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PARTICLE-HOLE STATE DENSITIES IN PRE-EQUILIBRIUM NUCLEAR REACTION MODELS

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Contents

Particle-hole state densities in pre-equilibrium nuclear reaction models

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Abstract. The particle-hole level and state densities required to calculate the cross sections of pre-equilibrium nuclear reactions are reviewed. Using the equidistant spacing model, explicit expressions are found for the total density of states and for the density of final accessible states. These are modified to take account of the restrictions due to the Pauli principle and the finite depth of the nuclear potential. The dependencies if the densities on spin, isospin, linear momentum and pairing are described. The effects of departures from the equidistant spacing model, particularly those due to shell structure, are also discussed. Some comparisons are made with realistic densities obtained by full combinatorial calculations Some recommendations are made concerning the best choices to be made for pre-equilibrium calculations, combining accuracy and convenience.

KEY WORDS: Particle-hole level densities; equidistant-spacing model; pre-equilibrium models; Fermi gas; intranuclear transitions; Darwin-Fowler method; level schemes; nuclear physics

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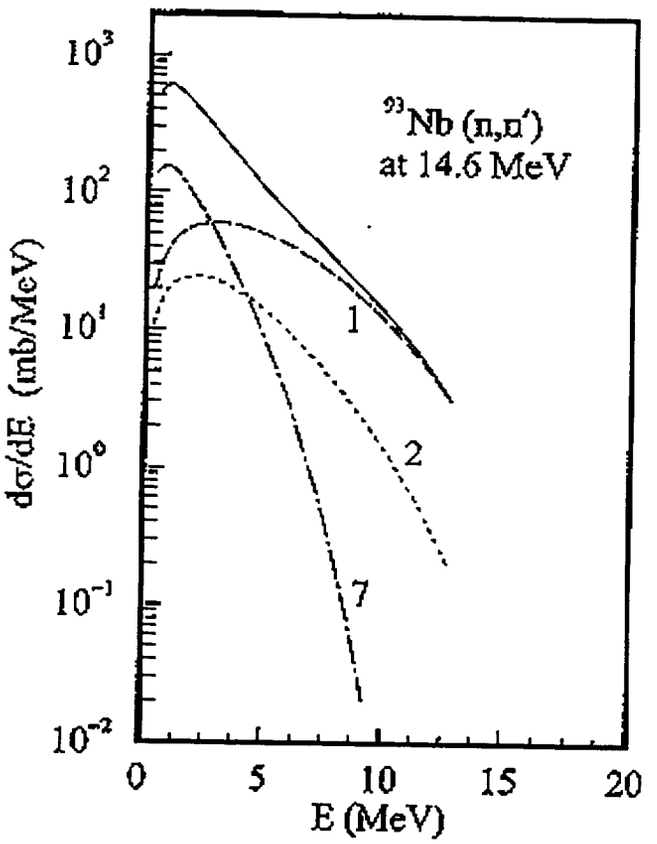

Figure 1. The calculated spectra of nucleons emitted from the 1st, 2nd and 7th stage of the ^{93}Nb(n,n') reaction at 14.6 MeV. These are shown as the dashed, dotted and dash-dotted lines, respectively. The total cross section (which includes the compound nucleus cross section that dominates at small energies) is shown by the full line (Herman *et al* 1992).

the transit time, about 10^{-22} s to 10^{-20} s depending on the incident energy and target nucleus, whereas the compound nucleus stage takes about 10^{-18} s to 10^{-16} s.

The number of excitons during the compound nucleus stage fluctuates about a mean value, and although the probability of emission per unit time is extremely small, the total compound nucleus cross-section is often greater than the pre-equilibrium cross section. This is shown in figure 2 for the interaction of 14 MeV neutrons by titanium. For small exciton numbers the cross-section decreases, and then passes through a minimum at about $n = 5$, which marks the transition from pre-equilibrium to compound nucleus emission. At higher energies, the switch from pre-equilibrium to the equilibrium emission occurs at correspondingly higher exciton numbers. The broad peak at higher exciton numbers corresponds to compound nucleus emission. A time-dependent picture of the development of a nuclear reaction with respect to the exciton numbers participating is

Particle-hole state densities in pre-equilibrium nuclear reaction models

1. Introduction

For many years it was assumed that nuclear reactions take place in two stages, a direct stage that occurs in the time it takes the projectile to cross the target nucleus and a compound stage when emission takes place from the fully equilibrated nucleus. Theories enabling the cross-sections for emission of particles from both stages to be calculated have been developed, and are extensively used to analyze experimental data. More extensive measurements showed, however, that some cross-sections cannot be understood in this way, and this is interpreted as evidence for the emission of particles after the direct stage but before the establishment of statistical equilibrium, These pre-equilibrium reactions, as they are called, have been extensively studied (Gadioli and Hodgson 1992).

The first model of pre-equilibrium reactions, the exciton model, was formulated by Griffin (1966) and used to calculate the energy distributions of the emitted particles. The model was developed by Blann (1971), Gadioli *et al* (1973) and others and used to analyze a wide range of experimental data (Blann 1975, Gadioli *et al* 1976, Gruppelaar *et al* 1986, Zhivopistsev *et al* 1987, and Gadioli and Hodgson 1992). Subsequently the quantum-mechanical theories (Feshbach *et al* 1980, Tamura *et al* 1982, Nishioka *et al* 1986, 1988a and 1989) made it possible to calculate the angular distributions of the emitted particles.

In the pre-equilibrium theories of nuclear reactions it is assumed that the excitation process takes place by successive nucleon-nucleon interactions in a series of stages. Each interaction produces a particle-hole $(p-h)$ pair and each particle and hole is called an exciton. The first few states are therefore $2p1h$, $3p2h$, and so on. The number of excitons $n = p + h$ and the stages are labelled by s, so that $n = 2s + 1$. Usually each nucleon-nucleon interaction produces another exciton, but occasionally a particle receives enough energy for it to be emitted; these are the pre-equilibrium reactions. The energy distribution of these pre-equilibrium particles changes as the reaction proceeds: as expected, those emitted in the earlier stages have on the average more energy than those emitted in later stages, as shown in figure 1 for 14.6 MeV neutrons on ^{93}Nb.

In general, the cross-section for pre-equilibrium emission falls rapidly as the incident energy is shared among the nucleons of the target nucleus, and emission becomes increasingly unlikely. Eventually the compound nucleus attains statistical equilibrium and emits particles very slowly until this is no longer energetically possible. These times are on a nuclear scale: the pre-equilibrium stage takes place in a time of the order of

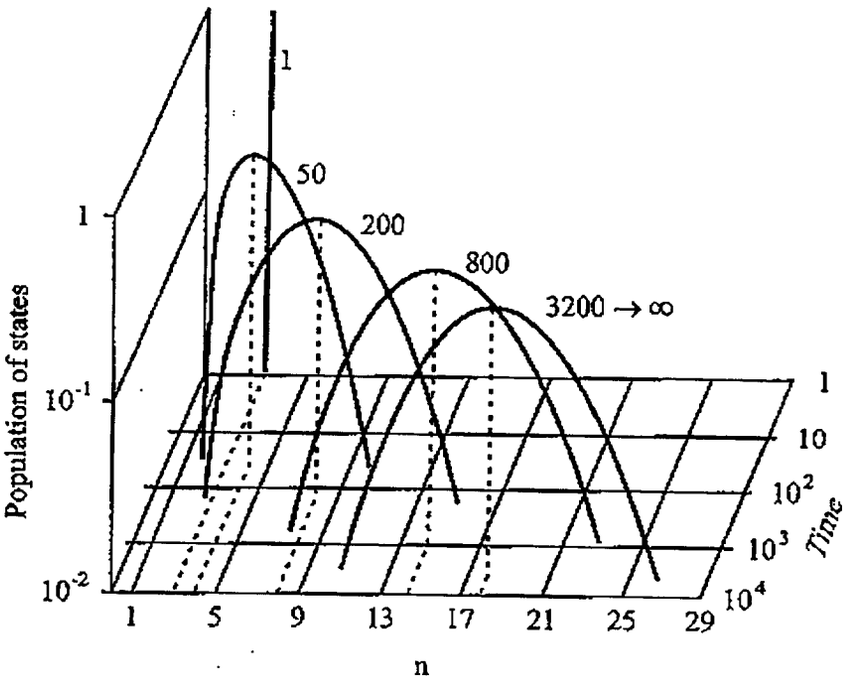

Figure 3. Three-dimensional plot showing the time evolution of a reaction given by the master equation of the exciton model (without the emission term). Each curve shows the distribution of exciton numbers n at the time indicated (in relative units). The distribution broadens as the reaction proceeds (Blann 1974).

golden rule

$$\lambda^{\pm}(n,E) = \frac{2\pi}{\hbar}|M|^2\omega_{\mathrm{f}}^{\pm}(n,E)\ , \tag{3}$$

where $|M|^2$ is the mean squared transition matrix element and $\omega_{\mathrm{f}}^{\pm}$ are the densities of the final accessible states, where the $\pm$ refers to the $\Delta n = \pm 2$ transitions. Here, the final accessible states are those which can be reached by a single two-body interaction from the initial configuration, and their number is generally (much) less than the total number of states with given exciton number and excitation energy.

In order to calculate these cross-sections and the intranuclear transition rates it is necessary to know the density of possible states $\omega(p,h,E)$ (defined as the number of states per unit energy between $E - \frac{1}{2}\Delta E$ and $E - \frac{1}{2}\Delta E$, where the energy interval ΔE includes a large number of states) as a function of the numbers of particles and holes and the excitation energy E. This is essentially a combinatorial problem, subject to some restrictions that will be mentioned later on. The number of states increases very rapidly with n, as shown for some illustrative examples in table 1. This increase is so rapid that it is impracticable, even for quite small exciton numbers, to evaluate the number of states by a numerical algorithm. Analytical methods are therefore essential, and these form the main subject of the present review.

These calculations must be carried out for all values of J, and then the total sum

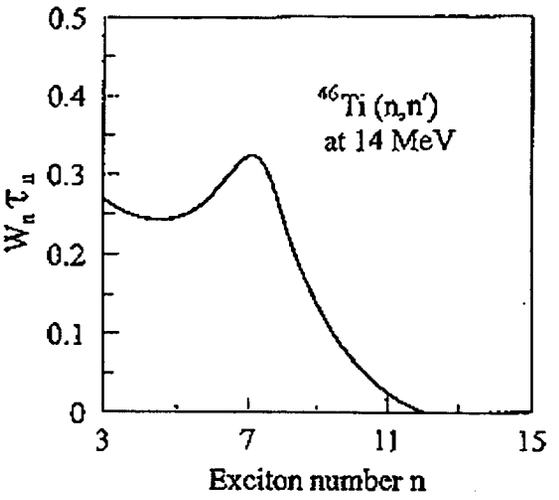

Figure 2. The neutron emission cross section as a function of exciton number for 14 MeV neutrons on Ti (Chatterjee and Gupta 1981).

shown in figure 3.

The rate of emission $\lambda_x(n, E, \varepsilon)$ of a nucleon x of energy ε from a state of $n = p+h$ excitons in a nucleus of excitation energy E, neglecting spin variables (apart from the statistical weight factor $(2s_x + 1)$) (see, e.g., Gadioli and Hodgson 1992, pp. 19 and 235) is given by the product of a statistical weight factor, the cross-section $\sigma^*_{\mathrm{INV}}(\varepsilon)$ of the inverse reaction and the ratio of two state density functions $\omega(p, h, E)$

$$\lambda_x(n, E, \varepsilon) = \frac{2s_x + 1}{\pi^2\hbar^3}\mu_x\varepsilon\sigma^*_{\mathrm{INV}}(\varepsilon)\frac{\omega(p-1, h, U)}{\omega(p, h, E)} , \tag{1}$$

where μ_x is the nucleon reduced mass and $U = E-B_x-\varepsilon$ is the energy of residual nucleus which is produced in an $(n-1)$-exciton state. Some authors include an additional factor which takes account of the charge composition of the excitons with respect to the ejectile (see below). A similar equation can be obtained for cluster emission; this depends on the assumptions made about the complex particle creation and emission.

The particles emitted from each exciton state have different energy distributions, as shown in figure 1. The resulting particle spectra therefore depend on the contributions from separate exciton states. Thus, for the energy spectra

$$\frac{\mathrm{d}\sigma_x}{\mathrm{d}\varepsilon} = \sigma_{\mathrm{R}}\sum_n \tau(n)\lambda_x(n, E, \varepsilon) , \tag{2}$$

where σ_{R} is the cross-section for the formation of the composite system, and $\tau(n)$ is the time spent by a nucleus in the n-exciton state. These quantities depend on the equilibration process, whose course, consequently, depends on the intranuclear transition rates $\lambda^{\pm}(n, E)$ between two neighbouring exciton numbers, i.e. $n \to (n \pm 2)$. They can be obtained either using a known interaction in given potential, or using the Fermi

2. The equidistant-spacing model

As a first approximation, we consider the equidistant-spacing model (ESM) suggested by Bethe (1936). In this model all the single-particle levels are equally spaced in energy. In the ground state of a nucleus with mass number A, all the A lowest-lying levels are occupied and the highest one specifies the Fermi level. The excitation energy of a nucleus is distributed in some way among the excited particles above and the empty holes below the Fermi level. The ESM is a great simplification, but nevertheless it is very useful and gives quite a good energy dependence of the total density of states and at the same time shows the basic properties of the densities and how and where they originate. The principal advantages of the equidistant spacing model is that it facilitates the analytical calculation of the exciton state densities. If a more realistic spacings are used, this becomes practically impossible. The following sections give expressions for the state densities subject to various restrictions.

2.1. Densities of states without Pauli principle

Strutinski (1959) and Ericson (1960) showed that the equidistant spacing model (ESM) gives

$$\omega(n,E) = \omega(p,h,E) = \frac{g^n E^{n-1}}{p!h!(n-1)!}\ , \tag{4}$$

where p (h) is the number of particles (holes), $n = p + h$ is the total exciton number. If g is the single-particle level density,

$$\omega(1,0,E_p) = \omega(0,1,E_h) \ = \ g\ . \tag{5}$$

The formula (4) can then be obtained by successive application of the recursion relations (see Gadioli *et al* 1973)

$$\begin{aligned} \omega(p,0,E_p) &= \frac{1}{p}\int_0^{E_p} g\omega(p-1,0,U)\mathrm{d}U \\ \omega(0,h,E_h) &= \frac{1}{h}\int_0^{E_h} g\omega(0,h-1,U)\mathrm{d}U \end{aligned} \tag{6}$$

and finally

$$\omega(p,h,E) = \int_0^E \omega(p,0,U)\omega(0,h,E-U)\mathrm{d}U\ . \tag{7}$$

The last equation specifies the generally valid procedure used to obtain particle-hole densities from those for particles and for holes separately. The particle and hole densities in (7) need not necessarily be obtained from the ESM*.

