Passenger Railway Optimization

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It all started . . .

. . . back in the Greece

- *c. 600 BC - A basic form of the railway, the rutway, - existed in ancient Greek and Roman times, the most important being the ship trackway Diolkos across the Isthmus of Corinth. Measuring between 6 and 8.5 km, remaining in regular and frequent service for at least 650 years, and being open to all on payment, it constituted even a public railway, a concept which according to Lewis did not recur until around 1800. The Diolkos was reportedly used until at least the middle of the 1st century AD, after which no more written references appear.*
- Timeline of railway history; From Wikipedia, the free encyclopedia

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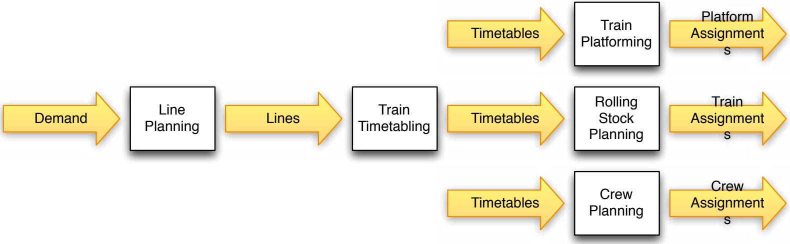
Figure : Source – Google Maps

Railway Planning Horizon



Day 0

Figure : Canadian Soldiers Building a Light Railway, [ca.1918][1]

[1] – source: Archives of Ontario, Canadian Expeditionary Force Albums

References

A. Caprara, L. G. Kroon, M. Monaci, M. Peeters, and P. Toth, *Passenger railway optimization*, Handbooks in Operations Research and Management Science (C. Barnhart and G. Laporte, eds.), vol. 14, Elsevier, 2007, pp. 129–187 (English).

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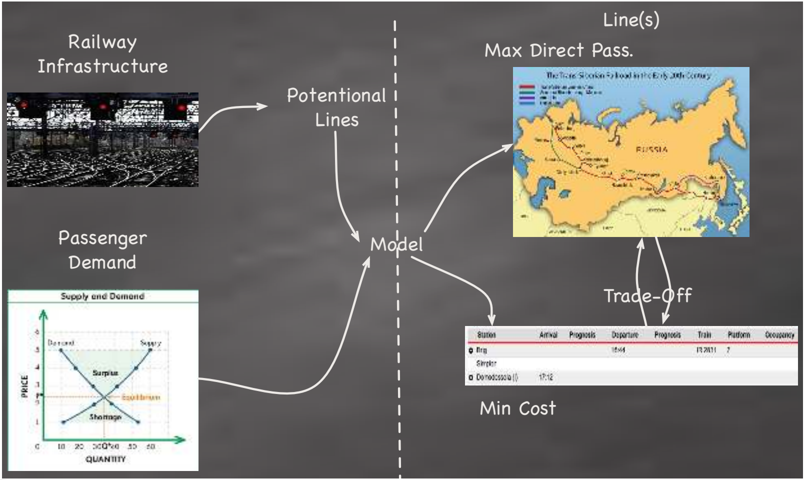
Line Planning Problem (LPP)



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LPP Model I

$G = (V, E)$	–	undirected graph G representing the railway network
$v \in V$	–	set of stations
$e \in E$	–	set of edges representing the tracks between stations
$p \in P$	–	set of unordered pairs of stations ($p = (p_1, p_2)$) with positive demand
d_p	–	number of passengers, that want to travel between stations p_1 and p_2
E_p	–	set of edges on the shortest path between stations p_1 and p_2
$d_e = \sum_{p:e \in E_p} d_p$	–	the total number of passengers, that want to travel along edge e
$l \in L$	–	set of potential lines (assumed to be known a priori)
E_l	–	set of edges of the line l
$f \in F$	–	set of potential frequencies
$c \in C$	–	set of available capacities
$i \in I$	–	set of indices representing combination of assigned capacity c and frequency f to a line l
k_i	–	operational cost for a combination i (*e.g.* train driver, conductor(s), carriage kilometers

$$x_i = \begin{cases} 1 & \text{if and only if line } l_i \text{ is to be operated with a frequency } f_i \text{ and capacity } c_i, \\ 0 & \text{otherwise.} \end{cases}$$

d_{lp} – number of direct passengers traveling on line l between the pair of stations p

