Passivity-based control of a class of Blondel-Park transformable electric machines*

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In this paper we study the viability of extending, to the *general rotating electric machine's* model, the passivity-based controller method that we have developed for induction motors. In this approach the *passivity* (energy dissipation) properties of the motor are taken advantage of at two different levels. First, we prove that the motor model can be decomposed as the feedback interconnection of two passive subsystems, which can essentially be identified with the electrical and mechanical dynamics. Then, we design a torque tracking controller that preserves passivity for the electrical subsystem, and leave the mechanical part as a "passive disturbance". In position or speed control applications this procedure naturally leads to the well known cascaded controller structure which is typically analyzed invoking time-scale separation assumptions. A key feature of the new *cascaded control paradigm* is that the latter arguments are obviated in the stability analysis. Our objective in this paper is to characterize a class of machines for which such a passivity-based controller solves the *output feedback torque tracking* problem. Roughly speaking, the class consists of machines whose nonactuated dynamics are well *damped* and whose electrical and mechanical dynamics can be suitably *decoupled* via a coordinate transformation. The first condition translates into the requirement of approximate knowledge of the rotor resistances to avoid the need of injecting high gain into the loop. The latter condition is known in the electric machines literature as Blondel-Park transformability, and in practical terms it requires that the air-gap magnetomotive force must be suitably approximated by the first harmonic in its Fourier expansion. These conditions, stemming from the construction of the machine, have a clear physical interpretation in terms of the couplings between its electrical, magnetic and mechanical dynamics, and are satisfied by a large number of practical machines. The passivity-based controller presented here reduces to the well known indirect vector controller for current-fed induction machines. Our developments constitute an extension, to voltage-fed machines, of this *de facto* standard in industrial applications. Furthermore, our analysis provides it with a solid theoretical foundation.

1. Introduction

One of the main driving forces of control theory has traditionally been the characterization of classes of systems for which a certain control objective is achievable. This allows us to identify the system structural constraints—usually expressed in terms

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of system invariants—which are compatible with the desired performance. A fairly complete theory along this line is now available both for linear [7], [36] and nonlinear systems [24], [52]. From the practical viewpoint the interest in identifying the systems invariants is to be able to attach to them some *physical interpretation* that can be used in the controller design. Although in the linear case this characterization is readily available and practically useful, for instance in the form of effective transmission delays or sensor actuator couplings, their nonlinear counterparts developed to date seem to be less suitable for practical applications. A good example of this state of affairs concerns the fundamental problem of *stabilizability*. Stabilizable systems have been identified via geometric conditions for linearization [24], [52], [45] or transformation to particular (triangular, pure feedback) forms [49], [30], [46]. (See [26], [59], [30], [46] for recent surveys and other characterizations of stabilizable systems). In the authors' opinion there are several shortcomings to these results. First, the geometric conditions seem to be hard to verify in practical applications and are usually unrelated with the physical constraints of the system. Second, the control laws typically require full state measurement, hence the inclusion of observers for their practical implementation. Finally, since they are derived neglecting the systems physical constraints, they are often not well defined in all the operating regimes.

An alternative approach to systems stabilization that, to a certain extent, overcomes the aforementioned shortcomings is to relate stabilizability of the system to the possibility of rendering it *passive* via feedback. This idea, first advanced in an adaptive control context in [60], has been explored in[1], e.g., [54], [9], [62], [27]. The relevance of using passivity as a building block for control systems design stems, not just from the important role this concept plays in systems stability analysis and its close connection with the systems physics, but also in the *invariance* of this property *vis a vis* feedback interconnection [13]. This aspect is particularly relevant in our work where the key steps in the controller design are the decomposition of the system into passive subsystems, and the choice of a feedback control that preserves the passivity of the closed loop.

