Passive Finite Dimensional Repetitive Control of Robot Manipulators

Josip Kasac, Branko Novakovic, Dubravko Majetic and Danko Brezak

***Abstract*—In this paper a new class of finite dimensional repetitive controllers for robot manipulators is proposed. The global asymptotic stability is proved for the unperturbed system. The passivity-based design of the proposed repetitive controller avoids the problem of tight stability conditions and slow convergence of the conventional, internal model-based, repetitive controllers. The passive interconnection of the controller and the nonlinear mechanical systems provides the same stability conditions as the controller with the exact feed-forward compensation of robot dynamics. The simulation results on a three degrees of freedom spatial manipulator illustrate the performances of the proposed controller.**

***Index Terms*—Repetitive control, passive system, manipulators, robot dynamics, stability**

I. Introduction

An important subject in the control of mechanical systems is tracking periodic reference signals and attenuating periodic disturbances. Many tracking systems, such as computer disk drives [1], rotation machine tools [2], or robots [3], have to deal with periodic reference and/or disturbance signals. A promising control approach to achieving the tracking of periodic reference signals is learning control or repetitive control.

In most of the conventional approaches to robot trajectory control, including parametric adaptive control, it is necessary to compute in real time the so-called inverse dynamics equations of the robot or regression matrix. However, due to the model uncertainties, it is difficult to derive the exact description of the system. Also, using neural networks for learning feed-forward control has some drawbacks: slow convergence and relatively large tracking errors.

There have been many studies on the topic of repetitive control for controlling mechanical systems in an iterative manner. In contrast with the conventional approaches to robot trajectory control, repetitive control schemes are easy to implement and do not require the exact knowledge of the dynamic model.

Repetitive controllers can be classified as being either internal model-based or external model-based [4]. Controllers using the internal model are linear and have periodic signal generators [5], [6]. In the external model controllers the disturbance model is placed outside the basic feedback loop [3], [7].

The internal model controllers are based on a delayed integral action of the form $(1-\exp(-sT))^{-1}$ which produces an infinite number of poles on imaginary axes. However, the asymptotic convergence can only be guaranteed under restrictive conditions in the plant dynamics - zero relative degree or direct transmission term. These conditions are generally not satisfied in robot control applications because they imply acceleration measurement. Further, the positive feedback loop used to generate the periodic signal decreases the stability margin. So, the repetitive controller is likely to make the system unstable. To enhance the robustness of these repetitive control schemes, the repetitive update rule is modified to include the so-called Q-filter [5], [6]. Unfortunately, the use of the Q-filter eliminates the ability of tracking errors to converge to zero. Therefore, the trade-off between stability and tracking performance has been considered to be an important factor in the repetitive control system.

Another problem is that, due to infinite dimensional dynamics of delayed line, a large memory space is required for digital implementation of the control law. To overcome this problem, in [8] a finite dimensional approximation of delayed line is proposed in the form of a cascade connection of N harmonic oscillators and one integrator.

The advantages of the internal model controllers are that they are linear, making analysis and implementation easier. The disadvantages are that the stability is almost entirely governed by the feedback loop of the repetitive compensator. The frequency response of the system is altered and robustness to noise and unmodelled dynamics is reduced.

The external model controllers are based on the feedforward compensation of inverse dynamics. The disturbance model is adjusted adaptively to match the actual disturbance. The central idea in [3] is that the disturbance can be represented as a linear combination of basis functions like Fourier series expansion. In this way, an adaptive control law with regressor matrix containing basis functions is obtained. In [7] unknown disturbance functions are represented by integral equations of the first kind involving a known kernel and unknown influence functions. The learning rule indirectly estimates the unknown disturbance function by updating the influence function.

The main advantage of the external model approach is that there is no significant influence on the stability conditions of the control system. The map between the feedforward function error and the tracking errors is strictly passive. Thus, the control system is robust to the imprecise estimation of the robot inverse dynamics. The disadvantage is that the analysis and implementation are more complex than for the internal model-based algorithms.

In this paper a new class of internal model-based repetitive controllers for robot manipulators is proposed. The proposed finite dimensional repetitive controller is founded on the passivity-based design and has a structure in the form of a parallel connection of N linear oscillators and one integrator. The passive interconnection of the proposed controller with

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The authors are with the Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, HR-10000 Zagreb, Croatia (e-mail: josip.kasac@fsb.hr, branko.novakovic@fsb.hr, dubravko.majetic@fsb.hr, danko.brezak@fsb.hr).

nonlinear mechanical systems has the same stability conditions as the controller with the exact feed-forward compensation of robot dynamics [9], [10].

Throughout the paper we use the notation: $\|x\| = \sqrt{x^T x}$ for the Euclidean norm of the vector $x \in \mathbb{R}^n$, $\lambda_M\{A\}$ and $\lambda_m\{A\}$ for the maximal and minimal eigenvalues, respectively, of the symmetric positive definite matrix A.

This paper is organized as follows. Robot dynamics and its main properties are presented in Section II. In Section III a class of finite dimensional internal model-based repetitive controllers is introduced and conditions for global asymptotic stability are established in Section IV. The passivity properties of the proposed controllers are considered in Section V. The simulation results are presented in Section VI. The concluding remarks are emphasized in Section VII.

