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PARTON RECOMBINATION MODEL INCLUDING RESONANCE PRODUCTION

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ABSTRACT:

Possible effects of resonance production on the meson inclusive distribution in the fragmentation region are investigated in the framework of the parton recombination model. From a detailed study of the data on vector-meson production, a reliable ratio of the vector-to-pseudoscalar rates is determined. We then examine the influence of the decay of the vector mesons on the pseudoscalar spectrum and find the effect to be no more than 25% for $x > 0.5$. The normalization of the non-strange antiquark distributions are still higher than those in a quiescent proton. The agreement between the calculated results and data remains very good.

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PARTON RECOMBINATION MODEL INCLUDING RESONANCE PRODUCTION

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I. Introduction

The parton model has recently been extended to describe the production of hadrons with small p_T in the fragmentation region of the incident particle.[1 - 6] In the framework of a simple quark-antiquark recombination model the description of the shapes and ratios of hadron spectra is quite successful. In this model a fast (valence) quark, whose momentum distribution is assumed to be the same as revealed in deep inelastic scattering, recombines with a slow (sea) anti-quark to produce the detected meson. In fitting the data, one is led to a description of the momentum distribution of the non-strange sea which is significantly higher in normalisation than that observed in deep inelastic scattering.[2,3]

Of course when a quark and antiquark recombine in this model, there is no reason why they should not form a resonance which in turn decays, producing the detected particle. It has been realised by several authors [3,5,6,7] that this resonance "contamination" should be included in the model and some rough estimates [5,7] of the effect of ρ production have indicated corrections of 20-30% for $x > 0.5$.

Independent of the considerations of any specific model, it is important to know what proportion of pions, for example, are produced directly or from resonance decays. A detailed quark model calculation [8] predicted that 90% of all produced pions should be decay products of resonances and there sometimes appears to be experimental confirmation for dominance by vector-meson production.[9,10] However one expects the proportion of contamination to be highest in the central region, where the pion cross-section is largest and so this proportion may well be much smaller in the fragmentation regions.

In this paper, we make a detailed estimate, using the recombination model, of the effects of vector-meson production on the $\pi^{\pm}$ and $K^{\pm}$ fragments from the proton. We carefully examine all the data on inclusive ρ^0, K^* production in order to fix up the probability of the $q\bar{q}$ forming a vector-meson or directly a pseudoscalar meson. We investigate the effects of varying the angular distributions of the vector-mesons, of which we consider ρ, ω and K^*. Nucleon resonance decays can be easily shown to have a negligible effect on the meson spectrum.

At $x = 0.5$ we find that vector-meson decays account for about one-third as many pions or kaons which are directly produced. As expected, this proportion rises as x decreases and the meson spectrum given by the recombination model is thereby enhanced for x below 0.5. Although one cannot find any tendency of the experimental x distributions to have a slope consistent with such an amount of resonance contamination at small x, one does not really expect the recombination model to be valid in that region. The inclusion of the resonance decay component certainly improved the π^-/ρ^0 ratio. As x decreases the ratio increases insufficiently rapidly compared with data if that component is not included.

One interestion question is whether the inclusion of resonance contamination can affect the amount of non-strange sea quarks which the recombiration uses to fit the experimental data. It has been conjectured [3] that resonance effects could significantly reduce the ratio of the nonstrange/strange sea components. We find that this ratio is not very different when resonance effects are switched on but that it is possible to make an adequate fit to the data for $x > 0.5$ with nonstrange and strange sea components each reduced by roughly one-half.

In the next section we write down the formalism for describing the resonance contamination. In section III we make a detailed evaluation of the experimental situation and in the final section we make fits the various data and make our conclusions.

II. Recombination Model with Resonance Contributions

In the recombination model [2] the inclusive spectrum for a meson M_{ij} (i,j are the quark labels) fragmenting from an incident hadron is

$$H_{M_{ij}}(x) = \frac{E}{\sigma_{Tot}} \frac{d\sigma}{dx_L} = \int F_{ij}(x_1,x_2)\ R_{ij}(x_1,x_2,x)\ \frac{dx_1}{x_1}\frac{dx_2}{x_2} \qquad (1)$$

where x, x_1, x_2 are the momentum fractions of the produced meson and of the partons. In (1) the idea of impulse approximation has been used. Although no large momentum transfer is involved, it is justified on the grounds of short-range correlation among the partons. $F_{ij}(x_1,x_2)$ is the joint distribution of two partons, i quark at x_1 and j antiquark at x_2.

In Ref. 2 the simple factorizable form for F_{ij} was adopted, i.e.

$$F_{ij}(x_1,x_2) = F_i(x_1)F_j(x_2)\rho(x_1,x_2) \tag{2}$$

where

$$\rho(x_1,x_2) = \beta(1 - x_1 - x_2) \tag{3}$$

β being a constant. The simple form of (2) is commensurate with the simple picture of the meson production process in terms of the recombination of a quark and an antiquark. A possible interpretation of (1) and (2) is that j represents an "effective parton" which carries the necessary quantum number to form the meson M_{ij} but may in itself contain gluons. Eq. (2) is a crude way of estimating the momentum distribution in a kinematical region where little is known about such joint probabilities. In Ref. 4 an attempt is made to modify (2) to a form that satisfies the momentum sum rules by using the independent emission model. In our view such a modification, while suggesting an interesting possibility for F_{ij}, does not improve the credibility of the model. Although the integrations in (1) extends over all values of x_1 and x_2 between 0 and 1, the integrand is physically meaningful at best over a limited region of the integration space, which, one hopes, makes the dominant contribution to the integral. In that limited region the requirement of sum rules (involving integration over the whole phase space) may not be meaningful for F_{ij}. The necessity of an enhanced sea [2,3] is an indication that the usual parton distributions in the central plateau are inadequate for our purpose. Thus we shall continue to use (2) and relinquish the possibility of determining the normalization of F_{ij} on account of the unknown constant β.

