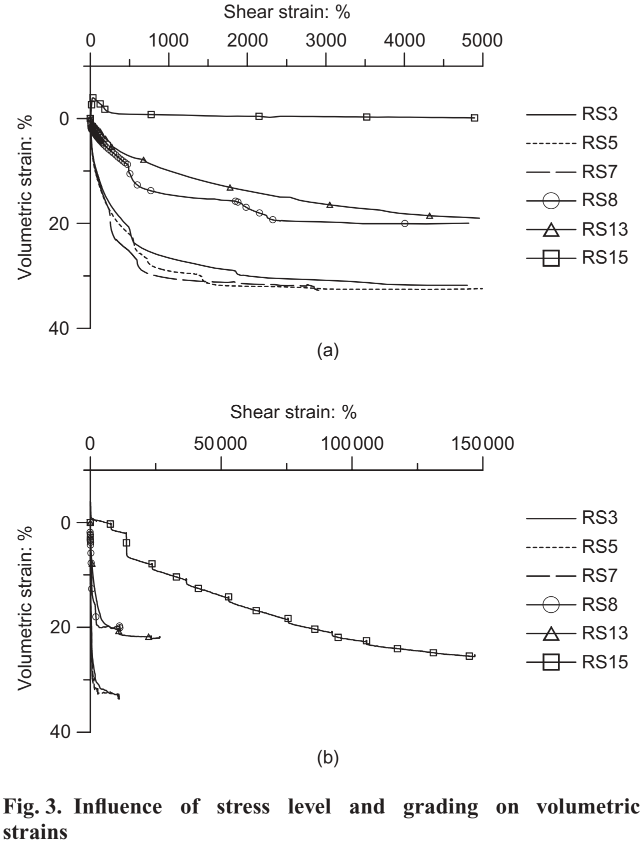


Fig. 3. Influence of stress level and grading on volumetric strains

(2002) the vertical stress could be controlled with reasonable accuracy, but here greater difficulty was experienced in achieving a particular vertical stress, particularly at the higher stress levels. This was first because the side friction was found to be much more variable in the ring shear apparatus used, and second because at the very large strains reached the side friction was found to vary significantly during the test.

In Fig. 3(b) it can be seen that the tests at lower stress levels (RS8, RS13, RS15) take much larger strains for the volumetric strain to stabilise, with about 20 000% required for test RS13 at 288–386 kPa. For test RS15 at 60–77 kPa the volumetric strain initially shows the dilation that would be seen at smaller strain levels in triaxial tests, but then the volume change becomes compressive, although very slow, and it had just about stabilised when the test was terminated at 147 000%. At low shear strains the volume change is highly dependent on the confining stress, with, as expected, low stress level tests dilating and those at higher stresses compressing. However, the large volumetric compression that appears at very large shear strains for all stress levels as a result of particle breakage seems to be much less dependent on stress level.

Tests RS3, RS5 and RS7 all have similar final volumetric strains. Test RS3 was at a slightly lower stress level, but on a slightly looser sample than test RS5, factors that may have counteracted each other. However, test RS7, which is on a sample with a grading that is again uniform, but in this case finer, reaches a similar final ε_{v}. Test RS8, which is on a well-graded sample, also reaches a constant ε_{v} at a similar shear strain to the other tests, but the ultimate value is lower.

In Fig. 4 the particle size distributions are shown with both the usual linear and logarithmic percentage passing axes for all tests conducted with a vertical stress in the range 650–930 kPa. The variation in vertical stress undoubtedly adds to the data scatter, but this range represents the extreme stresses, and most tests have an average stress in the range 750–850 kPa. Despite the variation of vertical stress a number of interesting features can be observed. Data are also repeated here from the tests at smaller shear strains by Luzzani & Coop (2002) for completeness, one of which (LCSB1) at the smallest strains was conducted in a shear box rather than a ring shear apparatus. The ‘no shearing’ curve represents the particle grading of the soil subjected to one-dimensional loading to a vertical stress of 800 kPa in the oedometer.

