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ABSTRACT

Thermal equilibrium among hadrons in a nuclear fireball is assumed. Pion and nucleon total and differential cross sections are calculated. The pion differential cross section shows structure due to the decay of resonances. For neutron rich projectile-target combinations there is a net conversion of neutrons to protons.

I. INTRODUCTION

In a recent paper[1] it was found that the gross features of the proton inclusive spectra in relativistic heavy ion collisions could be described by a nuclear fireball model. The model has three essential ingredients: geometry, to calculate the number of nucleons in the fireball; kinematics, to calculate the velocity of the fireball and its excitation energy; and thermodynamics, to describe the decay of the fireball. At 250 and 400 MeV/nucleon beam energy pions and resonances were not included in the fireball, and the nucleons were given a Maxwell distribution. At 2100 MeV/nucleon two fireballs were assumed and an effective temperature was calculated using the Hagedorn mass spectrum.[2] However the fit to the data is questionable.[3]

A recent paper[4] has shown that it may be possible to test the geometric aspect of the nuclear fireball model by observing the bremsstrahlung emitted in the collision. In this paper the implications of the thermodynamic aspect of the model are more fully explored. Only the one-fireball model will be considered. The extension of the method to two fireballs will be obvious.

II. THERMODYNAMICS OF THE FIREBALL

In the nuclear fireball model the projectile and target nuclei are taken to be uniform density spheres. For a given impact parameter the fireball consists of those nucleons whose extrapolated straight line trajectories intersect the other nucleus. The baryon number and charge of the fireball are thus determined by geometry. The mass and velocity of the fireball are then determined uniquely by kinematics.

We assume that enough interactions occur during the initial formation and subsequent expansion of the fireball that thermal equilibrium occurs among all the hadrons consistent with the conservation laws.

The assumptions made here are essentially the same as those in the work of Chapline, Johnson, Teller and Weiss.[5] However there are three significant differences between their work and this one. The first is that they consider only central collisions, whereas we shall integrate over impact parameters to obtain cross sections. The other differences will be pointed out as we go along.

As discussed by Chapline et al., a prerequisite for the establishment of a thermal hadronic system is that the mean free path ℓ_i of any particle be much less than the size R of the system. $\ell_i^{-1} = \sum_j \sigma_{ij}\rho_j$ where σ_{ij} is the cross section and ρ_j the number density. Typically $\sigma \approx$ 25-100 mb for nucleon-nucleon and pion-nucleon scattering at these energies. Nuclear fireball densities may exceed twice normal nuclear density (0.15 fm^{-3}). The condition $R \gg \ell$ leads to $A^{1/3} \gg 0.5 \sim 2.0$, where A is the number of nucleons in the fireball. $A \gtrsim 50$ may suffice. Thus the model will break down for peripheral collisions. To judge the applicability of the model to a given projectile-target combination we might evaluate A at the most heavily weighted impact parameter.[1]

A second point of departure from Chapline et al. is the choice of the volume of the fireball when it decays. They choose it so that the baryon density is twice normal nuclear density, independent of the number of mesons produced. However, Pomeranchuk[6] has observed that one should not choose a fireball volume independent of the number of hadrons it contains and then expect to use noninteracting gas formulae to describe them. If the hadron density is high there will be many interactions. As the system expands its density will decrease and so will the number of interactions. Due to the short range nature of the strong interactions one might expect that some critical density ρ_c will be reached after which most of the particles effectively cease to interact. This hadron density is expected to be of the order of $\left(\frac{4}{3}\pi m_\pi^3\right)^{-1}$. Of course the use of a critical density is only an approximation but it makes the problem much more tractable. For a further discussion of this point see the review by Feinberg.[7]

In the spirit of Hagedorn we will use noninteracting gas formulae to describe each hadron type which we expect to be a statistically significant component of the fireball when it decays. Hagedorn's mass spectrum is not directly applicable here because, for instance, the number of protons and neutrons will not in general be equal. The distribution of particles of type i in momentum space is:

$$\frac{d^2N_i}{dp^3} = \frac{(2S_i+1)V}{(2\pi)^3}\left(\exp\left(\frac{\sqrt{p^2+m_i^2} - \mu_i}{T}\right) \pm 1\right)^{-1} \tag{1}$$

$$\rightarrow \frac{(2S_i+1)V}{(2\pi)^3} \exp\left(\frac{\mu_i - m_i}{T}\right)\exp\left(\frac{-p^2}{2mT}\right) . \tag{1'}$$

The arrow indicates the classical limit. Here S_i is the spin, $m_i \neq 0$ is the mass and μ_i the chemical potential. V is the volume of the fireball at the instant of decay and $\pm$ refers to fermion/boson. We use $\hbar = c = k = 1$. The total number of particles of type i is[8] :

$$N_i = \frac{(2S_i+1)V\, m_i^2\, T}{2\pi^2} \sum_{n=1}^{\infty} \frac{(\mp)^{n+1}}{n} \exp\left(\frac{n\mu_i}{T}\right) K_2\left(\frac{nm_i}{T}\right) \tag{2}$$

$$\rightarrow (2S_i+1)V\left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right) \quad . \tag{2'}$$

The average energy E_i, including rest mass, of a particle of type i is[8]:

$$N_i E_i = \frac{(2S_i+1)V\, m_i^3\, T}{2\pi^2} \sum_{n=1}^{\infty} \frac{(\mp)^{n+1}}{n} \exp\left(\frac{n\mu_i}{T}\right)\left(K_1\left(\frac{nm_i}{T}\right) + \frac{3T}{nm_i} K_2\left(\frac{nm_i}{T}\right)\right) \tag{3}$$

$$\rightarrow N_i (m_i + \frac{3}{2} T) \ ; \tag{3'}$$

the K_n are Bessel functions.

