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Paschen's law for a hollow cathode discharge

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An expression for the breakdown voltage of a one-dimensional hollow cathode discharge has been derived. The breakdown condition which corresponds to Paschen's law contains, in addition to the first Townsend coefficient, and the secondary electron emission coefficient two parameters which characterize the reflecting action of the electric field and the lifetime of the electrons in the discharge. The breakdown voltage for a hollow cathode discharge in helium was calculated and compared to that of a glow discharge operating under similar conditions.

Hollow cathode discharges break down at lower voltages and carry order of magnitude higher currents compared to conventional glow discharges of similar dimensions and gas parameters. There are several mechanisms which may be responsible for these hollow cathode effects.[1] Among them are the secondary emission due to cathode bombardment by photons and by metastables, the enhancement of ionization due to stepwise ionization, the presence of sputtered particles, and the pendulum motion of fast electrons. Their relative importance is still under discussion. Experimental and numerical studies, however, indicate that the pendulum effect plays a major role in the electrical breakdown and the sustainment of a hollow cathode discharge.[2–6] Based on this premise, we have derived an expression for the breakdown voltage of hollow cathode discharge between plane parallel electrodes. The analytical model allows us to predict trends and approximate magnitudes of gas discharge parameters, and can therefore be used to explore the parameter range of hollow cathode discharges before using a more sophisticated numerical approach.[6]

The effect of pendulum electrons on charge generation can be understood by considering the various processes which are related to the pendulum motion in a one-dimensional hollow cathode geometry (Fig. 1). In this structure, two plane parallel cathodes are separated by a distance $2d$. The anode, located directly in the middle between the cathodes, consists of a grid with a transmission, T. For $T=0$, the geometry resembles two glow discharges back to back. In this geometry, electrons which have traversed one cathode sheath and crossed the anode plane are reflected back by the opposite cathode sheath and so forth. They are able to ionize as long as their energy exceeds the ionization energy. If the secondary electrons are generated in a region of high electric field strength, they may gain sufficient energy to produce ternary electrons. The deeper the electrons penetrate into the opposite sheath, i.e., the larger their mean-free path, the more likely the effect. Therefore, the effectiveness increases with decreasing pressure. The effectiveness of the hollow cathode effect is also dependent on the lifetime of the high-energy electrons within the discharge which is determined by electron capture at the electrodes and through losses in directions perpendicular to the hollow cathode field. In the one-dimensional system (Fig. 1), only capture of electrons at the anode needs to be considered.

According to this model, the electron current in the hollow cathode consists of two components: The current density of electrons moving towards the anode, j_1, superimposed by a current density of electrons reflected back from the opposite cathode layer, j_2. The spatial dependence of the two currents is described by

$$\frac{dj_1}{dx}=\alpha_1(x)j_1+\beta(x)j_2, \tag{1a}$$

$$\frac{dj_2}{d(-x)}=\alpha_2(x)j_2-\beta(x)j_2, \tag{1b}$$

where x is the distance from the cathode, with $0\leqslant x\leqslant d$. Townsend's first ionization coefficients for the two electron currents are α_1 and α_2. The reflecting action of the electric field is characterized by the parameter β. The ratio of the electron currents at the anode is determined by the anode transmission T:

$$j_2(d)=Tj_1(d). \tag{2}$$

The distortion of the electrical field E due to space charges can be neglected during breakdown, meaning that E is constant. The ionization and reflection coefficients, however, are varying with position, due to the nonequilibrium behavior of the electrons. For only slight spatial variations, the coefficients $\alpha_1(x)$, $\alpha_2(x)$, and $\beta(x)$ can be replaced by their spatial averages, a_1, a_2, and b, then the solutions of Eq. (1) are

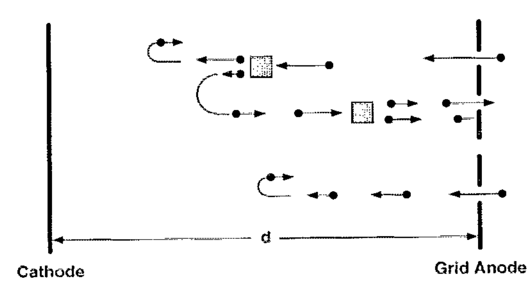

FIG. 1. Processes contributing to the hollow cathode effect, such as reflection of electrons in the sheath electric field, generation of secondary and ternary electrons, and capture of electrons at the anode grid are shown schematically for the left-hand part of the hollow cathode system.

