

## Particle Trajectories in Model Current Sheets

### 2. Applications to Auroras Using a Geomagnetic Tail Model

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Individual particle trajectories are determined analytically and numerically in two possible configurations of electric and magnetic fields in the geomagnetic tail. The models are based on reconnection models incorporating a neutral point with associated neutral or current sheet and on the observed neutral sheet in the geomagnetic tail. Both models contain magnetic field lines oppositely directed on either side of a current sheet, with some field line connection through the sheet and with an electric field perpendicular to the magnetic field and parallel to the sheet. The models differ in the rate of variation of a magnetic field component perpendicular to the neutral sheet and hence in the rate of field line crossing of the neutral sheet. For the two models, particles are accelerated and turned toward the earth within the neutral sheet and are ejected from the neutral sheet with small pitch angles to a magnetic line of force, with energies of tens of kilovolts. For the first model, a dipole and tail model, electrons are ejected at about  $150 R_E$  and protons about  $50 R_E$  back in the tail. For the second model, an extended tail model, electrons are ejected at about  $500 R_E$ , and protons at about  $400 R_E$ . Proton auroras would be expected about  $\frac{1}{2}^\circ$  lower latitude than electron auroras, and isotropic fluxes should be measurable out to distances of the order of  $2.5 R_E$  from the earth. Extremely thin sheets of incoming particles are produced, about 1 km for electrons between 1 and 10 keV. These results are obtained from an approximate, nonadiabatic theory and are verified by machine computations. To map the thin output sheets onto the earth, a three-dimensional dipole and tail model is used for the numerical computations of many proton trajectories. Thin output sheets of accelerated particles are found using Liouville's theorem. These thin sheets or spatially intense regions are near the auroral zones when mapped onto the earth; they move to lower latitudes on the earth with an increase in the strength of the tail field, and their thickness is roughly proportional to the thermal velocity of the particles incident on the tail. The geomagnetic tail may sometimes be quite long without field-line merging and may sometimes be shorter with merging. These models may therefore be useful in the description of auroral acceleration whenever the merging process is going on. The models may be applicable to other situations where neutral points or sheets may exist, such as the day-side magnetospheric current sheet, the interplanetary field, solar flares, etc. If  $H_\alpha$  auroral emission occurs as suggested by Eather, these results imply that alphas should be found equatorward of precipitating protons with about twice the proton energy.

#### INTRODUCTION

This paper is concerned with the possibility that the geomagnetic tail and more specifically, the neutral sheet in the tail, accelerates particles from the solar wind and is the immediate source of some auroral particles.

It was originally Hoyle's [1949] suggestion that auroral particles may be accelerated near the earth at magnetic neutral points formed by the addition of an interplanetary magnetic field to the earth's dipole field. Dungey [1953]

showed that the magnetic field and plasma should quickly collapse to a sheet-like configuration about an x-type neutral point, with magnetic field energy being given to particles and plasma. If one adds a uniform interplanetary magnetic field of arbitrary direction to a dipole field representing the earth's magnetic field, then in general two neutral points are formed. Dungey [1962] argued that a closed topological surface would be formed about the earth for the case of an interplanetary field with a northward component (antiparallel to the earth's dipole), and an open topology would be formed with a southward interplanetary field component. Dungey also argued that for the open

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topology particles could be accelerated in the sheet-like regions about the neutral points, and these particles would then follow the magnetic field lines to the earth forming the auroral zones.

For acceleration of particles, the key point in Dungey's idea is that *the electric field*, which a stationary observer would see as the solar wind sweeps by, and which disappears in the frame of the moving solar wind, does not disappear in the neighborhood of the neutral point, while merging or reconnection is taking place. Charged particles can therefore be accelerated by this electric field in the sheet-like region about the neutral point where the magnetic field becomes very weak.

It is now known that the earth has a quite extended magnetic tail, with a 'neutral sheet' separating regions of oppositely directed magnetic fields [Ness, 1965]. Speiser and Ness [1967] find the neutral sheet to be well developed on all orbits of the IMP 1 satellite beyond about  $15 R_E$  (geocentric distance in earth radii) in the geomagnetic tail. 'Well developed' in this sense implies that the normal magnetic field component is less than or of the order of 20% of the tail field. The sheet appears to begin at about  $10 \pm 3 R_E$  near the geomagnetic equatorial plane; it appears to be thicker toward dawn than midnight, and there is a normal magnetic field component through the sheet that is between 5 and 20% ( $1-4 \gamma$ ) of the tail field near the dawn edge of the tail, and less than 5% ( $1 \gamma$ ) near the noon-midnight meridian plane. For the IMP 1 measurements, at geocentric distances less than  $30 R_E$ , no definite detection of a neutral point is found.

The acceleration process discussed in this paper can be effective for protons if the magnetic field within the neutral sheet is less than about  $1 \gamma$  and if it is of the order of  $0.01 \gamma$  or less for electrons, and if an electric field exists across the tail. Magnetospheric electric fields have yet to be measured; thus their existence is open to debate. Electric fields appear to be necessary, however, for producing the high-latitude DS current system [Dungey, 1962; Taylor and Hones, 1965; Obayashi, 1966].

Assuming that magnetic lines of force are equipotentials, it is therefore likely that electric fields will be found throughout the magnetosphere. Furthermore, Bratenahl and Hirsch [1966] have demonstrated the existence of an

electric field in the laboratory with an experimental neutral point discharge. They see the sheet-like collapse about an  $x$ -type neutral point with accompanying field line merging.

Many measurements of fields and particles in the magnetosphere and magnetospheric tail have now been made. Cahill's [1964] magnetic field results using data from Explorer 14 complement Ness' results and show the development of the tail structure in closer to the earth.

Heppner [1967], with measurements from OGO, has correlated sudden tail field depressions with a sudden onset of a negative bay. He finds a correlation between the events, but the bay sometimes precedes the depression measured at the satellite. Dungey [1966] has attempted to describe this connection as a sudden turning on of the merging process in the tail. Those cases where the bay is seen first on the ground may then imply a propagation time from the position of reconnection, which is shorter to the ground than to the satellite position. Further measurements are certainly needed. Rothwell [1966] finds little correlation between the energetic electron 'islands' measured by Anderson [1965] and the occurrence of high-latitude magnetic bays, although it certainly is difficult to get simultaneous measurements at the same local time. It may be argued that the islands should instead be correlated with higher energy events. Reid and Parathasarathy [1966] find that there is a detailed correlation of some ground-based radiowave absorption records with the energetic islands seen by Anderson.