Comparing tests RS2, RS3 and RS4 with the particle size distributions from tests that reached smaller shear strains, it is evident that, just as ε_{v} stabilised at around 2000–4000% shear strain, so does the grading. Initially the grading changes rapidly with increasing shear strain, but the change from test RS2 (2860%) to test RS3 (11 100%) is very small.

Using a linear percentage passing axis gives a gradings curve that rotates around the largest particle size, but remains concave upwards. The rotation around the maximum particle size is similar to observations that McDowell & Bolton (1998) made for the compression of sands. With a logarithmic axis it is clear that the gradings curve evolves towards a linear distribution. McDowell & Bolton also identified that the particle size distribution of sands undergoing one-dimensional compression evolves with a fractal distribution, which results in a linear particle size distribution on the double logarithmic graph. They found that in compression the gradient was generally around 0·5, corresponding to a fractal dimension of 2·5. Here there again appears to be a tendency towards a fractal distribution, but with a slightly lower final gradient of 0·43 for test RS3 and so a fractal dimension of 2·57.

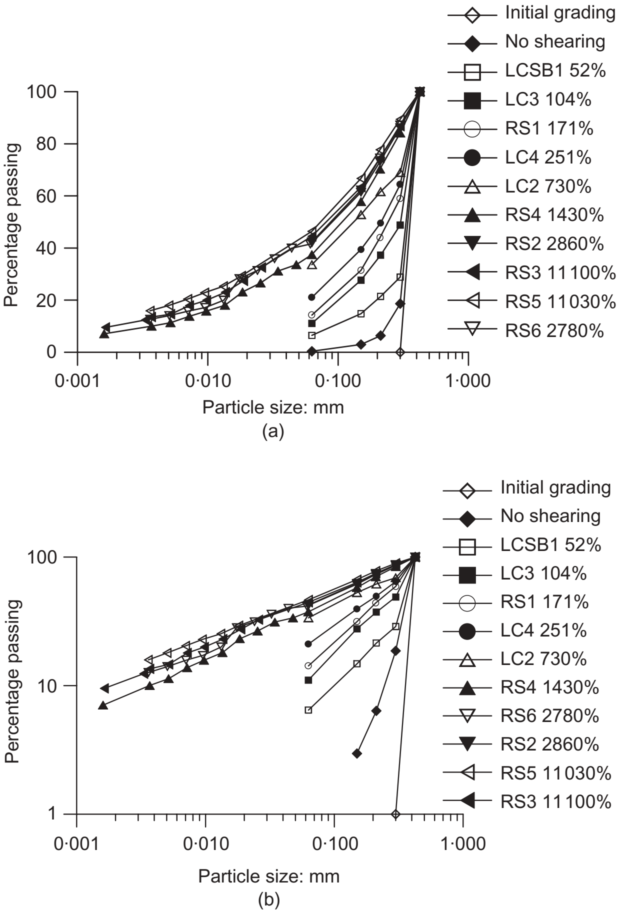


Fig. 4. Evolution of particle size distribution for Zone 2 for tests with σ'_{v} in range 650–930 kPa (LC, test by Luzzani & Coop (2002); SB, shear box test): (a) semi-logarithmic axes; (b) double logarithmic axes

From the tests at lower stresses (Figs 5 and 6) it is clear that again there is a tendency at the largest strains towards a unique final gradings curve. However, the final gradings curves are different for each stress level, so that not only does the final amount of breakage reduce with stress level, but also the final gradings curve tends to become more curved even when double logarithmic axes are used.

Figure 7 shows the final gradings curves for the two tests investigating the influence of initial grading. From Fig. 7(a) it can be seen that for the initially well-graded sample the final gradings curve for test RS8, which reached 11 710% shear strain, is slightly higher than that of the initially poorly graded samples. The difference is small, but significant when compared with the scatter of data from tests RS3 and RS5, which are those tests on the original 0·3–0·425 mm grading that are at the same stress level and are considered to have reached a stable final grading.