To have some feeling for typical values of the densities of particle-hole states in real pre-equilibrium problems, we use also some knowledge beyond the scope of the present review, namely the range of energies and nuclei where the standard formulations of the pre-equilibrium models are well justified. These arguments have been given by Agassi

* An updated version of this approach appeared also in a recent paper by Bogila *et al* (1996a)

Table 1. Some of typical values of the numbers of states and their densities for some important configurations for two selected nuclei and energies calculated using the Ericson formula (see the next section). The equilibrium exciton number is $\overline{n}$.

Quantity	A=50 E=15 MeV	A=200 E=100 MeV
$\overline{n}$	9	43
n_{max}	15	77
No. of states		
3 excitons	830	590 000
$\overline{n}$ excitons	1.1×10^6	8.9×10^{41}
Density of states (MeV^{-1})		
3 excitons	3 200	9.1×10^6
$\overline{n}$ excitons	4.1×10^6	1.4×10^{43}

corresponds to the level density that is obtained by thermodynamical arguments (Gadioli and Hodgson 1992, ch. 2; Bethe 1936 and 1937). This provides a useful constraint on the exciton density calculations.

Many formulae for the particle-hole state densities have been used in calculations of the cross sections of pre-equilibrium nuclear reactions. The earlier calculations used simple models of the particle-hole excitations (Strutinski 1959, Ericson 1960, Williams 1971), but many improvements have been made taking into account the finite depth of the nuclear potential (Běták and Dobeš 1976), spin dependence of the decay (Obložinský 1987, Obložinský and Chadwick 1990) and the conservation of linear momentum (Mädler and Reif 1980, Chadwick and Obložinský 1991 and 1992). These models are described in sections 2 and 3.

The success of the pre-equilibrium theories makes it desirable to standardize all parts of the calculations, and the expressions used for the particle-hole densities are among the more important. The more sophisticated calculations of particle-hole densities are available only in specialized papers, and in addition it is not always clear which is the best formula to use in a particular practical calculation. The aim of the present work is therefore to collect together and review the main results of many partial studies and to present them in a coherent form suitable for numerical calculations.

It is possible in principle to calculate the exciton state density by counting all possible configurations numerically. Even for small exciton numbers this requires very large computing times. Furthermore, it fails to give the insight that can be attained by analytical methods, however approximate. Nevertheless, we mention these calculations and discuss their possible applications in the last section of the review.

2.2. Densities of states with Pauli principle

The inclusion of the Pauli principle greatly complicates the calculation of the density of states. In the languague of pre-equilibrium models, the Pauli principle requires that no two excitons of the same type are allowed to be in the same state, which implies that they cannot have the same energy.

The state densities have been obtained under these conditions by Böhning (1970). He studied the partition of an integer M into *no more* than N integer parts. This problem was essentially solved by Euler (1753). Let the energies of particle levels to be $1/g$, $2/g$, $3/g$, ... and let there be N particles with total excitation energy E. The minimal energy needed due to the Pauli principle is $N(N+1)/(2g)$, so that the "energy space" available for excitation is

$$M = gE - \frac{1}{2}N(N+1) \; . \tag{11}$$

The number of partitions $p_N(M)$ of N particles with energy M satisfies the recurrence relation (Williams 1971)

$$p_N(M) = p_{N-1}(M) + p_N(M-N) \tag{12}$$

with

$$\begin{aligned} p_N(0) &= 1 \\ p_0(M) &= \delta_{0M} \end{aligned} \tag{13}$$

For $N \geq M$, $p_N(M)$ is independent of N and so is degenerate.

The total number of particle-hole states $r(p,h,E)$ is obtained by folding those of the particles and holes, as in (7)

$$r(p,h,E) = \sum_{m=0}^{M} p_p(m) p_h(M-m) \; . \tag{14}$$

The structure of (12) suggests that the resulting number of states and therefore also the densities for $M \gg N$ (as is the real case for nuclear reactions) can be approximated by a polynomial in M of order $(N-1)$. Neither (12) nor (14) are continuously increasing functions of energy, and they can remain constant even if the energy M is increased by several integer units.

While the recipe for obtaining the state densities within the ESM as given by Böhning is correct and precise, it is seldom used; simple analytical formulae or the relatively precise realistic densities that are easily obtained with modern computers are generally preferred.

A feasible method to obtain an analytical formula was suggested by Williams (1971). Though the results are only approximate, it is quite accurate and it is easily programmed for numerical calculation. The derivation is based on the Darwin-Fowler method for non-localized particles (see Münster 1969) with subsequent use of the Cauchy theorem in conjunction with the saddle-point approximation.

et al (1975), who showed that the necessary conditions for the statistical assumptions to be valid are

$$T_{\mathrm{P,n}} \gg (T_{\mathrm{coll,n}}, T_{\mathrm{dec,n}}) \gg T_{\mathrm{NN}} \ , \tag{8}$$

where $T_{\mathrm{P,n}}$ is the Poincaré recurrence time (the time needed for the statistical system to return to its original, very nonequilibrium configuration) for the n-exciton configuration, $T_{\mathrm{coll,n}}$ the average time interval between two successive interactions, $T_{\mathrm{dec,n}}$ the time of the decay of given n-exciton state by emission of either particle or gamma and T_{NN} the duration of the nucleon-nucleon interaction. If we express the conditions (8) in terms of energies, nuclei and their single-particle densities, we find that they are satisfied for nuclei with $A \gtrsim 40$ and $E \gtrsim 15$ MeV. The upper energy limit is given by the possible appearance of non-nucleonic degrees of freedom, which limits the simple approach to about 150 MeV excitation energy.

Table 1 gives the numerical values (rounded) for the most probable (or "equilibrium") exciton number $\overline{n}$ ♯, the maximum exciton number n_{max} allowed by the Pauli principle and the corresponding numbers of states and densities for the 3-exciton state (the most important one for the nucleon emission under usual conditions) and the equilibrium exciton number $\overline{n}$, which dominates the equilibrium (compound nucleus) emission. The number of states at n_{max} is very small again, much less than at 3 excitons, and small departures from this tiny value can be fully attributed just to the approximate nature of the formulae. The table 1 shows the numbers of states for nuclei and energies close to the lowest and highest "corners" of the applicability of the model.

The total density of states which has to be compared with classical compound nucleus (i.e. equilibrium) theories is

$$\omega(E) = \sum_{\substack{n \\ \Delta n = 2}} \omega(n, E) \ , \tag{9}$$

where the summation is over all possible exciton states. The exciton number changes in steps of two because $n = 2s + 1$ (see Introduction). The maximum exciton number is limited in practice by the Pauli principle (since two identical excitons cannot share the same state) and, in the extreme case of very high energies, by twice the total number of nucleons in a nucleus. (We do not discuss the applicability of the model at such energies here.)

The density of final accessible states needed for the calculation of intranuclear transition rates (Williams 1970, corrected for the indistinguishability of excitons of the same type by Ribanský *et al* 1973) is given by

$$\begin{aligned} \omega_{\mathrm{f}}^{-} &= \frac{1}{2} gph(n-2) \\ \omega_{\mathrm{f}}^{+} &= g\frac{(gE)^2}{2(n+1)} \ , \end{aligned} \tag{10}$$

where the superscripts + and − refer to the $\Delta n = \pm 2$ transitions, respectively.

♯ We have used a more refined expression for state densities than that given by Ericson to obtain $\overline{n}$, namely that given in the next paragraph.

Table 2. Typical values of the numbers of states and their densities for some important configurations for two selected nuclei and their energies calculated using the Williams formula with $g = A/13$ (MeV^{-1}). For comparison, the total density of states is also given.

Quantity	A=50 E=15 MeV	A=200 E=100 MeV
No. of states		
3 excitons	800	590 000
$\overline{n}$ excitons	3.0×10^5	1.4×10^{39}
Density of states (MeV^{-1})		
3 excitons	3 100	9.1×10^6
$\overline{n}$ excitons	1.1×10^6	2.0×10^{40}
Total density of states (MeV^{-1})	2.8×10^6	7.2×10^{40}

This term arises partially from the minimal energy α_{ph} and partially as the result of the integration over the saddle in the complex plane when evaluating eq. (20)†.

In the more general case of different spacings g and $\tilde{g}$ for particles and holes, the density of n-exciton states becomes (Dobeš and Běták 1976)

$$\omega(p,h,E) = \frac{g^p \tilde{g}^h (E - A_{ph})^{p+h-1}}{p!h!(p+h-1)!}\,\Theta(E - \alpha_{ph})\,, \tag{24}$$

where

$$\alpha_{ph} = \frac{1}{2g}(p^2+p) + \frac{1}{2\tilde{g}}(h^2-h) \tag{25}$$

and

$$A_{ph} = \frac{1}{4g}(p^2+p) + \frac{1}{4\tilde{g}}(h^2-3h) \tag{26}$$

As in the preceding section, we make some comparisons in table 2 for two "case studies", with densities calculated using the Williams formula (21). The densities for low exciton numbers remain practically at their previous values, whereas in the vicinity of the equilibrium exciton number $\overline{n}$ the difference is essential (and is still much greater at higher exciton numbers) due to the rapidly growing correction term for the Pauli principle A_{ph}.

One can see how good approximation to the ESM (21) is from figure 4 taken from the original Williams' paper. The consistency of the particle-hole state densities (21) summed over all exciton numbers with the classical compound nucleus level density (see (9) has been verified by Williams (1971) (figure 5) and later on for more sophisticated cases by other authors (Avrigeanu and Avrigeanu 1989, Zhang 1993). The comparison of Williams (1971) is shown in figure 5.

† The Θ function on the r.h.s. of (21) is often neglected when the density formula is used. One must keep in mind, however, the range of variables for which the formula has been derived since this determines its range of validity.

Let $a_1, a_2, \ldots$ be the occupation numbers of the single-particle states and $\varepsilon_1, \varepsilon_2, \ldots$ the corresponding energies. Then,

$$\begin{aligned} \sum_t a_t &= f \\ \sum_t a_t \varepsilon_t &= E_f \ , \end{aligned} \tag{15}$$

where f stands for a given type of exciton (i.e. particles or holes).

In the following, let us consider the ESM of for the particle and hole energies, which are $1/g$, $2/g$, $3/g$, ...for particles and and 0, $1/g$, $2/g$, ...for holes. The system partition function is

$$Z_{\text{sys}} = Z_p Z_h \tag{16}$$

where Z_p and Z_h are the partition functions for particles and holes, respectively. They are subject to the conditions (15), so Z_p for particles is given by the coefficient of $x^f y^{E_f}$ in the expansion of the function

$$\mathcal{Z}_p = \prod_{i=1}^{\infty} \left(1 + x y^{i/g}\right) \tag{17}$$

and similarly for holes (Münster 1969, p. 108; Williams 1971)

$$\mathcal{Z}_h = \prod_{k=0}^{\infty} \left(1 + x y^{k/g}\right) \ . \tag{18}$$

These generating functions may be rewritten in the form

$$\mathcal{Z}_p = 1 + \sum_{m=1}^{\infty} \frac{x^m \ y^{\frac{1}{2}m(m+1)/g}}{\prod_{i=1}^{m}(1 - y^{i/g})} \tag{19}$$

and similarly for holes. The number of states for particles and holes (see (14)) is

$$r(f, \mathcal{E}) = \frac{1}{(2\pi i)^2} \oint \oint \mathcal{Z}_f \cdot \frac{\mathrm{d}x \ \mathrm{d}y}{x^{f+1} y^{\mathcal{E}+1}} \ . \tag{20}$$

Here, $\mathcal{E}$ is the energy of system of f fermions, where f stands for either p or h.

The integrals may be evaluated by the saddle-point method, and the resulting density is

$$\omega(p, h, E) = \frac{g^{p+h}(E - A_{ph})^{p+h-1}}{p!h!(p+h-1)!} \ \Theta(E - \alpha_{ph}) \ , \tag{21}$$

where Θ is the Heaviside (step) function and

$$\alpha_{ph} = \frac{1}{2g}[(p^2 + p) + (h^2 - h)] \tag{22}$$

is the minimal energy needed to put p particles and h holes in the levels taking account of the Pauli principle and the correction term which lowers the energy in (21), and

$$A_{ph} = \frac{1}{4g}[(p^2 + p) + (h^2 - 3h)] \ . \tag{23}$$

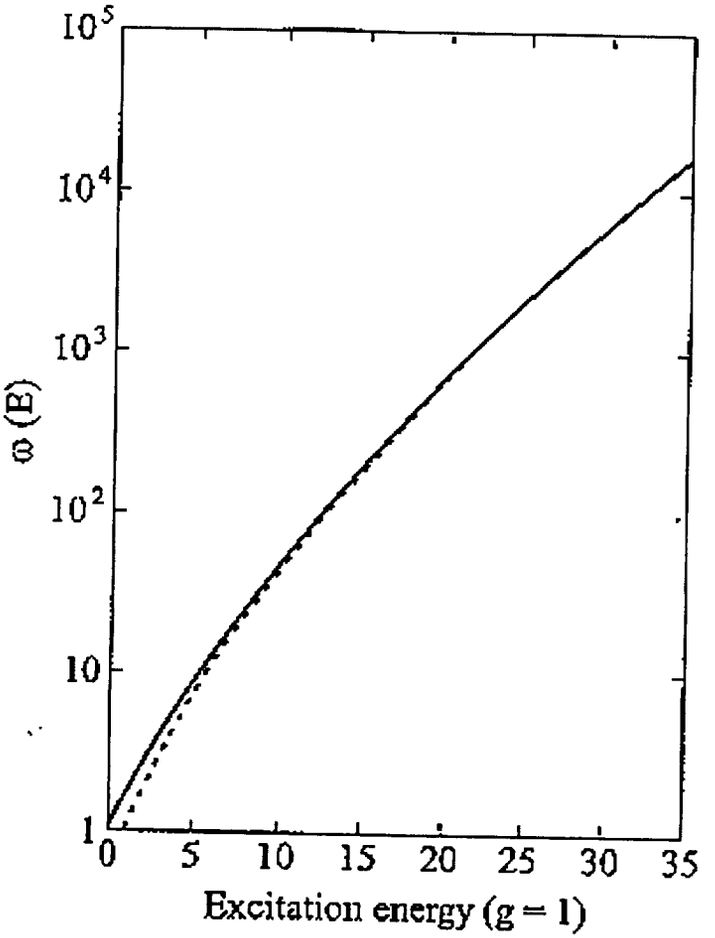

Figure 5. Comparison of total density of states as obtained summing particle-hole densities (9, 20) and from the CN formula $\omega(E) = \exp(\pi\sqrt{\frac{2}{3}gE}/(\sqrt{48}gE)$ (Williams 1971).