If auroral particles are to have their source in the magnetospheric tail, are there satellite observations showing particles coming down field lines near the auroral zones? McDiarmid and Burrows [1965] using data from the Alouette 1 satellite at 1000 km, show occasional spikes of high-intensity electrons coming down field lines (energies above 40 keV, fluxes approaching  $10^9 \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$ ) occurring in a narrow latitudinal range at latitudes above the trapping region.

Sharp *et al.* [1965] at Lockheed have reported seeing thin regions of precipitating electrons over the auroral regions and recently (J. Evans, personal communication, 1966) may also have seen thin beams of incoming protons. O'Brien [1964] observing precipitating auroral electrons using Injun 3 data concludes that most

accelerating processes take place high above the satellite, accelerating electrons preferentially parallel to the magnetic field lines. There may not be complete unanimity, however, that auroral particles come down field lines from large distances. *Mozzer and Bruston* [1966], for example, appear to have evidence of the acceleration of auroral protons below 300-km altitude.

Are there energetic particles observed near the neutral sheet in the geomagnetic tail? *Murayama* [1966] has analyzed the IMP 1 energetic electron data [*Fan et al.*, 1966] and finds that they do see larger electron intensities near the neutral sheet.

*Anderson and Ness* [1966] correlating energetic electron and magnetic field data on the IMP 1 satellite find a broad region of magnetic depression within which the high-intensity spikes of energetic electrons are seen. Within this broad region (which may be called the plasma sheet) is usually found the relatively narrow neutral sheet. *Bame et al.* [1967] observe the broad plasma sheet region with detectors on the Vela satellites. They see a dawn/dusk asymmetry, with a thicker spatial distribution toward dawn. *Murayama* [1966] also sees a thickening toward dawn, and *Speiser and Ness* [1967] find the same characteristic for the imbedded neutral sheet. These results appear to be consistent with the idea of an electric field in the neutral sheet from dawn to dusk.

Previous theoretical arguments have included some particle trajectory analyses about a neutral line [*Weiss and Wild*, 1964; *Chapman and Kendall*, 1964], and *Parker* [1957] has investigated particle motion about a neutral sheet. *Parker* found that regardless of the initial particle configuration, stability soon results, with the current given by curl  $\mathbf{B}$  just as in classical hydromagnetics. These analyses did not, however, include an electric field resulting from the merging process. *Coppi et al.* [1966] suggest that a collisionless pinch instability of a neutral sheet can transform magnetic energy into kinetic energy and could yield characteristic times similar to auroral observations.

If the magnetic field  $\mathbf{B}$  at the center of the neutral sheet goes to zero, the cyclotron radius increases without bound, so no matter how slowly  $\mathbf{B}$  may vary,  $R/L(L$ , a characteristic system length) will be large, and adiabatic theory cannot be used. Particle trajectories must

therefore be either determined analytically from the equations of motion or computed numerically. If  $\mathbf{B}$  is small but nonzero at the center of the sheet,  $R/L$  must be determined to see if adiabatic theory can be used.

In the following section, particle trajectories are calculated applying the results of the non-adiabatic analytic theory [*Speiser*, 1965*b*, subsequently referred to as Part 1] to two models of possible fields in the geomagnetic tail. It is shown that the results are consistent with some auroral observations, such as the thinness of arcs, and proton and electron dawn/dusk asymmetries and latitudinal separations. A section follows with numerical results of a more complicated model that allows mappings of the output sheets onto the earth to be made. These results, then, are based on simple models of the geo-electric-magnetic tail field. The formidable problem of a completely self-consistent solution has not been attempted here, but is examined by *Dungey and Speiser* [1967].

#### SUMMARY OF ANALYTICAL SOLUTIONS

To obtain analytical solutions to the particle differential equations of motion, two simple models were discussed in Part 1. The first model was that of a strictly neutral sheet, the magnetic field being parallel to the sheet and varying linearly with distance across the sheet. Such a field reverses across the sheet and is zero at the center of the sheet. The second model differs from the first only in the addition of a small magnetic field component normal to the sheet. This component implies some field-line connection through the sheet and gives the field lines a finite radius of curvature in the sheet. This model is an approximation to the field near an  $x$ -type neutral point where the normal field component decreases toward the neutral point. For both models, an electric field normal to the magnetic field and parallel to the sheet was assumed. (see Figure 1). Such an electric field is based on *Dungey's* [1953, 1958] theoretical arguments on the discharge at an  $x$ -type neutral point as discussed in the previous section. The magnitude of the electric field is chosen from the currently best accepted values of the potential across the polar cap, which is required to drive the observed high-latitude current systems [*Obayashi*, 1966]. At the time of a bay *Obayashi* estimates that a typical magni-

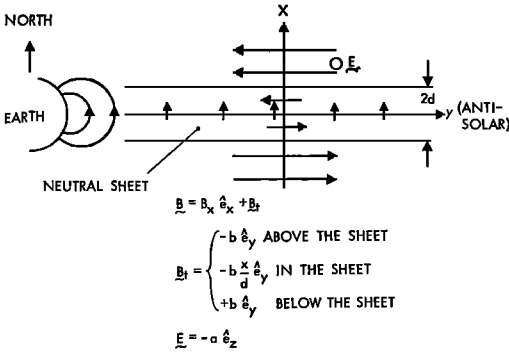


Fig. 1. Dipole and tail model (in the meridian plane containing the earth-sun line). The thickness of the neutral sheet is  $2d$ , the magnetic field strength outside the sheet is  $b$ .  $B_x$  or  $\eta b$  is determined from the dipole field at a particular location, and the electric field strength is  $a$ . Arrows are magnetic field components. For a 'strictly neutral' sheet, let  $B_x$  go to 0.

tude of the potential is of the order of 50 kilovolts. In what follows, a potential of 70 kilovolts across a tail of width  $40 R_E$  is actually used. Field lines above the ionosphere are assumed to be equipotentials so that this potential can be mapped into the tail.

Of course from Dungey's reconnection theory the electric field arises from the merging process, and the mapping takes place from the tail to the polar cap, with the resulting enhancement of the ionospheric currents.

The results of Part 1 using the fields of Figure 1 are summarized as follows.