The recombination function was assumed in Ref. 2 to be

$$R_{ij}(x_1,x_2,x) = \alpha \frac{x_1 x_2}{x^2} \delta\left(\frac{x_1}{x} + \frac{x_2}{x} - 1\right) \tag{4}$$

on the basis that it should have the same functional dependence on x_1/x and x_2/x as that for the valence partons in a meson. Counting rule is used to determine the factor x_1x_2/x^2 in (4). It is important to recognise that R_{ij} specifies the range of interaction (in rapidity) among the partons that contribute to the formation of the meson. The range would be longer if R_{ij} vanishes as $(x_1x_2)^a$ with $0 < a < 1$. It would stretch over into the wee region of the central plateau if a were zero, and divergent result would ensue for (1). Thus a = 1 is consistent with the usual short-range correlation, which in turn is crucial for the justification of the impulse approximation employed in (1).

To generalize the recombination function to the case that includes the contribution from vector mesons, we write

$$R_{ij}(x_1,x_2,x) = c_{ij}\, R^M_{ij}(x_1,x_2,x)$$

$$+ \sum_k c_{ik}\, d^i_{kj} \int R^V_{ik}(x_1,x_2,y)\, D^{M/V}\left(\frac{x}{y}\right) \frac{x}{y^2}\, dy \qquad (5)$$

The first term represents the direct recombination into the detected meson and the second term represents production and decay of the vector meson. R^M_{ij} and R^V_{ij} are defined by the r.h.s. of Eq. (4) with the coefficients α_M and α_V respectively; α_V includes the spin factor 3 and the mass effect.

The c_{ij}, d^i_{kj} are just the squares of Clebsch-Gordon coefficients while $D(x/y)$ describes the nature of the vector-meson decay V_{ik} into the pseudoscalar meson M_{ij} with the normalisation

$$\int_0^1 D(z)\, dz = 1 \qquad (6)$$

The scaled variable $z = x/y$ is simply related to the "helicity" angle θ_h which is the direction of the decay product, in the vector-meson rest-frame, with respect to the initial hadron direction:

$$\cos\theta_h = \frac{m_V}{q}\left(z - \tfrac{1}{2} + \Delta\right) \qquad (7)$$

where q is the momentum of the detected meson in the vector-meson rest frame, and Δ is a constant depending on the masses involved. Thus if we know the angular distribution for the relevant vector-meson, which is usually translated into estimates of the density-matrix elements for the decay, the $D(z)$ is fully determined. Unfortunately, while there are very good experimental data for the ρ, ω, K^* density-matrices produced in the incident meson direction, there is virtually no experimental information on the backward production process which is the relevant kinematic region here.

In the appendix, we write down expressions for D(z) for the two-body decays $\rho \to 2\pi$, $K^* \to K\pi$, and $\omega \to 3\pi$ for various possibilities of the helicity angular distribution, which we use in the section IV.

Feeding in expressions (4) to equation (5), we get

$$R_{ij}(x_1,x_2,x) = c_{ij}\alpha_M \frac{x_1 x_2}{x} \delta(x_1 + x_2 - x)$$

$$+ \sum_k c_{ik} d^i_{jk} \alpha_V x_1 x_2 x \int_x^1 D^{M/V}(\tfrac{x}{y})\ \delta(x_1 + x_2 - y) \frac{dy}{y^3} \qquad (8)$$

Substituting into (1), using Eqs. (2),(3) gives

$$H_{M_{ij}}(x) = c_{ij}\alpha_M \beta \frac{(1-x)}{x} \int_0^x dx_1\ F_i(x_1)\ F_j(x - x_1)$$

$$+ \sum_k c_{ik} d^i_{jk} \alpha_V \beta\, x \left\{ \int_0^x dx_1 \int_{x-x_1}^{1-x_1} dx_2 + \int_x^1 dx_1 \int_0^{1-x_1} dx_2 \right\} \frac{(1 - x_1 - x_2)}{(x_1 + x_2)^3} \cdot$$

$$\cdot\ F_i(x_1)\ F_k(x_2)\ D^{M/V}\left(\frac{x}{x_1 + x_2}\right) \qquad (9)$$

III. The Data

We shall be comparing the expressions (9) for $pp \to \pi^{\pm}X$ and $K^{\pm}X$, including the effects of ρ, ω and K^* resonances, with the data at high energies on those reactions which cover the large x region. There are two recent experiments on $\pi^{\pm}$, $K^{\pm}$ inclusive production in the proton fragmentation region; a Fermilab-Illinois collaboration [11] at p_{lab} = 100, 200 and 400 GeV/c and an ISR experiment by the CHLM collaboration [12] at $\sqrt{s}$ = 45 GeV.

The expression (9) refers to the p_T^2 integrated quantity $(E/\sigma_{TOT})d\sigma/dp_L$ and since both experiments measure data only at specific p_T values and over different ranges of x at each p_T, we must first interpolate and extrapolate the data to do the p_T^2 integration. To do this we performed a fit to the data of each experiment in turn, using a parametrisation

$$E\frac{d^3\sigma}{dp^3} = \sum_{i=1}^{N} \frac{A_i(1-x)^{D_i}}{(1+p_T^2/C_i)^{1+B_i}} \tag{10}$$

and then

$$\frac{E}{\sigma_{TOT}}\frac{d\sigma}{dp_L} = \frac{\pi}{\sigma_{TOT}} \sum_{i=1}^{N} \frac{A_i C_i}{B_i} (1-x)^{D_i} \tag{11}$$

We found N = 3 sufficient to give a good fit. There is a significant difference in the shapes of the resulting spectra, say for $\pi^\pm$, between the two experiments which is apparent in a direct comparison of the data on $Ed^3\sigma/dp^3$ at $p_T = 0.75$ GeV even allowing for a shift in normalisation for either set of data. This is demonstrated in Fig. 7 where the two sets of data show a different x dependence. Since we have no prejudice against either set we make separate fits and extract two interpolated spectra, using Eqs. (10) and (11), for each detected meson to compare with the model in the next section.

IV. Comparison of Model with Data and Conclusions

Since the fast quark which recombines with an antiquark to form the detected meson is assumed to have the same momentum fraction distribution as seen in deep inelastic scattering, but at "low" Q^2 before the onset of scaling violation, we take a parametrisation for the valence quark from a good fit to that data, in our case we take that given by Field and Feynman.[13] We shall parametrise the sea-quark distributions by simple power law behaviours, i.e.

$$F_{\bar{u}}(x) = u_o(1-x)^{n_u}, \quad F_{\bar{d}}(x) = d_o(1-x)^{n_d}, \quad F_{\bar{s}}(x) = F_s(x) = s_o(1-x)^{n_s} \tag{12}$$

and adjust the parameters u_o, n_u etc. by fitting the inclusive data. This has been the procedure in previous attempts to compare the recombination model with the inclusive data[2,3]. Because we are including vector-meson resonances, we have, in addition to the parameters α_π, α_K which govern the probability of a specific $q\bar{q}$ pair forming a pion or kaon, additional parameters α_ρ, α_ω, α_{K^*} which are the corresponding quantities for the vector-mesons, whose spin factor is included in the definitions of the α's.