II. Robot Dynamics

The dynamic model of the n-link rigid-body robotic manipulator with revolute joints is represented by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u + d, \tag{1}$$

where q is the $n \times 1$ vector of robot joint coordinates, $\dot{q}$ is the $n \times 1$ vector of joint velocities, u is the $n \times 1$ vector of applied joint torques and forces, $M(q)$ is the $n \times n$ inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of centrifugal and Coriolis torques, $g(q)$ is the $n \times 1$ vector of gravitational torques and forces, obtained as the gradient of the robot potential energy $U(q)$

$$g(q) = \frac{\partial U(q)}{\partial q}, \tag{2}$$

and d is the vector of external disturbances and unmodeled static nonlinearities.

We assume that the matrix $C(q,\dot{q})$ is defined using the Christoffel symbols, so that the matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric. This implies that $\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T$.

The dynamic model (1) for the robot manipulator with revolute joints has the following properties that are used for the stability analysis [11], [10].

Property 1. The inertia matrix $M(q)$ is a positive definite symmetric matrix which satisfies

$$\lambda_m\{M\}\|\dot{q}\|^2 \le \dot{q}^T M(q)\dot{q} \le \lambda_M\{M\}\|\dot{q}\|^2. \tag{3}$$

Property 2. There exist positive constants k_M, k_{C1}, k_{C2} and k_g so that for all $x, y, z, v, w \in \mathbb{R}^n$, we have

$$\|M(x)z - M(y)z\| \le k_M\|x-y\|\,\|z\|, \tag{4}$$

$$\begin{aligned}\|C(x,z)w - C(y,v)w\| \le\; & k_{C1}\|z-v\|\,\|w\| + \\ & + k_{C2}\|z\|\,\|x-y\|\,\|w\|,\end{aligned} \tag{5}$$

$$\|g(x) - g(y)\| \le k_g\|x-y\|, \tag{6}$$

$$\|C(x,y)z\| \le k_{C1}\|y\|\,\|z\|, \tag{7}$$

where the values of the parameters k_g, k_{C1}, k_{C2} and k_M can be estimated by

$$\begin{aligned} k_g &= n\left(\max_{i,j,q}\left|\frac{\partial g_i(q)}{\partial q_j}\right|\right), \\ k_{C1} &= n^2\left(\max_{i,j,k,q}|c_{ijk}(q)|\right), \end{aligned} \tag{8}$$

$$\begin{aligned} k_{C2} &= n^3\left(\max_{i,j,k,l,q}\left|\frac{\partial c_{ijk}(q)}{\partial q_l}\right|\right), \\ k_M &= n^2\left(\max_{i,j,k,q}\left|\frac{\partial M_{ij}(q)}{\partial q_k}\right|\right), \end{aligned} \tag{9}$$

where $c_{ijk}(q)$ are Christoffel symbols.

III. Control Problem Formulation

A. Finite-Dimensional Repetitive Controller

The periodic reference trajectory $q_d(t)$ with the period T can be represented in the form of Fourier series

$$q_d(t) = a_0 + \sum_{k=1}^{\bar{N}}[a_k\cos(k\omega t) + b_k\sin(k\omega t)], \tag{10}$$

where $\omega = \frac{2\pi}{T}$ is the fundamental frequency, and a_0, a_k and b_k are known constant vectors.

We consider the control law given by

$$u = -K_P\tilde{q} - K_D\dot{\tilde{q}} - k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}} - K_I z_0 - \sum_{k=1}^{N} Q_k\dot{z}_k, \tag{11}$$

$$\ddot{z}_k + k^2\omega^2 z_k = Q_k(\dot{\tilde{q}} + \alpha\tilde{q}), \qquad k = 1,...,N, \tag{12}$$

$$\dot{z}_0 = \dot{\tilde{q}} + \alpha\tilde{q}, \tag{13}$$

where $\tilde{q} = q - q_d$, $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$ are the joint position error and velocity, respectively, q_d is the time periodic desired joint position represented by (10), K_P, K_D, K_I and Q_k $(k = 1,...,N)$ are the $n \times n$ constant positive-definite diagonal matrix, $k_D^{(1)}$, α is positive constant and N is the number of harmonic oscillators.

The desired joint position q_d is assumed to be twice continuously differentiable. In other words, we assume that there exist finite upper bounds on the norm of the desired velocity and acceleration, denoted by $\|\dot{q}_d\|_M$ and $\|\ddot{q}_d\|_M$. The nonlinear derivative term $k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}}$ in the control law (11) is introduced to ensure the global asymptotic stability of the closed-loop system [12].

The parallel interconnection of the N harmonic oscillators (12) and the integrator (13) represents the internal model of the periodic reference signal $q_d(t)$ including higher order harmonics which are induced by nonlinear robot dynamics, so that the condition $N \ge \bar{N}$ must be satisfied.

B. Residual Robot Dynamics

The dynamic model of the robot manipulator (1) can be rewritten in the following form

$$M(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + h(\tilde{q},\dot{\tilde{q}}) = u + d - f(q_d,\dot{q}_d,\ddot{q}_d), \tag{14}$$

where

$$\begin{aligned} h(\tilde{q},\dot{\tilde{q}}) =\; & [M(q) - M(q_d)]\ddot{q}_d + [C(q,\dot{q}) - C(q_d,\dot{q}_d)]\dot{q}_d + \\ & + g(q) - g(q_d), \end{aligned} \tag{15}$$

is the so-called *residual robot dynamics* introduced in [13], [14] and

$$f(q_d,\dot{q}_d,\ddot{q}_d) = M(q_d)\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + g(q_d), \tag{16}$$

represents the unknown *desired robot inverse dynamics*.