The statement of thermal equilibrium implies certain relations among the chemical potentials. For instance, $p + n \leftrightarrow n + n + \pi^+$ implies that $\mu_{\pi^+} = \mu_p - \mu_n$. Chapline et al. do not consider chemical potentials.

For a given impact parameter we can calculate the mass M, charge Q and baryon number B of the fireball. The unknown quantities which are to be determined are the proton and neutron chemical potentials, the temperature and the volume at the critical density. They are found by solving the equations for the conservation of energy, charge and baryon number, and the constraint that the number density be ρ_c:

$$M = \sum_i N_i E_i \ , \tag{4}$$

$$Q = \sum_i N_i Q_i \ , \tag{5}$$

$$B = \sum_i N_i B_i \ , \tag{6}$$

$$\rho_c = \frac{1}{V} \sum_i N_i \quad . \tag{7}$$

Strange particle production will not be considered in this paper but it is clear how to include it.

III. DEPENDENCE ON THE CRITICAL DENSITY

To examine the dependence on the value chosen for the critical density consider for simplicity the collision of two identical nuclei, with a proton/neutron ratio of one, in the CM. Assume that only pion creation is significant. Then $\mu_p = \mu_n \equiv \mu_N$ and $\mu_{\pi+} = \mu_{\pi o} = \mu_{\pi-} = 0$. To solve eqs. (4) - (7) for μ_N, T and V it is convenient to eliminate V by dividing (4) by (6) and (5) by (6). Noting that $N_p = N_n \equiv \frac{1}{2} N_N$ and $N_{\pi+} = N_{\pi o} = N_{\pi-} \equiv \frac{1}{3} N_\pi$ the equation for Q/B is an identity. In the classical approximation μ_N can be eliminated from the remaining pair of equations to give an equation for T:

$$\rho_c (M/B - m_N - \tfrac{3}{2} T) = 3(m_\pi T/2\pi)^{3/2} \exp(-m_\pi/T)$$

$$\times \ (M/B - m_N + m_\pi) \quad . \tag{8}$$

We find that if $\rho_c \to 0$ then $T \to 0$ and $N_\pi/N_N \to (M/B - m_N)/m_\pi$. This means that if the system (fireball) expands indefinitely while still in thermal equilibrium all the energy not tied up in nucleon masses will be converted to pions. At the other extreme if $\rho_c \to \infty$ then $T \to \frac{2}{3}$ $(M/B - m_N)$ and $N_\pi/N_N \to 0$. This means that all the excitation energy is converted to nucleon thermal motion.

Figure 1 displays the temperature, as a function of the critical density at several beam energies, from a computer solution of the exact equations including relativity and fermion/boson statistics. Figure 2 shows the corresponding pion/nucleon ratio, which is also equal to the ratio of the single-particle inclusive cross sections. (Contributions from evaporation of target and projectile spectator pieces, if any, are neglected.) Note that the temperature increases and the number of pions decreases as the critical density is increased. As the beam energy increases the dependence of T and N_π/N_N on ρ_c becomes stronger. This is expected because if the beam energy was low enough no pions would be created and the temperature would be independent of the critical density.

Also shown in the figures are the values of the density for nuclear matter, normal nuclei and pionic clusters.[9] We would expect that $0.04 \lesssim \rho_c(\mathrm{fm}^{-3}) \lesssim 0.12$. The value $\rho_c = 0.05$ fm^{-3} will be used in the remainder of the paper.

IV. INCLUSION OF RESONANCES

After the pion the next significant component of the fireball should be the Δ(1232). To examine its influence consider again the example of the preceding section. In addition to the chemical potentials already mentioned there are

$$\mu_{\Delta^{++}} = \mu_{\Delta^{+}} = \mu_{\Delta^{o}} = \mu_{\Delta^{-}} = \mu_N \qquad (9)$$

and so

$$N_{\Delta^{++}} = N_{\Delta^{+}} = N_{\Delta o} = N_{\Delta^{-}} \equiv \frac{1}{4} N_\Delta \quad . \qquad (10)$$

Again eliminate V by dividing (4) by (6) and (5) by (6). The equation involving Q/B is an identity. The remaining equations must be solved numerically.

Figure 3 shows the temperature as a function of beam energy both with and without the delta. Including the Δ lowers the temperature by 5 MeV or less. Figure 4 shows the ratios of the number of pions and deltas to the baryon number of the fireball. These ratios are independent of impact parameter. When the fireball reaches critical density we expect the particles to effectively cease to interact, and so the deltas decay into pions plus nucleons. The number of observed pions is thus $N_\pi + N_\Delta$. The ratio of this number to the baryon number is also the ratio of the single particle inclusive cross sections, σ_π/σ_N. Note that this ratio is practically the same whether or not the delta is included. A measurement of σ_π should not be expected to yield much information about the fireball other than the critical density. (σ_N is determined solely by geometry in this model.)