2.3. Densities of final accessible states with Pauli principle

The calculation of the densities of final accessible states is a straightforward, though rather complicated generalization of what has just been shown above. In the following, we allow for different numbers of particles and holes, and also for different energies, by a method due to Dobeš and Běták (1976). To do this, let $a_1, a_2, \ldots$ be the occupation numbers of the one-fermion states of the initial and $b_1, b_2, \ldots$ those of the final configuration, respectively. In this way, we also allow for transitions, as both the configurations may be different. Let $\varepsilon_1, \varepsilon_2, \ldots$ be the energies of the corresponding single-fermion states and $\mathcal{E}_\mathrm{i}$ and $\mathcal{E}_\mathrm{f}$ those of the initial and final states. Let us define the occupation function

$$\begin{aligned} \gamma(a_t) &= 1 \quad \text{for } a_t = 0, 1 \\ \gamma(a_t) &= 0 \quad \text{for } a_t < 0 \text{ or } a_t > 1 \,. \end{aligned} \tag{27}$$

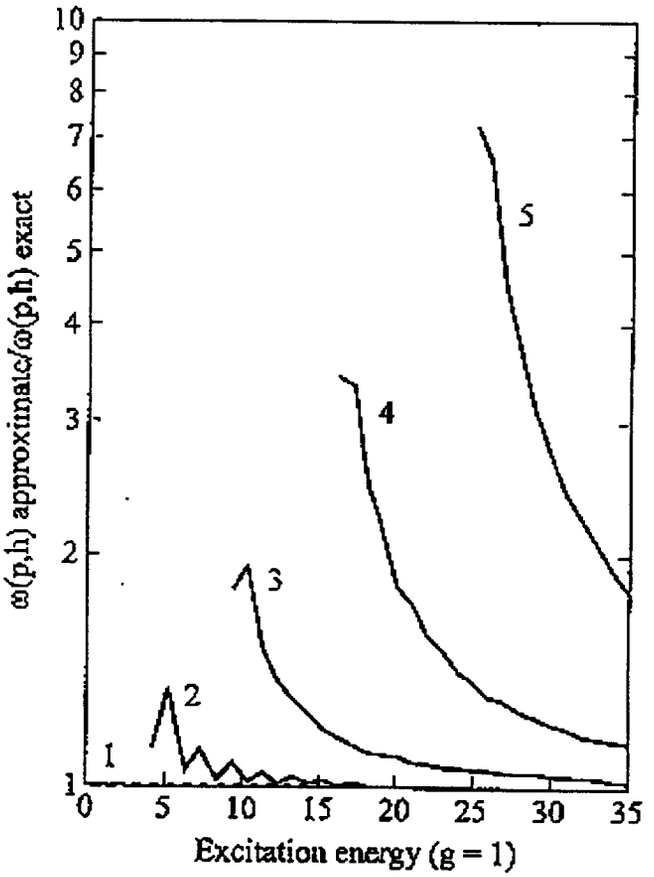

Figure 4. Comparison of approximate particle-hole state density computed from (20) and the exact values within the ESM. The graph shows the ratio of the aproximate to the exact value, and the numbers next to each curve identify the number of particles and holes, taken to be equal here. Note that the energy is given in units of the ESM spacing, not in MeV. (Williams 1971).

Baguer *et al* (1989) used the computer code REDUCE to obtain corrections to all available orders due to the Pauli principle. They obtained to so-called "exact" formulae, including all possible energy terms, but still however containing the approximations of the Darwin-Fowler method. As the Böhning results cannot be expressed as polynomials, the densities of Baguer *et al* fluctuate with energy in a very interesting semi-periodic manner (see figure 6).

There is one very important difference between the properties of the densities (21) and of the more refined approaches published later on (Běták and Dobeš 1976, Obložinský 1986, Baguer *et al* 1989, Anzaldo-Meneses 1995) and those suggested by Ericson (4), namely that they do not extend at given excitation energy to infinite exciton numbers but are cut off at some maximal value $n_{\max}$.

the generation function is

$$\mathcal{Z}(x,y,z,u,w) = \sum_{(a,b)} \prod_t \gamma(a_t)\gamma(b_t) x^{a_t} y^{a_t\varepsilon_t} z^{b_t\varepsilon_t} u^{b_t-a_t} w^{|b_t-a_t|} , \tag{29}$$

and the total number of configurations

$$C_{kj}(f,\mathcal{E}_\mathrm{i},\mathcal{E}_\mathrm{f}) = \frac{1}{(2\pi i)^5} \oint\oint\oint\oint\oint \mathcal{Z} \cdot \frac{\mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}u\ \mathrm{d}w}{x^{f+1} y^{\mathcal{E}_\mathrm{i}+1} z^{\mathcal{E}_\mathrm{f}+1} u^{j-k+1} w^{j+k+1}} . \tag{30}$$

The integrations over u and w in (30) are easily performed even in a general case. The rest of the calculation must be done for a specific scheme of particle and hole states.

The number of realisations of final accessible states obtained using (30) (Dobeš and Běták 1976) is

$$\begin{aligned} C^+(p,h,E_\mathrm{i},E_\mathrm{f}) = \int\int \quad & \left[C^{12}(p,E_\mathrm{i}-U_\mathrm{i},E_\mathrm{f}-U_\mathrm{f}) C^{01}(h,U_\mathrm{i},U_\mathrm{f}) \right. \\ & + C^{01}(p,E_\mathrm{i}-U_\mathrm{i},E_\mathrm{f}-U_\mathrm{f}) C^{12}(h,U_\mathrm{i},U_\mathrm{f}) \\ & \left. + C^{01}(p,E_\mathrm{i}-U_\mathrm{i},E_\mathrm{f}-U_\mathrm{f}) C^{01}(h,U_\mathrm{i},U_\mathrm{f}) \right] \mathrm{d}U_\mathrm{i}\ \mathrm{d}U_\mathrm{f} \end{aligned} \tag{31}$$

and similarly for the other possible processes.

The densities of final accessible states on the energy shell $E_\mathrm{i} = E_\mathrm{f} = E$ are

$$\begin{aligned} \omega_\mathrm{f}^+(p,h,E) &= C^+(p,h,E,E)/\omega(p,h,E) \\ \omega_\mathrm{f}^0(p,h,E) &= C^0(p,h,E,E)/\omega(p,h,E) \\ \omega_\mathrm{f}^-(p,h,E) &= C^+(p-1,h-1,E,E)/\omega(p,h,E) . \end{aligned} \tag{32}$$

The equations (31) and (32) are valid in general. For simplicity, we use $g = 1$ in the following formulae. The explicit results for the densities of final accessible states when only the highest power of energy is retained, are (Dobeš and Běták 1976)

$$\begin{aligned} \omega_\mathrm{f}^-(p,h,E) &\approx \frac{1}{2}ph(n-2)\left[1 - \frac{n-1}{8(n-2)(E-A_{ph})}((p-1)(p-2)+(h-1)(h-2))\right] \\ \omega_\mathrm{f}^0(p,h,E) &\approx \frac{E-A_{ph}}{2n}\left(p(p-1)\left[1-\frac{n(p+2)}{2(E-A_{ph})}\right] + p(p-1)\left[1-\frac{n(h+2)}{2(E-A_{ph})}\right]\right. \\ &\qquad \left. +4ph\left[1-\frac{n(n-1)}{4(E-A_{ph})}\right]\right) \\ \omega_\mathrm{f}^+(p,h,E) &\approx \frac{(E-A_{ph})^2}{2(n+1)}\left[1-\frac{n+1}{n(E-A_{ph})}\left(\frac{5}{8}p(p-1)+\frac{5}{8}h(h-1)+ph+\frac{1}{2}n\right)\right] . \end{aligned} \tag{33}$$

The expression for ω_f^- is identical to that obtained by Obložinský *et al* (1974) by a much simpler and sufficiently transparent method where, unfortunately, the same form of the Pauli principle correction is assumed for different exciton states.

More complete expressions than that given by (33) (up to the second order of corrections) for the densities of final accessible states were obtained by Isaev (1985). However, even the (33) are not often used and they are replaced by simpler though less precise results.

For practical use, an even more straightfoward approximation is available. If we ingore the influence of the Pauli principle on the $\Delta n = -2$ intranuclear transitions where

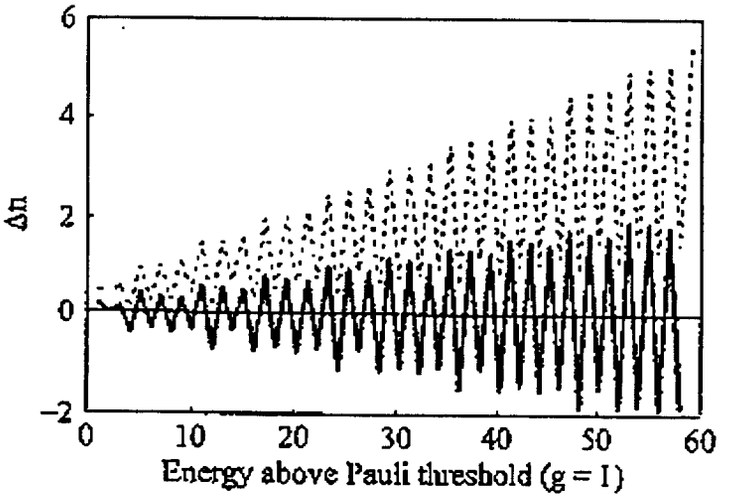

Figure 6. Comparison of the energy-dependence of the density of states within the ESM, as given by the Williams expression (20) (dashed line) and by "exact" formula of Baguer *et al* (full line), with respect to the exact (Böhning) results for the case of 4 particles. The energy here is that above Pauli threshold α_{ph}. At lower particle numbers, both formulae just oscillate around zero. When considering the deviations, one must remember that the absolute number of the densities at the right edge of the plot (at $gE = 60$) is 1900, so that even the largest oscillation in the Williams case is only about 0.25%.

We define the total number of possible configurations with f fermions in the initial state and $(f+j-k)$ fermions in the final state as $C_{kj}(f, \mathcal{E}_\mathrm{i}, \mathcal{E}_\mathrm{f})$. Here, k and j are the numbers of fermions which undergo the intranuclear transition; k is the number of states that are unoccupied after the transition and j the number that become occupied, respectively. Thus, $k = j = 0$ for the simple calculation of the density of states without any transition. The typical number of single-particle states considered in pre-equilibrium reactions (i.e. the maximum value of t) is several hundreds or thousands, whereas we have to deal with huge sets $\{a\}$ of different distributions (the typical dimension of such a set may be 10^{10} or even much more, depending on the complexity of state and the corresponding energy).

The constraints are

$$\begin{aligned}
\sum_t a_t &= f \\
\sum_t a_t \varepsilon_t &= \mathcal{E}_\mathrm{i} \\
\sum_t b_t \varepsilon_t &= \mathcal{E}_\mathrm{f} \\
\sum_t (b_t - a_t) &= j - k \\
\sum_t |b_t - a_t| &= j + k \,,
\end{aligned} \tag{28}$$

and neglecting the factors $\tilde{g}(lh-j)$ in the energy terms on the r.h.s. of (37) we finally arrive at the easily usable formula

$$\omega(p,h,E) = g^p \tilde{g}^h \sum_{l=0}^{h} \left(\begin{array}{c} h \\ l \end{array} \right) (-)^l \, \Theta(E - \alpha_{ph} - lE_{\mathrm{F}}) \frac{(E - A_{ph} - lE_{\mathrm{F}})^{n-1}}{p!h!(p+h-1)!} \, . \tag{40}$$

For approximate calculations, the above equation can be further reduced to

$$\omega(p,h,E) \approx g^{p+h} \sum_{l=0}^{h} \left(\begin{array}{c} h \\ l \end{array} \right) (-)^l \, \Theta(E - lE_{\mathrm{F}}) \frac{(E - lE_{\mathrm{F}})^{n-1}}{p!h!(p+h-1)!} \, . \tag{41}$$

The associated transition rates can be derived in the same manner as the preceding section for the case of an infinite well (Běták and Dobeš 1976). The final formulae are rather lengthy and not very suitable for practical use. For such cases, they can be replaced by

$$\omega_{\mathrm{f}}^{\pm}(p,h,E;E_{\mathrm{F}}) \approx \omega_{\mathrm{f}}^{\pm}(p,h,E;E_{\mathrm{F}}=\infty) \frac{\omega(p \pm 1, h \pm 1, E; E_{\mathrm{F}})}{\omega(p \pm 1, h \pm 1, E; E_{\mathrm{F}}=\infty)} \, , \tag{42}$$

where $\omega_{\mathrm{f}}^{\pm}(p,h,E;E_{\mathrm{F}}=\infty)$ are the rates for the infinite well, given e.g. by (32). For a sufficiently wide range of energies, this expression agrees within 5 per cent with the exact one (Běták and Dobeš 1976).

Distinguishing between bound and unbound states in the theory of pre-equilibrium decay (Feshbach 1973), corresponding to the multi-step compound and multi-step direct reactions, led to an increased demand for explicit expressions for state densities for bound states, for use in multi-step compound theories (Feshbach *et al* 1980, Tamura *et al* 1982, Nishioka *et al* 1986, 1988a and 1989).

Though these densities are only a special case of (37) or (40), with the same type of energy constraint as was for holes (holes are not allowed to have higher energy than is the Fermi energy, E_F) to be applied also to particles (their energy must not exceed now the particle binding energy B), they have not been used immediately. Stankiewicz *et al* (1985) derived the densities with the help of integral equations of the type given by (6) and (7) neglecting the Pauli principle. Kalbach (1981) consistently distinguished active and passive particles and holes and finally Obložinský (1986) gave an analogue to the (40). His expression reads

$$\begin{aligned} \omega(p,h,E) = g^p \tilde{g}^h \sum_{k=0}^{p} \sum_{l=0}^{h} \left(\begin{array}{c} p \\ k \end{array} \right) \left(\begin{array}{c} h \\ l \end{array} \right) (-)^l \, \Theta(E - \alpha_{ph} - kB - lE_{\mathrm{F}}) \\ \times \frac{(E - A_{ph} - kB - lE_{\mathrm{F}})^{n-1}}{p!h!(p+h-1)!} \, , \end{aligned} \tag{43}$$

or, approximately

$$\omega(p,h,E) \approx g^{p+h} \sum_{k=0}^{p} \sum_{l=0}^{h} \left(\begin{array}{c} p \\ k \end{array} \right) \left(\begin{array}{c} h \\ l \end{array} \right) (-)^l \, \Theta(E - kB - lE_{\mathrm{F}}) \frac{(E - kB - lE_{\mathrm{F}})^{n-1}}{p!h!(p+h-1)!} \, . \tag{44}$$

Later on, similar equations have been reported also by some other groups (Zhang and Yang 1988, Mao and Guo 1993).

it is least important, the steady-state equilibrium conditions give directly (Ribanský *et al* 1973)‡

$$\omega_{\mathrm{f}}^{+}(p,h,E)=\frac{g}{2(n+1)}\frac{[g(E-A_{p+1,h+1})]^{n+1}}{[g(E-A_{ph})]^{n-1}} \tag{34}$$

in close analogy with (10).