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LPP Model II

$$\max \quad w_1 \cdot \sum_{l \in L} \sum_{p \in P} d_{lp} - w_2 \cdot \sum_{i \in I} k_i \cdot x_i \qquad (1)$$

$$\text{s.t.} \quad \sum_{i \in I : l_i = l} x_i \leq 1, \qquad \forall l \in L, \qquad (2)$$

$$\sum_{i \in I : e \in E_{l_i}} f_i \cdot c_i \cdot x_i \geq d_e, \qquad \forall e \in E, \qquad (3)$$

$$\sum_{p \in P : e \in E_p} d_{lp} \leq f_i \cdot c_i \cdot x_i, \qquad \forall l \in L, \forall e \in E, \qquad (4)$$

$$\sum_{l \in L : E_p \subset E_l} d_{lp} \leq d_p, \qquad \forall p \in P, \qquad (5)$$

$$x_i \in \{0, 1\}, \qquad \forall i \in I, \qquad (6)$$

$$d_{lp} \geq 0, \qquad \forall l \in L, \forall p \in P. \qquad (7)$$

Train Timetabling Problem (TTP)

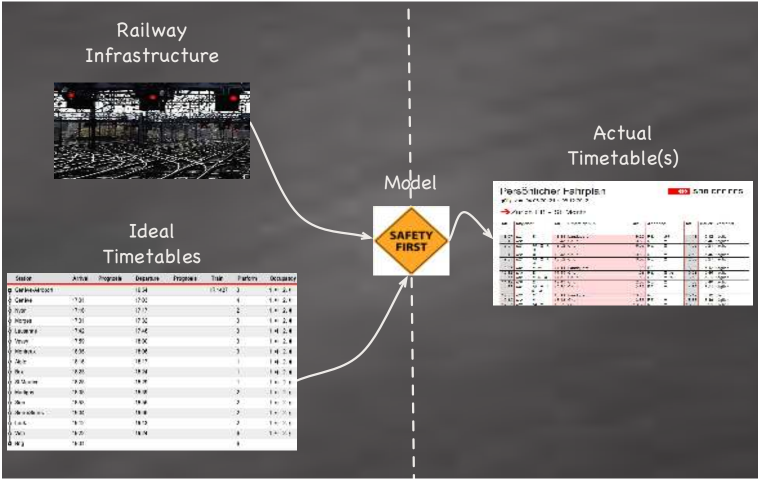
Non-Cyclic

- departs differently over the time horizon
- prior knowledge of the timetable needed
- lower cost

Cyclic

- departs at every cycle
- good for "unplanned" user
- higher cost

Non-Cyclic TTP



Non-Cyclic TTP Model I

$G = (V, A)$	–	directed acyclic multigraph G
$v \in V$	–	set of nodes
$a \in A$	–	set of arcs
$t \in T$	–	set of trains
$a \in A^t$	–	subset of arcs used by train t
σ	–	source node
τ	–	sink node
p_a	–	profit of arc a
$\delta_t^+(v)$	–	set of arcs in A^t leaving the node v
$\delta_t^-(v)$	–	set of arcs in A^t entering the node v
$\mathbb{C}$	–	family of maximal subsets C of pairwise incompatible arcs

$$x_a = \begin{cases} 1 & \text{if and only if the path in the solution associated with train } t \text{ contains arc } a, \\ 0 & \text{otherwise.} \end{cases}$$

Non-Cyclic TPP Model II

$$\max \sum_{t\in T}\sum_{a\in A^t} p_a \cdot x_a \tag{8}$$

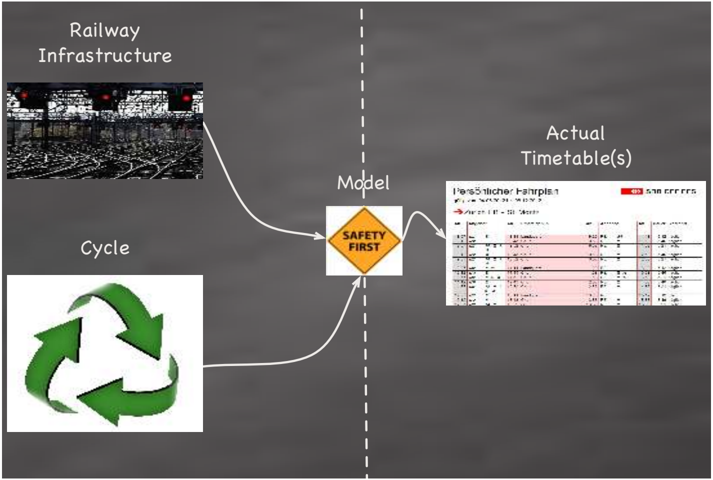
$$\text{s.t.} \quad \sum_{a\in\delta_t^+(\sigma)} x_a \leq 1, \qquad \forall t \in T, \tag{9}$$

$$\sum_{a\in\delta_t^-(v)} x_a = \sum_{a\in\delta_t^+(v)} x_a, \qquad \forall t \in T, \forall v \in V \setminus \{\sigma, \tau\} \tag{10}$$

$$\sum_{a\in C} x_a \leq 1, \qquad \forall C \in \mathbb{C}, \tag{11}$$

$$x_a \in \{0, 1\}, \qquad \forall a \in A. \tag{12}$$

Cyclic TTP


Railway
Infrastructure
Cycle
Model
SAFETY
FIRST
Actual
Timetable(s)
Persönlicher Fahrplan