To calculate the contribution of the deltas to the pion and nucleon differential cross sections it is helpful to make use of the Lorentz invariance of $E\,d^3N_i/dp^3$, where $E = \sqrt{p^2 + m_i^2}$. In the rest frame of the delta it is:

$$E \frac{d^3N_i}{dp^3}(\vec{p}) = \frac{\delta(E - E_o)}{4\pi p_o} \quad . \tag{11}$$

Here i refers to pion or nucleon, E_o is its energy in this frame and p_o its momentum. Go to the rest frame of the fireball and integrate over all deltas:

$$E \frac{d^3N_i}{dp^3}(\vec{p}) = \frac{1}{4\pi p_o} \int \frac{d^3N_\Delta}{dp_\Delta^3} \, \delta(E' - E_o) dp_\Delta^3 \tag{12}$$

where $E' = (E_\Delta E - \vec{p}_\Delta \cdot \vec{p})/m_\Delta$. The result of the integration is:

$$E \frac{d^3N_i}{dp^3}(\vec{p}) = \frac{Vm_\Delta T^2}{\pi^3 p_o p} \left(\sum_{n=1}^{\infty} \frac{(\mp)^{n+1}}{n^2} e^{-nx} (nx+1) \right. \tag{13}$$

$$\left. + \frac{\mu_\Delta}{T} \ln(1 + e^{-x}) \right) \Bigg|_{x_+}^{x_-} \quad ,$$

$$Tx_\pm = \frac{m\Delta}{m_i^2} (EE_o \pm pp_o) - \mu_\Delta \quad . \tag{14}$$

In practice only the first one or two terms in the series are significant. The contribution of the deltas to the number density in momentum space is to be contrasted with the thermal contribution, Eq. (1).

To compute the differential cross sections note that for the identical projectile – target combinations we have been considering the only dependence on the impact parameter b in eqs. (1) and (13) is through V. A convenient relationship is:

$$V(b) = \frac{B(b)}{\rho_c}\left(1 + \frac{N_\pi}{B}\right) \quad . \tag{15}$$

The dependence on b is shown explicitly. Integrating over impact parameters gives:[10]

$$\int_0^{2R} 2\pi b\, V(b)\, db = 2\pi R^2 \frac{A}{\rho_c}\left(1 + \frac{N_\pi}{B}\right) \quad . \tag{16}$$

A is the mass number of the projectile (= target) nucleus and R is its radius. $4\pi R^2$ is the total reaction cross section in this model.

To illustrate the behavior Fig. 5 is a plot of the Lorentz invariant differential cross section $E\, d^3\sigma/dp^3$ for pions and nucleons in the CM for the reaction $^{40}Ca + {}^{40}Ca$ at 1050 MeV/nucleon beam energy. We take $R = 4$ fm. The calculation was done including nucleons and pions in one case, and nucleons, pions and deltas in the other. Inclusion of the delta affects the nucleon spectrum very little. However it does change the shape of the pion spectrum by introducing a hump centered near 150 MeV pion kinetic energy. This effect is caused by the decay of the deltas which contribute a significant number of observed pions. If one believed that all observed pions came from the decay of resonances then the pion curve would drop to zero at zero kinetic energy. Thus experimental observation of just the general shape of the pion spectrum should be sufficient to decide upon the origin of the observed pions.[11] If the pion spectrum was very anisotropic in the CM then this model of pion production would be ruled out of course.

V. NEUTRON TO PROTON CONVERSION

An interesting question is whether or not thermal equilibrium of the fireball will change the observed ratio of protons to neutrons. Does $\sigma_p/\sigma_n = 46/73$ for $^{238}U + ^{238}U$ independent of beam energy? To answer this question consider the collision of two equal mass nuclei but with an arbitrary proton/neutron ratio. (Target and projectile could have different charge also.) As discussed in the last section it does not matter much whether or not we include the delta if we are only interested in the number of observed pions and nucleons. For simplicity neglect the delta.

The quantities which need to be determined are T, V, μ_p and μ_n. Now:

$$\mu_{\pi^+} = \mu_p - \mu_n \ ,$$
$$\mu_{\pi^0} = 0 \ , \tag{17}$$
$$\mu_{\pi^-} = \mu_n - \mu_p \ .$$

It is convenient to divide (4) by (6) and (5) by (6) to eliminate V. The equation involving Q/B is no longer an identity. Figure 6 shows the result of a computer solution to the equations at a beam energy of 1050 MeV. The proton/neutron ratio is larger after thermalization than it was before. Also the number of positive pions is less than the number of neutral pions, which in turn is less than the number of negative pions. This phenomenon can be understood in the following way. If one didn't worry about charge conservation the assumption of thermalization would require that $N_p/N_n = 1$ and $N_{\pi^-}/N_{\pi^+} = 1$. However charge is conserved and so both equalities cannot be satisfied simultaneously (unless the initial proton/neutron ratio is one). There is a compromise by making each ratio as close to one as possible. Using (17) in the classical approximation (2') we see that $N_{\pi^+}/N_{\pi^0} = N_{\pi^0}/N_{\pi^-} = N_p/N_n$. This relation will not be true when fermion/boson statistics are used. As the beam energy increases the number of pions will increase and so the proton/neutron ratio will increase.

This neutron to proton conversion also has implications for the production of light nuclei such as hydrogen and helium isotopes. Several models[12,13,14] have been suggested to account for their existence. All of the models use the distribution of protons and neutrons in momentum space as input. A neutron to proton conversion will certainly affect their calculations. For example the ratio $^3He/^3H$ should increase with beam energy.

VI. CONCLUSION

In this paper we made the assumptions: (1) that all hadrons are in thermal equilibrium in a nuclear fireball model; (2) that all strong interactions can be turned off when the hadron number density reaches some critical value; and (3) that noninteracting gas formulae can be applied at this critical density. From these assumptions we are able to calculate the total and differential cross sections for all observed particles. The only parameter in the model is the critical density at which the strong interactions are turned off, and its expected value varies by a factor of three.