First, we fix up the value of α_ρ/α_π by looking at the data on inclusive ρ^0 production. We assume that ρ^0 will be produced only by direct recombination, not as decay products of higher mass resonances. We have just the first term of Eq. (9) with α_M replaced by α_ρ.

There exists now a great deal [14] of data on ρ^0 and K^* inclusive production. In Fig. 2, we take the data in the proton fragmentation region and show the situation for the ratio of the processes $ap \rightarrow \pi^- X$ and $ap \rightarrow \rho^0 X$ for $a = \pi^+, \pi^-, K^-, p$. There is a clear tendency for the ratio to fall with increasing x, as expected from the different charges of the π and ρ, but the precise magnitude of the ratio is not well determined. As far as the model is concerned, this ratio is not sensitive to the details of the sea-quark distributions etc. but only to the ratio α_ρ/α_π. Also it is sensitive to the effect of including the vector-mesons in the production of pions. The best fit is obtained by taking $\alpha_\rho/\alpha_\pi = 0.45$ which means, crudely speaking, that a given quark-antiquark pair is twice as likely to directly recombine into a π as into a ρ. The ratio of inclusive production of ρ^0 to K^{*+} is essentially determined by the product of the ratios α_ρ/α_{K^*} and d_o/s_o. This latter ratio is usually taken to be around 10 and if we look at the experimental ρ^0/K^{*+} ratio given by Böckmann [14] we find that this is consistent with taking $\alpha_{K^*} = \alpha_\rho$, which is in line with our assumption $\alpha_\pi = \alpha_K$. Likewise we shall assume $\alpha_\omega = \alpha_\rho$.

Next we turn to the inclusive $\pi^\pm$, $K^\pm$ distributions themselves, and as pointed out in the previous section we make separate comparisons with the Fermilab-Illinois data [11] and the CHLM data [12] from the ISR. In Fig. 3, we show a comparison with the former where we took the anti-quark distributions corresponding to the parameters

$$u_o = 0.45,\quad n_u = 7;\quad d_o = 0.60,\ n_d = 7;\ s_o = 0.057,\ n_s = 4.5. \qquad (13)$$

As we have stated before the valence quark distributions were taken from Field and Feynman [13]. There is, of course, the free parameter β which decides the overall normalisation for all detected mesons. This has been adjusted so as to give the best fit for the curves corresponding to the inclusion of resonance "contamination". We also show the curves corresponding to direct production of π, K only. We see that the α_V/α_M ratio determined above implies that at $x = 0.5$, for example, about 26% of all π^- are the result of vector-meson decays (97% of the decay π^- come from ρ^0). As expected, when x becomes smaller the proportion of resonance contamination increases significantly. The recombination model however is expected to be less valid in the region $x < 0.5$ and this is reflected in the deviation of the agreement with data as x decreases. In fact the "switching on" of the resonance contamination pronounces the divergence even further at these small x values.

In Fig. 4 we compare with the ISR data [12], taking the same anti-quark parametrisation as above but with β renormalised down by 20% to give the best overall fit. It is noticeable that, apart from the $x > 0.85$ region for π^+, the agreement for the pion distributions is very good - much better than with the Fermilab-Illinois data [11] in Fig. 3.

As we see, the effect of including vector-meson decays does not lead to any dramatic change in the recombination model. It was hoped [3] that such inclusion would allow the strange and non-strange sea distributions to become more alike. We find that production ratios are only slightly affected by resonance contamination - the π^+/K^+ ratio changes by only 6% at $x = 0.2$ and the π^-/K^- ratio changes by about 15%. In order to get a reasonable fit we still need to take $s_o/u_o \sim 0.1$ and $n_u - n_s \simeq 2.5$, as in previous fits without resonance contributions. The magnitude of the non-strange sea is effectively determined by the K^+/K^- ratio. For example, Duke and Taylor found that to fit the K^+/K^- ratio the non-strange sea had to be so enhanced that it accounted for more than half the proton's total momentum. In our fit we have not paid particular emphasis to the region $x < 0.4$ and the result is that we can allow a smaller non-strange sea - the parameters of Eq. (13) correspond to 27% of the proton's momentum being carried by the non-strange sea. But this is not significant; it has very little to do with resonance contributions, more with the fact that we do not seriously attempt to fit the K^+/K^- ratio in a region where the model may not apply. It also means that the recombination model with F_{ij} as given in Eq. (2) may not be reliable for the production of K^- since two sea quarks are involved and the factorizability of their distributions is questionable.

We point out that in the above fits we took the angular distributions of the vector mesons, in their own rest-frame, to be isotropic. This we did out of ignorance. As pointed out in section II there is virtually no determination of the density matrices for vector mesons in the proton fragmentation region. Nevertheless, it is important to know how sensitive are our results to the assumption of isotropy.

We carry out an exercise where we take three possibilities for the decay distribution; $\cos^2\theta_h$, isotropic, and $\sin^2\theta_h$ and compare with an analysis of direct and indirect pion production by Grässler et al. [14] at 16 GeV/c. They attempt to estimate in their bubble-chamber experiment what proportion of the π^- produced in π^+p interactions are the results of ρ^0 decay. In Fig. 5, we show their π^- direct and indirect cross-sections in the proton fragmentation region together with the results of the recombination model for the same quantities. One thing is clear - the amount of resonance contamination is very sensitive to the nature of the resonance (in the case ρ^0) decay. At $x = 0.5$, the indirect π^- production from ρ^0 decay varies by a factor 4 depending on which extreme choice we make for the angular distribution; $\cos^2\theta_h$ or $\sin^2\theta_h$. Thus it is important to measure the ρ density matrix elements in the backward direction so that we have some guide to the nature of the angular distribution. Judging from the comparison in Fig. 5, there is perhaps an indication favouring a $\cos^2\theta_h$ type decay for the ρ, although it may be dangerous to draw any definite conclusion especially at this comparatively low energy.