Our motivation in this paper is to contribute, if modestly, to the development of a stabilization theory for physical systems that incorporates at a fundamental level the systems physical structure. To achieve this objective we restrict ourselves to a particular, but practically very important, class of systems—*the general rotating electric machine* [47], [71], and establish conditions for stabilizability which are interpretable in terms of the systems physics. Our main contribution is the definition of a class of machines such that the *output feedback torque tracking* problem can be solved via a passivity-based controller. Roughly speaking, the class consists of machines whose nonactuated (rotor) dynamics are *suitably damped* and whose electrical and mechanical dynamics can be "partially" *decoupled*, via a coordinate transformation. Machines satisfying the latter condition are known in the electric machines literature as Blondel-Park transformable [41]. In practical terms it requires that the air-gap magnetomotive force be suitably approximated by the first harmonic in a Fourier expansion. These two conditions, stemming from the construction of the machine, have a clear physical interpretation in terms of materials and the couplings

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[1]Some efforts to reinterpret from a passivity perspective the currently popular "backstepping technique" [32], [31] derived for systems with special forms are reported in [43], [35], [30]. See also [59] for some interesting connections between input to state stability and dissipativity.

between its electrical, magnetic and mechanical dynamics, and are satisfied by a large number of practical machines.

The idea of *passivity-based* controller designs is to reshape the systems natural energy and inject the required damping in such a way that the control objective is achieved. Expected advantages of this approach are the enhanced robustness properties and lack of (controller calculation) singularities, properties which stem from the fact that cancellation of systems nonlinearities is avoided. The technique has its roots in classical mechanics [21], [2] and was introduced in control theory in the seminal paper [66], see also [58] for an early reference. This method has been instrumental in the solution of several robotics [55], [3], [37], [1], induction motor [16], [56] and power electronics [63] problems which were untractable with other stabilization techniques. See also the recent interesting book [68].

The passivity-based controller design we present here is the extension of a class of Blondel-Park transformable machines of the one used in [16] for induction motors, and proceeds as follows. First, we carry out a decomposition of the system dynamics as a *feedback interconnection of passive subsystems* where the outputs of the forward subsystem are the measurable and the regulated outputs. A lemma that gives conditions on the energy function under which this is possible is given. Second, we design an inner feedback loop that, via the injection of a nonlinear damping term, ensures the controlled subsystem defines a *strictly* passive map from *control* signals to *measurable* outputs. Third, the passivity-based technique is applied to this subsystem leaving the feedback subsystem as a "passive perturbation". As explained in [16] this last step involves the definition of the desired closed-loop energy function whose associated (target) dynamics evolves on a subspace of the state space insuring zero error tracking[2]. The overall procedure leads to a *nonlinear dynamic output feedback* controller that ensures global asymptotic torque tracking with internal stability.

In position or speed control applications an outer loop is added to the torque controller [56]. As pointed out in [39], there appears to be universal agreement that this structure, (with a fast inner current loop that can be regarded as creating impressed currents to the stator windings necessary to achieve the desired torque specified by the outer loops), is the most effective control scheme for electrical drives. This procedure naturally leads to the well known cascaded controller structure which is typically analyzed invoking time-scale separation assumptions. A key feature of the *new cascaded control paradigm* is that stability is now established without these arguments. See [57] for further discussions on this topic.

The remainder of the paper is organized as follows. In Section 2 we present a simple, but important, passive systems feedback decomposition lemma. The class of systems considered in the paper—the generalized electric machine's model—is given in Section 3, and in Section 4 we formulate the control problem and describe the design procedure. Conditions for strict passifiability of the electrical subsystem are established in Section 5. In Section 6 and 7, we explain how the passivity based approach is used to achieve current and torque tracking, and in Section 8 we present the main results in this paper. Examples and experimental results are given in Section 9 and Section 10. Finally, concluding remarks are given in Section 11.

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[2]It is worth pointing out that to overcome the singularity problems on a backstepping-based controller for induction motors the authors in [12] borrowed this last step from [56].