1. Particles of either sign incident on the sheet oscillate about the sheet owing to the reversal of the magnetic field.

2. For the 'strictly neutral' sheet,  $B_x = 0$ , a particle oscillates about the sheet, gains energy from the electric field, and is 'shot out' the sides of the sheet ( $-\hat{e}_x$  direction for protons,  $+\hat{e}_x$  for electrons).

3. For small  $B_x > 0$ , as a particle oscillates about the sheet, it gains energy from the electric field and is turned into the  $-\hat{e}_y$  direction (toward the earth) by  $B_x$ . (See Figure 1.)

4. A particle will oscillate until it has been turned so much by  $B_x$  that  $\dot{z}$ , its velocity in the  $\hat{e}_z$  direction, changes sign. At that time the particle is ejected from the neutral sheet. (The Lorentz force  $\mathbf{V} \times \mathbf{B}$  changes sign.) The ejection

time is  $\tau = \pi/(q/m)B_x$ . ( $q$  and  $m$  are the particle's charge and mass.)

5. The velocity at ejection is almost entirely in the  $-\hat{e}_y$  direction if  $B_x \ll B_{tail}$ . (The ejection velocity  $dy/dt(\tau) = -2|E|/B_x$ .) Thus the ejection pitch angle ( $\alpha$ ) will be small if  $B_x$  is small.

6. In the moving system where the electric field is transformed away, the motion is seen as an oscillation in  $x$  about the neutral sheet combined with a circular drifting of the trajectory in the neutral sheet (the  $y$ - $z$  plane). The neutral sheet effectively uncouples the circular drift from the oscillation about the sheet (although the oscillation is coupled to the circular drift).

#### ANALYTICAL APPLICATIONS

The electric field strength in the tail,  $a$ , is assumed to be about 0.3 volts/km as discussed above. Since particles gain energy by drifting across the tail while oscillating about the neutral sheet (item 3 above), an electric field of different magnitude will affect the ejection velocity (item 5 above), and will limit the maximum attainable particle energy.

The magnetic field strength,  $b$ , is taken to be  $20 \gamma$  ( $1 \gamma = 10^{-5}$  gauss =  $10^{-9}$  weber/m<sup>2</sup>). Ness [1965] finds the field to be from 10 to  $30 \gamma$ , and about  $40 \gamma$  at the time of a magnetic storm.

*Model 1 for the normal component,  $B_x$ .* The other parameter,  $\eta$ , that we need to know is the ratio of the magnetic field component perpendicular to the neutral sheet,  $B_x$ , to  $b$ , the solar-antisolar field strength outside of the neutral sheet. As a first case, we will assume that  $B_x$  is furnished by the earth's dipole field as sketched schematically in Figure 1. This assumption is certainly artificial and merely provides a model for the rate of crossing of field lines through the neutral sheet. Another model for  $B_x$  is discussed at the end of this section.

Using the results of Part 1, Table 1 can be constructed. Figure 2 shows a sketch of particle trajectories for this model [Speiser, 1967].

The analytical study, Part 1, was based on the assumption that  $\eta$  is a constant. From Table 1, for protons at  $50 R_E$  it is seen that the particle drifts  $25 R_E$  toward the earth before it is ejected from the sheet. Using the dipole model,  $\eta$  would change by a factor of 8, so the above-mentioned approximation does not seem very good. The larger  $\eta$  would serve to turn the pro-

TABLE 1. Application of the Analytic Results for a Dipole and Tail Model

Particle Enters the Neutral Sheet in the Tail at:	$\eta$ $B_z$ (dipole) ( $= \frac{1}{4} \gamma$ ) $b$	Proton Energy at Ejection	Electron Energy at Ejection	$y_p(r)$ , Distance protons travel toward earth before ejection	$y_e(r)$ , distance electrons travel toward earth before ejection	$z_p(r)$ , distance protons travel across tail before ejection	$z_e(r)$ , distance electrons travel across tail before ejection	$\alpha$ , maximum pitch angle of particles at ejection
50 $R_E$	$1.25 \times 10^{-2}$ [ $B_z = (1/4) \gamma$ ]	30 keV	16 eV	$-25 R_E$	-80 km	$-16 R_E$	50 km	$0.4^\circ$ * $4^\circ$ †
150 $R_E$	$4.63 \times 10^{-4}$  ( $B_z = 10^{-3} \gamma$ )	70 keV (maximum potential across the tail)	12 keV	Protons drift completely across the tail	$-10 R_E$	Protons drift completely across the tail	6 $R_E$	$0.01^\circ$ * .1°† $5^\circ$ ‡

$a = 0.3$  v/km,  $b = 20 \gamma$ ,  $a = |\mathbf{E}|$ ,  $b = |\mathbf{B}_{tail}|$   
 Input boundary conditions:  
 \* Assuming  $|x_0/u - 2| = 1$ .  
 † Assuming  $|x_0/u - 2| = 10$ .  
 ‡ Assuming  $|x_0/u - 2| = 400$ .

ton in a tighter circle, and the proton would be ejected sooner and would gain less energy than indicated in the table. However, if a trajectory is broken into a number of small segments, over each of which  $\eta$  is approximately a constant, then Part 1 can be used over each segment. For the above example, when the proton has drifted in to about 40  $R_E$ , it has already gained about 20 keV, so the inconstancy of  $\eta$  does not affect the results as much as would be at first supposed. The reason for this is that the biggest part of the energy gain is during the first part of the trajectory before the particle has been turned much toward the earth.

In Part 1 the qualitative behavior of the oscillation about the neutral sheet has been determined, but the details have not been determined analytically. Knowledge of the output pitch angle requires the detailed knowledge of all of the velocity components at output.  $dx/dt$  was estimated as of the order of  $x_0$ , and  $\dot{z}$  was estimated as of the order of 0, since ejection occurs when  $\dot{z}$  changes sign. The pitch angles in Table 1 are therefore shown for several values of  $x_0$ , making the above assumptions. Speiser [1967] shows a more detailed analysis of the motion within the sheet.

*Trajectory computations using Model 1 for  $B_z$ .*  
 As a check on these approximate analytic results, many proton trajectories have been computed, solving the equations of motion using a Runge-Kutta numerical integration algorithm. Using  $B_z = 1/4 \gamma$  (from the earth's dipole field at

50  $R_E$ ) and the values of  $a$  and  $b$  as used in Table 1, it is found that ejection pitch angles lie between 0 and  $6^\circ$  for incoming velocities of 0-200 km/sec.