2.4. Finite potential depth and bound-state constraints

The importance of the finiteness of the nuclear potential well on the pre-equilibrium decay was first pointed out by Blann (1972). He gave the correction for the two exciton configurations, which mostly influence the calculation of the pre-equilibrium nucleon emission, namely $2p1h$ and $1p1h$. Under simplifying assumptions such as the neglect of the Pauli principle, expressions can be obtained easily by the integration procedure already given in (6).

The correct derivation of the densities of states taking account of the Pauli principle should however be made using the methods of statistical physics that give (20). The hole generation function including the finiteness of the nuclear potential well is not that given by (18), but (Běták and Dobeš 1976)

$$\mathcal{Z}_h=\prod_{k=0}^{\tilde{g}E_{\mathrm{F}}-1}\left(1+xy^{k/\tilde{g}}\right)\ , \tag{35}$$

where E_{F} is the Fermi energy. This leads to

$$\mathcal{Z}_h=1+\sum_{n=1}^{\infty}x^n y^{\frac{1}{2}n(n-1)/\tilde{g}}\prod_{k=1}^{n}\frac{1-y^{E_{\mathrm{F}}-(n-k)\tilde{g}}}{(1-y^{k/\tilde{g}})}\ . \tag{36}$$

Hence, one straightforwardly finds

$$\omega(p,h,E)=g^p\tilde{g}^h\sum_{l=0}^{h}\sum_{j}C_h^{lj}(-)^l\,\Theta(E-\alpha_{ph}-lE_{\mathrm{F}}+\tilde{g}(lh-j))\times\frac{(E-A_{ph}-lE_{\mathrm{F}}+\tilde{g}(lh-j))^{n-1}}{p!h!(p+h-1)!} \tag{37}$$

with

$$\begin{aligned}C_h^{00}&=1\\ C_h^{lj}&=\sum_{\substack{0<k_1<\ldots<k_l<h\\ k_1+k_2+\ldots+k_l=j}}1\qquad\text{for } lh-\frac{1}{2}l(l-1)\le j\le\frac{1}{2}l(l+1)\\ C_h^{lj}&=0\qquad\text{for } j<\frac{1}{2}l(l+1)\text{ and for } j>lh-\frac{1}{2}l(l-1)\ .\end{aligned} \tag{38}$$

Using

$$\sum_j C_h^{lj}=\binom{h}{l} \tag{39}$$

‡ The first densities of final states with some correction due to the Pauli principle have been published by Cline (1972b). The form of the Pauli principle correction is not given there correctly.

one-fermion formulation with inclusion of some effective charge factor. In these two-fermion calculations, however, one has to fix the relative strength of different types of intranuclear transitions and there is very little information from the experimental data which could usefully distinguish among various suggested approaches.

In practice, therefore, another way is used, namely the introduction of some effective charge factor in the emission rates (different charge factors were proposed by various authors, see below).¶ This method is only an effective and approximate one, and must be understood as such. Strictly speaking, the appearance of the effective charge factor in the emission rates is an intrusive element in the pre-equilibrium calculations, as it violates the proper equilibrium limit, some physicists therefore deny its presence in the emission rates.

The philosophy behind the charge factor is probably best justified in the case of factor K introduced by the Milano group (Braga-Marcazzan *et al* 1972 for the lowest emitting state, Birattari *et al* 1973, and finally Gadioli Erba and Sona 1973, see also Gadioli and Hodgson 1992, par. 2.5.4). Let us for simplicity consider the case of a (p,n) reaction. The incoming proton may interact with either a proton or a neutron, thus creating a 3-exciton configuration of type $2\pi\tilde{\pi}$ or $\pi\nu\tilde{\nu}$. Neutron emission is impossible from the first possible 3-exciton configuration and it leads to a 2-exciton configuration of a $\pi\tilde{\nu}$ type in the residual nucleus from the latter one. The ratio of densities on the r.h.s. of (1) in this case is therefore

$$\frac{\omega(1,0,0,1,U)}{\omega(2,1,0,0,E)+\omega(1,0,1,1,E)}\,, \tag{47}$$

in reality, whereas in the one-fermion formulation of the pre-equilibrium decay one has

$$\frac{\omega(2,1,U)}{\omega(2,1,E)}\,. \tag{48}$$

If we therefore use the simpler one-fermion formulation, we have to multiply the emission rate in this case by the ratio K of both the expressions (47) and (48). This ratio may be easily evaluated assuming $g_\pi = \tilde{g}_\pi = g_\nu = \tilde{g}_\nu = g/2$ and neglecting the Pauli principle correction. This procedure yields $K_\nu(3) \approx 0.667$.

It is easy to prove that (see Gadioli and Hodgson 1992, par. 2.5.4, p. 87)

$$K_\pi + K_\nu = 2 \tag{49}$$

for every exciton number, independently of the incident projectile. In the very same way, one can also derive the charge factor K for cluster (e.g. α) emission.

The charge factor K of the Milano group clearly assumes that *i)* intranuclear transitions to more complex states are governed only by the available phase space, i.e. the matrix element of a transition $\langle|M|^2\rangle$ is independent of the initial proton/netron exciton composition and is the same for an excitation of a proton as for a neutron; *ii)* that the never-come-back approximation may be used for its derivation, i.e. that there

¶ Similarly, the charge factors were suggested and are used also in other models, e.g. in the hybrid model (Blann 1973 and 1975). Their connection to the particle-hole state densities is not emphasized and we shall not go discuss them here.

2.5. One- and two-fermion densities

So far, we have divided excitons only into particles and holes. The real nucleus, however, is not composed of nucleons of just one type, but of neutrons and protons. We should, therefore, make a further distinction in our calculations. The formulae given above are usually refered to as one-fermion ones to distinguish them from the two-fermion case, where we consider the neutrons separately from the protons §.

A straightforwad way to do this has been already introduced in the derivation of densities by Williams (1971). Let us denote the proton particles (holes) as π ($\tilde{\pi}$) and the neutron ones as ν ($\tilde{\nu}$). The total number of excitons is n is it was in the one-fermion case, $n = \pi + \tilde{\pi} + \nu + \tilde{\nu}$. The two-component density is now

$$\omega(\pi, \tilde{\pi}, \nu, \tilde{\nu}, E) = g_\pi{}^\pi \tilde{g}_\pi{}^{\tilde{\pi}} g_\nu{}^\nu \tilde{g}_\nu{}^{\tilde{\nu}} \frac{(E - B_{\pi\tilde{\pi}\nu\tilde{\nu}})^{n-1}}{\pi!\tilde{\pi}!\nu!\tilde{\nu}!(n-1)!} \tag{45}$$

with

$$B_{\pi\tilde{\pi}\nu\tilde{\nu}} = \frac{1}{4g_\pi}(\pi^2 + \pi) + \frac{1}{4\tilde{g}_\pi}(\tilde{\pi}^2 - 3\tilde{\pi}) + \frac{1}{4g_\nu}(\nu^2 + \nu) + \frac{1}{4\tilde{g}_\nu}(\tilde{\nu}^2 - 3\tilde{\nu}) \tag{46}$$

The two-fermion densities with shell corrections were published by Berger and Martinot (1974). Their calculation stems from the same grounding as that of Williams (1971) and Dobeš and Běták (1976), but it avoids the use of the Cauchy residue theorem. At modest energies (up to about 80 MeV), the results agree well with those of Williams, but at significantly higher energies and high exciton numbers, Williams formula seems to be an overestimation. The method of Berger and Martinot is used also to study the shell effects, which are found to be significant in some cases.

The complete two-fermion calculations can be found in paper by Gupta (1981), who used the two-fermion master equations to describe the equilibration in the case when neutrons and protons are distinguished. The two-fermion version of the exciton model was studied in detail by Dobeš and Běták (1983). They finalized the form of the transition rates for all occuring transitions (in the two-fermion case, there are more different transitions than only $\Delta n = \pm 2$, as it was in the one-fermion formulations) and investigated the different matrix elements governing the intranuclear transitions. Subsequent important refinements can be found in Kalbach (1986) and Herman *et al* (1989), but these papers are essentially concerned with other aspects of the densities and will be treated in detail in other sections of this review.

One can treat some selected problems of the two-fermion approach (see Běták *et al* 1974, Běták and Dobeš 1979, Avrigeanu *et al* 1997), or make calculations taking full account of two-fermion nature of the problem (Gupta 1981, Dobeš and Běták 1983, Kalbach 1984 and 1986, Jingshang 1994 and Fu 1994).‖ These calculations are very time-consuming and the agreement with the data is of the same quality as in the

§ Rather frequently, one meets the terms "one-component" and "two-component". That is unfortunately not used consistently in all papers, as some denote the two-fermion case as "four-component" and consequently the one-fermion one as "two-component".

‖ We do not consider here two-fermion calculations which go beyond the ESM model, as they will be treated in section 5.

emission. The general behaviour of the Q factor is similar to that of the K factor of the Milano group. It tends to the value of 1 at equilibrium, but there is no normalization condition of type (49) which should be valid throughout the reaction.

A closely related factor $\overline{Q}$ has been obtained from comparison of the one- and two-fermion results by Gupta (1981). His factor for $Z = N$ coincides with that of Kalbach, but differs in the other cases [+].

When using the one-fermion formulation with some form of the charge factor, we must be aware of the introduced shortcomings. Apart from that, the numerical results are nearly indistinguishable, up to a necessary renormalization of the transition matrix element $\langle |M|^2 \rangle$ by a factor of two in the case of the $\mathcal{R}$ factor with respect to the three others[*].

The proper approach would be a real two-fermion calculation, but this is very seldom perfomed, and we find only the one-fermion calculations in practice. Because of the disadvantages of all charge factors proposed until now, and especially for the purity of the model approach and to keep the well-established equilibrium limit, many calculations are performed (though in one-fermion version of the model) without any effective charge factor.

3. Densities with spin, isospin and linear momentum

3.1. Spin dependence – general

In the compound nucleus (i.e. equilibrium) case, the nucleon emission rates are (Gadioli and Hodgson 1992, p. 256)

$$\lambda_{\pi,\nu}([E,J] \xrightarrow{\epsilon} [U,S]) = \frac{1}{h}\frac{\rho(U,S)}{\rho(E,J)} \sum_{j=|S-1/2|}^{S+1/2} \sum_{l=|J-j|}^{J+j} T_{lj}(\epsilon) \ , \tag{52}$$

where the emission proceeds from a composite system of excitation energy E and spin J to the residual nucleus of an energy U and spin S, and l is the orbital momentum of the emitted particle. The densities are assumed to be factorized (see, e.g., Kikuchi and Kawai 1968 (par. 1.7, p. 32), Bohr and Mottelson 1969 (p. 155 and App. 2B), Gadioli and Hodgson 1992 (par. 2.1, p. 40))

$$\rho(E,J) = \omega(E)\frac{2J+1}{2\sqrt{2\pi}\sigma^3}\exp\left(-\frac{(J+1/2)^2}{2\sigma^2}\right) \ . \tag{53}$$

[+] One more charge factor has been suggested by Zhang (1990, 1993), yielding in practice close results to the two Q factors given above.

[*] The effective rerplacement of the two-fermion densities by the one-fermion ones also includes the assumption of the relative interaction of $\pi\pi$, $\pi\nu$, and νnu pairs. Thus Gadioli *et al* (1971) and Gupta (1981) assume equal strength of all these interactions, Cline (1972a) and Kalbach (1977) excitations proportional to N/A and Z/A, Blann *et al* (Blann and Mignerey 1972, Blann 1973) proportional to $\sigma_{\pi\nu}/\sigma_{\nu\nu}$, Dobeš and Běták (1983) use the same assumption and approximate it by a value of 3 and recently Kalbach (1995c) assumes the strength of the $\pi\pi$ and $\nu\nu$ interaction to be by a factor of 1.7 higher than that of the $\pi\nu$ one. In her former paper (Kalbach 1986), she relates the absolute values of the matrix elements in the one- and the two-fermion case, the former one being roughly one half of the latter.

is no configuration mixing and/or feeding from more complex states eventually reached earlier in the reaction, and — and what is the most serious one — *iii)* that we are fully justified to replace the single-fermion state density in the denominator of (48) by the sum of all two-fermion configurations (of the same exciton number) entering the denominator of (47), i.e. that we have the right to effectively consider also the exciton configurations which do not contribute to the emission at all in the specific case. In addition, there are *iv)* some simplifying assumptions (neglect of the Pauli principle and assuming of all g's equal) that enable the charge factor K to be expressed in a convenient closed form. All the above assumptions, however, may be a subject to discussion and none of them is of a basic importance. From the physical point of view we consider the K factor to be the clearest and best justified of all charge factors developed.

Another charge factor $\mathcal{R}$ was suggested by Cline (1972a). Its philosophy is different from that of the K factor. For illustration, we shall consider the same reaction as above, namely (p,n). If we assume that the interaction between the neutron and the proton is of the same strength, the incoming proton excites another proton with probability Z/A, and a neutron with probability N/A. In the 3-exciton state, we have 2 particles (and 1 hole). It follows that of these two particles there are $1+Z/A$ protons and N/A neutrons, on average. As we have two particles, the probability of getting a neutron is $N/(2A)$, and this is a factor which should be taken to multiply the one-fermion emission rates to account for proper charge composition. For states significantly different from the initial configuration and close to the equilibrium we assume that the charge composition of excitons which is the same (proportional to) as that of the composite system, which gives N/A for the neutron and Z/A for the proton emission. Obviously,

$$\mathcal{R}_\pi + \mathcal{R}_\nu = 1 \tag{50}$$

at each stage of the reaction. The philosophy of the $\mathcal{R}$ factor can be easily put into closed formulae for both nucleon and cluster emission, and we refer the reader to the original source in this case (Cline 1972a).

The equilibrium limit of $\mathcal{R}$ for nucleon emission in the case of $Z = N$ is clearly $\frac{1}{2}$ (and e.g. $\frac{3}{8}$ for the α-particles). This implies a disagreement with the equilibrium emission expression by the same factor introduced artificially from outside and is therefore not justified. This handicap is of much deeper nature than the objections raised against the K factor above.

To remove the improper equilibrium limit of the $\mathcal{R}$ factor, Kalbach (1977) introduced another charge factor Q. It has a proper compound nucleus limit of 1, and the explicit expression is

$$Q_\beta(p) = \left(\frac{A}{Z}\right)^{\pi_\beta} \left(\frac{A}{N}\right)^{\nu_\beta} \frac{\pi_\beta!\nu_\beta!}{p_\beta!}\mathcal{R}_\beta(p)\ , \tag{51}$$

where $\mathcal{R}$ is the charge factor introduced earlier, p_β is the mass number of the emitted particle consisting of π_β protons and ν_β neutrons. As is easily seen, the ratio of the proton to the neutron emission is amplified by a factor of N/Z with respect to that obtained using $\mathcal{R}$ charge factor for the lowest exciton state contributing to the nucleon

and the angular momentum density of pair states

$$F(j_3) = \sum_{j_1 j_2} (2j_1+1) R_1(j_1) (2j_2+1) R_1(j_2) \begin{pmatrix} j_1 & j_2 & j_3 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 . \tag{61}$$

An approximation to these formulae has been given by Bogila *et al* (1995b), which also facilitates this kind of calculations with smaller computers.