The finer but uniformly graded sample (RS7, Fig. 7(b)) reached a final gradings curve that was again above that of the other samples at this stress level. In compression, McDowell & Bolton (1998) showed that the gradings curves at a given stress level for samples with similar initial uniformity of grading, but different absolute particle sizes, were all parallel. In Fig. 7(b) a reference line has been drawn that is approximately parallel to an average of the final gradings for tests RS3 and RS5, but passing through the maximum initial particle size of 0·3 mm for test RS7. It can be seen that the final gradings curve lies quite close to this.

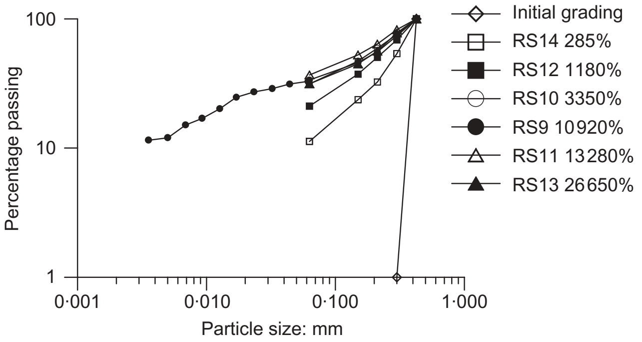


Fig. 5. Evolution of particle size distribution for Zone 2 for tests with σ_v' in range 248–386 kPa

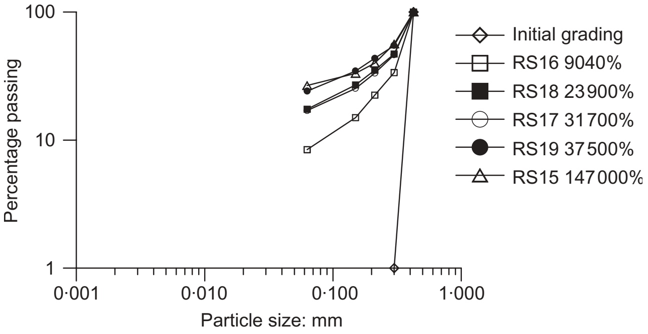


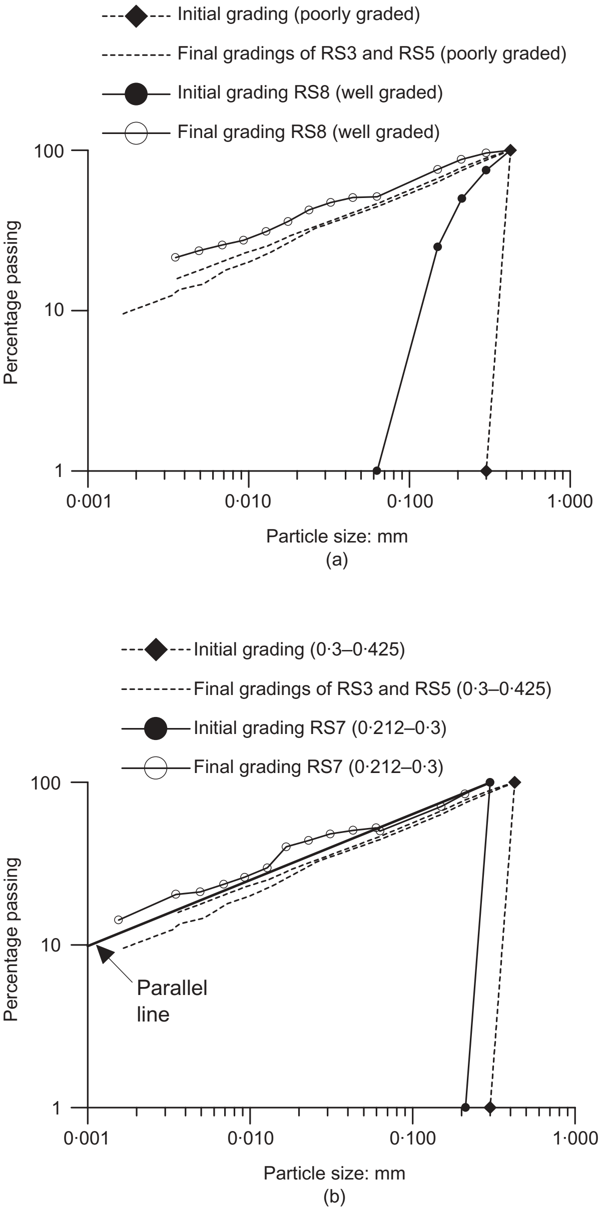
Fig. 6. Evolution of particle size distribution for Zone 2 for tests with σ_v' in range 60–97 kPa