From Table 1, at 50  $R_E$  if  $x_0 \approx 12 u \approx 200$  km/sec, a pitch angle of about  $4^\circ$  is expected and is therefore in general agreement with the computed value. However, the computations do

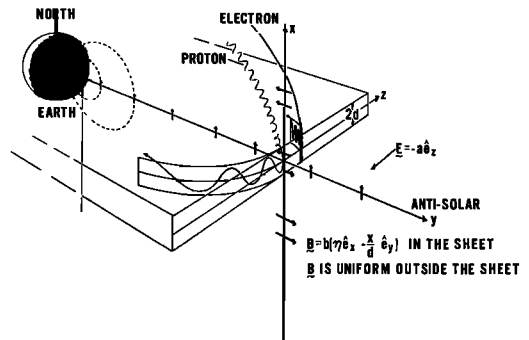


Fig. 2. Sketch of particle trajectories using the fields of Figure 1. Both protons and electrons oscillate about the sheet accelerating in opposite directions, and are turned in the same direction by the small magnetic field component perpendicular to the sheet. The direction turned is toward the earth if this perpendicular component is northward as indicated, otherwise into the tail if the component is southward. When the particles are turned  $90^\circ$ , they are ejected from the neutral sheet. Electrons come out much sooner than protons, with the same velocity as protons, and hence gain less energy. The dimensions shown are illustrative and not to scale.

not show the pitch angle to be related to  $\dot{x}_0$  in as simple a manner as found from the qualitative arguments for determining the pitch angle in Part 1.

The analysis in Part 1 of the energy gained, the turning of the trajectory toward the earth, the trapping in the neutral sheet, and subsequent ejection when  $\dot{z}$  changes sign, is confirmed by the trajectory computations. The qualitative behavior, oscillation frequency, amplitude variation, etc., about the neutral sheet also agrees with the theory. At ejection, however, the perpendicular velocity components  $\dot{x}$  and  $\dot{z}$ , and hence the pitch angle, depend on where the particle is and what its velocity is when  $\dot{z}$  changes sign. If the particle is close to the peak of its last oscillation when  $\dot{z}$  changes sign, it will have a small  $\dot{x}$  and thus small pitch angle; if it is close to  $x = 0$ , its  $\dot{x}$  will be large and will increase until ejection and will thus imply a large pitch angle.

Figure 3 is an isometric projection of a machine computation of a proton trajectory using the fields of Figure 1 and the constants as in Table 1. The initial velocities at the neutral sheet ( $x = 600$  km) are indicated. The comparisons of the theoretical predictions of Part 1 with the computed values are indicated on Figure 3.

*Model 2 for  $B_z$ .* Dungey [1965] predicts a tail length of the order of  $1000 R_E$ , so a neutral point may exist at about  $500 R_E$ . Another possible model for the perpendicular component,  $B_z$ , would therefore be one that goes linearly from about  $1 \gamma$  at about  $50 R_E$  to 0 at  $500 R_E$ . This model is sketched in Figure 4. The limiting field line from the neutral point as sketched is attached to the auroral zone in agreement with Dungey's [1961] open model. Table 1 can be used for the application of the analytical results to this model, with the only modification being the first entry, that is the distance back in the tail at which the particle enters the neutral sheet. For the first column  $B_z$  is  $\frac{1}{4} \gamma$  at  $y = 388 R_E$ , so protons of about 30 keV will come from this region and electrons of about 12-keV energy will come from the region where  $B_z \approx 10^{-3} \gamma$  or from about  $495 R_E$ .

From conservation of flux we can find the latitudinal separation at the earth of the two field lines that come from  $388 R_E$  and  $495 R_E$ . That is:

$$\int_{\text{earth}} \mathbf{B} \cdot d\mathbf{A} = \int_{\text{tail}} \mathbf{B} \cdot d\mathbf{A} \quad (1)$$

or

$$\pi B_0 R_E^2 \int_{23^\circ}^{\theta_1} \sin \theta d\theta = \int_{388 R_E}^{495 R_E} B_z(y) T dy \quad (2)$$

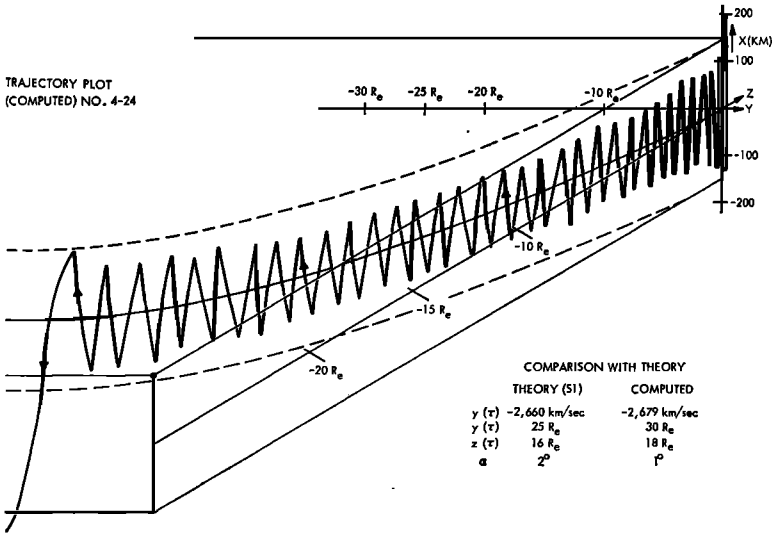


Fig. 3. Isometric trajectory plot of a proton trajectory in the dipole and tail model, see Figure 1,  $B_z = 1/4 \gamma$ ,  $b = 20 \gamma$ ,  $d = 600$  km,  $a = 1/3$  v/km. For this model the earth would be at  $-50 R_E$ . Initial conditions:  $x = 600$  km,  $y = z = 0$ ,  $\dot{x} = -60$  km/sec,  $\dot{y} = 15$  km/sec,  $\dot{z} = 10$  km/sec.  $\tau$  is the ejection time, and  $\alpha$  is the ejection pitch angle.

