# Passive (self-pumped) phase conjugate mirror: Theoretical and experimental investigation

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We report the results of a theoretical and experimental investigation of a passive (self-pumped) phase conjugate mirror. This device is based on real time holography in materials which allow a spatial phase shift between the refractive index grating and the light interference pattern. An imaging experiment is reported showing the phase conjugating nature of the device. The holographic medium used was a single crystal of barium titanate.

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In a recent article[1] we reported the demonstration in our laboratory of several novel optical oscillator configurations including a unidirectional ring resonator and a passive (self-pumped) phase conjugate mirror (PPCM). These devices are based on real time holography in nonlinear media such as barium titanate[2] and strontium barium niobate.[3] The PPCM consists of a nonlinear medium lying in a resonator bounded by two ordinary mirrors (Fig. 1). Practical interest in this device arises because a distinct disadvantage of conventional phase conjugate mirrors (PCM's) is that two high quality pumping beams must be provided by an external laser. The PPCM, on the other hand, pumps itself and has, for example, been used by us as a compact, simple phase conjugate mirror in a phase conjugate resonator laser with dynamic intracavity distortion correction capability.[4] The subject of this letter is a theoretical and experimental investigation of the PPCM.

The phenomenon of light amplification by two-beam coupling in photorefractive crystals is well known,[5] and is the basis of the buildup of oscillation in the PPCM when the initial oscillation strength in the $M_1$-$M_2$ cavity (Fig. 1) is zero. The $c$ axis of the crystal is oriented so that light in beam 1 is amplified by two-beam coupling from input beam 4 and is fed back by successive reflections from mirrors $M_2$ and $M_1$. Oscillation continues to build up until steady state is reached for beams 1 and 2 which are now pumping the crystal as a phase conjugate mirror for input beam 4. The problem addressed here is the reflectivity of this device (i.e., the intensity of beam 3 divided by the intensity of beam 4 at $z = 0$). Because the pumping beams are derived from and fed by the signal beam 4 itself, use of the undepleted pumps approximation is not possible. Thus, we use the analysis of Ref. 6 where we derived the reflectivity of a holographic phase conjugate mirror without assuming undepleted pumps. There, the boundary conditions were the input intensities of the pumping beams $I_1(0)$ and $I_2(l)$, and the probe beam $I_4(0)$. After some further development of the theory to take account of the new boundary conditions, the intensity reflectivities $M_1$ and $M_2$ of the cavity mirrors 1 and 2, respectively, we arrive at the following expression for the intensity reflectivity $R$:

$$R = \frac{(\Delta + 1)^2 |T|^2}{M_2 |\Delta T + [\Delta^2 + (\Delta + 1)^2/M_2]^{1/2}|^2}, \qquad (1)$$

where

$$\Delta = I_2(l) - I_1(0) - I_4(0), \qquad (2)$$

$$T = \tanh\{(\gamma l/2)[\Delta^2 + (\Delta + 1)^2/M_2]^{1/2}\}, \qquad (3)$$

and $\gamma l$ is a coupling strength characteristic of the medium.[7] In these equations we have normalized all intensities by the conserved total average intensity $I_0 = I_1(z) + I_2(z) + I_3(z) + I_4(z)$. $\Delta$ is given by the solution(s) of the equation

$$M_1 M_2 = \left| \frac{T + [\Delta^2 + (\Delta + 1)^2/M_2\,]^{1/2}}{\Delta T + [\Delta^2 + (\Delta + 1)^2/M_2\,]^{1/2} + (\Delta + 1)T/M_2} \right|^2 . \qquad (4)$$

In some cases this equation has multiple roots, giving rise to multistability.

While in photorefractive materials $\gamma l$ is independent of the total average light intensity $I_0$, in other media, such as atomic vapors, $\gamma l$ is typically proportional to the total average light intensity. Since this intensity is conserved[6] the theory presented here is easily applicable to these other materials provided, of course, the other assumptions hold.

We show in Fig. 2 a contour plot of the reflectivity $R$ as a function of $M_1$ and $M_2$ for a particular value of the coupling strength $\gamma l = -3$ (i.e., with the $\pi/2$ phase shift typical of photorefractive materials). We see that towards the left of this plot the reflectivity can be multivalued, and also that when $M_2$ is high reflectivity remains high even when $M_1$ is

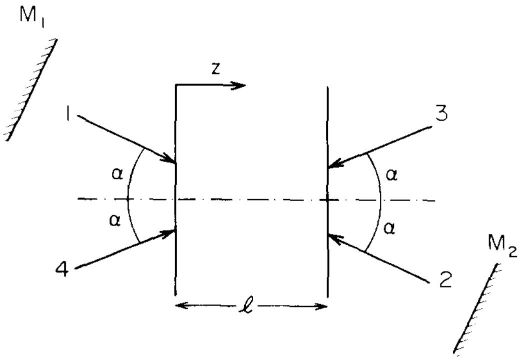

FIG. 1. Geometry of passive phase conjugate mirror. The probe is beam 4 and its phase conjugate is beam 3. The two pumping beams, 1 and 2, oscillate in the cavity bounded by mirrors $M_1$ and $M_2$.