3.2. Spin cut-off

The first expression for the spin-dependent particle-hole density was already given in the original work of Williams (1971). Generally, the spin cut-off parameter is a product of the mean number of unpaired particles and holes ν_{ph} and an averaged square of the projection of the individual spins on the axis $\langle m^2 \rangle$ (Ericson 1960)

$$\sigma^2 = \nu_{ph} \langle m^2 \rangle . \tag{62}$$

According to Ericson (1960), the number of unpaired particles and holes increases with temperature t in the compound nucleus theory,

$$\nu_{ph} = g\, t . \tag{63}$$

Williams (1971) took the same relation as a starting point to express the spin cut-off in the particle-hole scheme. As the temperature at a given excitation energy E can be easily expressed via the exciton number using the state density

$$t = \left[\frac{\mathrm{d} \ln \omega(p,h,E)}{\mathrm{d}E} \right]^{-1} , \tag{64}$$

we straightforwardly get

$$\sigma_n^2 = \frac{gE - \frac{1}{8}n^2}{n-1} \langle m^2 \rangle . \tag{65}$$

The form of (65) assumes equal numbers of particles and holes. From (65) the limiting value of σ_n^2 for small n, the most significant one for the pre-equilibrium emission, is

$$\frac{gE}{n-1} \langle m^2 \rangle , \tag{66}$$

which is clearly wrong for $n = 1$ and also implies that the lower the number of particles and holes, the higher the number of unpaired particles and holes — an obvious contradiction. For these reasons the Williams formula for the spin-dependent density has never been used in real calculations.

Combining the (62) and (63) from the classical theory of the compound nucleus we get the spin cut-off in the form

$$\sigma^2 = \langle m^2 \rangle g t = g \langle m^2 \rangle \sqrt{\frac{E - E_0}{a}} , \tag{67}$$

where E_0 is the pairing correction (see below). The spin projection is $\langle m^2 \rangle \propto A^{2/3}$ with the proportionality contant varying from 0.146 (Jensen and Luttingen 1952) to 0.24 (Facchini and Saetta-Menichella 1968).

where σ is the spin cut-off parameter of the compound nucleus.

Similarly, in the pre-equilibrium case one has (Shi *et al* 1987)

$$\lambda_{\pi,\nu}([E,J,n] \xrightarrow{\epsilon} [U,S,n-1]) = \frac{1}{h}\frac{\rho(n-1,U,S)}{\rho(n,E,J)}\mathcal{R}_{\pi,\nu}(n)\sum_{j=|S-1/2|}^{S+1/2}\sum_{l=|J-j|}^{J+j} T_{lj}(\epsilon) \ . \tag{54}$$

The particle-hole state density $\rho(n,E,J)$ is

$$\rho(n,E,J) = \frac{g(gE-A_{ph})^{n-1}}{p!h!(n-1)!}R_n(J) \ , \tag{55}$$

where g is the single-particle level density, and A_{ph} is the correction term due to the Pauli principle. In the limiting case of infinite number of levels, the spin distribution is

$$R_n(J) = \frac{2J+1}{2\sqrt{2\pi}{\sigma_n}^3}\exp\left(-\frac{(J+1/2)^2}{2{\sigma_n}^2}\right) \ , \tag{56}$$

where σ_n is the spin cut-off parameter. This form is in practice assumed and used also for finite numbers of levels. This distribution is normalized so that

$$\sum_J (2J+1)R_n(J) \approx 1 \ , \tag{57}$$

so that although it contains the factor σ_n^3, the main effect of increasing σ_n^3 is to broaden the distribution rather than to decrease the overall magnitude.

We need to distinguish between the cut-off parameter for a fully equilibrated compound nucleus and that appropriate for the pre-equilibrium stage.

The densities of particle-hole states with spin are the basis for pre-equilibrium angular-momentum dependent calculations. For the transition rates, one usually assumes that they are factorized. For the $\Delta n = -2$ transitions, the densities of the final accessible states are (Obložinský 1987, Obložinský and Chadwick 1990)

$$\omega_{\mathrm{f}}^{+}(n,E,J) = \omega_{\mathrm{f}}^{+}(n,E)X_{nJ}^{\downarrow} \ , \tag{58}$$

with the energy part $\omega_{\mathrm{f}}^{+}(n,E)$ equal to the spin-independent density of the final accessible states for such a transition (e.g., (10) or (34), and the spin part is (Obložinský 1987)

$$X_{nJ}{}^{\downarrow} = \frac{1}{R_n(J)}\sum_{j_4 Q} R_1(Q)\tilde{F}(Q)R_{n-1}(j_4)\Delta(Qj_4J) \ . \tag{59}$$

The density of the inverse process is obtained via the steady-state equilibrium conditions. In (59), the R's are the spin parts of the density of states, J is the spin of the state (it is conserved during the transition) and is decomposed into the spin of the inert "core", j_4 and Q, the spin of the exciton initiating given transition (j_1 to j_3 are spins of the excitons created in such a transition). The proper relation of all spins is expressed by $\Delta(Qj_4J)$, which is 1 for $|Q-j_4| \le J \le Q+j_4$ and 0 otherwise and by functions

$$\tilde{F}(Q) = \sum_{j_3 j_5}(2j_5+1)R_1(j_5)(2j_3+1)F(j_3)\begin{pmatrix} j_5 & j_3 & Q \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}^2 , \tag{60}$$

A similar relation has been reported by Reffo and Herman (1982), Plyuiko (1978), and Gardner and Gardner (1989). The proportionality constants, when referred to explicitly, were 0.28 (Reffo and Herman) and 0.16 (Plyuiko). The original method of Reffo and Herman (1982) based on the BCS calculations, has been tailored to low exciton numbers, whereas the formula of Feshbach *et al* (1980), on the other hand, is more appropriate at $n \approx \overline{n}$.

In their subsequent papers, Herman and Reffo (1987b and 1992) used their combinatorial code to obtain so-called realistic level densities (see section 5.3). From the fit to the levels produced by their computer code, the energy dependence of the spin cut-off is weak. They found that the energy-averaged spin cut-off can be approximated by

$$\sigma_n^2 = cnA^{2/3} + 0.1A^{2/3} + 4 \tag{73}$$

with $c = 0.22$. This is approximately

$$\sigma_n^2 = cnA^{2/3} \tag{74}$$

with $c \approx 0.26$. If we want to keep at least some energy dependence, we can write

$$c = 0.24 + 0.0038E \ . \tag{75}$$

However, σ_n^2 for 2-exciton configurations of type $2p0h$ ($1\pi 1\nu$) are about 5 units higher than those for the $1p1h$ ($1\pi 1\tilde{\pi}$ or $1\nu 1\tilde{\nu}$) ones; this effect reduces to 2 units for 4-exciton configurations and is negligible for 6 excitons and more. With c given by (75),

$$\sigma_n^2 = n \cdot (0.24 + 0.0038E)A^{2/3} \ . \tag{76}$$

Herman and Reffo (1992) also studied the effects of the shell structure. Generally, the Williams formula is found to be good, though at lower energies some differences may be significant. However, collective phenomena like deformation are clearly absent in the Williams formula.

The detailed study of spin effects by Chadwick and Obložinský (1992) used

$$\sigma_n^2 = n\left(\frac{2m\varepsilon_{\rm av}}{3}\right) \ , \tag{77}$$

which in the case of the equidistant scheme gives

$$\varepsilon_{\rm av} = \frac{2p(p+1)}{ng}\frac{\rho(p+1,h,\tilde{E})}{\rho(p,h,\tilde{E})} - \frac{\tilde{g}}{n} + E_{\rm F} \ , \tag{78}$$

where

$$\tilde{E} = E - (p-h)E_{\rm F} \ . \tag{79}$$

Finally, one obtains

$$\sigma_n^2 = n \cdot 0.282A^{2/3} \ , \tag{80}$$

very close to the results of Herman and Reffo.

Bogila *et al* (1992) studied both the exciton dependence of the spin cuf-off as well as the influence of the Pauli principle. The formula (69) identifies $\langle m^2 \rangle$ with σ_s^2, the

The difficulties of the Williams suggestion (65) were overcome by Ignatyuk and Sokolov. They (Sokolov 1972, Ignatyuk 1973, Ignatyuk and Sokolov 1973 and 1978, Ignatyuk 1983) *derived* the spin-dependent density from the general expression for the statistical sum for the particles. In the small-momentum approximation (projection of total momentum of nucleus $M \ll \langle m \rangle (p+h)$) they obtained for the spin cut-off factor

$$\sigma_n^2 = \langle m^2 \rangle \frac{(p+h)^2}{p+h-1} \approx \langle m^2 \rangle (p+h) \ . \tag{68}$$

The case of Fermi particles (which is the real situation) is more complicated. Using the saddle-point method (and again for the case of small momenta), the spin cut-off parameter takes the form

$$\sigma_n^2 = \frac{2g}{\beta} \langle m^2 \rangle (1 + \mathrm{e}^{-\gamma}) \ , \tag{69}$$

where β and γ are determined by a set of coupled equations (see the original papers for details). The cut-off σ_n^2 determined in this way is an increasing function of energy, at least for small n. In some models (e.g. for the Fermi gas), it reaches its maximum $\sigma_n^2 \approx \sigma^2$ close to $n = \overline{n}$ (Ignatyuk and Sokolov 1978) and decreases thereafter. For a global understanding of pre-equilibrium phenomena, however, the differences between different models are not so essential, so we concentrate our attention to the $n < \overline{n}$ (or even $n \ll \overline{n}$) region‡.

As an approximation to the method of Ignatyuk and Sokolov (1973), Fu (1986) solved these equations. Introducing a critical temperature T_c ($= 2\Delta_0/3.5$, where Δ_0 is the ground state pairing gap (see below)) and the corresponding most probable exciton number at this temperature, n_c, and spin cut-off, $\sigma_\mathrm{c}^2 = gT_\mathrm{c}\langle m^2 \rangle$, he obtained

$$\sigma_n^2 = \sigma_\mathrm{c}^2 \ln 4 \left(\frac{n}{n_\mathrm{c}}\right) \left(\frac{E - E_\mathrm{thr}}{E}\right)^x \tag{70}$$

with x being a polynomial of the second order in $\sqrt{n/n_\mathrm{c}}$, and E_thr is the threshold energy needed for realization of given state (see Fu 1984). In the case of no pairing and equidistant levels, it coincides with the quantity α_{ph} (eq. (22)) in the Williams formula.

However, even these expressions are often considered to be inconvenient for practical use and a dependence of the type (72) is preferred.

According to Feshbach *et al* (1980), a relation similar to (70) holds also in the case of states with specified exciton number; the resulting exciton spin cut-off is proportional to the exciton number,

$$\sigma_n^2 = c \cdot t \cdot \frac{n}{\overline{n}} \ . \tag{71}$$

If we neglect the linear term in the relation between the temperature and excitation energy and take it simply in the form $E = at^2$, the exciton-dependent spin cut-off can be expressed as

$$\sigma_n^2 = 0.16 n A^{2/3} \ . \tag{72}$$

‡ The formalism of Ignatyuk and Sokolov can be applied also straightforwardly to the two-fermion case (e.g. Fu 1992).

the other hand, if isospin is conserved, there is a chance to see the $T_>$-state decay. For the $T_>$ states, the neutron emission is hindered because the residual states of the lowest isospin are forbidden. This implies a significant relative enhancement of the proton emission.

To calculate isospin effects in nuclear reactions it is therefore necessary to determine how much the isospin is conserved or mixed, and what is the isospin symmetry energy. The evaluation of the particle-hole densities taking account of isospin assumes that there is one-to-one correspondence between states of the same isospin T in isobaric nuclei,

$$\begin{aligned}\omega(p,h,E,T,T_z=T) &= \omega(p,h,E+E_{\rm sym}(T,T-1),T,T-1) \\ &= \omega(p,h,E+E_{\rm sym}(T,T-2),T,T-2) = \ldots\ , \end{aligned} \quad (86)$$

where $E_{\rm sym}$ is the symmetry energy, i.e. the excitation energy of the lowest state (of any exciton number) of isospin T in the nucleus considered. The semi-empirical mass formula yields here in a simple case of volume plus surface symmetry energy (see Kalbach 1995c)

$$E_{\rm sym}(T,T_z) = (110A^{-1} - 133A^{-4/3})(T^2 - {T_z}^2) \qquad [\text{MeV}]\ . \quad (87)$$

If we write the analogue of (24) with of isospin included (omitting now all other details, like pairing and the influence of the irregularities in the level scheme due to the shell structure etc.), the corresponding particle-hole density is

$$\omega(p,h,E,T) = \frac{g^n(E-A_{ph})^{n-1}}{p!h!(n-1)!} f_T(p,h,T)\ , \quad (88)$$

where the Pauli correction term is now (Kalbach 1995b)

$$A_{phT} \approx A_{ph} + E_{\rm sym}(T,T_z) \quad (89)$$

The function f_T in (88) is the correction factor for states with good isospin, which is taken from Kalbach (1993). If isospin is asumed to be completely mixed, the symmetry energy $E_{\rm sym}$ is zero and the correction factor f_T is unity.

3.4. Linear momentum

Mädler *et al* (1978) suggested the use of linear momentum to describe the angular distributions. The angular distributions arise from a multiplicative factor in the emission rates, i.e. the basic expression for the emission is the same as before, but with an additional angle-dependent factor (normalized to 1 after integration over angle). In practice, they use an average value for the single particle linear momentum, which is derived using very simple assumptions for a quantity dependent on the exciton number.

In their subsequent paper (Mädler and Reif 1980), they switched to the total linear momentum, calculated from the partition function (of statistical mechanics). A closed formula is given only for the limiting case of zero initial velocity, though the recipe is more general. Finally the linear momentum is decomposed into $p_{||}$ and $p_\perp$.

The last paper in their series (Mädler and Reif 1982) uses the same basic idea, but with a more consistent statistical derivation. The momentum is kept as a single (vector)

value for the single-particle states. The energy dependence of σ_s^2 is found to be (from rather general considerations)

$$\sigma_s^2(\varepsilon) = \sigma_s^2(E_\mathrm{F})\varepsilon/E_\mathrm{F} \ , \tag{81}$$

which implies for particles

$$\sigma_p^2(\varepsilon) = \langle m^2 \rangle (p + E/E_\mathrm{F}) \tag{82}$$

and

$$\sigma_h^2(\varepsilon) = \langle m^2 \rangle (h - E/E_\mathrm{F}) \tag{83}$$

for holes.