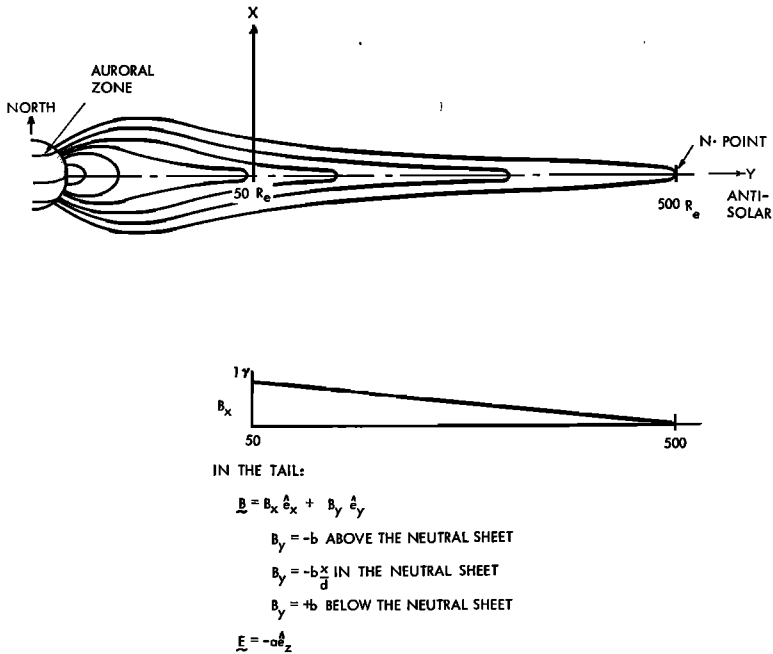


Fig. 4. Extended tail model. Field lines are sketched for the above model, where  $B_0$  is assumed to vary as indicated, going from  $1 \gamma$  at  $50 R_E$  to 0 at  $500 R_E$ .

where  $B_0 \approx 60,000 \gamma$  (assumed constant if  $\theta_1$  is small),  $B_x(y)$  is taken from Figure 4, and  $T$ , the tail width, is assumed about  $40 R_E$ . For the above example,  $\theta_1$  comes out to be about  $23.4^\circ$ ; thus for this model of the fields, protons are ejected along field lines that intersect the earth's surface about  $0.4^\circ$  lower latitude than do the field lines along which energetic electrons are ejected. For the dipole and tail model,  $\theta_1 = 23.7^\circ$ , so  $\theta_1$  is not very sensitive to the model used.

From the analytic study, Part 1, it was found that the velocity of the particles ejected from the neutral sheet varies inversely as  $B_x$ . For electrons, using  $|E|_{\text{tail}} = 0.3 \text{ v/km}$ ,  $|B|_{\text{tail}} = 20 \gamma$ , we have

$$W = (1.2/B_x^2)$$

where  $W$  is the energy in ev, and  $B_x$  is in gammas. For the model used in Figure 4, we have

$$B_x = (500 - y/450)\gamma \quad (3)$$

with  $y$  measured in earth radii ( $R_E$ ). Therefore the ejected electron energy as a function of distance back in the tail is

$$W = \frac{2.4 \times 10^5 \text{ ev}}{(500 - y)^2} \quad (4)$$

and the equivalent proton energy is found by multiplying the right-hand side of equation 4 by 1,836, the proton-to-electron mass ratio. Both expressions for the energy are valid until the maximum potential across the tail has been gained (see Table 1). The colatitude is found as a function of distance in the tail by conserving flux as before, and from equation 4 the electron energy can be found as a function of colatitude, and this is:

$$\theta - \theta_0 = 0.14/W \quad (5)$$

where the angles are in radians for  $W$  in ev, and  $\theta_0$  is taken to be about  $23^\circ$ . From equation 5 the latitudinal separation on the earth for ejected electrons from 1 to 10 kev, for example, is found to be about  $1.3 \times 10^{-4}$  radians, which corresponds to a beam width of about 0.8-km thickness at the earth.

#### FIELD-LINE LOADING

M. P. Nakada (personal communication, 1965) has suggested that field-line loading may

be an important problem for any auroral theory in which the particle source is located far away. That is, fluxes of particles with energies and intensities large enough for auroras may have more energy density than magnetic field energy density if they come from weak regions of magnetic field and if these fluxes are isotropic in the weak field region.

*O'Brien* [1964] observed that the average energy flux of electrons with energies greater than 1 keV precipitated in the auroral zone is about  $4 \text{ ergs cm}^{-2} \text{ sec}^{-1}$ . He also notes that fluxes may occasionally be as high as  $2000 \text{ ergs cm}^{-2} \text{ sec}^{-1}$ , [*McIlwain*, 1960; *O'Brien and Laughlin*, 1962]. Assuming these fluxes are of 5-keV electrons, the above numbers imply fluxes of  $6 \times 10^8$  to  $3 \times 10^{11}$  electrons  $\text{cm}^{-2} \text{ sec}^{-1}$ , and thus densities of 0.01 to 60 electrons  $\text{cm}^{-3}$ . *Omholt* [1963] estimates densities as high as 1000 electrons  $\text{cm}^{-3}$  at the time of an intense aurora.

Liouville's theorem should be valid between the earth and the particle source in the tail, and if it is assumed that the source distribution is isotropic in velocity space, then the spatial density would be the same at the source and at the earth. For a 15- $\gamma$  tail field, the magnetic energy density is about  $10^{-9} \text{ erg/cm}^3$ , but the particle energy density corresponding to the high fluxes quoted by *O'Brien* and the large densities from *Omholt* is larger by at least two orders of magnitude.

Therefore it seems unreasonable to assume that the distribution function is isotropic at the source. *O'Brien* [1966] in fact concludes, 'The very intense particle fluxes that cause bright nighttime auroras and spikes are concentrated in a cone only a few degrees wide centered around the local  $B$  in the equatorial plane. If the energization processes are due to mechanisms such as magnetic merging in the tail, with the conversion of magnetic energy to particle kinetic energy, then this flux directionality should be made a requisite of the postulated mechanism.' From Table 1 it is seen that this mechanism does indeed produce particles concentrated in a cone a few degrees wide about the local  $B$ .

#### DISCUSSION OF ANALYTICAL APPLICATIONS

The significant results of the previous applications of the analytic theory Part 1 to a tail model of the earth are that particles are ac-

celerated, turned in toward the earth, and ejected from the neutral sheet with small pitch angles to the magnetic lines of force. This ejection at small pitch angles may be important for the prevention of 'field-line loading' as discussed in the previous section. Certainly more experimental observations on particle fluxes and energies and magnetic field configurations are needed before some of these questions can be answered.