QUANTIFICATION OF PARTICLE BREAKAGE

For some of the tests on 0·3–0·425 mm samples in the stress range 650–930 kPa, the relative breakage has been quantified for all three zones, and the data from these tests are shown in Fig. 8. The initial value of B_r of 0·057 at zero shear strain is that caused by a vertical stress alone of about 800 kPa, from an oedometer test. There is significantly greater breakage in Zone 2 than in either Zone 1 or Zone 3, and the breakage in Zones 1 and 3 is similar. However, the breakage in all three zones stabilises at around 2000–4000%, which is similar to the strains required for the volumetric strain to stabilise at this stress level.

In Fig. 9 the ratio of the volumetric strain to the relative breakage has been calculated, and, as Luzzani & Coop had observed for lower strain levels, the ratio remains constant, confirming that the compressive volumetric strain is directly related to the particle breakage. The ratio is not much affected by either the uniformity of the initial grading (RS8) or the absolute particle size (RS7).

The evolution of the percentage passing within each particle size interval is illustrated in Fig. 10. The data points shown at 1% are those from the oedometer test, but as a logarithmic strain scale has been used for clarity, these points have had to be plotted at a finite strain, thereby making the assumption, for the sake of the plot, that there is no breakage in the first 1% of shearing. None of the tests investigated breakage in the medium strain range, so the trends shown up to 100% shear strain are uncertain. However, the data for larger strains from the ring shear tests illustrate some interesting features. The percentage passing for the coarsest interval (0·3–0·425 mm) reduces rapidly, whereas that for the particles passing 0·063 mm sieve increases from almost zero at the start of the test, so that the



(a)

(b)

Fig. 7. The influence of initial grading: (a) effect of range of initial particle sizes; (b) effect of absolute value of particle size

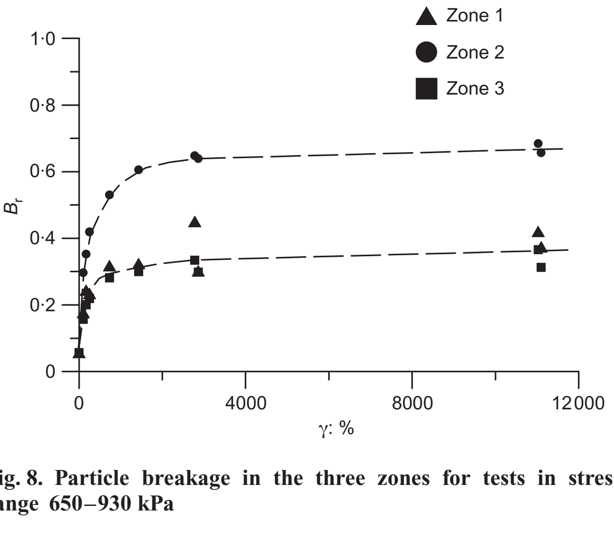


Fig. 8. Particle breakage in the three zones for tests in stress range 650–930 kPa