In conclusion, we would claim that any realistic model for describing inclusive particle production, such as the recombination model, has to incorporate the effects of particles produced from resonance decays. Some doubts have been expressed about the success of the recombination model in the past because this contribution had been neglected. We have found that this contribution can be incorporated and does not spoil the phenomenological success of the model in the region $x > 0.5$. For smaller x, when resonance effects become more significant, the agreement with data worsens but it can be argued that the model is unreliable there anyway. In any case, the model can be used as a framework for analysing what proportion of mesons do arise from the decays of higher states. The conclusions we reached were that, in the fragmentation region of the proton, the proportion is less than or equal to about onequarter though the precise amount will depend on the angular distribution of the relevant decay. The normalisation of the "effective" sea quarks is still higher than the antiquark distribution of the quiescent proton probed in deep inelastic scattering. This is reasonable in view of the gluon conversion in hadronic collisions. But our effective sea is not as enhanced as that found in Ref. 3. The model remains to be improved regarding the production of K^- and $\bar{p}$.

APPENDIX

THE VECTOR-MESON DECAY FUNCTION D(z)

First, let us consider the decay of ρ meson to two pions, one of which is the detected meson. It is easy to show that

$$\cos\theta_h = \frac{m_\rho}{q}(z - \tfrac{1}{2}); \quad z = x/y \tag{A.1}$$

where q is the "q-value" of the decay - i.e. the pion momentum in the ρ rest-frame, and θ_h, the helicity angle, is the direction of the pion relative to the proton measured in the ρ rest-frame. Since we have virtually no information on the density-matrix elements for the ρ meson in the proton fragmentation region, we shall choose three possibilities corresponding to an isotropic, $\cos^2\theta_h$, $\sin^2\theta_h$ angular distribution i.e.

$$D(z) = \left\{ \begin{array}{ll} 2q/m_\rho , & \text{(isotropic)} \\ \left(\frac{3}{2}\right)(m_\rho/q)^3 \ (z - \tfrac{1}{2})^2, & (\cos^2\theta_h) \\ \left(\frac{3}{4}\right)(m_\rho/q) \left[1 - (m_\rho/q)^2 \ (z - \tfrac{1}{2})^2\right], & (\sin^2\theta_h) \end{array} \right\} \text{ for } (\tfrac{1}{2} - q/m_\rho) \leq z \leq (\tfrac{1}{2} + q/m_\rho) \tag{A.2}$$

Next, let us consider the decay of K^* to K and π, where the pion is the detected particle. We then have

$$\cos\theta_h = \frac{m_{K^*}}{q}(z - \tfrac{1}{2} + \Delta) \tag{A.3}$$

where
$$\Delta = (m_K^2 - m_\pi^2)/2m_{K^*}^2 \tag{A.4}$$

and the corresponding expressions in (A.2) are trivially modified to include Δ. When we consider K^* decay but the kaon is the detected particle, the sign of Δ is reversed and the corresponding expressions to (A.2) follow trivially.

In considering ω decaying to 3 pions, one of which is the detected meson, we first look at the situation as if the remaining two pion system, "ρ", had a fixed mass. We then get an expression for $\cos\theta_h$ as in Eq. (A.3) but with

$$\Delta = (m^2_{"\rho"} - m^2_\pi)/2m^2_\omega \tag{A.5}$$

We then integrate over three-body phase-space and obtain

$$D(z) = \int_{2m_\pi}^{m_\omega - m_\pi} r(m_{"\rho"})\,dm_{"\rho"} D\left[\cos\theta_h(m_{"\rho"})\right] \Bigg/ \int_{2m_\pi}^{m_\omega - m_\pi} r(m_{"\rho"})\,dm_{"\rho"} \tag{A.6}$$

where $\quad r(m_{"\rho"}) = pq/m_\omega m_{"\rho"}$.

We illustrate the functions D(z) for the case of isotropic decay and for the case of the detected meson being a pion in Fig. 6. Note that, as expected the region of smaller $z = x/y$ becomes the more dominant region as the partner of pion in the decay becomes heavier.

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FIGURE CAPTIONS

1. Variation of the data of references (11) and (12) at $p_T = 0.75$ GeV/c. In each case the relative difference of the data from an arbitrary function $f(x) = 3.4(1 - x)^{3.34}$ mb. GeV^{-2} is plotted.

