tories. This is due to the multipole structure of the daughter trajectories in the CHKZ representation. In this case, the condition Eq. (27) is not satisfied.

However, inclusion of cuts in the CHKZ representation does not change our results. This is true for the rather general case where cuts are constructed from moving poles.

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[8]The constant $k_{l,m}$ is defined by the following equation:

$$k_{l,m}=\sum_{r=0}^{l} C_{l,r}\sum_{\mu=0}^{r}\binom{r}{\mu} a^{\mu} \sum_{\lambda=0}^{2r-2\mu+1} \binom{2r-2\mu+1}{\lambda} q^{\lambda} \times p^{2r-2\mu+1-\lambda} d_{2r-2\mu-\lambda+1,m},$$

where $C_{l,r}$ and $d_{l,m}$ are given by

$$P_l(y)=\sum_{r=0}^{l} C_{l,r} y^r \text{ and}$$

$$y^l=\sum_{m=0}^{l} d_{l,m} P_m(y).$$

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PHYSICAL REVIEW D VOLUME 9, NUMBER 5 1 MARCH 1974

Particle ratios in energetic hadron collisions*

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We construct a simple statistical model in order to estimate, for very energetic collisions, the ratios of particles with various quantum numbers produced with center-of-mass momenta less than a few GeV. Two conclusions are that (1) isobar decay can account for a large fraction of the SU(3) violation observed experimentally, and (2) at high transverse momentum the dominance of pions diminishes.

In this paper we construct an extremely simple statistical picture of hadron production in the central region of rapidity. It illustrates how several features of the produced-particle spectrum in hadron collisions can be understood without resorting to sophisticated dynamical models. While we are not sure whether this is a model which can be used for serious quantitative study, we do believe it contains qualitative features of some generality.

Subsequent to carrying out this work, we realized that Anisovich and Shekhter[1] have developed a very similar picture. We recommend that the interested reader compare that work with this. Not surprisingly our versions differ in various ways: The initial assumptions differ somewhat, and comparison of our results with theirs provides an indication of the sensitivity of the basic idea to details. Also, Anisovich and Shekhter considered not only the central rapidity region (for low-$p_\perp$ secondaries) but the target and projectile fragmentation regions as well. On the other hand, we have applied the idea to the production of particles of high and low $p_\perp$ in the central rapidity region.

The model is very simple. Imagine that, as a result of the high-energy collision, the interaction region is a chaotic reservoir of partons: quarks and antiquarks. A quark leaving the zone of interaction must pick up at least one neighbor parton from the reservoir in order to make a hadron. Assuming there are equal numbers of quarks and antiquarks of each variety in the reservoir, there is a 50% chance that the q grabs a $\bar{q}$. We assume that at this point the $q\bar{q}$ system is satisfied and departs for infinity as a meson M. In the other half of the cases the qq system must grab again, and there is again a 50% chance of satisfaction, now as a qqq baryon state B. Proceeding thus, we have the following configurations:

Configuration	M or B	Probability
$q\bar{q}$	M	$\frac{1}{2}$
qqq	B	$\frac{1}{4}$
$qq\bar{q}\bar{q}$	M	$\frac{1}{8}$
$qq\bar{q}qq$	B	$\frac{1}{16}$
...	...	...

Evidently the ratio M/B equals 2. (Dynamics of a more sophisticated model could modify the value of this ratio, probably tending to enhance it, but perhaps this is a good first approximation. For instance, Anisovich and Shekhter used the ratio $M/B=6$ because they considered the configuration $qq\bar{q}\bar{q}$ to be two mesons and $qq\bar{q}qq$ to be a meson and baryon, etc.) Continuing our policy of making the simplest possible assumption to see how far it gets us, we assume that the M and B are produced in the SU(6) $\underline{35}$ and $\underline{56}$ representations, according to statistical weights. Then computation of the particle ratios in the central rapidity plateau (integrating over $p_\perp$) is reduced to bookkeeping. The ledger is presented in Table I. Combining the totals of Table I(a) (weighted by $\frac{2}{3}$) with the totals of Table I(c) (weighted by $\frac{1}{3}$) gives the following numbers of particles per emitted hadron:

Particle	π^+	π^0	K^+	K_L	η	p	n
Number emitted	0.5	0.53	0.11	0.09	0.02	0.09	0.07

The substantial deviation from exact SU(3) symmetry comes about because much of the statistical weight lies in ρ, K^*, Δ, and Y^*, all of which pro-