[a]Deceased.

$$j_1(x)=\exp(a_1x)\{j_1(0)+j_2(d)(b/b^*)\times\exp[-(b-a_2)d][\exp(b^*x)-1]\}, \quad (3a)$$

$$j_2(x)=j_2(d)\exp[-(b-a_2)(d-x)], \quad (3b)$$

with $b^*=b-(a_1+a_2)$. For breakdown to occur, the electrons emitted from the cathode through ion impact need to generate a sufficient number of ions through ionization in the gas space to at least sustain their flux at the cathode. This condition reads

$$j_e(0)=\gamma j_i(0)=\gamma(M-1)j_e(0). \quad (4)$$

where γ is the secondary electron emission and $j_e(0)$ and $j_i(0)$ are the electron and ion current, respectively, at the cathode. The electron multiplication M is defined as the number of electrons leaving the discharge at the electrodes per electron emitted at the cathode:

$$M=j_{e,\text{out}}/j_e(0)=1+1/\gamma. \quad (5)$$

For $T=0$, describing a glow discharge, the electron current leaving the discharge, $j_{e\text{out}}$, is the forward electron current at the anode, $j_1(d)$. In the case of a transmission T different from zero, which is characteristic for a hollow cathode discharge configuration, the electron multiplication is determined by the electron current into the anode, $j_1(d)-j_2(d)$, plus the current carried by electrons which reach the cathode against the retarding force of the electric field. The multiplication for the planar hollow cathode geometry therefore reads

$$M=M_{\text{hc}}=[j_1(d)-j_2(d)+j_2(0)]/j_1(0). \quad (6)$$

With Eqs. (2), (3), and (6), we obtain

$$M_{\text{hc}}=\frac{(1-T)\exp(a_1d)+T\exp(-b^*d)}{1-(b/b^*)T[1-\exp(-b^*d)]}. \quad (7)$$

Combining Eqs. (5) and (7) yields the breakdown condition for a one-dimensional hollow cathode discharge:

$$\gamma=\frac{1-(b/b^*)T[1-\exp(-b^*d)]}{(1-T)\exp(a_1d)+T\exp(-b^*d)-1+(b/b^*)T[1-\exp(-b^*d)]}. \quad (8)$$

This breakdown condition contains five parameters instead of two, α and γ, for a glow discharge. Two additional parameters are the transmission of the anode T, a measure for the lifetime of the electrons in the discharge, and b, the reflection coefficient for electrons entering the opposite cathode layer. The reflection coefficient b can be interpreted as the reciprocal of the average penetration depth of pendulum electrons into the opposite cathode sheath. Also, instead of one ionization coefficient for the glow discharge, two coefficients, a_1 and a_2, are needed to describe the ionization due to the forward and reverse electron flux in the hollow cathode structure.

The coefficients, $\alpha_1(x)$, $\alpha_2(x)$, and $\beta(x)$, were computed by means of a Boltzmann code. Two coupled Boltzmann equations for the electrons moving in and opposite to the anode direction were solved using a finite difference method for helium as working gas.[7] The pendulum effect was considered by introducing a reflection term in the right-hand side of the Boltzmann equations. The reflection term contains the number of electrons which enter the linear volume element dx (or are generated in dx through ionization) with an energy less than $eEdx$, and therefore are reflected due to the effect of the electric field E; e is the electron charge. For all collisional processes, only forward scattering was considered.

The results of this calculation are shown in Fig. 2 for helium at a pressure of $p=1$ Torr. The gap distance d is 1 cm, and the applied voltage is 400 V. The anode transmission was assumed to be $T=0.5$. The spatially averaged values of $\alpha_1(x)$, $\alpha_2(x)$, and $\beta(x)$ are $a_1=0.740$, $a_2=0.552$, and $b=2.238$ cm^{-1}. The root-mean-square deviations of a_1, a_2, and b, from α_1, α_2, and are β are approximately 20%. The error introduced by averaging is largest near the anode because of the larger density of electrons at this electrode compared to the cathode region.