The function $f(q_d, \dot{q}_d, \ddot{q}_d)$ is a periodic function with the same fundamental frequency as $q_d(t)$ and can be represented by the infinite Fourier series expansion

$$f(q_d, \dot{q}_d, \ddot{q}_d) = \bar{a}_0 + \sum_{k=1}^{\infty} [\bar{a}_k \cos(k\omega t) + \bar{b}_k \sin(k\omega t)], \quad (17)$$

where $\bar{a}_0$, $\bar{a}_k$ and $\bar{b}_k$ are unknown constant vectors.

The following property of the function $h(\tilde{q}, \dot{\tilde{q}})$ is important in the subsequent stability analysis.

Property 4. By defining

$$\begin{aligned} c_1 &= k_g + k_M \|\ddot{q}_d\|_M + k_{C2} \|\dot{q}_d\|_M^2, \\ c_2 &= k_{C1} \|\dot{q}_d\|_M, \end{aligned} \quad (18)$$

the norm of residual dynamics (15) satisfies (see [9], [10])

$$\|h(\tilde{q}, \dot{\tilde{q}})\| \leq c_1 \|\tilde{q}\| + c_2 \|\dot{\tilde{q}}\|. \quad (19)$$

From the inequality (19) follows

$$\begin{aligned} -(\dot{\tilde{q}} + \alpha \tilde{q})^T h(\tilde{q}, \dot{\tilde{q}}) \ \leq \ & \alpha c_1 \|\tilde{q}\|^2 + c_2 \|\dot{\tilde{q}}\|^2 + \\ & + \ (c_1 + \alpha c_2) \|\tilde{q}\| \ \|\dot{\tilde{q}}\|. \end{aligned} \quad (20)$$

The parameters c_1 and c_2 can be estimated on the basis of the Fourier representation of the desired periodic signal (10)

$$\begin{aligned} c_2 &\leq k_{C1} \sum_{k=1}^{\bar{N}} k\omega (\|a_k\| + \|b_k\|), \\ c_1 &\leq k_g + k_M \sum_{k=1}^{\bar{N}} k^2 \omega^2 (\|a_k\| + \|b_k\|) + \frac{k_{C2} c_2^2}{k_{C1}^2}, \end{aligned} \quad (21)$$

where we used the properties $|\sin(k\omega t)| \leq 1$ and $|\cos(k\omega t)| \leq 1$.

Remark 1. One of the simplest motion control schemes for the system (14) is PD control with feedforward compensation [9], [10]: $u = -K_P \tilde{q} - K_D \dot{\tilde{q}} + f(q_d, \dot{q}_d, \ddot{q}_d)$. The implementation of the above-mentioned control law requires the exact knowledge of the matrices $M(q)$, $C(q, \dot{q})$ and the vector $g(q)$. However, due to the model uncertainties, it is difficult to derive the exact dynamic model of the robot manipulator. The main idea of using the controller (11)-(13) is the model-free feedback compensation of the periodic function $f(q_d, \dot{q}_d, \ddot{q}_d)$. This idea will be clarified through the derivation of the error equations of the closed-loop system.

C. Error Equations

Introducing the change of variables $\tilde{z}_k = z_k - z_k^*$, $k = 0, 1, ..., N$, with

$$\begin{aligned} z_0^* &= -K_I^{-1} \bar{a}_0, \\ z_k^* &= k^{-1} \omega^{-1} Q_k^{-1} [\bar{b}_k \cos(k\omega t) - \bar{a}_k \sin(k\omega t)], \end{aligned} \quad (22)$$

the following error equations are obtained

$$M(q)\ddot{\tilde{q}} + C(q, \dot{q})\dot{\tilde{q}} + h(\tilde{q}, \dot{\tilde{q}}) = \bar{u} + w, \quad (23)$$

$$\bar{u} = -K_P \tilde{q} - K_D \dot{\tilde{q}} - k_D^{(1)} \|\tilde{q}\| \dot{\tilde{q}} - K_I \tilde{z}_0 - \sum_{k=1}^{N} Q_k \dot{\tilde{z}}_k, \quad (24)$$

$$\ddot{\tilde{z}}_k + k^2 \omega^2 \tilde{z}_k = Q_k (\dot{\tilde{q}} + \alpha \tilde{q}), \qquad k = 1, ..., N, \quad (25)$$

$$\dot{\tilde{z}}_0 = \dot{\tilde{q}} + \alpha \tilde{q}, \quad (26)$$

where we used the Fourier series expansion (17) of the function $f(q_d, \dot{q}_d, \ddot{q}_d)$ and the property $\ddot{z}_k^* + k^2 \omega^2 z_k^* = 0$, which follows from (22) for $k = 1, ..., N$. The function w in (23) has the following form

$$w = d - \sum_{k=N+1}^{\infty} [\bar{a}_k \cos(k\omega t) + \bar{b}_k \sin(k\omega t)]. \quad (27)$$

where the second term represents the error in the estimation of the desired robot inverse dynamics that consists of the harmonics of $N + 1$ order.