Notation

See also [13]. $\mathcal{R}$—field of real numbers; $\|\cdot\|$—Euclidean norm; $(\cdot)^T$—transposition; $\mathcal{I}_n - n \times n$ identity matrix; $e^{(\cdot)}$—matrix exponential;

$$\frac{d}{dt}(\cdot) = (\dot{\cdot})\text{—total time derivative;}$$

$\mathcal{L}_2^n, \mathcal{L}_\infty^n, \mathcal{L}_{2e}^n, \mathcal{L}_{\infty e}^n$—spaces of n-dimensional square integrable, essentially bounded functions and their extensions; a causal system $\Sigma: \mathcal{L}_{2e}^n \mapsto \mathcal{L}_{2e}^n$ is said to be *passive* (resp. *output strictly passive*) if $\exists \alpha \geq 0$ (resp. > 0) and $\exists \beta \in \mathcal{R}$ such that

$$\int_0^\gamma u^T(t)(\Sigma u)(t)\,dt \geq \alpha \int_0^\gamma \|\Sigma u\|^2 dt + \beta, \ \forall u \in \mathcal{L}_{2e}^n, \ \forall \gamma \geq 0.$$

2. Passive subsystems feedback decomposition

We consider nonlinear electromechanical systems described by the Euler-Lagrange equations of motion [47]

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}(q, \dot{q})}{\partial \dot{q}}\right] - \frac{\partial \mathcal{L}(q, \dot{q})}{\partial q} = Q \tag{2.1}$$

where[3] $\mathcal{L} = \mathcal{J}^*(\dot{q}, q) - \mathcal{V}(q)$ is the system Lagrangian, $\mathcal{J}^*: \mathcal{R}^{2n} \to \mathcal{R}$ is the total system coenergy-state function, $\mathcal{V}: \mathcal{R}^n \to \mathcal{R}$ is the total system energy-state function, $q \in \mathcal{R}^n$ are the generalized coordinates and $Q \in \mathcal{R}^n$ are the external (dissipative and control) forces. We assume that the system energy-state function (potential energy) is bounded from below.

From the well known energy balance equation of Euler-Lagrange systems [68]

$$\underbrace{H(t) - H(0)}_{\text{stored energy}} = \underbrace{\int_0^t \dot{q}^T Q\,ds}_{\text{dissipated + supplied}}$$

where

$$H := \left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)^T \dot{q} - \mathcal{L}$$

is the overall system's total energy, it follows immediately that the system (2.1) defines a passive operator $\Sigma: \mathcal{L}_{2e}^n \to \mathcal{L}_{2e}^n: Q \mapsto \dot{q}$.

Lemma 2.1.

Assume the Lagrangian of (2.1) can be decomposed in the form

$$\mathcal{L} = \mathcal{L}_e(q_e, \dot{q}_e, q_m) + \mathcal{L}_m(q_m, \dot{q}_m)$$

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[3]To simplify the notation, here and throughout the rest of the paper we will omit the arguments of the functions, giving them explicitly only when they are first defined.

where $q := [q_e^T, q_m^T]^T$ with $q_e \in \mathscr{R}^{n_e}$ and $q_m \in \mathscr{R}^{n_m}$. Then, Σ can be represented as the negative feedback interconnection of two *passive subsystems*

$$\Sigma_e : \mathscr{L}_{2e}^n \to \mathscr{L}_{2e}^n : \begin{bmatrix} Q_e \\ -\dot{q}_m \end{bmatrix} \mapsto \begin{bmatrix} \dot{q}_e \\ y \end{bmatrix}$$

$$\Sigma_m : \mathscr{L}_{2e}^{n_m} \to \mathscr{L}_{2e}^{n_m} : (y + Q_m) \mapsto \dot{q}_m$$

where

$$y := \frac{\partial \mathscr{L}_e}{\partial q_m}$$

is the subsystems *coupling signal*, and $Q := [Q_e^T, Q_m^T]^T$ with $Q_e \in \mathscr{R}^{n_e}$, $Q_m \in \mathscr{R}^{n_m}$.