The effect of the Pauli principle is to decrease the number of exciton degrees of freedom, which becomes

$$n^* = n(1 - \alpha_{ph}/E) \ , \tag{84}$$

and therefore

$$\sigma_n^2 = n^* \langle m^2 \rangle = n(1 - \alpha_{ph}/E)\langle m^2 \rangle = n(1 - n^2/n_\mathrm{max}^2)\langle m^2 \rangle \ , \tag{85}$$

where n_max is the maximum exciton number allowed at given energy due to the Pauli principle. †

3.3. Isospin

In nuclear reactions, states with different values of isospin can be populated. The lowest lying states have the lowest possible isospin (equal to its z-component). At some higher excitation energy, a set of states with one additional unit of isospin, the $T_>$ states, will begin. The densities of both sets of states, $T_<$ and $T_>$, increase exponentially with the excitation energy, but since the $T_>$ states begin at higher energy, they will always be far less numerous than the $T_<$ states.

The implications of this for pre-equilibrium decay have been studied by Kalbach (Kalbach-Cline *et al* 1974, Kalbach 1993, Kalbach 1995b, 1995c). In proton- and ^{3}He-induced reactions on targets with isospin T_0, a fraction of $1/(2T_0 + 1)$ of the composite nucleus formation cross section forms states with the isospin $T = T_>$, whereas the remaining (usually major) fraction of the cross section, namely $2T_0/(2T_0 + 1)$ goes to the more abundant $T_<$ states. In neutron- and α-particle induced reactions, only states of the ground state isospin are populated.

The influence of these considerations on the particle emission depends on the extent of the isospin conservation in nuclear reactions. If the isospin is mixed completely, the decay will be essentially that of the very much more abundant $T_<$ states. This implies that the proton and the neutron emission proceed essentially in the same manner. On

† Anzaldo-Meneses (1997) gives a procedure for level density (based on the grand partition function) for "arbitrary periodic spectrum". It is suitable to obtain also the spin distributions (in the case of harmonic oscillator or similar "periodic" solution), but the particles are considered as **bosons** and therefore no equivalent of the Pauli principle correction is introduced.

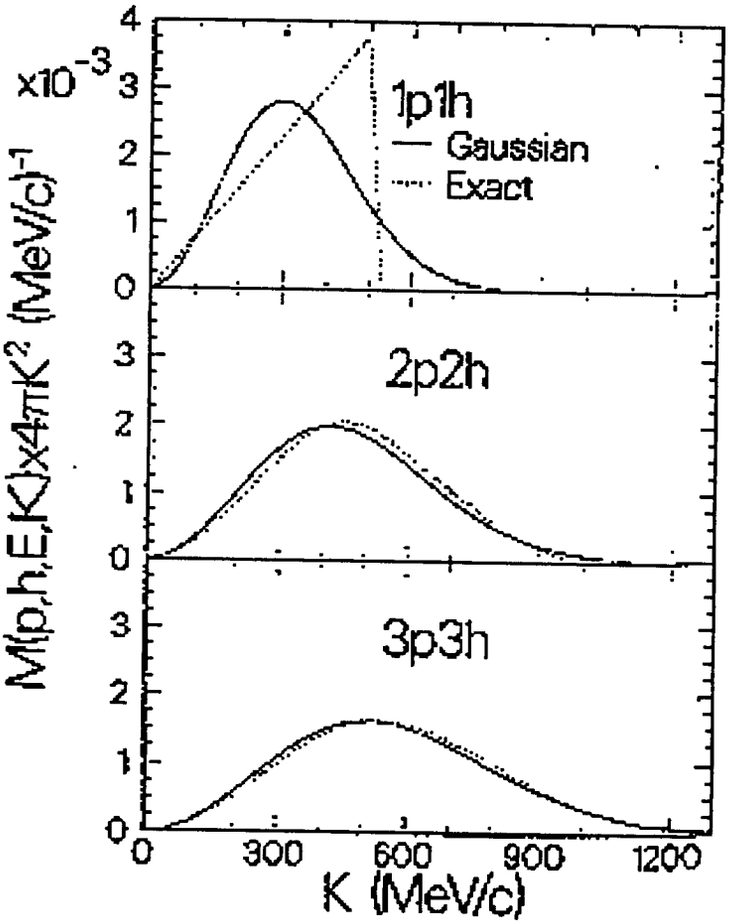

Figure 7. Comparison of the linear-momentum distribution (multipled by $4\pi K^2$) for exact and statistical Gaussian solutions of state densities with linear momentum, for an excitation energy of 5 MeV in a Fermi-gas nucleus. The accuracy of the Gaussian solution increrases rapidly with increasing n (Chadwick and Obložinský 1992).

significant only for the $1p1h$ configuration, where the real distribution is practically triangular. Everywhere else the Gaussian form of (96) can be successfully used (see figure 7). This important conclusion gives a practical and very useful recipe to calculate the angular distributions (for details, see Chadwick and Obložinský 1992). Recently, an approach to the level desnities with linear momentum and the corresponding angular distributions has been successfully reported for the Fermi gas case (Blann and Chadwick 1998).

4. Shell-structure effects within the equidistant scheme

The assumption of equidistant levels is a rather useful concept, but it is nevertheless oversimplified when compared to the real situation. A significant improvement may be achieved if we introduce at least some features of real nuclei. Probably the most profound

of them is the influence of pairing and possibly also the existence of irregularities, like the energy gaps as a manifestation of closed shells and subshells, in the single-particle level schemes.

4.1. Pairing

The formulae for the level density in the compound nucleus case are sharply increasing functions of the excitation energy. However, due to strong tendency of nucleons (of the same kind) to couple in pairs, one has to add an additional energy to break such pairs for nuclei with even number of nucleons of either type. Thus, this energy (in even-Z and/or even-N nuclei) is effectively not available to increase the density of levels. Very roughly, this pairing-energy correction is (see, e.g., Ignatyuk 1983)

$$E' = E - E^0_{\rm pair} \; , \tag{97}$$

where $E^0_{\rm pair}$ is the conventional pairing energy correction, i.e. zero for odd-odd nuclei, and a quantity close to $12A^{-1/2}$ MeV for odd-A nuclei and twice taht value for the even-even ones (Mashnik 1993).

Nowadays calculations commonly use the values of both pairing energy corrections for the neutrons and the protons together with the single particle level density as a parameter. The first extensive detailed study of that type by Gilbert and Cameron (1965) is possibly still now the most extensive (with large tables of the values) and the most frequently used one.

A slight modification of the above approach is the back-shifted Fermi-gas model. Therein, the corrections may be both positive and negative, and they are (in general) nonzero for all nuclei. Suitable sets of paremeters can be found in a paper of Dilg *et al* (1973) or more recently Iljinov *et al* (1992).

The simplest way to include the pairing correction in the particle-hole density is in exactly the same way as in the compound nucleus case, i.e. to take the pairing-energy correction (together with individual values for the single-particle level density) for each individual nucleus from the tables (Gilbert and Cameron 1965, Facchini and Saetta-Menichella 1968, Dilg *et al* 1973, Rohr 1984, Capote *et al* 1987, Iljinov *et al* 1992, Mengoni and Nakajima 1994) as the same values for all exciton numbers.

In fact, the first pre-equilibrium calculations were so rough that they reproduced only the main features of the emitted spectra and anything more detailed has been omitted. Soon afterwards, however, indications emerged that one should take account of the statistics of nuclei involved as well with details of the particle-hole level densities (Lee and Griffin 1972, Ignatyuk and Sokolov 1972, Grimes *et al* 1973). Later, the use of (at least) pairing became necessary also in these reactions.

As an improvement on the equidistant-spacing model in the vicinity of closed shells, a schematic model of shell structure was proposed for the compound nucleus case by Rosenzweig (1957). To simulate the bunching and degeneracy of nuclear states, Rosenzweig divided the equidistant spacing sequence of states into groups separated by gaps. As the final effect, the pairing energy is effectively modified by an amount which

depends on the average degeneracy of the levels in the neighbourhood of the Fermi level as well as on their average spacing therein. This idea has been later on applied to the pre-equilibrium decay by Cline (1971) and in a more consistent way by Běták (1975); the corresponding results showed the usefulness of such an approach. However, inclusion of this effect requires not well-defined quantities like the average degeneracy and the average level density near the Fermi level. In practice, the Rosenzweig effect is seldom considered in pre-equilibrium calculations.

4.2. Exciton-dependent pairing

A simple treatment of pairing corrections in the same way as in the compound-nucleus theory is useful, but for finer study one needs a more detailed approach (Ignatyuk and Sokolov 1973 and 1978). Let Δ_0 be the ground state pairing gap and $\Delta(E,n)$ the excited-state pairing gap. The ground-state pairing gap Δ_0 is readily given, e.g. Dilg *et al* (1973) use $\Delta_0 = 2\sqrt{117.6/(gA)}$ MeV; the excited state pairing gap Δ is calculated from the pairing theory (see, e.g., Moretto 1975) using Δ_0 and g. The exciton-number dependent pairing correction (which reduces the "available" excitation energy as in (97)) is (Ignatyuk and Sokolov 1973)

$$E_{\rm pair}(U,n) = \frac{g}{4}\left[\Delta_0^2 - \Delta^2(E,n)\right] \ . \tag{98}$$

The pairing correction $E^0_{\rm pair}$ for the total state density is equal to $E_{\rm pair}$ evaluated along the most probable exciton number $\overline{n}$ for $E \geq 3.15E^0_{\rm pair}$ and is (Ignatyuk and Sokolov 1973 and 1978, Fu 1986, 1992)

$$E^0_{\rm pair} = E_{\rm pair}(E,\overline{n}) = \frac{1}{4}g\Delta_0^2 \ . \tag{99}$$

For the two-fermion case of (45), the pairing corrections are sums over the neutrons and the protons, namely

$$E_{\rm pair}(E,n_\pi,n_\nu) = E_{\rm pair}(E_\pi,n_\pi) + E_{\rm pair}(E_\nu,n_\nu) \ , \tag{100}$$

formally the same as in the one-fermion case. With exception of very low excitation energies, the energy E_π can be expressed as

$$E_\pi = n_\pi E/n \ , \tag{101}$$

and similarly for E_ν. The neutron and proton parts of the spin cut-off take formally just the same expressions as in the one-component case, only with the total quantities (exciton number, excitation energy) replaced by those related to the given type of nucleons.

4.3. Active and passive holes

There is a long-standing problem about the proper treatment of the number of particles and holes in excited nuclei in connection with their statistics. Obviously, an even-even target nucleus is $0p0h$ in its ground state. Impacting nucleon can be considered on one

hand as a $1p$ excitation, but on the other hand we may equally require that all nuclei have $p = h$ throughout all their development. The problem is of very little importance for nuclei in the vicinity of closed shells (closed shell nuclei *must* be of $p = h$ at all stages, i.e. $0p0h$ in their ground state) and for the case of both projectile and ejectile being nucleons. But if we are far off these conditions (especially for reactions with clusters either on input or emitted), the whole problem emerges more profoundly.

It has been addressed already by Lee and Griffin (1972), and in much complete study by Kalbach (1975), who introduced the shell-shifted equi-spacing model with active and passive particles and/or holes. In pre-equilibrium calculations we are interested only in particles (holes) which represent degrees of freedom; that is those which have permutable excitation energy. "Passive" particles (holes) which are fixed adjacent to the Fermi level are not counted. However, they contribute to the Pauli energy. If the Fermi surface is taken to be half-way between the last filled and first vacant single-particle states in the nucleon ground-state configuration, then the ground state is always of the type $p = h = 0$ if we count also the passive excitons, and correspondingly $p = h$ for every excited state (under the same condition). Usually, the passive particles (holes) are not counted. For a composite system formed by nucleon bombardment, there is usually a passive hole at the Fermi level, so that $p > h$. Thus, the usual starting configuration of $n = 1$ can be classified as $1p + 1$ passive hole and $n = 3$ $(2p1h)$ as $2p + 1h + 1$ passive hole.

The original idea has been further developed (Kalbach 1987, 1989, 1995a and 1995b). If we keep just the leading term, the correction is (Kalbach 1987)

$$A_{\mathrm{Kalb}}(p,h) = \frac{q^2}{g} - \frac{p(p+1) + h(h+1)}{4g} \,, \tag{102}$$

where $q = \max(p,h)$. Similar question has been raised by Zhang and Yang (1988), but their results are correct only for $p = h$.

4.4. Surface effects

The weakly bound nucleons from open shells near the Fermi level are the ones that are easily excited. The finite-depth formula for the particle-hole density (41) makes it possibile to study such behaviour. Already Gmuca and Ribanský (1980) used the effective depth of the nucleon potential well as a parameter and they obtained harder spectra of emitted nucleons and therefore better agreement to the data.

The more attention we devote to the surface region, the more we need accurate level densities in that region. Kalbach (1985) introduced an additional correction near the Fermi surface. She started from the energy dependence of the Fermi gas level densities,

$$g_{\mathrm{FG}}(\varepsilon) = k_{\mathrm{F}}\sqrt{\varepsilon} = g_0\sqrt{\varepsilon/E_{\mathrm{F}}} \,, \tag{103}$$

where k_{F} is the Fermi momentum (see (107) below). Obviously, the energy ε is measured from the bottom of the potential well, and not from the Fermi energy. If the average

particle energy is $\overline{\varepsilon_p}$ (and similarly that for holes $\overline{\varepsilon_h}$), the average single-particle level density (for particles) is

$$g = g(\overline{\varepsilon_p}) = g_0\sqrt{\frac{E_\mathrm{F} + \overline{\varepsilon_p}}{E_\mathrm{F}}} \tag{104}$$

and the average single-hole level density

$$\tilde{g} = g(\overline{\varepsilon_h}) = g_0\sqrt{\frac{E_\mathrm{F} - \overline{\varepsilon_h}}{E_\mathrm{F}}} \ . \tag{105}$$

In the case of an infinite potential well (or of excitation energy E less than the Fermi energy E_F) we have

$$\overline{\varepsilon_p} = \overline{\varepsilon_h} = E/n \ , \tag{106}$$

and a more complicated estimate (due to corrections introduced in (37) or (40)) can be obtained for other potentials (Kalbach 1985, Chadwick and Obložinský 1992, Avrigeanu and Avrigeanu 1994, Blann and Chadwick 1998). These refinements improve the fit to the data.

The role of surface effects has been revitalized by studies of the imaginary part of the optical potential for pre-equilibrium processes (Sato and Yoshida 1994) and in a recent study by Avrigeanu *et al* (1996).

5. Departures from the ESM

5.1. General features

All the considerations till now have been made within the equidistant spacing scheme, though in some cases with additional terms describing some facets of the behaviour of realistic nuclei. The equidistant spacing model works suprisingly well, but it is nevertheless an oversimplification.