Remark 2. From the equation (27) we can conclude that the tracking error has zero harmonic content at the repetitive frequency and its harmonics up to N (where N is the number of harmonic oscillators in the controller). Also, the bound on the tracking error decreases with N. In the limit $N \to \infty$ the above-mentioned model of the repetitive controller works as well as the ideal infinite dimensional model in achieving the perfect tracking of periodic reference signals [15]. Note that this conclusion is valid only for twice continuously differentiable periodic reference signals, and cannot be generalized to arbitrary periodic reference signals, as shown in [5]. In that ideal case the stationary state of the system (14), (11)-(13) is $q = q_d$, $\dot{q} = \dot{q}_d$, $z_k = z_k^*$, $k = 0, 1, ..., N$.

IV. Stability analysis

In this section we provide the proof for global asymptotic stability of the unperturbed systems (23)-(26), where $w = 0$, and in the subsequent section we prove the passivity properties of the proposed controller, which guarantee the robustness to the disturbances $w \neq 0$.

We consider the stability by the Lyapunov's direct method. First, we propose the appropriate Lyapunov function candidate. Then, global stability conditions on the controller gains are established. Finally, the LaSalle invariance principle is invoked to guarantee the asymptotic stability.

A. Construction of Lyapunov Function

Multiplying the equation (23) by the output variable $y_1 = \dot{\tilde{q}} + \alpha \tilde{q}$, the equation (25) by the output variable $y_2 = \dot{\tilde{z}}_k$, the equation (26) by the output variable $y_3 = K_I z_0$ and summing them all up, we get a nonlinear differential form which can be separated in the following way

$$\frac{d}{dt} V(\tilde{q}, \dot{q}, \tilde{z}_0, \tilde{z}_1, \dot{\tilde{z}}_1, ..., \tilde{z}_N, \dot{\tilde{z}}_N) = -W(\tilde{q}, \dot{q}), \quad (28)$$

(for more details see e.g. [12]) where $V = V_1 + V_2$ is the Lyapunov function candidate

$$\begin{aligned} V_1 \ = \ & \frac{1}{2} \dot{\tilde{q}}^T M(q) \dot{\tilde{q}} + \alpha \tilde{q}^T M(q) \dot{\tilde{q}} + \frac{1}{2} \tilde{q}^T K_P \tilde{q} + \\ + \ & \frac{1}{2} \alpha \tilde{q}^T K_D \tilde{q} + \frac{1}{3} \alpha k_D^{(1)} \|\tilde{q}\|^3, \end{aligned} \quad (29)$$

$$V_2 \ = \ \frac{1}{2} \tilde{z}_0^T K_I \tilde{z}_0 + \frac{1}{2} \sum_{k=1}^{N} \dot{\tilde{z}}_k^T \dot{\tilde{z}}_k + \frac{1}{2} \omega^2 \sum_{k=1}^{N} k^2 \tilde{z}_k^T \tilde{z}_k, \quad (30)$$

and $-W$ is its time derivative

$$\begin{aligned} W \ = \ & \dot{\tilde{q}}^T K_D \dot{\tilde{q}} + k_D^{(1)} \|\tilde{q}\| \dot{\tilde{q}}^T \dot{\tilde{q}} - \alpha \dot{\tilde{q}}^T M(q) \dot{\tilde{q}} - \alpha \dot{\tilde{q}}^T C(q, \dot{q}) \tilde{q} + \\ & + \ \alpha \tilde{q}^T K_P \tilde{q} + (\dot{\tilde{q}} + \alpha \tilde{q})^T h(\tilde{q}, \dot{\tilde{q}}). \end{aligned} \quad (31)$$

B. Stability Conditions

The following step is to determine conditions for the positive definiteness of the function V and the positive semi-definiteness of the function W.

1) Positive definiteness of the Lyapunov function: The Lyapunov function V can be rearranged as follows

$$V = V_2 + \frac{1}{2}\left(\dot{\tilde{q}} + \alpha\tilde{q}\right)^T M(q)\left(\dot{\tilde{q}} + \alpha\tilde{q}\right) - \frac{1}{2}\alpha^2\tilde{q}^T M(q)\tilde{q} + + \frac{1}{2}\tilde{q}^T(K_P + \alpha K_D)\tilde{q} + \frac{1}{3}\alpha k_D^{(1)}\|\tilde{q}\|^3. \quad (32)$$

Using the property (3) we get

$$V \geq \frac{1}{2}\left(\lambda_m\{K_P\} + \alpha\lambda_m\{K_D\} - \alpha^2\lambda_M\{M\}\right)\|\tilde{q}\|^2 + + \frac{1}{2}\lambda_m\{M\}\|\dot{\tilde{q}} + \alpha\tilde{q}\|^2 + \frac{1}{2}\lambda_m\{K_I\}\|\tilde{z}_0\|^2 + + \frac{1}{2}\sum_{k=1}^{N}\|\dot{\tilde{z}}_k\|^2 + \frac{1}{2}\omega^2\sum_{k=1}^{N}k^2\|\tilde{z}_k\|^2, \quad (33)$$

that will be satisfied when

$$\lambda_m\{K_P\} + \alpha\lambda_m\{K_D\} > \alpha^2\lambda_M\{M\}. \quad (34)$$

2) Negative semi-definiteness of the time derivative of Lyapunov function: The following step is the derivation of the condition which ensures that the time derivative of Lyapunov function is a negative semi-definite function, i.e., $W \geq 0$.