Up to several GeV beam energy the most important contributions to the fireball are the pions, nucleons and deltas. The nucleon cross sections are independent, and the pion cross sections nearly independent, of the inclusion or exclusion of the delta. The slope of the nucleon differential cross section is made slightly steeper by the inclusion of the delta, whereas the shape of the pion differential cross section is changed significantly. For neutron rich projectile-target combinations there is a net conversion of neutrons to protons and a larger number of negative than positive pions. This conversion will affect the relative abundance of various isotopes of light nuclei.

A serious approximation in the model is that the transition from thermal equilibrium to a freely expanding system of particles is made instantaneously. For long-lived particles this is not too much of a problem but for short-lived particles it may be. For instance, the doubling time of the fireball volume is the same order of magnitude as the lifetime of the $\Delta(1232)$.

The possibility of a transparency between target and projectile at higher energy leading to two fireballs was not considered, although it is clear how to include it. Transparency affects the kinematics, not the thermodynamics of the model. Finally when making a detailed comparison with experiment for the spectra of nucleons and light nuclei it may be necessary to take account of the evaporation of the target and spectator pieces. Also the spectra of nucleons from the fireball need to be corrected for depletion due to the production of light nuclei.

VII. ACKNOWLEDGMENTS

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FOOTNOTES AND REFERENCES

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11. It should be noted that if $d^2\sigma/dEd\Omega$ were plotted instead all the curves would go to zero at zero kinetic energy and so the distinction among these three possibilities would be much less.

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FIGURE CAPTIONS

Fig. 1. Temperature of the fireball as a function of critical density for the collision of equal mass nuclei.

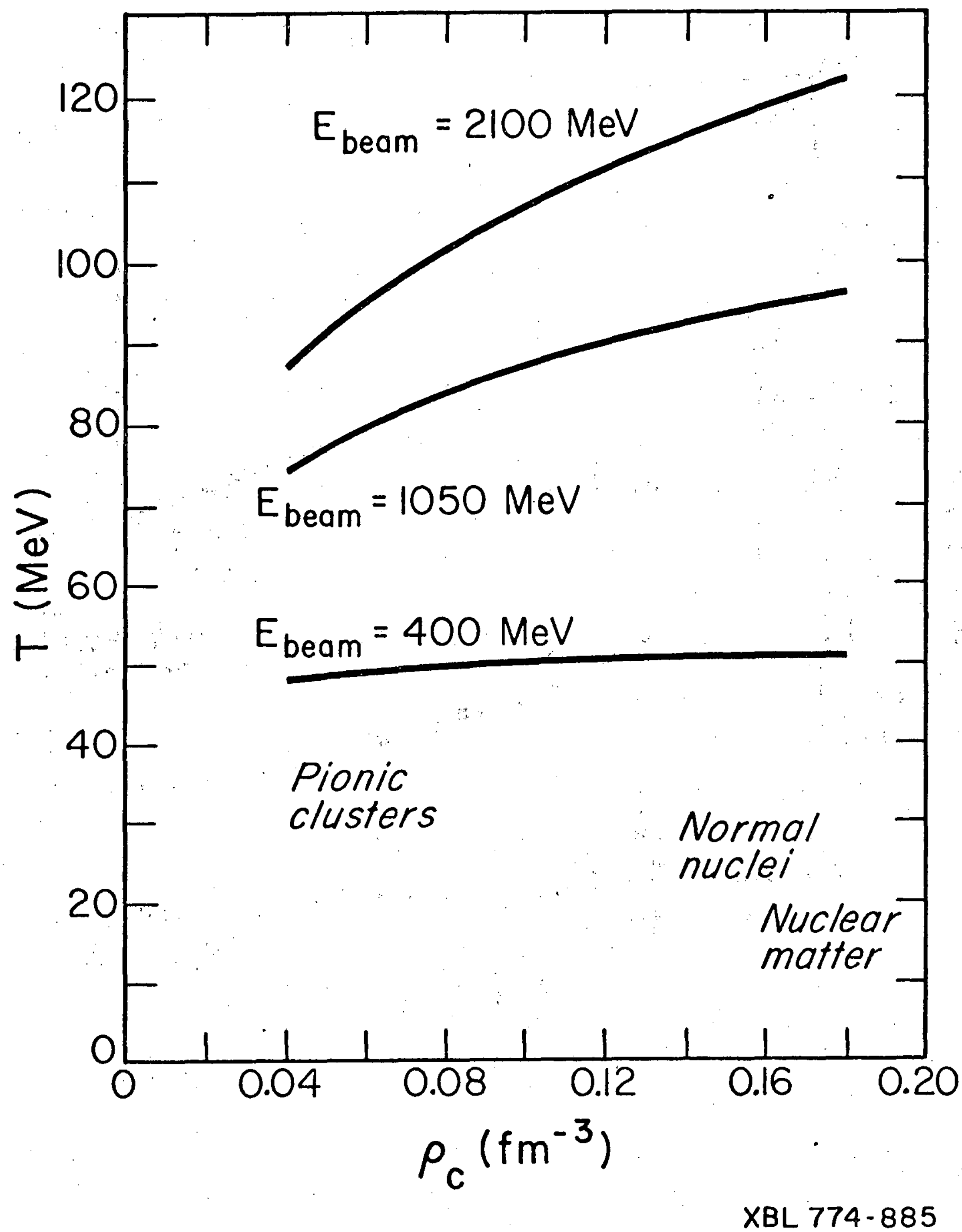
Fig. 2. Pion to nucleon ratio of the fireball as a function of critical density for the collision of equal mass nuclei.