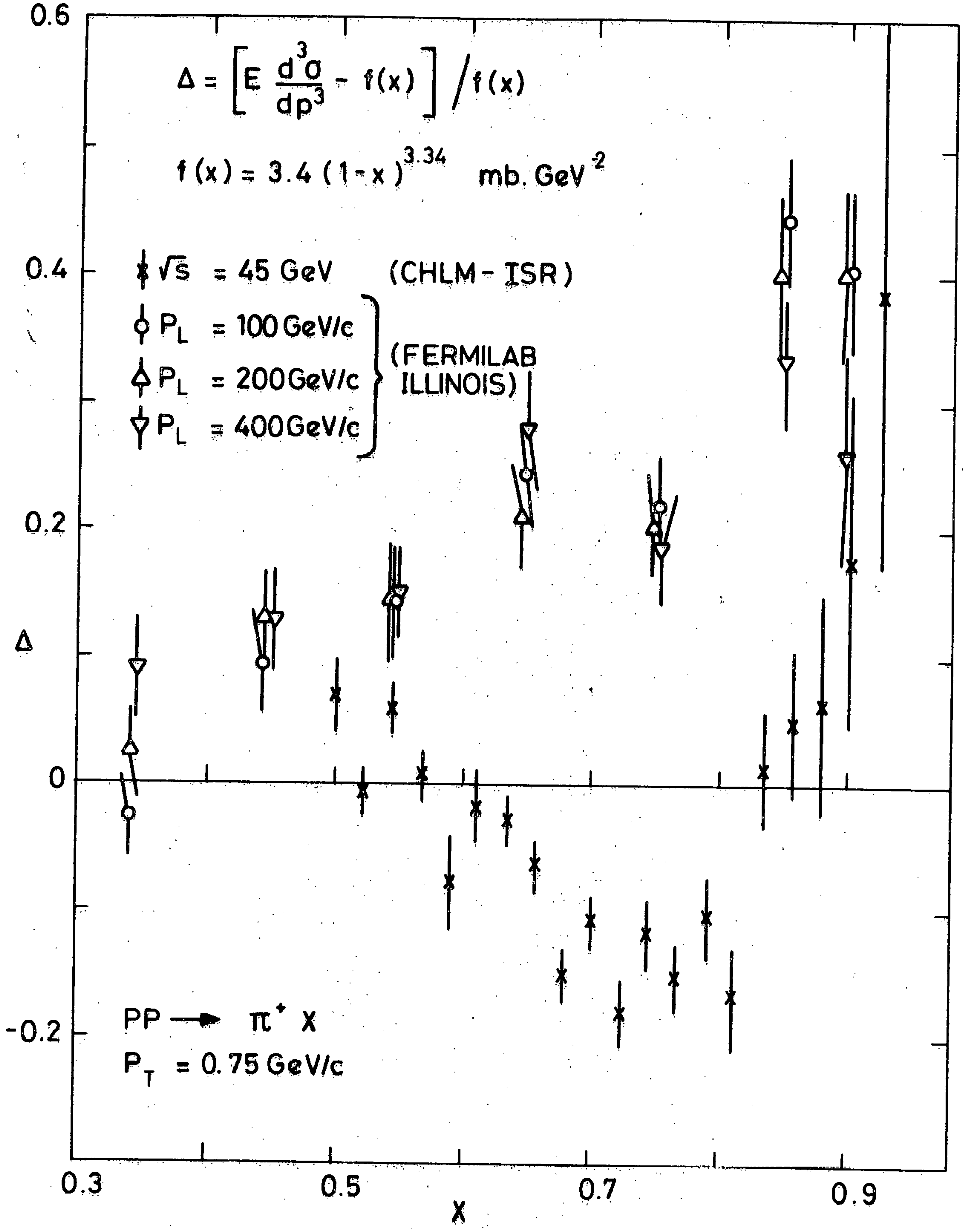
2. The ratio of the invariant cross-sections for the inclusive processes $ap \to \pi^- X/ap \to \rho^0 X$ for various choices of the beam particle and energies, plotted versus x in the proton fragmentation region. The two curves correspond to $\alpha_\rho/\alpha_\pi = 0.45$ in the recombination model, with and without the vector-meson contribution to the π distribution.

3. x distributions for $pp \to \pi^\pm X$, $K^\pm X$. The shaded areas are interpolations of the data of reference (11). The solid lines are fits from the recombination model, with parameters given in the text, including the effects of vector-meson resonances ρ, ω, K^*. The dashed lines correspond to dropping the resonance terms.

4. x distributions for $pp \to \pi^\pm X$, $K^\pm X$. The shaded areas are interpolations of the data of reference (12). The solid lines are fits from the recombination model, with parameters given in the text, including the effects of vector-meson resonances ρ, ω, K^*. The dashed lines correspond to dropping the resonance terms.

5. Comparison of the π^- x-distribution, in the fragmentation region of the proton, for the full inclusive process and for the pions which result from decays of ρ^0 mesons. The data are from Grassler et al. [(14)]. The solid line is the recombination model estimate, including vector-meson contributions (assuming isotropic decay). The dashed lines show the indirect inclusive distribution for π^- from ρ^0 decay assuming three possibilities for the angular distribution; $\cos^2\theta_h$, isotropic, and $\sin^2\theta_h$.

6. The function D(z) which describes the vector-meson decay distributions for the case of the detected meson being a pion. The distributions correspond to an isotropic angular distribution for ρ, K^* and ω decay.



0.6
0.4
0.2
0
-0.2
Δ
0.3
0.5
0.7
0.9
X
Δ = [E d³σ/dp³ - f(x)] / f(x)
f(x) = 3.4 (1-x)^3.34 mb. GeV^-2
√s = 45 GeV (CHLM - ISR)
P_L = 100 GeV/c
P_L = 200 GeV/c
P_L = 400 GeV/c
(FERMILAB ILLINOIS)
PP → π⁺ X
P_T = 0.75 GeV/c