First, notice that the upper bound on term $\dot{\tilde{q}}^T C(q,\dot{q})\tilde{q}$ in (31) can be estimated by

$$\dot{\tilde{q}}^T C(q,\dot{q})\tilde{q} \leq \alpha k_{C1}\|\tilde{q}\|\,\|\dot{q}\|\,\left\|\dot{\tilde{q}}\right\| \leq \leq k_{C1}\|\dot{q}_d\|_M\,\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\| + k_{C1}\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\|^2, \quad (35)$$

where we used the triangle inequality $\|\dot{q}\| \leq \left\|\dot{\tilde{q}}\right\| + \|\dot{q}_d\|$.

Applying the properties (3), (20) and (35), we get

$$W \geq (\lambda_m\{K_D\} - \alpha\lambda_M\{M\} - c_2)\left\|\dot{\tilde{q}}\right\|^2 + + (k_D^{(1)} - \alpha k_{C1})\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\|^2 + \alpha(\lambda_m\{K_P\} - c_1)\|\tilde{q}\|^2 - - (c_1 + 2\alpha c_2)\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\| \geq 0. \quad (36)$$

Finally, W can be bounded by a quadratic plus a cubic function

$$W \geq \begin{bmatrix} \|\tilde{q}\| \\ \|\dot{\tilde{q}}\| \end{bmatrix}^T R \begin{bmatrix} \|\tilde{q}\| \\ \|\dot{\tilde{q}}\| \end{bmatrix} + (k_D^{(1)} - \alpha k_{C1})\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\|^2, \quad (37)$$

with the matrix R given by

$$R = \begin{bmatrix} \alpha(\lambda_m\{K_P\} - c_1) & -\frac{1}{2}(c_1 + 2\alpha c_2) \\ -\frac{1}{2}(c_1 + 2\alpha c_2) & (\lambda_m\{K_D\} - \alpha\lambda_M\{M\} - c_2) \end{bmatrix}.$$

The function W is positive definite if the following conditions are satisfied

$$k_D^{(1)} > \alpha k_{C1}, \quad (38)$$

$$\lambda_m\{K_P\} > c_1, \quad (39)$$

$$\lambda_m\{K_D\} > \frac{(c_1 + 2\alpha c_2)^2}{4\alpha(\lambda_m\{K_P\} - c_1)} + \alpha\lambda_M\{M\} + c_2. \quad (40)$$

We can see that the condition (34) is trivially implied by the conditions (39)-(40). So, the conditions (38)-(40) are the final stability criterions which guarantee global stability. Finally, by invoking the LaSalle invariance principle, we conclude the asymptotic stability.

Remark 3. Note that the stability conditions (39)-(40), for $\alpha = 1$, are exactly the same as the local stability conditions in [10]. The global stability of the repetitive controller (11)-(13) is achieved by the nonlinear derivative term whose gain $k_D^{(1)}$ satisfies the condition (38). So, the proposed repetitive controller practically has no influence on the closed-loop stability. It is obvious from the stability conditions which do not contain the interaction gains Q_k and the integral gain K_I. This fact is a consequence of the passive interconnection between robot dynamics (1) and the repetitive controller (11)-(13).

V. PASSIVITY PROPERTIES OF REPETITIVE CONTROLLER

Unknown and unmodeled nonlinearities play an important role in the high-precision control of the robot manipulator. Friction is one of the most common nonlinearities present in mechanical systems. Despite the many efforts to characterize and estimate frictions, it is still difficult to identify and compensate them. Moreover, any cancellation of nonlinearities by feedback which is not exact, may produce undesirable closed-loop behavior like large tracking errors, limit cycles and stick-slip motion. In contrast with model-dependent controllers, the passivity-based controllers are robust to model uncertainties and external disturbances. In this section we prove the passivity properties of the proposed repetitive controller.

Consider dynamical systems represented by

$$\dot{x} = f(x,u), \quad (41)$$

$$y = h(x,u), \quad (42)$$

where $x \in \mathbb{R}^n$, $y, u \in \mathbb{R}^m$, $f(0,0) = 0$ and $h(0,0) = 0$. Moreover, $f(x,u)$ and $h(x,u)$ are supposed to be sufficiently smooth so that the system is well-defined.

Definition 1. (see [16]) The system (41)-(42) is said to be passive if there exists a continuously differentiable positive semidefinite function $V(x)$ (called the storage function) so that

$$u^T y \geq \dot{V}(x) + \epsilon\|u\|^2 + \delta\|y\|^2 + \rho\psi(x), \quad (43)$$

where ϵ, δ, and ρ are nonnegative constants, and $\psi(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is a positive definite function of x. The term $\rho\psi(x)$ is called the state dissipation rate. Furthermore, the system is said to be: lossless, if (43) is satisfied with equality and $\epsilon = \delta = \rho = 0$; input strictly passive, if $\delta = \rho = 0$ and $\epsilon > 0$; output strictly passive, if $\epsilon = \rho = 0$ and $\delta > 0$; state strictly passive, if $\epsilon = \delta = 0$ and $\rho > 0$.