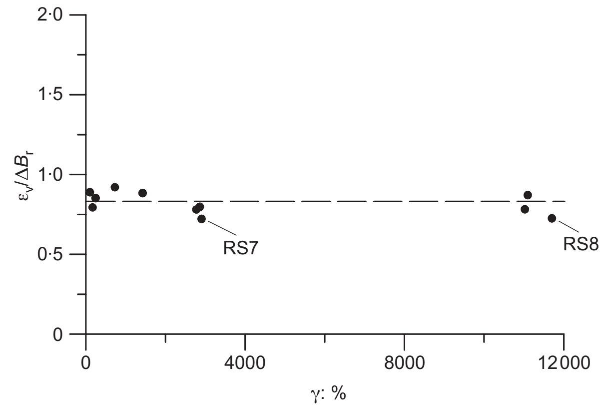


Fig. 9. Ratio of volumetric strain to change of relative breakage during shearing for tests in stress range 650–930 kPa

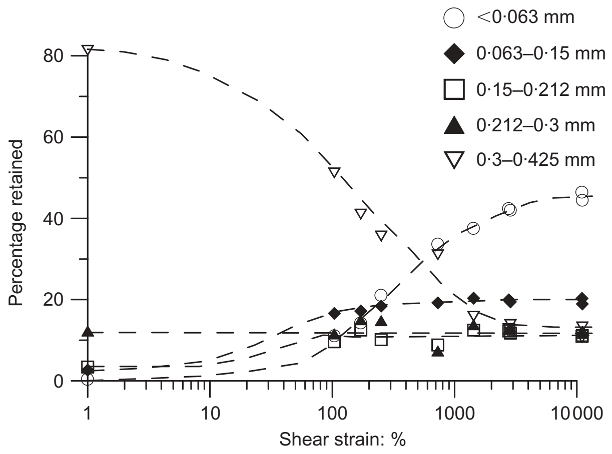


Fig. 10. Evolution of individual grain sizes for tests in stress range 650–930 kPa

two cross and stabilise, probably again in the region of 2000–4000%. It is, however, interesting to note that the changes to the intermediate gradings intervals are much smaller, and stabilise much more quickly, at around an order of magnitude smaller strain. For the sieve interval 0·212–0·3 mm the percentage passing remains practically constant throughout.

Figure 11 summarises all the relative breakage data for Zone 2 for all the tests. A separate trend can be seen for each stress range. The scatter of data for each stress level is less than might have been expected from the range of stresses because the range represents the extreme stresses, as discussed above. At low strains the data must tend towards the B_r value for oedometric compression only, and although there are no data in this region, for illustration a tentative curve has been sketched in for the tests in the 650–930 kPa range. At very large strains the value of B_r at each stress level stabilises, but again it is clear that the final degree of breakage reduces as the applied vertical stress reduces. The strains required for stabilisation increase rapidly as the vertical stress reduces, and are similar to those required for the volumetric strain to become constant. Comparing the initial voids ratios in Table 1 with the data in Fig. 11 it is difficult within the data scatter to pick out any trend for the effect of initial density.