FIG. 1

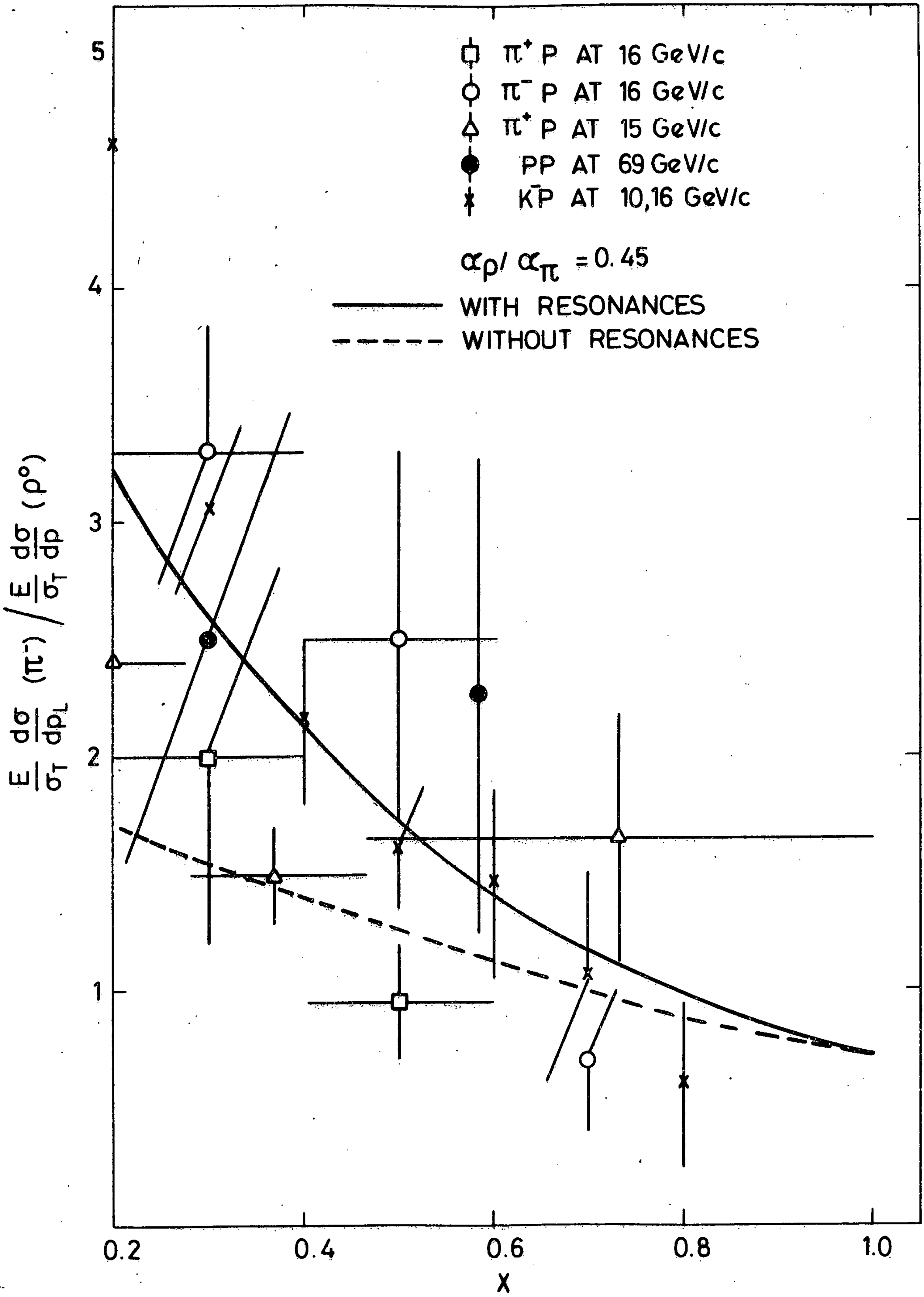
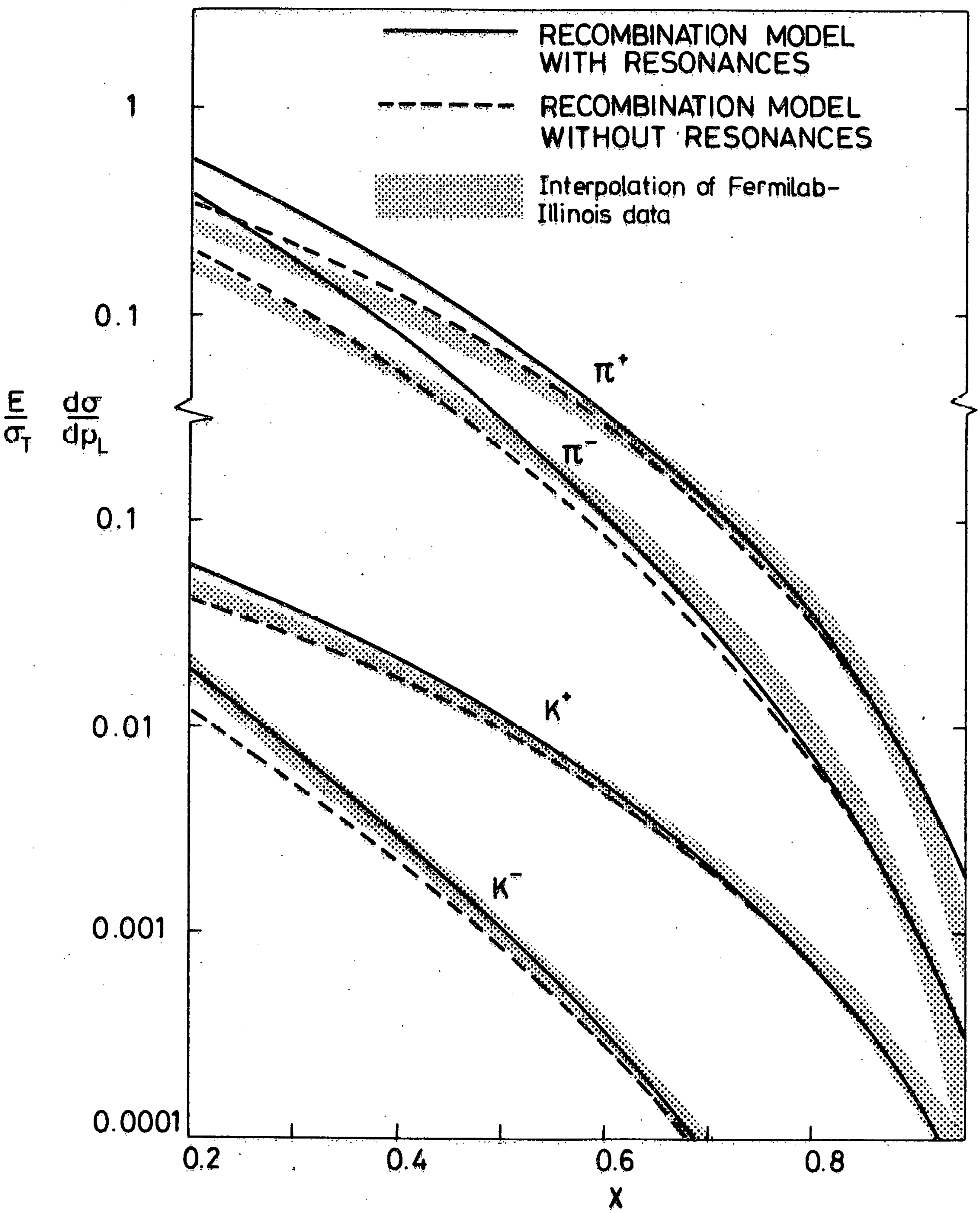
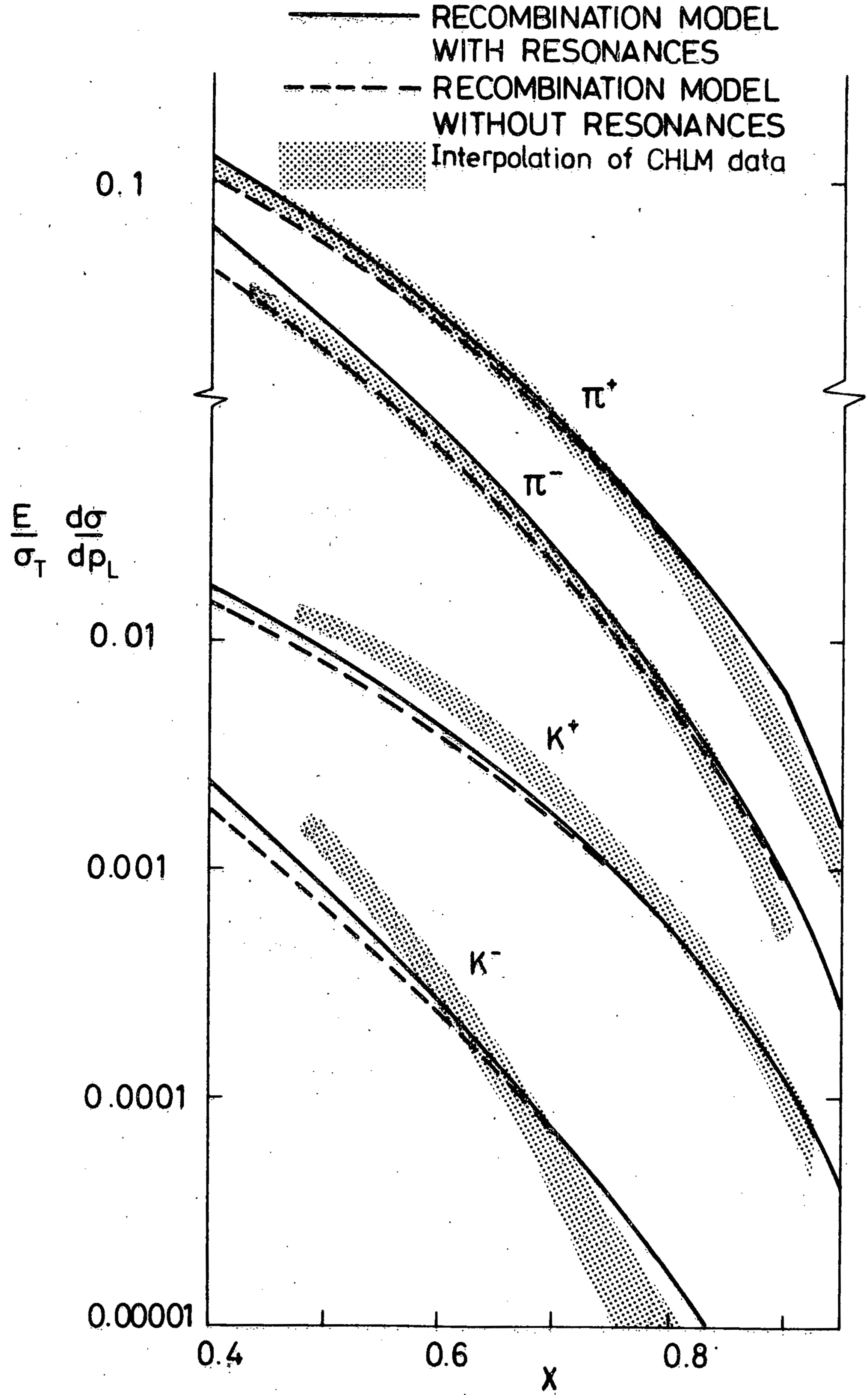