Proposition 1. The robot dynamics

$$M(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + h(\tilde{q},\dot{\tilde{q}}) = u_1 + w_1, \quad (44)$$

in a closed-loop with the nonlinear PD controller

$$u_1 = -K_P\tilde{q} - K_D\dot{\tilde{q}} - k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}}, \quad (45)$$

is state strictly passive from the input torque w_1 to the output $y_1 = \dot{\tilde{q}} + \alpha\tilde{q}$, with a radially unbounded positive definite storage function V_1 defined by (29) and the state dissipation rate is given by W,

$$w_1^T y_1 \geq \dot{V}_1(\tilde{q}, \dot{\tilde{q}}) + W(\tilde{q}, \dot{\tilde{q}}). \quad (46)$$

Note that V_1 is a positive definite and W is positive semidefinite function if the conditions (39)-(40) are satisfied.

Proposition 2. The system

$$\ddot{\tilde{z}}_k + k^2\omega^2 \tilde{z}_k = Q_k w_2, \qquad k = 1, ..., N, \quad (47)$$
$$\dot{\tilde{z}}_0 = w_2, \quad (48)$$

is lossless from the input w_2 to the output $y_2 = K_I z_0 + \sum_{k=1}^{N} Q_k \dot{\tilde{z}}_k$ with a radially unbounded positive definite storage function V_2 defined by (30),

$$w_2^T y_2 = \dot{V}_2(\tilde{z}_0, \tilde{z}_1, \dot{\tilde{z}}_1, ..., \tilde{z}_N, \dot{\tilde{z}}_N). \quad (49)$$

Note that V_2 is positive definite for any positive definite matrix K_I.

Proposition 3. The feedback interconnection between the system (44)-(45) and the system (47)-(48),

$$w_1 = -y_2 + w, \qquad w_2 = y_1, \quad (50)$$

is output strictly passive from the input torque w to the output $y_1 = \dot{\tilde{q}} + \alpha\tilde{q}$, with a radially unbounded positive definite storage function $V = V_1 + V_2$,

$$w^T y_1 \geq \dot{V} + \delta \|y_1\|^2, \quad (51)$$

where

$$\delta \leq \frac{a_1 a_2 - \frac{1}{4}a_3^2}{a_2 + \alpha^2 a_1 + \alpha a_3}, \quad (52)$$

and

$$\begin{aligned} a_1 &= \lambda_m\{K_D\} - \alpha\lambda_M\{M\} - c_2, \\ a_2 &= \alpha(\lambda_m\{K_P\} - c_1), \\ a_3 &= c_1 + 2\alpha c_2. \end{aligned} \quad (53)$$

Proof. Inserting $w_1 = -y_2 + w$, $w_2 = y_1$ in (46) and (49) we get

$$w^T y_1 \geq \dot{V}(\tilde{q}, \dot{\tilde{q}}, \tilde{z}_0, \tilde{z}_1, \dot{\tilde{z}}_1, ..., \tilde{z}_N, \dot{\tilde{z}}_N) + W(\tilde{q}, \dot{\tilde{q}}). \quad (54)$$

Further, inserting the inequality (36) with the notation (53) in (54), and comparing with (51) we get

$$\begin{aligned} w^T y_1 &\geq \dot{V} + a_1 \left\|\dot{\tilde{q}}\right\|^2 + a_2\|\tilde{q}\|^2 - a_3\|\tilde{q}\| \left\|\dot{\tilde{q}}\right\| \geq \\ &\geq \dot{V} + \delta\|\dot{\tilde{q}} + \alpha\tilde{q}\|^2, \end{aligned} \quad (55)$$

The final step is determining the parameter δ which satisfies the above mentioned inequality. By rearranging the inequality (55) and using the property of the scalar product $\tilde{q}^T\dot{\tilde{q}} \leq \|\tilde{q}\| \left\|\dot{\tilde{q}}\right\|$, we get the following inequality

$$\begin{aligned} &(a_1 - \delta)\left\|\dot{\tilde{q}}\right\|^2 + (a_2 - \alpha^2\delta)\|\tilde{q}\|^2 - (a_3 + 2\alpha\delta)\|\tilde{q}\| \left\|\dot{\tilde{q}}\right\| = \\ &= \begin{bmatrix} \|\tilde{q}\| \\ \|\dot{\tilde{q}}\| \end{bmatrix}^T \begin{bmatrix} (a_2 - \alpha^2\delta) & -\frac{1}{2}(a_3 + 2\alpha\delta) \\ -\frac{1}{2}(a_3 + 2\alpha\delta) & (a_1 - \delta) \end{bmatrix} \begin{bmatrix} \|\tilde{q}\| \\ \|\dot{\tilde{q}}\| \end{bmatrix} \geq 0 \end{aligned}$$

which is satisfied for

$$(a_1 - \delta)(a_2 - \alpha^2\delta) \geq \frac{1}{4}(a_3 + 2\alpha\delta)^2. \quad (56)$$

Solving the inequality (56) with respect to the parameter δ we get (52). Note that, from the conditions of positive definiteness of the matrix R in (37), the numerator on the right side of the inequality (52) is always positive.

Proposition 4. The feedback interconnection between the system (44)-(45) and the system (47)-(48) has the finite $\mathcal{L}_2$ gain $\gamma \leq \frac{1}{\delta}$ where δ is defined by (52). For proof see e.g. [16].