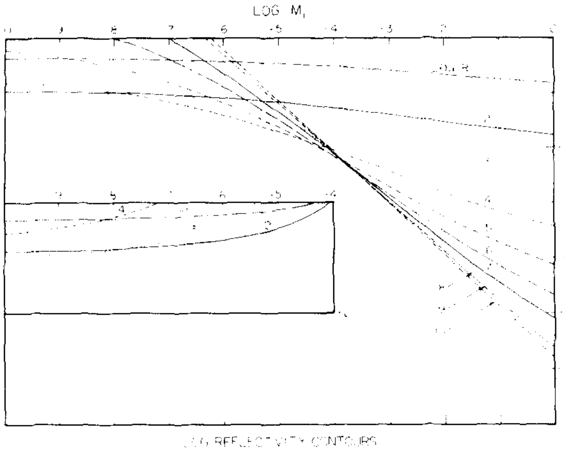

FIG. 2. Contour plots of the reflectivity of the passive phase conjugate mirror. Equality of contour levels in the region where the function is multivalued is indicated by equality of line form (dashes, dots, etc.) The coupling strength $\gamma l = -3$. For the sake of clarity some of the contours at low $M_1$ and high $M_2$ have been redrawn in an inset.

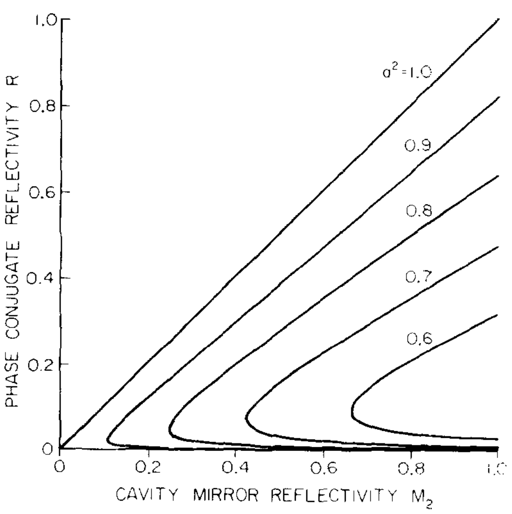

FIG. 3. Reflectivity of the passive phase conjugate mirror in the absence of mirror $M_1$, for several values of the parameter $a^2$. $a^2$ is related to the coupling constant $\gamma l$ by $\tanh(-\gamma l a/2) = a$.

small. In fact, we shall show that there is a threshold coupling strength above which it is possible to obtain finite reflectivities even in the absence of mirror $M_1$.

But first, we consider the threshold coupling strength for the buildup of oscillation from zero oscillation intensity in the $M_1$-$M_2$ crystal cavity. This corresponds to taking $I_1(0) = I_2(l) = 0$, that is, $\Delta = -1$. From Eq. (4) the threshold may be obtained as

$$M_1 M_2 = \exp[(\gamma + \gamma^*)l]. \tag{5}$$

This fits in well with the heuristic explanation of oscillation buildup given at the beginning of this section: the gain in the crystal simply has to be sufficient to overcome the losses due to the mirrors $M_1$ and $M_2$. Since the threshold depends only on the real part of the coupling strength, it follows that a nonlinear medium with no phase shift between the index grating and the interference pattern will not support operation beginning from zero oscillation strength. This is because of the absence in these materials of unidirectional two-beam coupling.

In addition, we see that no buildup of operation from zero oscillation strength is possible in the absence of mirror $M_1$ or $M_2$ even when $\gamma l$ does have a real part. However, by providing a seed beam in the $M_2$-crystal cavity it *is* possible in some cases to maintain oscillation in the absence of mirror $M_1$. The oscillation will not start by itself, but once initiated, it keeps going. In the theory $M_1 = 0$ implies [see Eq. (4)]

$$\tanh\{-(\gamma l/2)[\Delta^2 + (\Delta+1)^2/M_2]^{1/2}\} = [\Delta^2 + (\Delta+1)^2/M_2]^{1/2} \tag{6}$$

so that $\Delta$ may be found from the solution of the quadratic equation

$$\Delta^2 + (\Delta+1)^2/M_2 = a^2, \tag{7}$$

where $a$ is simply related to the coupling constant $\gamma l$ by

$$\tanh(-\gamma l a/2) = a. \tag{8}$$

The reflectivity can therefore be written in closed form as

$$R = \left(\frac{M_2^{1/2} \pm [a^2(1+M_2) - 1]^{1/2}}{M_2 + 2 \mp M_2^{1/2}[a^2(1+M_2)-1]^{1/2}}\right)^2 \tag{9}$$

so that the device is at threshold with reflectivity $R = R_t$

$$R_t = M_2/(M_2+2)^2 \tag{10}$$

when $a^2$ equals $a_t^2$

$$a_t^2 = 1/(1+M_2). \tag{11}$$

It is possible to show that of the two possible values of above-threshold reflectivity [Eq. (9)] only the one associated with the upper sign is stable. This mode of operation, without $M_1$ has been observed in our laboratory and has, in fact, been used in the PPCM as an end mirror for an argon ion laser.[4] In Fig. 3 we show the phase conjugate reflectivity as a function of the mirror reflectivity $M_2$ for various values of the parameter $a^2$. When $a^2 = 1$ ($-\gamma l = \infty$) the reflectivity of the phase conjugate mirror equals the reflectivity of the cavity mirror $M_2$. When $M_2 = 1$ $a_t^2 = 1/2$, so that the threshold value of $\gamma l$ for operation without $M_1$ is 2.493 [see Eq. (8)].