π⁺P AT 16 GeV/c
π⁻P AT 16 GeV/c
π⁺P AT 15 GeV/c
PP AT 69 GeV/c
K̄P AT 10,16 GeV/c
$\alpha_\rho / \alpha_\pi = 0.45$
WITH RESONANCES
WITHOUT RESONANCES
$\frac{E}{\sigma_T}\frac{d\sigma}{dp_L}(\pi^-) \Big/ \frac{E}{\sigma_T}\frac{d\sigma}{dp}(\rho^0)$
5
4
3
2
1
0.2
0.4
0.6
0.8
1.0
X

FIG. 2



RECOMBINATION MODEL WITH RESONANCES
RECOMBINATION MODEL WITHOUT RESONANCES
Interpolation of Fermilab-Illinois data
1
0.1
0.1
0.01
0.001
0.0001
$\frac{E}{\sigma_T} \frac{d\sigma}{dp_L}$
π^+
π^-
K^+
K^-
0.2
0.4
0.6
0.8
X

FIG. 3.



RECOMBINATION MODEL WITH RESONANCES
RECOMBINATION MODEL WITHOUT RESONANCES
Interpolation of CHLM data
0.1
0.01
0.001
0.0001
0.00001
$\frac{E}{\sigma_T} \frac{d\sigma}{dp_L}$
π^+
π^-
K^+
K^-
0.4
0.6
0.8
x

FIG. 4.