It was shown[7] (for voltages below 1 kV at a gap distance of 1 cm) that the pressure p and electric field E dependence of the spatially averaged parameters a_1, a_2, and b can be expressed similarly as for glow discharges:[8]

$$y_j=c_{1,j}p\exp[-c_{2,j}\sqrt{p/E}]; \quad y_j=a_1,a_2,b,a_G. \quad (9)$$

The subscript G refers to the glow discharge. In the pressure range of 0.5–2.0 Torr, the coefficients $c_{1,j}$ and $c_{2,j}$ are only slightly dependent on pressure: The root-mean-square-deviation of $y_j(p)$ from the computed values is less than 10%. The deviation increases with pressure. The calculated mean values for helium as a working gas are 1.84,

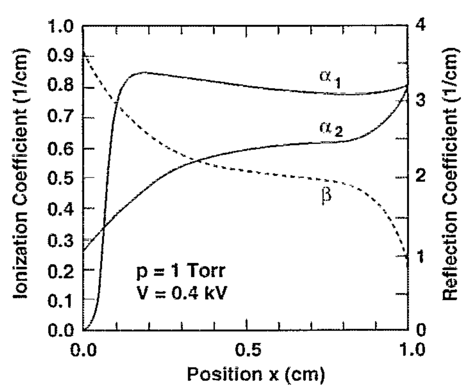

FIG. 2. The ionization coefficients, $\alpha_1(x)$ and $\alpha_2(x)$, for electrons moving into, and opposite to the direction of the anode, respectively, and the reflection coefficient, $\beta(x)$. The anode grid is at the position $x=1$.

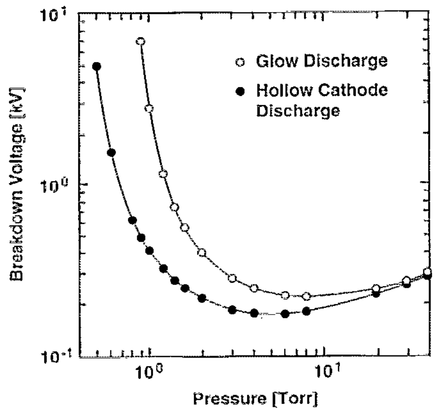

FIG. 3. Paschen curves for a one-dimensional hollow cathode discharge and a glow discharge in helium. The anode-to-cathode distance is $d=1$ cm.

2.00, 3.65, and 1.94 $(\text{cm Torr})^{-1}$ for $c_{1,j}$, and 16.93, 24.71, 9.90, and 15.54 $\sqrt{\text{V/cm Torr}}$ for $c_{2,j}$ for pressures of 0.5 Torr$\leqslant p \leqslant$2 Torr. The deviation of $c_{1,G}$ [1.94 $(\text{cm Torr})^{-1}$] from the corresponding value given in Ref. 8 [4.4 $(\text{cm Torr})^{-1}$] is probably due to our assumption of forward scattering only; $c_{2,G}$ differs from the given value 14.0 $\sqrt{\text{V/cm Torr}}$ in Ref. 8 by only approximately 10%.

By combining Eqs. (8) and (9), we obtain an expression for the breakdown voltage of a hollow cathode discharge, $V_{b,\text{hc}}=E\,d$, as a function of pressure p times distance d. The equation, which contains the breakdown voltage implicity in the ionization and reflection coefficients [Eq. (9)], can be considered as Paschen's law for a hollow cathode discharge.

In Fig. 3, the breakdown voltages for a hollow cathode discharge and a glow discharge in helium are plotted versus pressure for a gap length of $d=1$ cm. The secondary electron emission coefficient was assumed to be 0.3. Both curves have the typical Paschen curve shape: A linear increase of the breakdown voltage with pressure above a pressure times distance value of several Torr cm, and a steep increase in breakdown voltage at pressures below the Paschen minimum. However, for the hollow cathode geometry, this Paschen minimum is shifted towards lower pressure. Also, the absolute value of the breakdown voltage for the hollow cathode discharge is over the pressure range up to 20 Torr less than that for a glow discharge. The computed shift of the hollow cathode Paschen curve agrees with experimental observations.[9] With increasing pressure, the pendulum motion is suppressed—the hollow cathode discharge behaves more and more like a glow discharge.

The error introduced by the parameterization of the ionization and reflection coefficients is lowest for the pressure range between 0.5 and 2 Torr and for breakdown voltages below 1 kV. The spatial averaging of the coefficients and the assumption of forward scattering causes an overestimate of the breakdown voltage. However, even with all of these assumptions, the computed sparking potential for glow discharges at the critical point deviates from experimental values only by a factor of 1.5.[10]

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