These applications also predict proton auroras to be at lower latitudes than electron auroras (for the second model the latitudinal separation is about  $0.4^\circ$ ). *Omholt* [1963] says, 'There is often (perhaps always) a distinct dark region (up to  $1^\circ$  in latitude) between the "proton aurora" and the main forms.' *Sandford* [1966] says that on a day-to-day basis protons seem to come in a degree or so equatorward of discrete electron events.

Electrons in the energy range from 1 to 10 keV would be found in an extremely thin beam at the earth, i.e. about 1-km thickness. Equation 5 also predicts a hardening of the beam with latitude, but the electric drift between the ejection point and the earth has been neglected (this would tend to move the higher energies to slightly lower latitudes), and the self-consistency of such a thin beam has not been investigated. This result may therefore be incorrect.

The analytic results also predict a monochromatic beam at a given point in space, but those results are approximate and based on the assumption that the perpendicular component of magnetic field in the neutral sheet,  $B_z$ , is constant. For a linear variation of  $B_z$  as used here, a different spectrum should be found, but this requires computing and has not yet been done.

*O'Brien* [1964] suggests that a major experimental study should be made to determine the limits of isotropy in the incident beam and thus the cause of auroral precipitation. He finds that for electrons with energies greater than 40 keV, fluxes are isotropic to within 10% at 1000 km. The results of the present study imply fluxes from the tail within a cone of the order of 0.1 radian, and thus one would expect to measure isotropic fluxes out to about  $2.5 R_E$ .

The difference between the second model used (as in Figure 4) and the dipole and tail model lies in the distance from the earth at which par-



ticles are ejected from the neutral sheet. Indeed, any model giving a different variation of  $B_z$  (hence a different rate of crossing of field lines through the neutral sheet) will merely move the particle ejection points toward or away from the earth. If, however,  $B_z$  is negative, then particles will be turned away from the earth and will be ejected into the tail (in the antisolar direction). Close to the earth, however,  $B_z$  is positive, so it is likely that it becomes very weak far from the earth, and it would reverse if there is a neutral point somewhere in the tail. Such a reversal is likely if the interplanetary field has a southward component and the field has to eventually fit onto the interplanetary field. Thus, if there is a neutral point in the tail as indicated in Figure 4, and if the electric field is as indicated, particles will be shot in toward the earth between the earth and the neutral point, and will be shot away from the earth on the other side of the neutral point.

For particles to be trapped in the neutral sheet, it is only necessary that the magnetic field reverse across the sheet so that the magnetic force term in the Lorentz-force equation reverses across the sheet. The specific linear variation used in Part 1 lends itself to analytic solution and is probably valid over some portion of the sheet.

An important auroral observation is the existence of multiple arcs [Akasofu, 1965]. The models presented here predict an electron beam and a proton beam, or sheet, separated latitudinally, but not multiple electron output sheets, which would be necessary to produce multiple arcs. Chapman [1966] suggested that there may be multiple neutral points or lines in the geomagnetic tail each attached to a different earth-based field line, for the production of multiple arcs. If such a field topology did exist, the mechanism reported on here could work at each neutral point. However, it is not necessary that there be multiple neutral points to obtain multiple output sheets for this mechanism. If the normal magnetic field component within the neutral sheet becomes alternately large and small with increasing geocentric distance, but not necessarily going to zero, then multiple thin sheets of electrons can be produced. Such a changing field component might be produced by a variation in the neutral sheet current or

by instabilities. If the normal component varied between 0.1 and 0.01  $\gamma$ , for example, then multiple electron output sheets would be produced, but the single sheet of output protons would still be found equatorward of the multiple electron arcs.

That is, a multiplicity in the output proton distribution would develop only if the variation of the normal component was an order of magnitude or more larger.

### THREE-DIMENSIONAL DIPOLE AND TAIL NUMERICAL SOLUTION

Using a current sheet magnetic field model about an  $x$ -type neutral point, Speiser [1965a] presented some results of a numerical calculation of proton trajectories. Those results showed that accelerated protons emergent along magnetic lines of force, have their greatest intensity in a thin output sheet. Little was known at that time, however, about actual magnetospheric tail fields, so no connection was made from the assumed field lines to those connected at the earth.

The same computations can now be made to map the output sheets of accelerated particles onto the earth using a three-dimensional dipole and tail model. The first model used in the previous analytical section (Model 1 for  $B_z$ ) is thus a simplification of this model. This model is not a neutral point model, but it is similar to a neutral point model with the neutral point at infinity. The method of procedure is the same as used before [Speiser, 1965a] and will only be summarized here.

1. A three-dimensional dipole field is added to a tail field,  $B_t$ , as indicated in Figure 1. This field is certainly incorrect in the day-side magnetosphere and will only be used for calculations in the tail.

2. A proton trajectory is started at some point on an output plane ( $x_0 - z_0$  plane), with a velocity directly along the magnetic line of force through that point, and the trajectory is then numerically solved backward in time. After the particle has passed through the neutral sheet (backward in time), its velocity on the input side is noted. Assuming a Maxwellian distribution, a value for the distribution function can then be found on the input side.

3. Using Liouville's theorem (the distribution function is constant along a trajectory) an

intensity map over the output plane ( $x_0 - z_0$ ) may then be made. That is, defining a distribution as

$$f = f_{\max} e^{-(F^2/V_t^2)}$$

minima in the function  $F$  will imply maxima in the distribution function,  $f$ . (A Maxwellian distribution is chosen to illustrate the behavior,  $f_{\max}$  is the maximum value of  $f$  at  $F = 0$ ,  $V_t$  is a thermal velocity, and  $F^2 = (\mathbf{V} - \mathbf{u})^2$  where  $\mathbf{V}$  is the particle velocity and  $\mathbf{u}$  is the bulk flow velocity.) This use of Liouville's theorem is qualitative and only gives an upper limit to the output particle density that can be achieved. That is,  $N_0 = N_i(E_0/E)^{3/2}$ , where  $N_0$ ,  $E_0$  are output density and energy, and  $N_i$ ,  $E_i$  are the input density and energy. For  $E_i \approx 100$  ev from the solar wind, and  $E_0 \approx 10$  kev for auroral particles we see that as an upper limit the density of the auroral particles could be 1000 times the solar wind density.