Cyclic TTP Model I

$G = (N, A \cup A^s)$	–	graph G representing the railway network
$n, m \in N$	–	set of nodes
$a \in A$	–	set of regular tracks $a = (n, m)$
$a \in A^s$	–	set of single tracks $a = (n, m) = (m, n)$
$t \in T$	–	set of trains
$n \in N^t \subseteq N$	–	set of nodes visited by train t
$a \in A^t \subseteq A \cup A^s$	–	set of tracks used by train t
$(t, t') \in T_a$	–	set of all pairs of trains (t, t'), that travel along the track a in the same direction, where t' is the faster train
$(t, t') \in T_a^s$	–	set of all pairs of trains (t, t'), that travel along the single track a in the opposite direction, where t departs from n and t' departs from m
$t \in F_n^d, F_n^a$	–	set of all trains t, that have a fixed departure (arrival) at node n
$(t, t') \in S_n$	–	set of all train pairs $(t, t'), t < t'$, for which the departure times are to be synchronized at node n
$(t, t') \in C_n$	–	set of all train pairs $(t, t'), t < t'$, for which turn-around or connection constraint is required from train t to train t' at node n
b	–	cycle of the timetable
h	–	general headway upon departure and arrival at every node
r_a^t	–	time it takes to the train t to traverse the arc a
$\left[\underline{d_n^t}, \overline{d_n^t}\right]$	–	dwell time window of the train t at the node n
$\left[\underline{f_n^t}, \overline{f_n^t}\right]$	–	fixed arrival/departure window of the train t at the node n, in the case of completely fixed arrival/departure $\underline{f_n^t} = \overline{f_n^t}$
$\left[\underline{s_n^{tt'}}, \overline{s_n^{tt'}}\right]$	–	time window for the synchronization of trains t and $t]$ at node n
$\left[\underline{c_n^{tt'}}, \overline{c_n^{tt'}}\right]$	–	time window for the connection or turn around constraint between trains t and t' at node n

a_n^t	–	arrival time of train t at node n
d_n^t	–	departure time of train t from node n

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Cyclic TPP Model II

$$
\begin{aligned}
\max \quad & F(a,d) && && (13)\\
\text{s.t.} \quad & a_m^t - d_n^t = r_a^t && \text{mod } b, && \forall t \in T, a \in A^t \quad (14)\\
& d_n^t - a_n^t \in \left[\underline{d_n^t}, \overline{d_n^t}\right] && \text{mod } b, && \forall t \in T, \forall n \in N^t, \quad (15)\\
& d_n^{t'} - d_n^t \in \left[\underline{s_n^{tt'}}, \overline{s_n^{tt'}}\right] && \text{mod } b, && \forall n \in N, \forall (t,t') \in S_n, \quad (16)\\
& d_n^{t'} - a_n^t \in \left[\underline{c_n^{tt'}}, \overline{c_n^{tt'}}\right] && \text{mod } b, && \forall n \in N, \forall (t,t') \in C_n, \quad (17)\\
& d_n^{t'} - d_n^t \in \left[r_a^t - r_a^{t'} + h, b - h\right] && \text{mod } b, && \forall a \in A, \forall (t,t') \in T_a, \quad (18)\\
& a_n^{t'} - d_n^t \in \left[r_a^t + r_a^{t'} + h, b - h\right] && \text{mod } b, && \forall a \in A^s, \forall (t,t') \in T_a, \quad (19)\\
& d_n^t \in \left[\underline{f_n^t}, \overline{f_n^t}\right], && && \forall n \in N, \forall t \in F_n^d, \quad (20)\\
& a_n^t \in \left[\underline{f_n^t}, \overline{f_n^t}\right], && && \forall n \in N, \forall t \in F_n^a, \quad (21)\\
& a_n^t, d_n^t \in \{0, b-1\}, && && \forall t \in T, \forall n \in N_t. \quad (22)
\end{aligned}
$$

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Thank you for your attention.