Fig. 3. Influence of the Δ(1232) on the temperature of the fireball as a function of beam energy for the collision of equal mass nuclei.

Fig. 4. Influence of the Δ(1232) on the pion to baryon ratio in the fireball and on the observed pion to nucleon cross sections, as a function of beam energy for the collision of euqal mass nuclei.

Fig. 5. Influence of the Δ(1232) on the pion and nucleon differential cross sections for the reaction $^{40}Ca + ^{40}Ca$ at 1050 MeV.

Fig. 6. Observed charged and neutral particle ratios as a function of the initial proton to neutron ratio of the projectile-target system, for the collision of equal mass nuclei at 1050 MeV.



120
100
80
60
40
20
0
T (MeV)
E beam = 2100 MeV
E beam = 1050 MeV
E beam = 400 MeV
Pionic clusters
Normal nuclei
Nuclear matter
0
0.04
0.08
0.12
0.16
0.20
ρc (fm-3)
XBL 774-885

Fig. 1

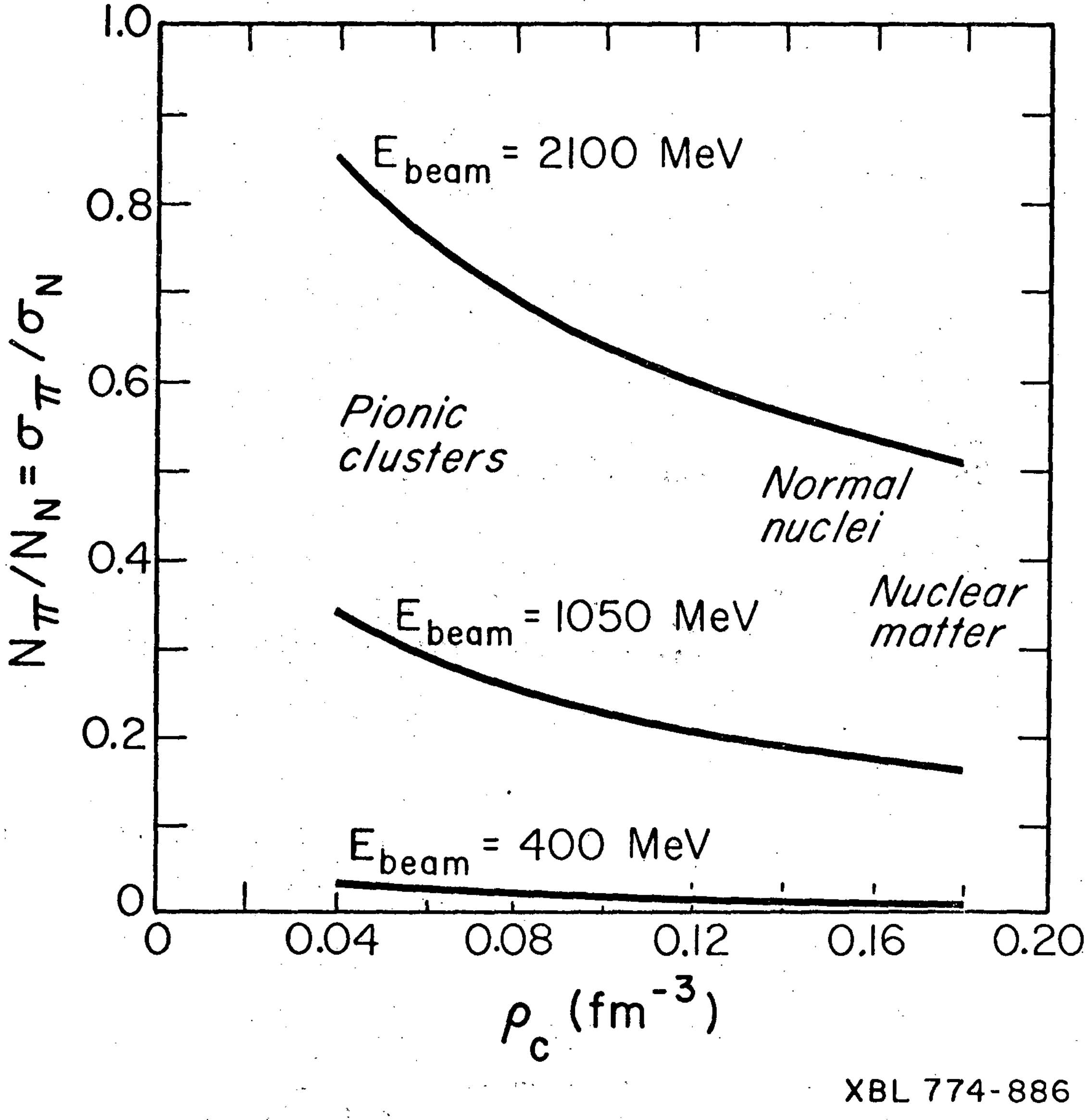

1.0
0.8
0.6
0.4
0.2
0
$N_\pi/N_N = \sigma_\pi/\sigma_N$
E_{beam} = 2100 MeV
Pionic clusters
Normal nuclei
Nuclear matter
E_{beam} = 1050 MeV
E_{beam} = 400 MeV
0
0.04
0.08
0.12
0.16
0.20
ρ_c (fm^{-3})
XBL 774-886