TABLE I. Particle spectra integrated over $p_\perp$: (a) The number of $\pi^+=\pi^-=\pi^0$, $K^+=K^-$, K_L, and η per emitted $\underline{35}$ meson, allowing for resonance and K_S decay. (b) The number of $\pi^+=\pi^-=\pi^0$, p, n, Λ^0, $\Sigma^+=\overline{\Sigma^-}$, $\Xi^0=\Xi^-$, and Ω^- per emitted $\underline{56}$, averaged over baryons and antibaryons (this table includes the effects of resonance decays but not of weak decays, and therefore we call it a "bubble-chamber" ledger). (c) The number of $\pi^-=\pi^+$, π^0, p, and n per emitted $\underline{56}$, averaged over baryons and antibaryons, after all decays have taken place (we call it a "spectrometer" ledger).

(a)

Parent	Stat. wt.	π^+/M	K^+/M	K_L/M	η/M
π,ρ,ω	15/35	0.65	0	0	0
K,K^*	16/35	0.42	0.25	0.25	0
ϕ	3/35	0.39	0.50	0.33	0
η	1/35	0	0	0	1.0
Weighted total		0.50	0.16	0.14	0.03

(b)

Parent	Stat. wt.	π^+/B	p/B	n/B	Λ^0/B	Σ^+/B	Ξ^0/B	Ω^-/B
N,Δ	20/56	0.27	0.25	0.25	0	0	0	0
Λ,Σ,Y^*	20/56	0.20	0	0	0.38	0.06	0	0
Ξ,Ξ^*	12/56	0.22	0	0	0	0	0.25	0
Ω^-	4/56	0	0	0	0	0	0	0.50
Weighted total		0.21	0.09	0.09	0.14	0.02	0.05	0.04

(c)

Parent	Stat. wt.	π^-/B	π^0/B	p/B	n/B
N,Δ	20/56	0.27	0.27	0.25	0.25
Λ,Σ,Y^*	20/56	0.52	0.50	0.27	0.20
Ξ,Ξ^*	12/56	0.81	1.06	0.33	0.17
Ω^-	4/56	0.83	1.33	0.33	0.17
Weighted total		0.52	0.60	0.28	0.21

duce pions in their decay. We see that after the resonance and weak decays have taken place the ratios are $K^-/\pi^- = 22\%$ and $\bar{p}/K^- = 89\%$, not desperately far from the experimental values of ~7% and ~100%, respectively.[2] Thus we draw our first conclusion: A large part of the observed SU(3) breaking in the central region may be due simply to isobar decay.[3]

A similar exercise can be carried out for inclusive distributions at high $p_\perp$. [In order to stay in the central rapidity plateau we must take $2p_\perp/\sqrt{s} \ll 1$, where $\pi^+/\pi^- \sim 1$, $p/\bar{p} \sim 1$, etc.] This time we will take account of the experimental fact[1] that the invariant cross section falls rapidly with

TABLE II. Particle spectra at high $p_\perp$, but in the central rapidity region ($2p_\perp/\sqrt{s} \ll 1$), resulting from $\underline{35}$ and $\underline{56}$ production with an invariant cross section falling as $p_\perp^{-n}$. Results are given for $n = 6, 8, 10$. We keep only contributions of 5% or more and therefore ignore three-body decays. (a) Relative numbers of $\pi^- = \pi^+ = \pi^0$, $K^- = K^+$, K_L, and η at a given $p_\perp$, per emitted $\underline{35}$ meson, allowing for resonance and K_S decay. (b) "Bubble-chamber" ledger for baryon parents: relative numbers of $\pi^- = \pi^+ = \pi^0$, $p = n$, $\Sigma^+ = \Sigma^-$, Λ^0, $\Xi^0 = \Xi^-$, and Ω^- at a given $p_\perp$, per emitted $\underline{56}$, averaged over parent baryons and antibaryons. (c) "Spectrometer" ledger for baryon parents: relative numbers of $\pi^- = \pi^+ = \pi^0$, p and n at a given $p_\perp$, per emitted $\underline{56}$, averaged over parent baryons and antibaryons.