4. Regions of greatest intensity may then be mapped onto the earth by solving for the trajectories forward in time from the output plane.

Figure 5 shows backward plots for three protons, with only the  $x$  component versus time. The strength of the tail field,  $b$ , is  $20 \gamma$ , the

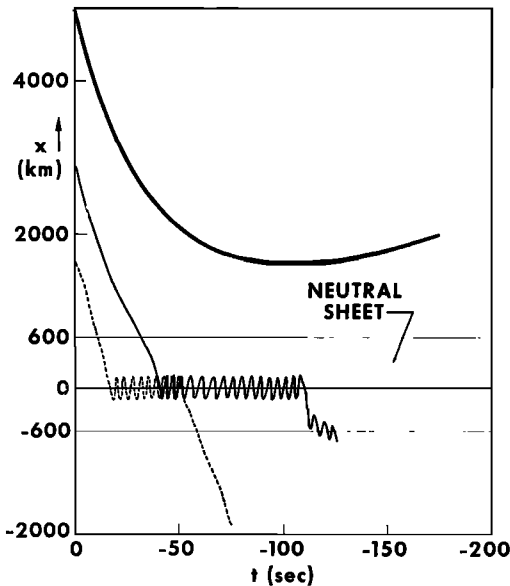


Fig. 5. Backward trajectory plots, starting at  $z_0 = 0.1 R_E$ ,  $y_0 = 26.5 R_E$ ,  $v_0 = 1664$  km/sec.  $x$  versus  $t$ ; zero pitch angle at  $t = 0$ .

$y$  and  $z$  starting positions are  $y_0 = 26.5 R_E$ ,  $z_0 = 0.1 R_E$ , and  $v_0 = 1664$  km/sec for all trajectories. It is seen that if the particle starts with too large an  $x_0$ , it follows a field line back for a time, and then the electric drift causes the particle to drift away from the neutral sheet (backward in time). Since the particle's energy changes only in the neutral sheet, its speed is the same on the input side ( $t \approx -170$  sec). A second particle starts at  $x_0 \approx 3000$  km, reaches the neutral sheet, and oscillates about 16 times before input. Its energy has thus changed very much, and the particle can have a small velocity at input. The remaining trajectory also reaches the neutral sheet, but does not stay in long before input ( $t \approx -50$  sec). Its energy has therefore not changed much, and its input velocity is not as small as the preceding particle. Using the ideas from Part 1, we can say that this latter particle did not stay so long in the neutral sheet because  $B_z$  was larger for it, and the particle was ejected sooner than the preceding particle.

If one therefore expects the majority of particles incident on the neutral sheet to have small velocities, then the middle trajectory of Figure 5 should indicate the region of the largest output intensity, since particles with nearby trajectories come from highly populated regions of velocity space. This is a crude explanation of the use of Liouville's theorem, and of why such a model produces the largest intensities of output particles in thin sheets.

Figure 6 shows a contour map of  $F$  on the  $x_0 - z_0$  plane where  $F$  is defined above, and minima in  $F$  are maxima in the distribution function.  $F$  does reach lows of about 100 km/sec in regions near the center of the 200-km/sec contours, and the breaks in the 200-km/sec contours are probably not real, i.e. they would probably disappear if a plot with a finer net were used.

Figure 7 shows the positions of the output sheets with large intensity ( $F < v_t$ , and  $v_t \approx 100$  km/sec) on the  $x_0 - z_0$  plane for various tail fields.

Figure 8 maps the intersections of Figure 7 forward onto the earth, using the dipole and tail model. These mappings look intriguingly like the auroral oval [Akasofu, 1966]. They go to higher latitudes away from midnight. They do in fact tend to close around on the day side,

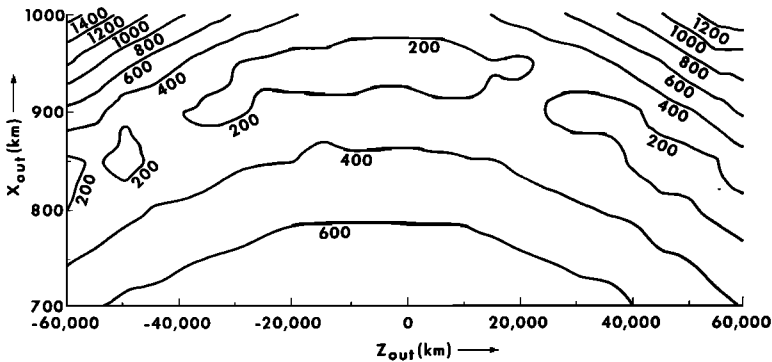


Fig. 6.  $F$  contours on the  $x_{out} - z_{out}$  plane in units of 200 km/sec,  $y_0 = 26.5 R_E$ ,  $V_0 = 1664$  km/sec,  $B_t = 80 \gamma$ .

but those points were not included since a more realistic field model should be used on the day side. O'Brien [1966] says that it is difficult to see how particles from the neutral sheet in the tail could form day-side auroras. However, from Dungey's [1961, 1962] arguments on the field topology and from mappings like this (Figure 8), it appears that some day-side field lines can certainly get dragged back into the tail and be closed across the neutral sheet. It is, of course, also possible that some day-side auroral particles come from the day-side current sheet where field line merging may also be taking place [Dungey, 1962], or from other processes as suggested by O'Brien.

A somewhat more realistic tail magnetic field model is used by Williams and Mead [1965]. They have added a current sheet in the tail that is cut off at about  $10 R_E$ , and this tends to produce a depression of the field in the 'cusp' region and drags field lines out into the tail from a lower latitude than does the simple dipole and tail model used here. Using their model, the output sheet with a  $20\text{-}\gamma$  field from Figure 7 would map onto the earth somewhere between  $65$  and  $70^\circ$  latitude at midnight rather than about  $73^\circ$  as in Figure 8. This would put the mappings into better agreement with the midnight position of the auroral zone.