Fig. 2

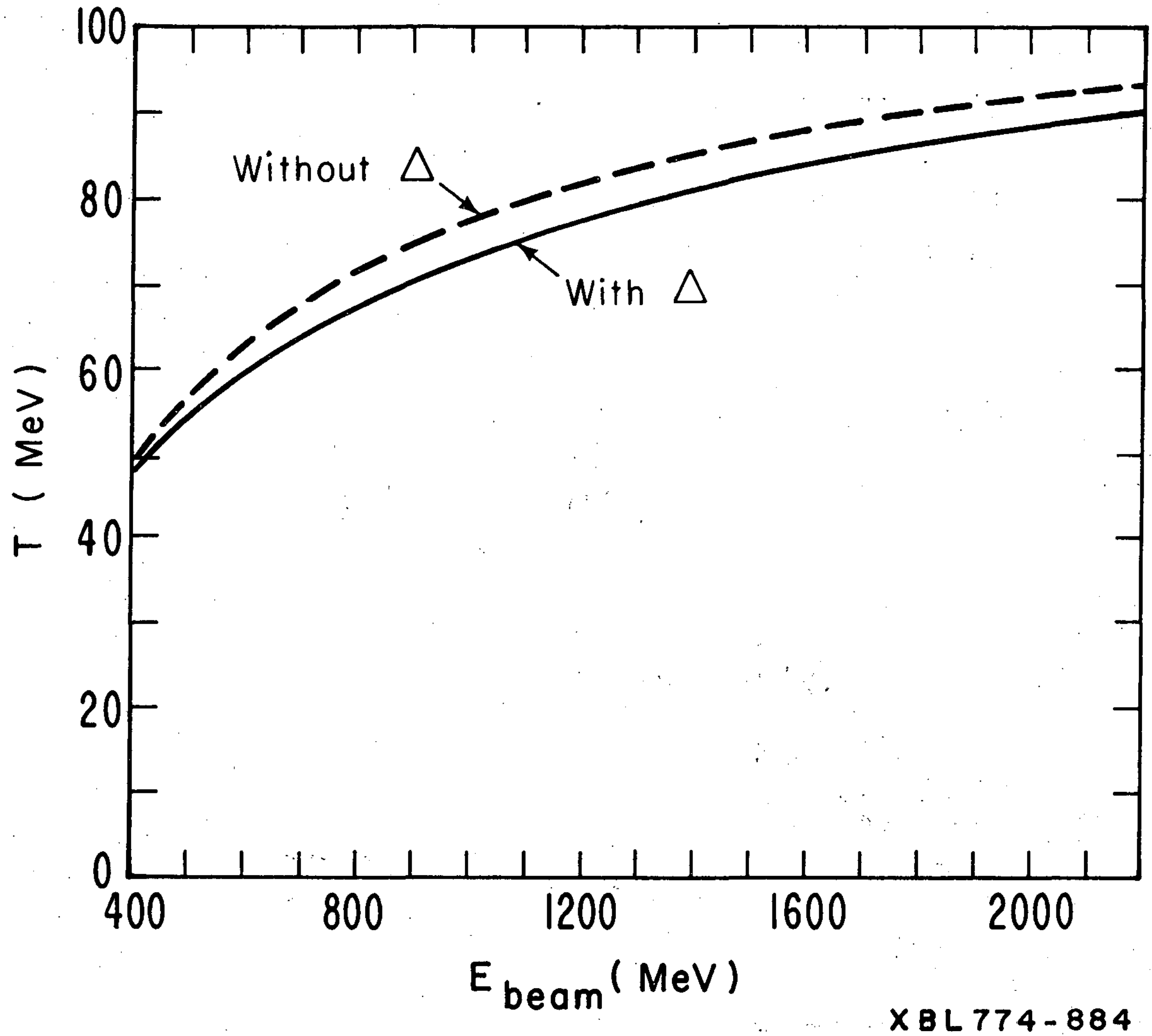