The L_2 gain, which characterizes the disturbance attenuation capabilities of the control scheme, can be used for the controller gains tuning. The L_2 gain γ is a function of the controller parameters α, $\lambda_m\{K_P\}$ and $\lambda_m\{K_D\}$. It can be seen that the function $\gamma = \gamma(\alpha, \lambda_m\{K_P\}, \lambda_m\{K_D\})$ has no isolated minimum regarding the parameters $\lambda_m\{K_P\}$ and $\lambda_m\{K_D\}$, i.e. the L_2 gain γ can be decreased by increasing the values of the gains K_P and K_D. On the other hand, for the given values of $\lambda_m\{K_P\}$ and $\lambda_m\{K_D\}$, there exists an isolated minimum regarding the parameter α.

VI. SIMULATION EXAMPLE

The manipulator used for simulation is a three degree of freedom spatial manipulator with revolute joints, considered in [17]. The manipulator parameters, which are taken from the first three links of the PUMA 560 robot [18], have the following values:

$I_1 = 0.35$ kg m^2, $m_2 = 17.4$ kg, $m_3 = 4.8$ kg,
$m = 0.5$ kg, $l_2 = 0.4318$ m, $l_3 = 0.4331$ m,

where I_1 is inertia of the first link, m_2 and m_3 are masses of the second and third links, m is load mass, l_2 and l_3 are lengths of the second and third links.

The parameters k_g, k_{C1}, k_{C2}, k_M and $\lambda_M\{M\}$, which can be estimated by using the expressions (8) and (9) derived in [10] and [11], have the following values: $k_g = 214.65$ kg m^2/sec^2, $k_{C1} = 63.86$ kg m^2, $k_{C2} = 383.20$ kg m^2, $k_M = 162.74$ kg m^2 and $\lambda_M\{M\} \leq 28.17$ kg.

The desired periodic reference trajectories are

$$q_{dj} = \frac{1}{2j} + \frac{1}{4}\sum_{k=1}^{3} \frac{j}{jk+1} \sin\left(k\omega t + \frac{\pi j}{2k}\right), \quad (57)$$

where $j = 1, 2, 3$ and $\omega = 1\ rad/s$. The numerical values of the parameters c_1 and c_2 can be calculated from the expression (21): $c_1 = 915.52$ kg m^2/sec^2, $c_2 = 62.61$ kg m^2/sec.

The controller gains α, K_P and K_D are chosen to minimize the $\mathcal{L}_2$ gain γ in accordance with the stability conditions (38)-(40) and control torques limitations due to actuator saturation. First, we chose the gain K_P in agreement with (38) as $K_P = \text{diag}\{1200, 1200, 1200\}$ N m. In that case, we can consider the $\mathcal{L}_2$ gain γ as a function of the parameters α and $\lambda_m\{K_D\}$, $\gamma = \gamma(\alpha, \lambda_m\{K_D\})$. The contour lines of this function, for different positive values of γ, are shown in Fig. 1. From the contour lines we can determine the minimal values of $\lambda_m\{K_D\}$ and the corresponding values of α which ensure the appropriate values of the $\mathcal{L}_2$ gain γ. Then, we chose the minimum possible $\mathcal{L}_2$ gain γ with the corresponding values of α and $\lambda_m\{K_D\}$, avoiding actuators saturation during the simulations. The controller gains α and K_D were finally set to $\alpha = 2.47$ sec^{-1}, $K_D = \text{diag}\{982.98, 982.98, 982.98\}$ N m sec, as shown in Fig. 1.

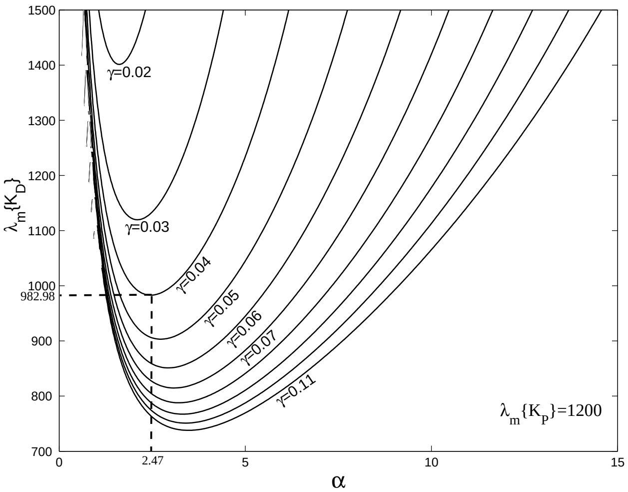

Fig. 1. The dependence of $\mathcal{L}_2$ gain γ on controllers parameters α, $\lambda_m\{K_P\}$ and $\lambda_m\{K_D\}$.