(a)

Parent	Stat. wt.	n	π^+/M	K^+/M	K_L/M	η/M
π,ρ	12/35	6	0.17	0	0	0
		8	0.14	0	0	0
		10	0.13	0	0	0
ω	3/35	...	0	0	0	0
K,K^*	16/35	6	0	0.11	0.11	0
		8	0	0.10	0.10	0
		10	0	0.09	0.09	0
ϕ	3/35	6	0	0.05	0.03	0
		8	0	0.02	0.01	0
		10	0	0.01	0.01	0
η	1/35	...	0	0	0	1.0
Weighted total		6	0.06	0.05	0.05	0.03
		8	0.05	0.05	0.05	0.03
		10	0.04	0.04	0.04	0.03

(b)

Parent	Stat. wt.	n	π^-/B	P/B	Λ^0/B	Σ^+/B	Ξ^0/B	Ω^-/B
N,Δ	20/56	6	0	0.14	0	0	0	0
		8	0	0.12	0	0	0	0
		10	0	0.10	0	0	0	0
Λ,Σ,Y^*	20/56	6	0	0	0.23	0.05	0	0
		8	0	0	0.19	0.05	0	0
		10	0	0	0.17	0.05	0	0
Ξ,Ξ^*	12/56	6	0	0	0	0	0.18	0
		8	0	0	0	0	0.16	0
		10	0	0	0	0	0.15	0
Ω^-	4/56	...	0	0	0	0	0	0.5
Weighted total		6	0	0.05	0.08	0.02	0.04	0.04
		8	0	0.04	0.07	0.02	0.04	0.04
		10	0	0.04	0.06	0.02	0.03	0.04

TABLE II *(continued)*

(c)

Parent	Stat. wt.	n	π^-/B	p/B	n/B
N, Δ	20/56	6	0	0.14	0.14
		8	0	0.12	0.12
		10	0	0.10	0.10
Λ, Σ, Y^*	20/56	6	0	0.10	0.08
		8	0	0.07	0.06
		10	0	0.05	0.04
Ξ, Ξ^*	12/56	6	0	0.01	0.01
		8	0	0	0
		10	0	0	0
Ω^-	4/56	···	0	0	0
Weighted total		6	0	0.09	0.08
		8	0	0.07	0.06
		10	0	0.05	0.05

increasing $p_\perp$, apparently as $p_\perp^{-n}$. The exponent may or may not be the same for M and B, but we continue to assume that the $\underline{35}$ and $\underline{56}$ are produced according to statistical weights. The bookkeeping for decaying resonances is a bit more tedious because of the steeply falling spectrum. If the probability of finding a child[4] C of momentum xP_μ, given a parent of momentum P_μ, is scale-invariant (a good approximation if $p_\perp \gg m_c$), i.e.,

$$\frac{dN_c}{dx} = g_c^P(x) , \tag{1}$$

then a simple calculation shows that the ratio of children to parents at a given $p_\perp$ is

$$\frac{\text{child}}{\text{parent}} = \int_0^1 dx\, x^{n-2} g_c^P(x) , \tag{2}$$

independent of $p_\perp$. In the two-body case $g_c(x)$ is very simple: zero for $x < x_c^l$ and $x > x_c^u$, and constant of value $1/(x_c^u - x_c^l)$ for $x_c^l < x < x_c^u$. If the parent mass is M and the children have masses m_1 and m_2, then if $m_1 = m_2 = m$, $x_1^l = x_2^l = m^2/M^2$ and $x_1^u = x_2^u = 1 - m^2/M^2$. Another interesting case is $m_1^2 \ll m_2^2$, when $x_1^l = 0, x_1^u = 1 - m_2^2/M^2$ and $x_2^l = m_2^2/M^2$, $x_2^u = 1$. For three-body decays $g_c(x)$ is generally quite small at high x. Inasmuch as n is empirically large ($\gtrsim 8$), the high-$p_\perp$ region is dominated by two-body decays in which the observed child is emitted forward in the parent center-of-mass frame.

The high-$p_\perp$ ledger is given in Table II for $n = 6$, 8, and 10. We see that high-$p_\perp$ pions from Δ decay, ρ decay, etc. are strongly suppressed. Consequently the fraction of kaons and baryons in the charged-particle spectrum increases from low $p_\perp$ to high $p_\perp$, and then stabilizes. For example, continuing to take $M/B = 2$ and assuming $n = 8$ for baryons and mesons, we find at high $p_\perp$ that $K^-/\pi^- \sim 95\%$, $\eta/\pi^0 \sim 57\%$, and $\bar{p}/K^-$(spectrometer) $\sim 68\%$, while if strange baryons are observed $\bar{\Lambda}^0/\bar{p} \sim 170\%$ and $\Omega^-/\bar{p} \sim 88\%$. It is important to remember, when comparing these results with data, that they are only applicable in the central rapidity plateau. At CERN ISR energies a plateau is not yet developed for the $\bar{p}$'s, so the asymptotic region has not quite been reached experimentally. Nonetheless the general trend, that at high $p_\perp$ there are relatively more kaons and baryons, is in line with the data.