We get much closer to the real situation if we use some improved scheme of levels. A starting point may be the Fermi gas or harmonic oscillator model; more sophisticated approaches benefit from a realistic scheme of single-particle levels. In principle, the general method of Dobeš and Běták (1976) (see Chapter 2.2) allows for an arbitrary level scheme, though only the ESM was considered originally. In some cases, the particle-hole state densities and particle-hole densities of final accessible states can be obtained as closed-form expressions. More often, however, it is not feasible to perform analytically all the integrations sketched in Chapter 2.2 and this difficulty is more pronounced in the case of the accessible final states than in simple calculations of particle-hole state densities. In such a case, only numerical results of combinatorial methods are generally available.

5.2. Soluble models (Fermi-gas and harmonic oscillator)

There are two popular and convenient models of the level schemes of the atomic nuclei, namely the Fermi gas and the harmonic oscillator models.

The Fermi-gas model assumes a square well potential, and in the ground state all states with momenta less than the Fermi momentum k_F are filled. The Fermi momentum can be calculated from the value of the Fermi energy,

$$k_F = \frac{3A}{2E_F^{3/2}} \tag{107}$$

or established using the data on nucleon scattering. The single-particle level density is no longer constant, but energy-dependent (see (103) and also figure 8).

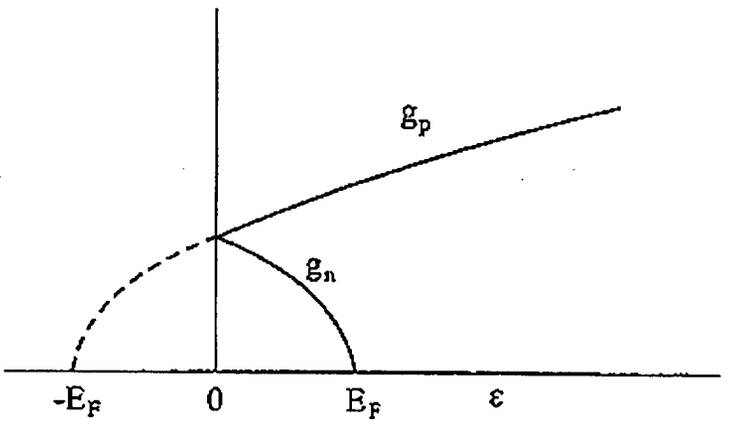

Figure 8. Energy variation of the Fermi-gas single-particle and single-hole level densities.

The harmonic oscillator is seldom used in its clear (infinite) form, but is usually truncated as to reproduce either the Fermi energy or the nuclear radius. For the states with energy less than the Fermi energy (or than the energy determining the nuclear radius), the energy dependence of the single-particle states is (Gadioli *et al* 1973)

$$g_{HO}(\varepsilon) = k_{HO}\varepsilon^2 \tag{108}$$

with

$$k_{HO} = \frac{3A}{E_F^3} \,. \tag{109}$$

In either case, one can couple the conventional value of the single-particle level density to the energy-dependent one in different ways. The usual one equates the single-particle level density of the ESM to the energy-dependent one at the Fermi energy. Alternatively, one can introduce some averaging constraint, such as, e.g., a requirement that g of the ESM and the Fermi energy E_F are related so as to place just A nucleons below the Fermi surface.

Gadioli *et al* (1973) derived the densities of states (or the transition rates) for the two cases just mentioned. In the case of the Fermi gas model, the rates of transitions induced by either a particle or a hole are easily evaluated and their leading term (when

a similar expansion is used for product of the exciton velocity, nuclear matter density and the mean cross section (see Kikuchi and Kawai 1968) is (Gadioli *et al* 1973)

$$\lambda_{1p}(\varepsilon) = \lambda_{1h}(\varepsilon) \propto (\varepsilon - E_F)^2 \ . \tag{110}$$

In the case of the (truncanted) harmonic oscillator, however, the results have been given only in graphical form.

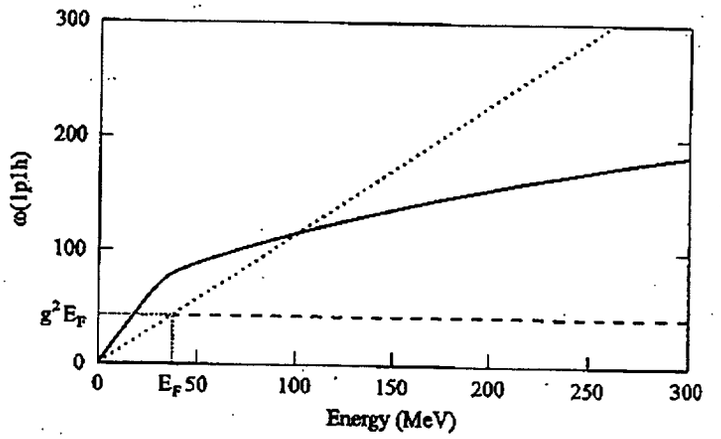

Figure 9. Comparion of the energy dependences of the state density within the ESM without and with the correction for the finitness of the potential well and the Fermi-gas one for the $1p1h$ state. The normalization of the single-particle level density in both cases is done so that the potential well accommodates just A nucleons from the bottom to the Fermi energy (Smith 1989).

The dependence (103) was a starting point for analytical studies of the Fermi gas level densities by Smith (1989), who compared the original Williams formula (21), the formula corrected for the finiteness of the nuclear potential well (40) and the results from the Fermi gas calculations (figure 9). Similarly, he calculated the ratio of densities directly entering the emission rates. For the two most important exciton configurations, the results are shown in figure 10.

Ghosh *et al* (1983) calculated semiclassicaly $1p1h$ and $2p2h$ level densities within the Thomas-Fermi approach. Generally, their results are numerical only, but in the case of harmonic oscillator, the analytical formulae are also presented. The method is applied in papers by Blin *et al* (1984 and 1986), who used the exact scheme of levels of the three-dimensional harmonic oscillator in the Thomas-Fermi approach‡. Their n-particle density is zero at nE_F (measured from the bottom of the well) and sharply increases above. In their latter work (1986), they also present calculations in Woods-Saxon potential for the simplest case of $1p1h$, and in the case of harmonic oscillator they also add possible temperature dependence. The subsequent paper (Hasse 1985/86) adds a possible angular momentum dependence, though evaluated only for a model, not for a real nucleus. Interestingly, the $1p1h$ density of non-zero angular momentum ($l = 10\hbar$

‡ The results of Blin *et al* have been further developed in papers by Zhang and Wu (1992a and 1992b)

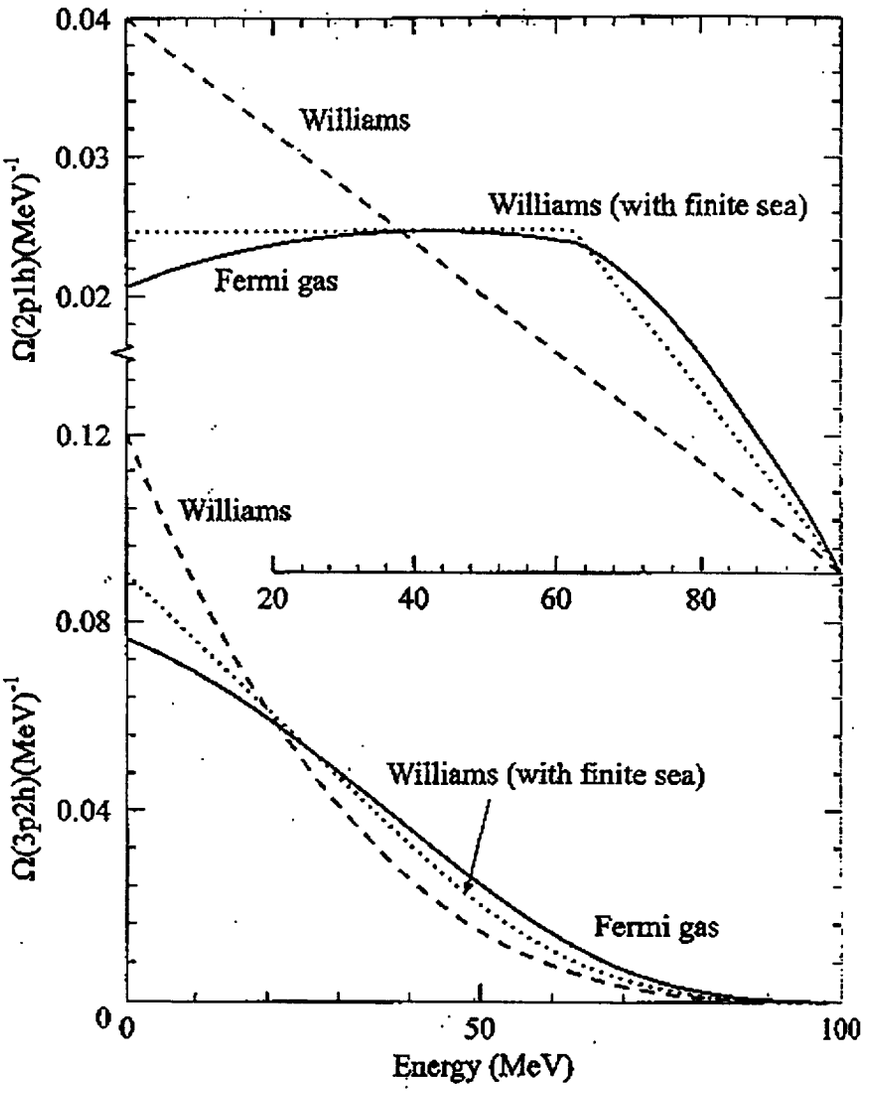

Figure 10. Ratio of densities directly entering the emission rates, namely $\Omega(p,h,E,\varepsilon) = \omega(1p,0h,\varepsilon)\omega(p-1,h,E-\varepsilon)/\omega(p,h,E)$ for $E = 100$ MeV and the two most important exciton configurations, namely $2p1h$ and $3p2h$ (Smith 1989).

for a hypothetical system of 200 particles) is composed of three parts: zero up to 25 MeV, a linear increase between 25 and 70 MeV, and constant thereafter.

Shlomo (1992) calculated the energy dependence of $g(\varepsilon)$ using the semiclassical Thomas-Fermi and local-density approximations and hence evaluated the particle-hole level density for a finite potential well (Shlomo *et al* 1995, Bogila *et al* 1995a, 1996a and 1996b). Partially, they repeat the results of Běták and Dobeš (1976), but by expanding the energy dependence of g near the Fermi surface in the power series,

$$g(\varepsilon) \approx g_F + g'_F(\varepsilon - \varepsilon_F) + \frac{1}{2}g''_F(\varepsilon - \varepsilon_F)^2 \qquad (111)$$

(the idea is very close to the approach of Kalbach (1985)) they obtain soluble non-ESM

cases. In practice, they use two potentials, namely the infinite square-well and the finite trapezoidal ($V = V_0$ for $r < R - D$, and $V = 0$ for $r > R + D$) potentials. In the former case,

$$g^{\mathrm{SQ}}(\varepsilon) \propto \sqrt{\varepsilon - V_0}\ , \tag{112}$$

(V_0 is negative), and the trapezoidal density is corrected by subtracting the free-gas level density for $\varepsilon > 0$. The final form in the latter case is

$$g^{\mathrm{TR}}(\varepsilon) \propto \sqrt{\varepsilon - V_0} \times \text{polynomial of 3rd degree in } (\varepsilon - V_0)\ . \tag{113}$$

The ratio of the "new" density of $p - h$ states with respect to the equidistant-spaciong model increases with the excitation energy (up to a factor of 4 to 10 in the case of selected exciton configurations (e.g. $4p0h$ in ^{40}Ca for the infinite trapezoidal potential at an excitation of 50 MeV). In the case of a square well, and also for a finite trapozoidal potential, the deviations are much smaller, and even decrease at higher excitations. Their latest paper in the series (Shlomo *et al* 1997) prepares the way to more general cases, as it gives the single-particle level density for the deformed potential well and also of the energy-dependent potential well, and the particle-hole level densities obtained by this group may be expected in their next publication.

Sato and Yoshida (1994) calculated the densities within the Thomas-Fermi approach; they are close to the results obtained within the ESM with the energy constraints (44).

Avrigeanu *et al* (1997) in a similar way evaluated the particle-hole state densities for several types of potentials (Woods-Saxon, Thomas Fermi with both infinite and finite potentials) and made some comparions with formulae of type (44) and for the classical Fermi gas.

5.3. Realistic densities

The attempt to combine a realistic level scheme of single-particle levels so as to obtain densities of excited states can be traced to the paper by Hillman and Grover (1969), but they did not specify the number of excited particles and holes. This has been achieved only in the pair of papers of Blann *et al* (Albrecht and Blann 1973, Williams *et al* 1973) and these particle-hole level densities became known as "realistic" particle-hole level densities. This term is not exact, however: the densities are really based on the realistic single-particle level scheme, but they usually do not include any perturbation of this level scheme due to nucleon interactions or any dependence of the single-particle level scheme on the excitation energy.

The papers of Blann *et al* , as well as those of their followers, (Zhivopistsev *et al* 1973, 1982 and 1987, Burtebaev *et al* 1978 and 1982, Galkin *et al* 1979, 1980 and 1984, Blekhman *et al* 1983) started with a level scheme of Woods-Saxon (Woods and Saxon 1954), Nilsson (1955), Seeger and Perisho (1967) or Seeger and Howard (1975), and by folding unperturbed states, they obtained densities with specified particle and hole numbers.

Originally, this (very time-consuming) process has been applied to get just the particle-hole state densities. Later on, special emphasis has been devoted to the role of the important shells for given nucleus (Grimes *et al* 1973a and 1976, Scobel *et al* 1984, Běták and Polhorský 1988), including the degeneracy of the levels and also different filling of (sub-)shells for various isotopes, and their influence on the spectra calculations has been studied (Blann *et al* 1985 and 1992). Somewhat off this main stream of interest, but using similar methods, have been the studies of the excitation energy behaviour of the total density expressed via the so-called realistic single-particle level densities (Mustafa *et al* 1992).