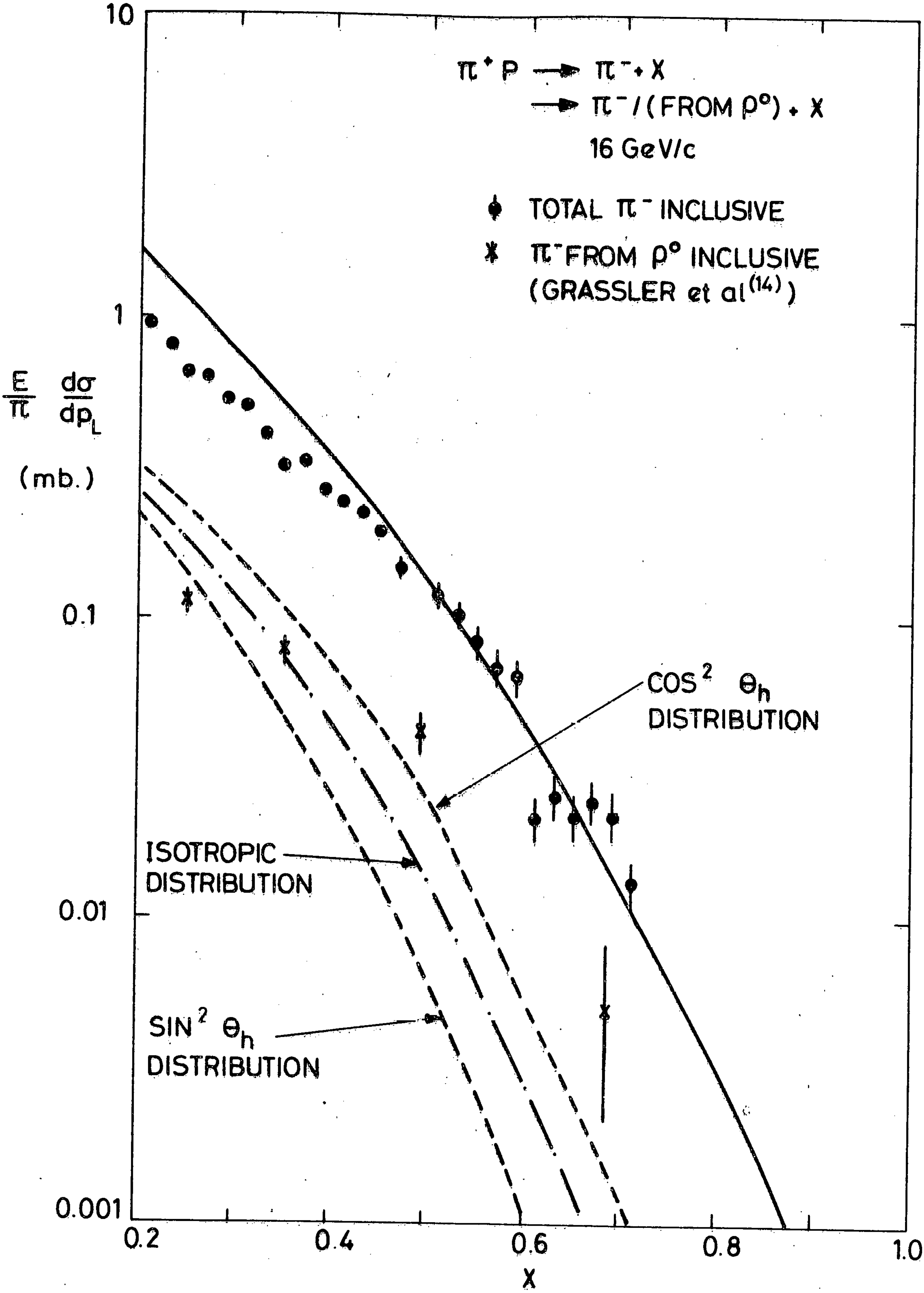

10
1
0.1
0.01
0.001
0.2
0.4
0.6
0.8
1.0
X
$\frac{E}{\pi}\frac{d\sigma}{dp_L}$
(mb.)
$\pi^+ p \rightarrow \pi^- + X$
$\rightarrow \pi^- / (\text{FROM } \rho^0) + X$
16 GeV/c
TOTAL π^- INCLUSIVE
π^- FROM ρ^0 INCLUSIVE
(GRASSLER et al[14])
$\cos^2\theta_h$ DISTRIBUTION
ISOTROPIC DISTRIBUTION
$\sin^2\theta_h$ DISTRIBUTION

FIG. 5.

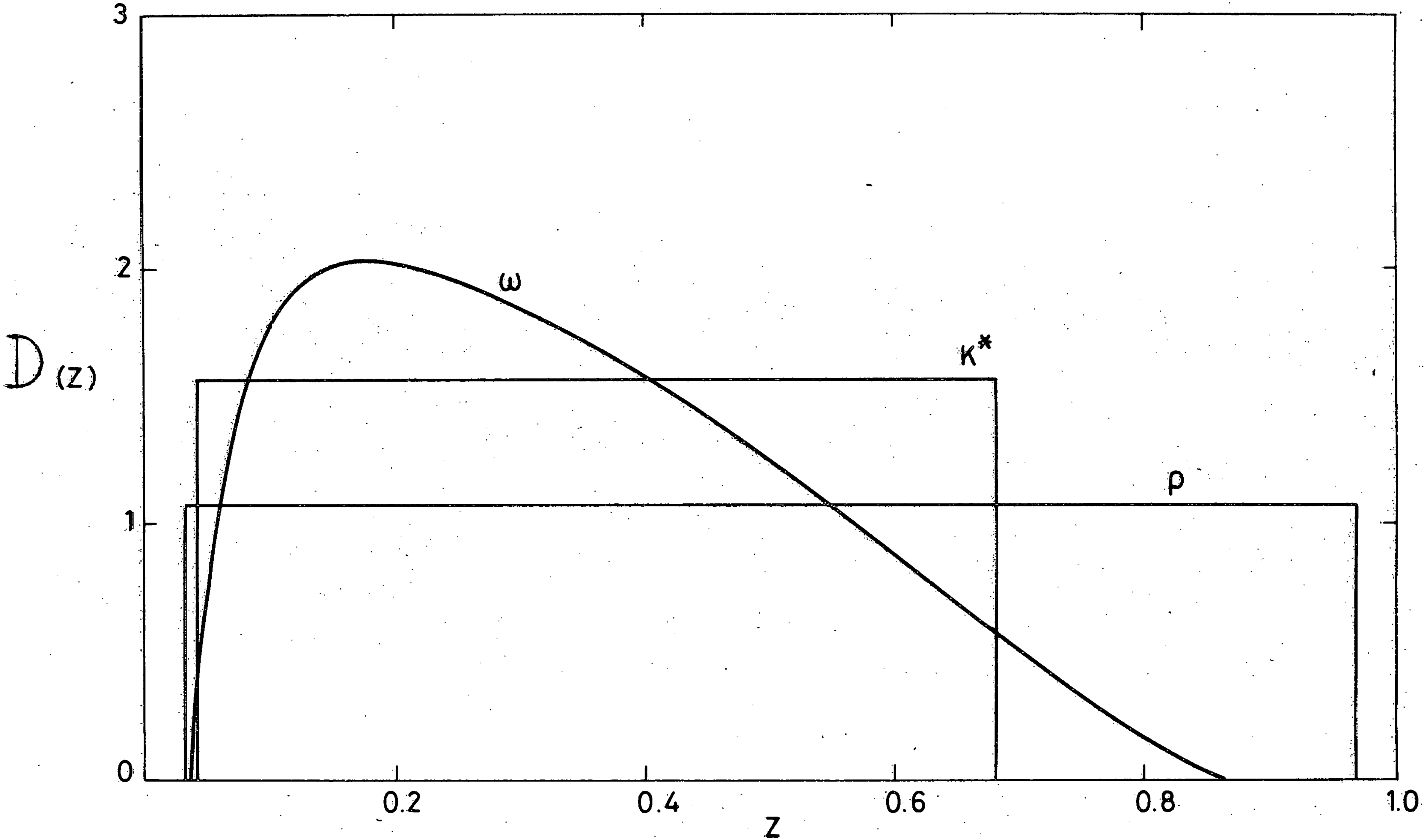


FIG. 6.