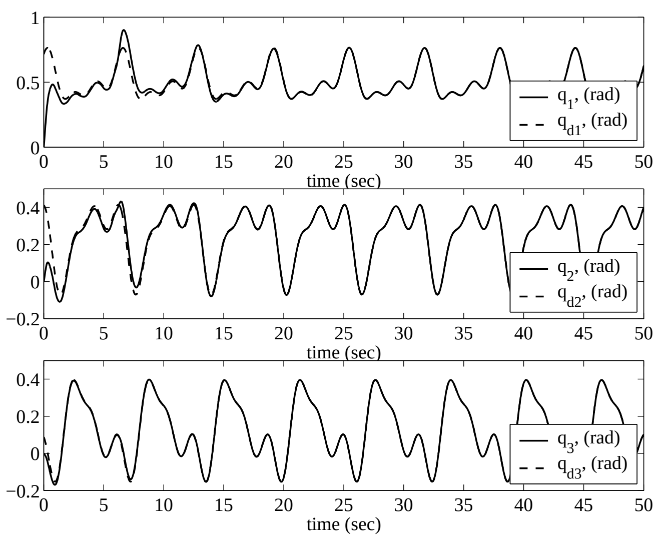

Fig. 2. The periodic reference signals and positions of robot manipulators.

Using the numerical values of α and c_2, the controller gain $k_D^{(1)}$ is chosen in agreement with (38): $k_D^{(1)} = 200\ \mathrm{N\,m\,sec}$. The gains K_I and Q_k, $k = 1, ..., N$ are not included in the stability conditions and $\mathcal{L}_2$ gain. Their values are determined during the preliminary simulations: $K_I = \mathrm{diag}\{150, 150, 150\}\ \mathrm{N\,m}$, $Q_k = \mathrm{diag}\{20, 20, 20\}$ for $k = 1, ..., N$. The number of oscillators is $N = 12$, and the fundamental frequency of oscillators is $\omega = 1\ rad/s$.

Fig. 2. shows a comparison of the positions of the robot manipulators and reference signals. In Fig. 3. we can see a comparison of the tracking errors for the repetitive controller (RC), linear PID controller (defined by (11)-(13) with $k_D^{(1)} = 0$ and $Q_k = 0$, $k = 1, ..., N$) and the computed torque (CT) controller $u = M(q)(\ddot{q}_d - K_P\tilde{q} - K_D\dot{\tilde{q}}) + C(q, \dot{q})\dot{q} + g(q)$, in the case of (a) compensated and (b) uncompensated viscous and dry friction $d = -F_V\dot{q} - F_C\mathrm{sign}(\dot{\mathrm{q}})$, where $F_V = F_C = \mathrm{diag}\{5, 5, 5\}\ \mathrm{N\,m}$. From the figure we can conclude that the PID controller can not asymptotically track the periodic reference signal. In contrast with the PID controller, the repetitive controller shows exponential convergence toward an arbitrary small tracking error which depends on the number of oscillators N. Although the model-based CT controller has faster convergence in the case of compensated friction, the RC controller has a significantly smaller tracking error in the case of uncompensated friction.

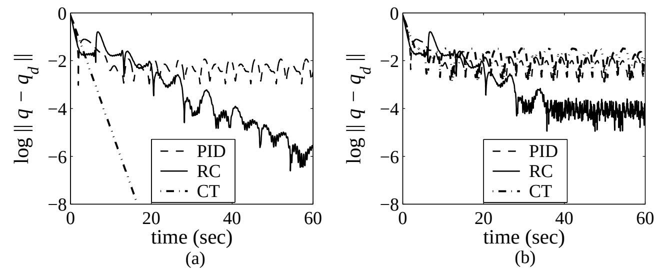

Fig. 3. A comparison of tracking errors for the repetitive controller (RC), linear PID controller and computed torque controller (CT) in the case of (a) compensated and (b) uncompensated viscous and dry friction.

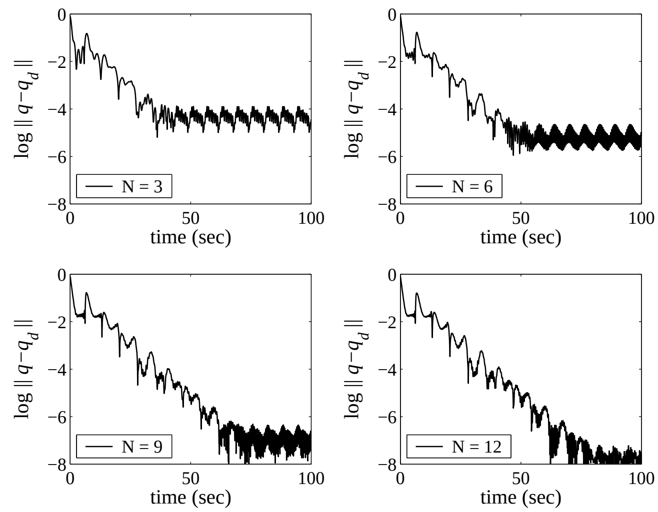

Fig. 4. The convergence rate and tracking errors, depending on the number of oscillators.

The dependence of the tracking error and convergence rate on the number of oscillators N is illustrated in Fig. 4. We can see that an increase in the number of oscillators decreases the tracking error. Further, from the figure we can see that the convergence rate is independent of the number of oscillators. So, there is no trade-off between convergence and accuracy, which is characteristic of most of the internal model-based repetitive controllers.

VII. Conclusions

In this paper a new class of the finite dimensional repetitive controllers for robot manipulators is proposed. The proposed repetitive controller connects the main advantage of the internal model controllers - implementation simplicity, with robustness based on the passivity of the external model controllers. The future work will be directed to the experimental verification of the proposed repetitive controller.

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