Evidently by leaving the ratio M/B and/or the ratio of strange to nonstrange quarks in the reservoir arbitrary we could considerably improve the agreement with experiment, if that were our purpose. Instead, we choose to abstract the following qualitative conclusions:

(1) In order to understand particle ratios it may be useful to take isobar production into consideration.

(2) Most of the SU(3) breaking in the central plateau can be accounted for by the known SU(3) violation in the decay of resonances.

(3) The spectrum at high $p_\perp$ is richer in kaons and baryons than that at low $p_\perp$, because a decaying isobar gives most of its momentum to the heavier child.

(4) There is a parent-child relation at high $p_\perp$: If the parent distribution falls as a power, $p_\perp^{-n}$, and the decay distribution of children relative to

the parent obeys Feynman scaling as in Eq. (1), then the distribution of children falls with the same power n. One implication of this parent-child relation is that if all high-$p_\perp$ hadrons are progeny (via scale-invariant cascade processes) of the *same* parent (e.g., quark parton or gluon), then they will all have the same power-law dependence on $p_\perp$. Hence measurement of the exponent n for various kinds of hadrons may test whether they can be regarded as produced from a single parent.

While we have focused on hadron-hadron collisions above, the same idea may have applicability to hadron production in high-energy e^+e^- annihilation and in the plateau regions in deep-inelastic electroproduction and neutrino interactions.

We thank David Horn for an interesting discussion.

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[1]V. Anisovich and V. Shekhter, Nucl. Phys. B55, 455 (1973). We are very grateful to the authors for bringing their work to our attention.

[2]A recent review with up-to-date references to the data is that of M. Jacob, CERN Report No. CERN-TH-1683, 1973 (unpublished).

[3]Many people have undoubtedly noticed this. We know D. Horn and S. Nussinov have reached similar conclusions (private communication).

[4]The perceptive reader will guess that this word has been chosen with care.

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Hadron distributions in the reactions hadron + hadron → lepton pair + hadrons and two virtual photons → hadrons, and tests of the parton model*

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The parton-model description of final-state hadron distributions can be characterized in terms of a few fragmentation regions and plateaus. We discuss such distributions in the reactions hadron + hadron → lepton pair + hadrons and virtual photon + virtual photon → hadrons; we also compare different distributions as tests of the parton model, and discuss kinematical conditions for the validity of the asymptotic descriptions of the distributions and the tests. From this discussion, we conclude that most of these asymptotic descriptions may be valid only at extremely high energies and a good nonasymptotic model is probably needed to describe these final-state hadron distributions at presently available energies.

The parton-model description of final-state hadron distributions has been discussed for various processes, namely deep-inelastic electroproduction,[1-3]

$$\text{lepton} + \text{hadron} \rightarrow \text{lepton} + \text{hadrons}, \qquad (1)$$

high-energy electron-positron annihilation,[1-3]

$$\text{electron} + \text{positron} \rightarrow \text{hadrons}, \qquad (2)$$

and deep hadron-hadron scattering,[4]

$$\text{hadron} + \text{hadron} \rightarrow \text{large-transverse-momentum hadron} + \text{other hadrons} \qquad (3)$$

in certain kinematical regions. Such distributions always involve some fragmentation regions and plateaus,[1-6] as illustrated in Figs. 1(a)–1(d), with the heights of the plateaus, as well as the shapes of the fragmentation regions, for different reactions related to one another. It is interesting to see if such descriptions also can be naturally extended to other reactions in terms of the existing fragmentation regions and plateaus. Furthermore, we indeed need to study some other reactions in order to provide more tests of the parton model. Except at extremely high energies, there is not enough phase space for the full structure of all these regions, and therefore various regions may overlap. Thus it is difficult to use the full structure of the hadron distribution as a test of the model. On the other hand, the overlapping regions can still be related to one another, and therefore