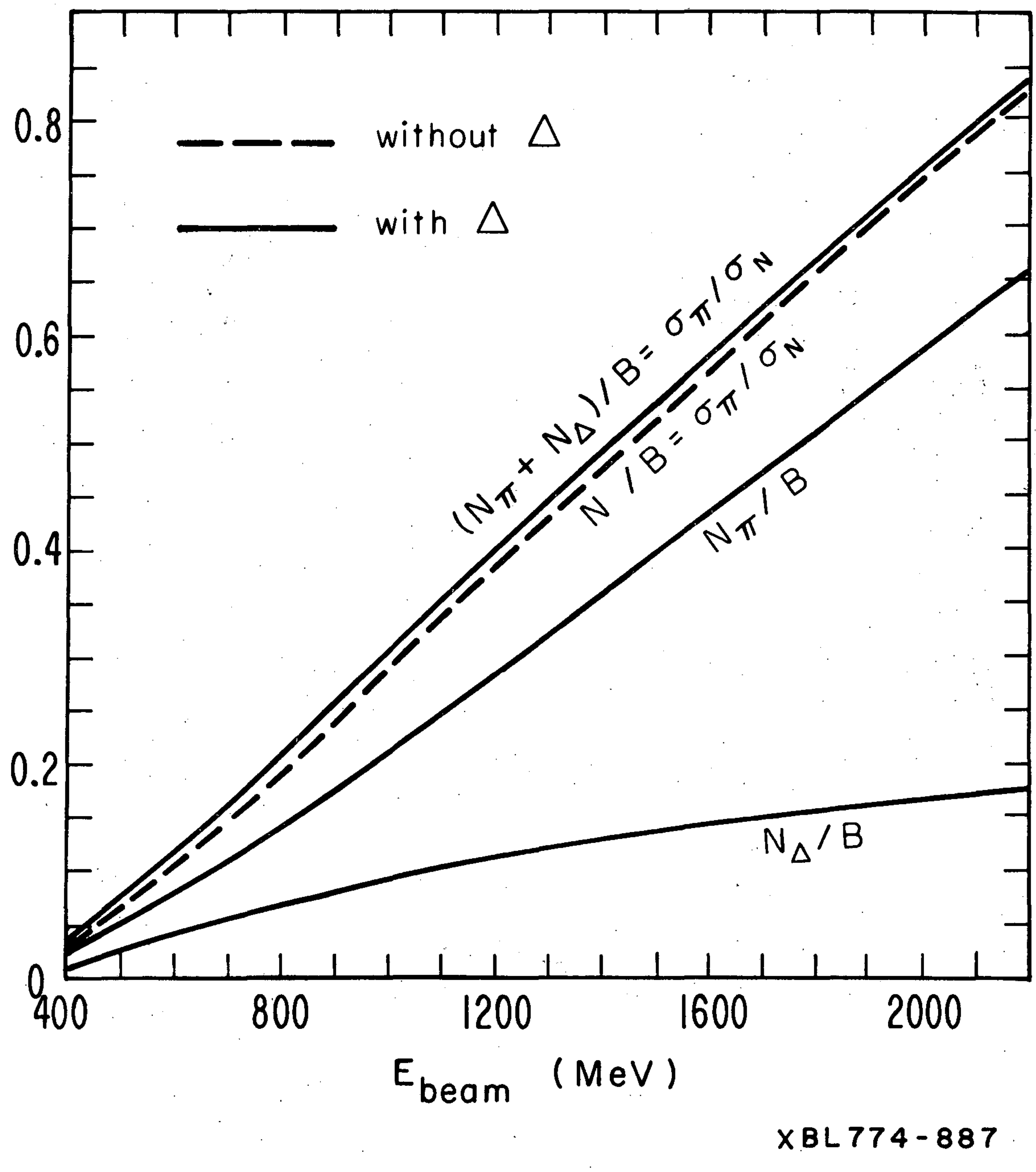
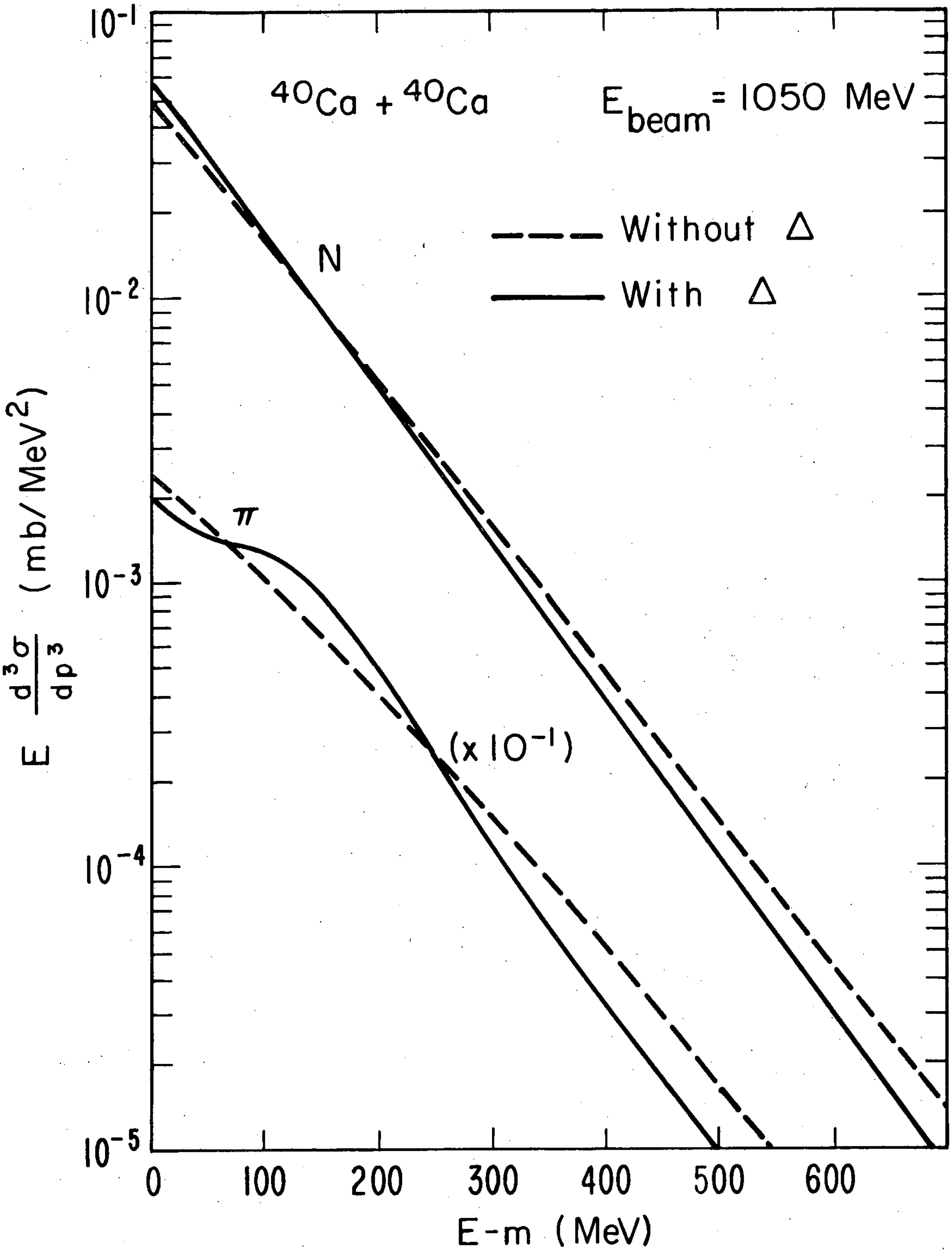