A theoretical discussion of the faithfulness of phase conjugation in the PPCM when the input beam contains spa-

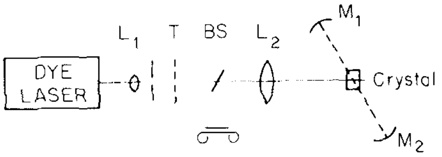

FIG. 4. Experimental arrangement used to demonstrate phase conjugation in the PPCM. The dye laser used was a Spectra Physics 380 ring laser with rhodamine 6G at 579.2 nm in single longitudinal mode. Using a cartesian coordinate system with the abscissa coincident with the beam direction, the locations of the elements measured in centimeters were 20× beam expander $L_1$ (−64,0), transparency $T$ (−55,0), beam splitter for observing phase conjugate reflection BS (−38,0), 14-cm focal length lens $L_2$ (−20,0), barium titanate crystal (0,0), 50-cm radius concave mirror $M_1$, (−28,11), 50-cm radius concave mirror $M_2$ (30, −13). The $c$ axis of the crystal pointed in the direction of the vector (0.94,0.35).

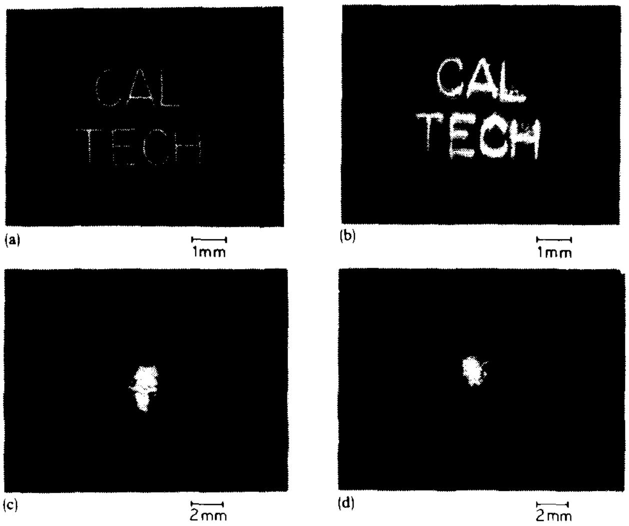

FIG. 5. Results of the phase conjugation experiment. (a) Transparency $T$, (b) phase conjugate beam picked off by beam splitter BS, (c) intensity pattern of the oscillation beam on mirror $M_1$, (d) intensity pattern of the oscillation beam on mirror $M_2$.

tial information will be the subject of future investigations in which consideration of the spatial characteristics of the self-induced pumping beams will be of prime importance. Current experimental results, are, however, encouraging both with regard to the effectiveness of the PPCM laser[4] and the imaging experiment described below which involved the apparatus of Fig. 4.

The expanded and spatially filtered output of a dye laser illuminated a transparency $T$ [Fig. 5(a)]. The beam then passed through lens $L_2$ and converged on the PPCM, consisting of a single poled crystal of barium titanate[8] lying in the $M_1$-$M_2$ resonator cavity. The reflected beam was picked off by a beam splitter (BS) and photographed at the location where a phase conjugate image would be expected. The result is shown in Fig. 5(b). The phase conjugating behavior of the PPCM is evident. Intensity patterns of the osillation beams in the $M_1$-$M_2$ cavity are shown in Figs. 5(c) and 5(d). These were photographs of the intensities at mirrors $M_1$ and $M_2$, respectively. There is no discernible relationship between the spatial modulation of these beams and that of the pumping input beam. The specklelike patterns at $M_1$ and $M_2$ are believed to be due to optical inhomogeneities in the crystal.

In summary we have developed a theory of passive (self-pumped) phase conjugation, yielding oscillation thresholds and phase conjugate reflectivities. An experiment showing the phase conjugating nature of the PPCM was described. We expect the PPCM to have important applications in the fields of laser design and phase conjugation in remote areas, where separate pumping beams are unavailable.

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[7] This coupling strength depends on parameters such as crystal orientation electro-optic tensor, and index grating wave vector [see B. Fischer, M. Cronin-Golomb, J. O. White, and A. Yariv, Opt. Lett. **6**, 519 (1981)]. When it is real, the phase shift between the light interference pattern and the refractive index grating is $\pi/2$. This is a behavior typical of photorefractive materials with no bias field. When $\gamma l$ is purely imaginary, the index grating is in phase with the light interference pattern, which is the case in media which have a local response such as atomic vapors.

[8] This crystal measured $7\times4.5\times4$ mm and was poled into a single domain so that the $c$ axis was parallel to the 4-mm side.