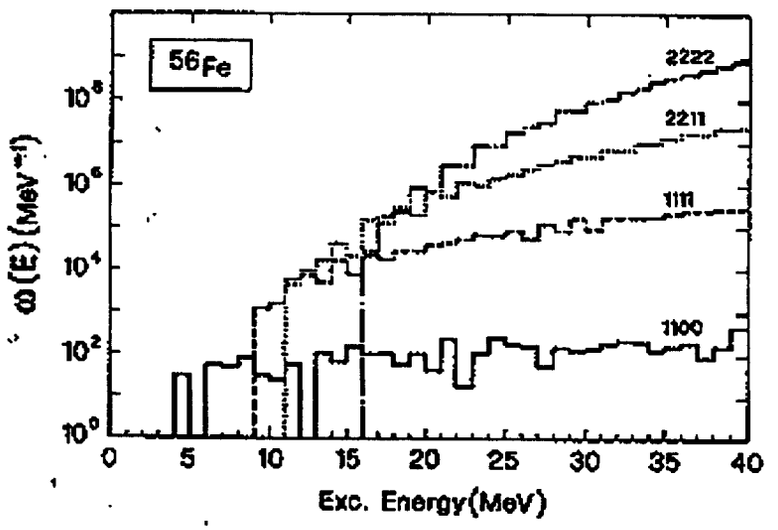

Figure 11. Energy dependence of a given exciton configuration, as obtained for ^{56}Fe using the realistic single-particle level scheme (with interactions switched on) (Herman and Reffo 1987b)

More recently, with the availability of better computers, it is possible to distiguish between neutrons and the protons and to include other quantum numbers, such as spin and parity (Orlik 1980, Herman and Reffo 1987a). The code ICAR of Herman and Reffo is the first one (and to our knowledge the only one) which in addition to pure combinatorics allows for a distortion of the single-particle level scheme due to the interaction of the excited nucleons within the BCS theory. An example for the case of ^{56}Fe nucleus, showing several low-exciton configurations, is in figure 11. Its predictions have been applied to nuclear reactions on both spherical and deformed nuclei (Herman and Reffo 1987b, 1988 and 1992)§. The parity distribution has been also considered and the value of the spin cut-off parameter (see section 3.2) obtained. At lower excitations, prevailing parity of the valence nucleons dominates and thus obviously varies from one nucleus to another, and with increasing excitation energy (above 30 or 40 MeV) the

§ Herman's and Reffo's code has been further updated by Capote *et al* (1994, 1997 and RIPL 1997/8).

ratio of positive- and negative-parity states is close to 1:1||.

A somewhat related is the paper of Giardina *et al* (1992), who analyzed an extensive set of (real) single-particle levels (for more than 4000 isotopes), and fitted them (for each isotope separately) to the form

$$g(E) = 2aE + b \tag{114}$$

above the Fermi level, and

$$\tilde{g}(E) = (c^2 + 4dE)^{-1/2} \tag{115}$$

below it, with a, b, c and d as fitting constants, and E the excitation energy. A similar dependence, namely

$$g(\varepsilon) = 2a\varepsilon + b \tag{116}$$

has been adopted for the single-particle states (ε is now the particle energy, and not that of a nucleus). They also suggested some procedure to yield a two-fermion particle-hole density within their approach. Their method, however, strongly depends on the fitting constants and has not been followed up further.

A complementary approach to the realistic particle-hole densities has been initiated by Jacquemin and Kataria (1986) and subsequently developed in a series of papers by Sato, Yoshida *et al* (Nishioka *et al* 1988b, Sato *et al* 1987, 1988, 1989, 1991, 1992a and 1992b, Yoshida *et al* 1995). It has been applied only to double magic nuclei, as ^{40}Ca and ^{208}Pb, where the interaction plays a dominant role. They studied the second moments for nuclear Hamiltonian matrix elements, and built the particle-hole density therefrom. In the case of the weak coupling limit, their results coincide with those of the ESM with both finite-well and bound-state constraints (44), whereas substantially different behaviour is obtained in the case of strong coupling limit. A difference between so-called true level densities (obtained straightforwardly) and the effective ones (which play their role in the transition rates) is marked, and also the effects of particle emission on level density has been studied (and found to be negligible). The first papers of the series give just the particle-hole level densities, while the particle-hole level densities with specified spin and parity are reported in the 1991 paper (Sato *et al* 1991). The influence of details of the interaction has been studied. A somewhat similar approach is also contained in a recent paper by Koning and Chadwick (1997), who distinguish between the "pure" density ω and that containing the interactions $\hat{\omega}$, always consistently in a two-fermion case. The contribution of the $1p1h$ configurations to the continuum is approximated by a Gaussian or alternatively by the "realistic calculations", and the resulting calculated spectra in this way are presented.

|| The importance of parity distributions for pre-equilibrium particle decay has been pointed out by Antalík (1982) and for the γ emission and in a more physically-grounded way by Obložinský (1990). A feasible way to obtain reasonable parity dependence of the particle-hole states has been established by Cerf (1993a). For more details of the method, see the next subsection.

5.4. A fast approach to realistic densities

The combinatorial approach to the realistic level densities is extremely time consuming, especially at higher exciton numbers, where the number of states grow very rapidly. In fact, one does not need all the densities of the exciton states expressed as realistic ones — the main influence comes from the relatively low exciton numbers. Even these became too time-consuming to be calculated every time. A reasonable way out has been developed in a series of papers by Cerf and Pichon (Cerf 1993a, 1993b, 1994a, 1994b, Cerf *et al* 1994, Pichon 1994), who invented a reasonable sampling of states using the Monte Carlo method, thus significantly reducing the computer time. Obviously, this introduces some statistical errors, but they can be easily controlled and even if one keeps them reasonably small, the computer time is reduced by several orders compared with the classical (combinatorial) approach. This enables the introduction of realistic densities into a whole set of standard calculations, though only in numerical form, so that the general trends are not so obvious as they are using closed formulae. In fact, the densities reported above are not those of particle-hole states, but rather the total densities. The method is nevertheless very suitable to be used also for specified particle and hole numbers (also including other quantum numbers, such as the distinction between the neutrons and the protons, and spin and parity). The use of the method is shown in figure 12, which shows the energy dependence of the average exciton number $\overline{n}$ on the excitation energy calculated for the ^{156}Eu nucleus (Cerf 1994c).

The distribution of parities is one of topics which could not be included straightforwardly within the ESM (usually, one supposes equal parities there), and has to be solved using realistic level densities. It has been illustrated for several configurations by Herman and Reffo (1992). At higher excitations (above 30 or 40 MeV), the parity distribution tends to half-to-half, but significant differences can be found at lower energies. These dependences are characteristic of the particular nucleus, and no general formula has been suggested. The statistical method of Darwin and Fowler in conjunction with the realistic level scheme used by Cerf (1993a) gives a usable procedure for the general case. It also includes a new quantity, the so-called *threshold for parity inversion*, which is the minimum energy needed to change the parity with respect to the ground state (i.e. all states with the excitation energy below this threshold keep the parity of the ground state).

The methods of Cerf *et al* have been further developed by Goriely (1996a, 1996b and 1997) and Hilaire (1997a and 1997b). The papers of Goriely deal only with the total densities of states, but their generalization to the particle-hole state densities is feasible. He uses the statistical model to improve the analytical approximations to the spin-dependent level density by a new method of estimating the shell and pairing effects. The influence of the shell structure on the excitation energy dependence (i.e. on the temperature) and on the spin distribution is evaluated using a semi-classical approximation to the single-particle state density, and the pairing effects are obtained analytically using the BCS theory. The new analytical formula found in this way agrees

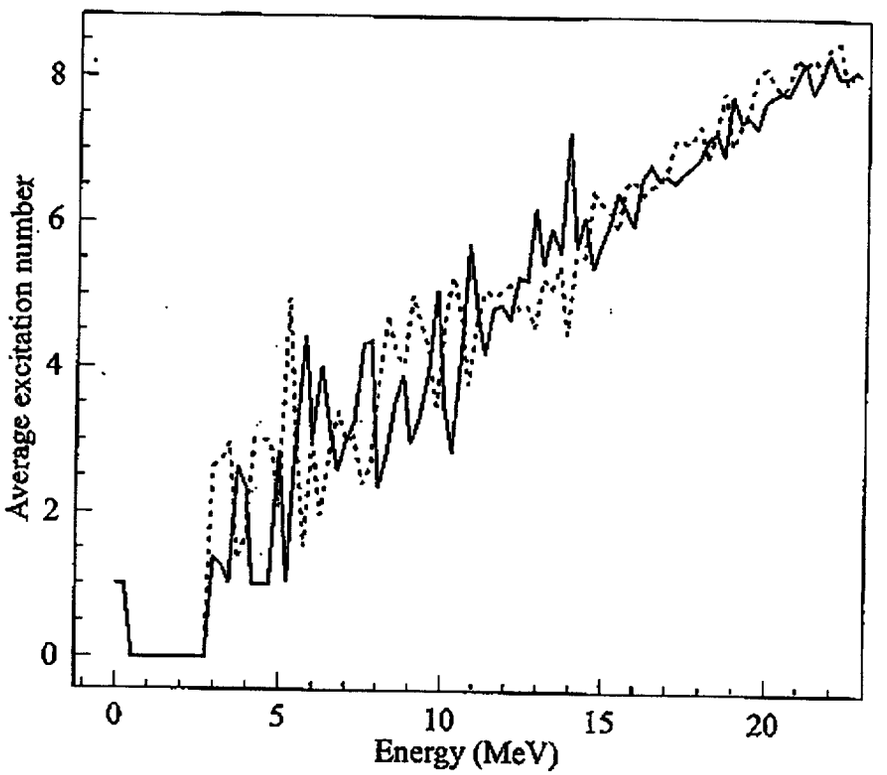

Figure 12. Energy dependence of the mean exciton number as obtained from the fast approach to realistic densities for ^{156}Eu (Cerf 1994c).

well with the exact numerical shell-model calculations.

In a similar way, Hilaire (1997a and 1997b) used a combinatorial method to calculate exact particle-hole and total level densities. This was first used to derive a generalization of Obložinský's formula (44) for particle-hole state densities within the context of the ESM. The comparison comes out to be excellent, apart from some deviations extremely close to $E \approx \alpha_{ph}$ (see figure 13). The method is then applied to the case of realistic single-particle level densities calculated from the Hartree-Fock-Bogoliubov equations with Gogny forces. The collective enhancement of the level denities due to the vibrational and rotational states is included and a detailed comparison is made with the experimental level densities¶. The energy dependence of the level density parameter is found to be well represented by the formula of Ignatyuk *et al* (1975) provided this is modified so that the asymptotic level density parameter behaves as $A/13$ and not $A/8$ and to take account of the slower vanishing of the shell effects with increasing energy.

¶ There is already an earlier attempt to include collective states into the particle-hole level densities, namely that of Deb and Zhivopistsev (1985). Though it improved the fit of the calculations to the data, the method used therein was not well justified. In the case of compound nucleus, the influence of collective states has been studied by Plyuiko *et al* (Ezhov and Plyuiko 1993, Plujko 1997)

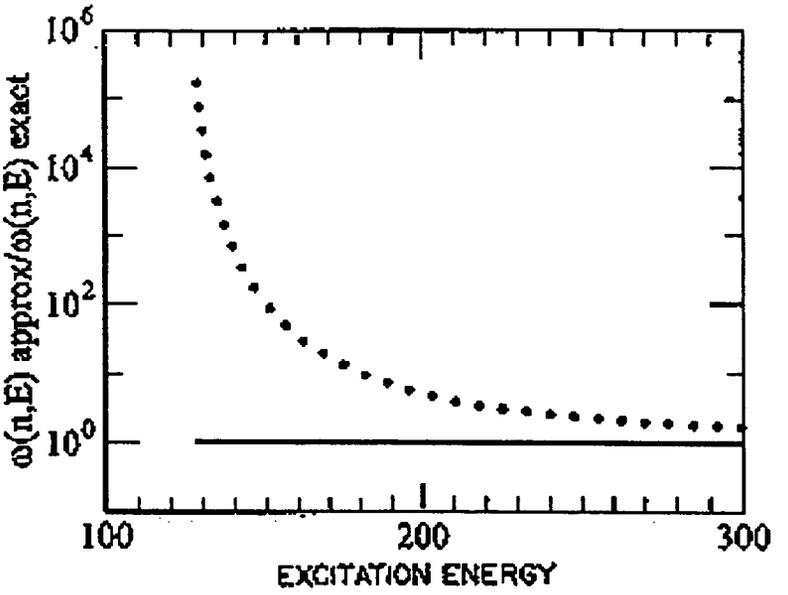

Figure 13. Comparison of the energy dependence of the exact state densities (within the ESM) and those from (42) for the 32-exciton configuration ($16p16h$) in ^{208}Pb. One must bear in mind that the mean exciton number $\overline{n}$ corresponding to the maximal energy considered is $\overline{n} = 20$ for energy of 350 (dimensionless units, which are used throughout this figure) and $\overline{n} = 13$ for the energy of 150 (dimensionless units), which is much in excess of the exciton configuration considered here (Hilaire 1997a).

6. Conclusions

Particle-hole state and level densities are extensively used in calculations of the cross-sections of pre-equilibrium reactions (Gadioli and Hodgson, 1992). These calculations require a knowledge of the contributiong reaction mechanisms, the optical potentials that give the wavefunctions of the incoming and outgoing particles, and the effective nucleon-nucleon interaction as well as the particle-hole densities. There is consequently considerable flexibility in the calculations, and this has often made it possible to obtain good fits to some rather limited data with very simplified assumptions and an inadequate understanding of the reaction mechanisms. Many pre-equilibrium reactions have contributions from the multistep compound and the multistep direct reaction mechanisms, and the cross-setions may also have substantial collective and compound nucleus components. Omission of one or more of these processes vitiated many early analyses, but now the way to calculate them is quite well understood (Demetriou *et al* 1996, Marcinkowski *et al* 1997, Hodgson 1995, 1996a and 1996b).

For accurate calculations it is thus now necessary to refine the other parts of the calculation. The optical potentials are usually taken from existing analyses of elastic scattering at similar energies by the same or nearby nuclei. It is however desirable to check their validity by analyzing data for several reactions over a range of incident

energies. The effective nucleon-nucleon interaction is usually taken to have the simple zero-range or Yukawa form, but now the inclusion of analyzing power data requires a spin-dependent interaction. Work is in progress using phenomenological forms and also effective interactions obtained from more fundamental G-matrix formalism (Lindsay 1995, Richter 1997).

In most analyses the particle-hole level densities have been taken to have the Williams form (21) with refinements by Běták and Dobeš (1976) and Obložinský (1986). The recent work of Koning and Chadwick (1997) and that of Marcinkowski *et al* (1997) show that it is necessary to use a two-fermion theory, distinguishing between neutrons and protons. More sophisticated formulae will only be used if it is no longer possible to fit the data with the simpler ones, or if it is possible to use the better formula without too much difficulty. At present, the simpler formulae are adequate for most cases. Though in some special cases, for example calculations of reactions to discrete states, this is no longer the case (e.g. transitions leading to discrete states calculated within the pre-equilibrium theory). The better formulae frequently require very large computing times, but this difficulty may be overcome by using the techniques of Cerf (1993a, 1993b, 1994a, 1994b and Cerf *et al* 1994), Goriely (1996a, 1996b and 1997) and Hilaire (1997a, 1997b). This will make it possible to study the usefulness of the more sophisticated expressions for particle-hole state and level densities.

Recently, the International Atomic Energy Agency has prepared a document (RIPL 1997/8), which recommneds suitable set of parameters and approaches to be used for nuclear reaction calculations, including also a short section on the particle-hole level densities. It includes the tools for comparison of various expressions used for the particle-hole level densities in pre-equilibrium calculations and also a possibility to calculate them in a microscopic theory using a realistic scheme of single-particle levels.

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