Eather [1966] argues that the helium  $\lambda 5876$  line due to precipitating alpha particles should be detectable in auroras. In a previous section, 'Model 2 for  $B_z$ ,' it was found that 30-keV protons would come from about  $400 R_E$ , while drifting about a half-width of the tail across the tail. For the same model, alpha particles will

drift the same distance across the tail at a distance of about  $350 R_E$  and will be ejected with twice the proton energy, 60 keV. Conserving flux as before, we find that the 60-keV alpha particles will arrive at the earth about  $\frac{1}{3}^\circ$  equatorward of the precipitating 30-keV protons, which is about the same latitudinal separation as that of the incoming proton and electron sheets predicted by this model. Eather suggests that a search be made for He  $I \lambda 5876$  emission to the north of, south of, and in the hydrogen emission zone. These results imply that, if it can be observed, it should be found to the south (northern hemisphere) with about the same spacing as that between proton and electron auroral forms.

#### SUMMARY AND CONCLUSIONS

A simple model is used for the possible electric and magnetic field configuration in the tail of the earth's magnetosphere. Satellite measurements out to about  $30 R_E$  have shown the general result of high-latitude field lines being pulled back in the tail, with the formation of a neutral or current sheet across which the field reverses direction. For the models discussed here, an electric field across the tail generally from dawn to dusk is assumed to exist, and a weak magnetic field component perpendicular to the neutral sheet and tending toward zero with distance is assumed inside the neutral sheet. These features are necessary if field-line merging or reconnection is going on at a neutral point in the tail. There may be times when the tail is long and merging is negligible, as suggested by Dessler [1965]. At those times

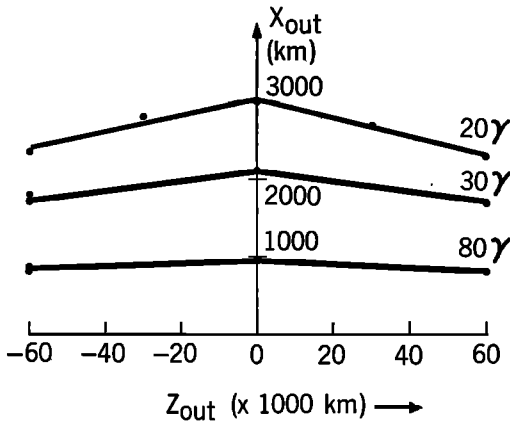


Fig. 7. Intersections of the output sheets ( $F < V_i$ ) with largest intensity with the  $x_0 - z_0$  plane ( $y_0 = 26.5 R_E$ ) for tail fields of 20, 30, and 80  $\gamma$ .

this acceleration process will not be effective. The self-consistency of these solutions and the effects of various instabilities should be investigated, but this has not yet been done.

Two models, a dipole and tail model (Figure 1) and an extended tail model (Figure 4) are used to apply the analytic theory Part 1. In both cases, electrons and protons incident on the neutral sheet are trapped, accelerated, turned toward the earth, and ejected from the neutral sheet with small pitch angles to a magnetic line of force. Such a flux directionality for bright nighttime auroras has been termed a necessary requisite of any tail acceleration mechanism by O'Brien [1966]. The main difference between the two models is the ejection point back in the tail. For the two models, electrons of the order of 10 keV are ejected at about 150 and 500  $R_E$ , respectively, and for protons of about 30 keV, the ejection distances are about 30 and 400  $R_E$ , respectively. The model produces extremely thin beams of incoming particles at the earth, about 1 km for electrons between 1 and 10 keV. Proton auroras are produced at latitudes about  $1/3^\circ$  lower than electrons, and isotropic fluxes should be expected out to about 2.5  $R_E$ .

Many trajectories of about 12-keV protons have been computed for a three-dimensional dipole and tail model to see how the fluxes map onto the earth. Using Liouville's theorem, regions of large intensity are found that are near the auroral zones when mapped onto the earth.

An increase of the tail field moves the mapping onto the earth to lower latitudes.

These features agree generally with some auroral observations, namely: the latitude of the auroral zones, the energies of auroral protons and electrons, the fluxes of auroral protons and electrons, the appearance of proton auroras at lower latitudes than electron auroras, a dawn/dusk asymmetry for electron/proton auroras, the movement of auroral forms to lower latitudes with an increasing tail field (hence solar wind pressure), the thinness of auroral forms, and the gross conjugacy of auroral events. The models predict precipitating alpha particles to be of the order of  $1/3^\circ$  equatorward of precipitating protons. Azford [1966] has suggested that turbulence or noise in the current sheet will be sufficient to upset these trajectories. Intuition suggests that there will be appreciable noise, probably generated by instabilities, but a completely noise-dominant situation is not necessarily expected on theoretical grounds and would not be consistent with the observational requirement of an anisotropic source and thin auroral arcs. Dungey and Speiser [1967] find that the particle trajectories without noise are useful for formulating the problem with noise. The major effect of noise should be to smear the energy distribution

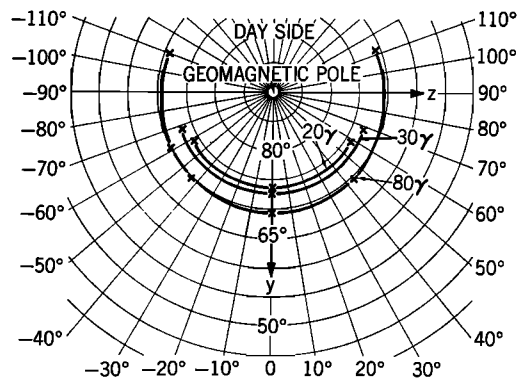


Fig. 8. Mapping of intersections of Figure 7 onto the earth using the simple dipole and tail model. These mappings move about  $5^\circ$  lower in latitude when Williams and Mead's [1965] field model is used. An asymmetry would be expected for higher energy particles, or for the same energy if the electric field were weaker, when a tail of finite width is considered. Protons would be shifted toward dusk and afternoon, and electrons would be shifted toward dawn and morning.

and increase the ejection pitch angles somewhat. Other instabilities and time dependencies may occur, and these may contribute to the dynamic nature of the aurora.

The models may be applicable to other situations where neutral points or sheets are thought to exist, such as the day-side magnetospheric current sheet, neutral points in the interplanetary field, the influence of Jupiter's satellite Io on Jupiter's radio emission [Warwick and Dulk, 1965] and solar flares. Since the models predict particles accelerated and ejected with the same velocity independent of mass, a comparison could be made, for example, of helium and proton energies in a flare (or in the geomagnetic tail) to see if they are in the ratio of the masses. If some of the recently observed 'red spots' on the moon [Kozyrev, 1959, 1963; Greenacre, 1963] are due to fluorescence, the particles could come from this mechanism while the moon is in or near the earth's tail.

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