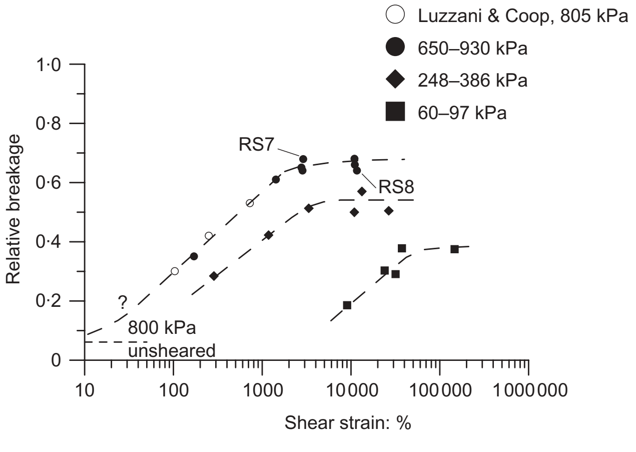


Fig. 11. Summary of all relative breakage data for Zone 2

MOBILISATION OF STRENGTH

Mobilised angles of shearing resistance have been calculated as $\tan^{-1}\tau/\sigma'_v$, where τ is the applied shear stress. These are plotted in Fig. 12 against a logarithmic strain scale. Whereas the peak ϕ'_{mob} at low strains depends on the density and stress level, after around a shear strain of about 30% all tests show an approximately constant value, with no tendency for any degradation of ϕ'_{mob} despite the severe particle breakage that the soil undergoes. Coop (1990) had shown that the critical state angle of shearing resistance for Dog's Bay sand was constant for all stress levels and did not reduce, as some authors had suggested, as a result of the particle breakage at higher confining stresses. From Fig. 12 it can now be seen that also particle breakage due to shearing has no significant effect on ϕ'_{mob}.

CONCLUSIONS

From this series of ring shear tests it may be concluded that particle breakage continues to very large strains indeed, far beyond those reached in triaxial tests. That particle breakage is accompanied by volumetric compression, and occurs even for tests at modest confining stresses. If an apparent critical or constant-volume state is seen in a triaxial tests, as Chandler (1985) and Baharom & Stallebrass (1998) assumed, it can only be as a result of counteracting dilative strains from particle rearrangement and compressive strains from particle breakage.

At very large strains a constant grading is reached, but that constant grading is dependent not only on the normal stress applied but also on the uniformity and absolute particle size of the initial grading. The mobilised strength is unaffected by the particle breakage.

The tests presented here were conducted on a carbonate sand. However, Luzzani & Coop (2002) found that even quartz sands at low stress levels were subjected to small

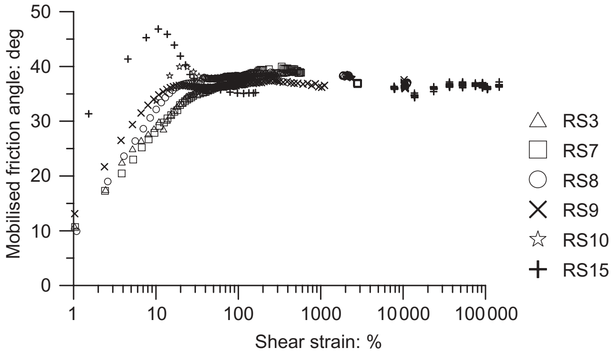


Fig. 12. Evolution of mobilised angle of shearing resistance with shear strain

amounts of particle breakage. It is perhaps logical that particle breakage cannot continue indefinitely, and so there must be final stable gradings for all sands, but the development of particle breakage with strain for other types of sand is too slow to allow the ring shear technique to be used to establish the final gradings because there would be excessive loss of soil through the rings.

These tests have shown conclusively that the 'critical state' that we see for a sand in a triaxial test is only a transient state of constant volume. Nevertheless, macroscopically it is still a valid constant-volume state for the strain levels reached in triaxial apparatus, and the critical state framework, based on triaxial tests, has been shown by others (e.g. Jovicic & Coop, 1997; Klotz & Coop, 2001) to be invaluable for understanding both the mechanics of sands and the behaviour of engineering structures in sands. When using such a framework, it should only be remembered that the critical state, as defined in a triaxial test, is not strictly rigorous, so that if the soil has the opportunity of shearing to very large strains or displacements, such as along a driven pile shaft, the behaviour of the sand might not be that expected from the critical state framework, and a continued particle breakage and volumetric compression would be seen, even if the mobilised angle of shearing resistance remained constant.

ACKNOWLEDGEMENTS

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NOTATION

B_p	breakage potential (Hardin, 1985)
B_r	relative breakage (Hardin, 1985)
B_t	total breakage (Hardin, 1985)
e	voids ratio
H_0	initial sample height
γ	engineers' shear strain
δh	horizontal displacement
δv	vertical displacement
ε_v	volumetric strain (or vertical strain in shear box or ring shear)
σ'_v	net vertical stress
τ	shear stress
ϕ'_{mob}	mobilised friction angle

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