100
80
60
40
20
0
T (MeV)
Without Δ
With Δ
400
800
1200
1600
2000
E beam (MeV)
XBL 774-884

Fig. 3



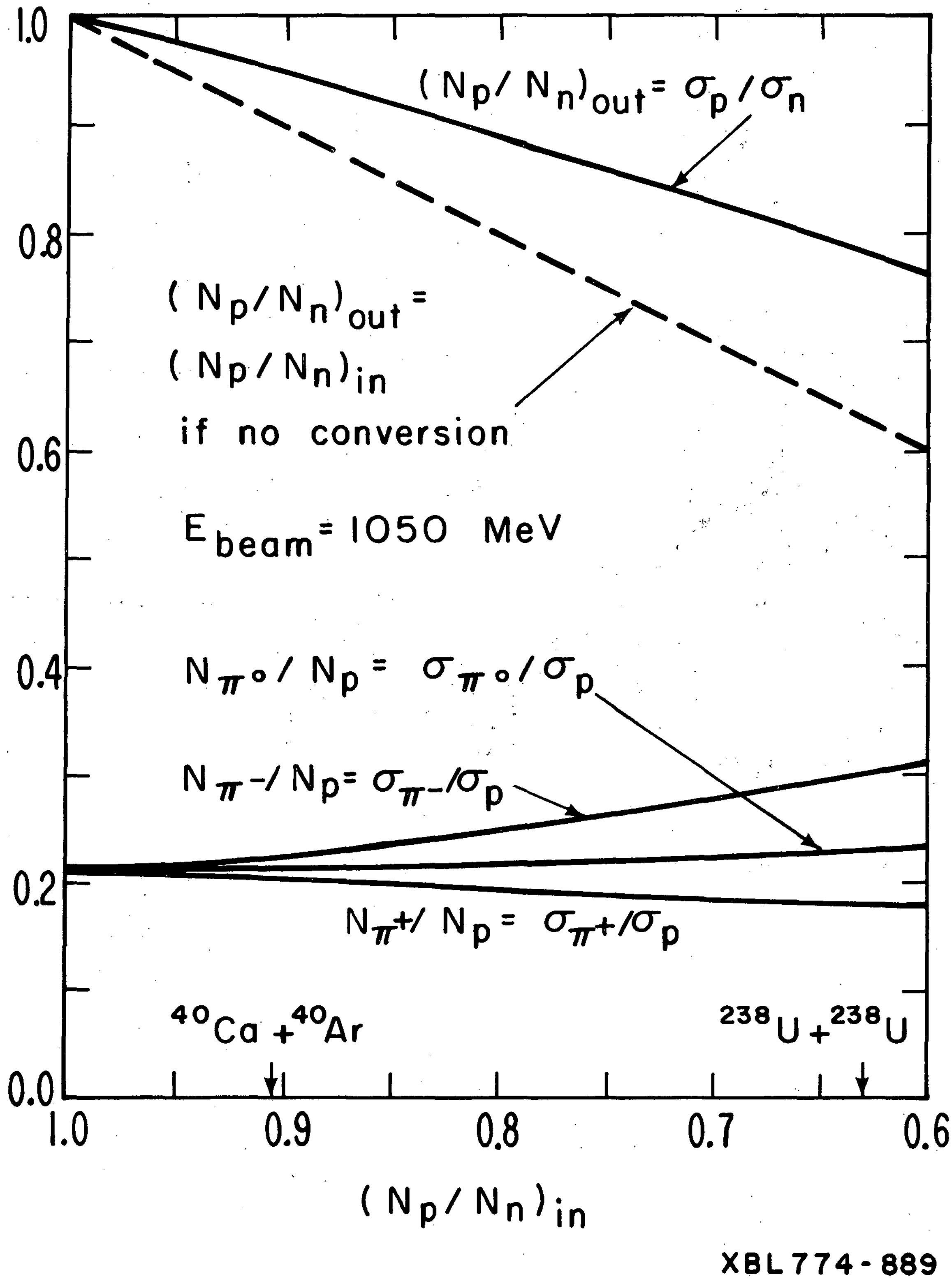
without Δ
with Δ
0.8
0.6
0.4
0.2
0
400
800
1200
1600
2000
(Nπ + NΔ)/B = σπ/σN
N /B = σπ/σN
Nπ/B
NΔ/B
Ebeam (MeV)
XBL774-887

Fig. 4



XBL774-888

Fig. 5



1.0
0.8
0.6
0.4
0.2
0.0
$(N_p/N_n)_{out} = \sigma_p/\sigma_n$
$(N_p/N_n)_{out} =$
$(N_p/N_n)_{in}$
if no conversion
$E_{beam} = 1050$ MeV
$N_{\pi^0}/N_p = \sigma_{\pi^0}/\sigma_p$
$N_{\pi^-}/N_p = \sigma_{\pi^-}/\sigma_p$
$N_{\pi^+}/N_p = \sigma_{\pi^+}/\sigma_p$
$^{40}Ca + ^{40}Ar$
$^{238}U + ^{238}U$
1.0
0.9
0.8
0.7
0.6
$(N_p/N_n)_{in}$
XBL 774-889

Fig. 6

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