Proof

Using the Euler-Lagrange procedure to derive the equations of motion, we get

$$\frac{d}{dt}\left[\frac{\partial \mathscr{L}_e}{\partial \dot{q}_e}\right] - \frac{\partial \mathscr{L}_e}{\partial q_e} = Q_e \tag{2.2}$$

$$\frac{d}{dt}\left[\frac{\partial \mathscr{L}_m}{\partial \dot{q}_m}\right] - \frac{\partial \mathscr{L}_m}{\partial q_m} = Q_m + y \tag{2.3}$$

Evaluating the total time derivative of $\mathscr{L}_e$ we get

$$\dot{\mathscr{L}}_e = \frac{\partial \mathscr{L}_e^T}{\partial q_e}\dot{q}_e + \frac{\partial \mathscr{L}_e^T}{\partial \dot{q}_e}\ddot{q}_e + \frac{\partial \mathscr{L}_e^T}{\partial q_m}\dot{q}_m \tag{2.4}$$

Noting that

$$\frac{\partial \mathscr{L}_e^T}{\partial \dot{q}_e}\ddot{q}_e = \frac{d}{dt}\left[\frac{\partial \mathscr{L}_e^T}{\partial \dot{q}_e}\dot{q}_e\right] - \frac{d}{dt}\left[\frac{\partial \mathscr{L}_e^T}{\partial \dot{q}_e}\right]\dot{q}_e$$

inserting this into (2.4), using (2.2) and rearranging the terms, we obtain

$$\frac{d}{dt}H_e = Q_e^T\dot{q}_e - \dot{q}_m^T y$$

where

$$H_e(q_e, \dot{q}_e, q_m) := \frac{\partial \mathscr{L}_e^T}{\partial \dot{q}_e}\dot{q}_e - \mathscr{L}_e$$

Using the arguments of Section 9.1 in [11] we can show that H_e is the total energy of the subsystem Σ_e. Integrating from 0 to γ and setting $\beta_e := -H_e(0)$ proves the passivity of Σ_e.

A similar procedure can be used to establish the passivity of Σ_m, using the energy function

$$H_m = \frac{\partial \mathscr{L}_m^T}{\partial \dot{q}_m}\dot{q}_m - \mathscr{L}_m$$

and (2.3).

3. General Rotating Machine

2.1. *Model*

In this paper we treat the general rotating machine considered in [41], (see also [74],

[72]). It consists of a total of n_e windings on stator and rotor and we assume ideal symmetrical phases and sinusoidally distributed phase windings. The permeability of the fully laminated cores is assumed to be infinite and saturation, iron losses, end winding and slot effects are neglected. Linear magnetic materials are considered, and we assume that all parameters are constant and known[4].

Under the assumptions above, application of Gauss' law and Ampere's law leads to the following affine relationship between the *flux linkage* vector $\lambda = [\lambda_1, ..., \lambda_{n_e}]^T$ and the *current* vector $\dot{q}_e = [\dot{q}_1, ..., \dot{q}_{n_e}]^T$

$$\lambda = D_e(q_m)\dot{q}_e + \mu(q_m) \tag{3.1}$$

where $q_m \in \mathcal{R}$ is the *mechanical position* of the motor, $D_e = D_e^T > 0$ is the $n_e \times n_e$ multiport inductance matrix of the windings and the vector μ represents the flux linkages due to the possible existence of *permanent magnets*. Both D_e and μ are bounded and periodic in q_m.

If we define as *generalized coordinates* of the system the total amounts of moving electric charge that has passed any given point on the different phase windings, q_i, $i = 1, ..., n_e$, and the angular position of the rotor q_m, we can compute the magnetic-field coenergy (with $'$ denoting the variable of integration) as [47]

$$\mathcal{K}_e = \sum_{i=1}^{n_e} \int_0^{q_i} \lambda_i(\dot{q}_i') = \frac{1}{2}\dot{q}_e^T D_e \dot{q}_e + \mu^T \dot{q}_e$$

and the mechanical kinetic coenergy as

$$\mathcal{K}_m = \frac{1}{2}D_m \dot{q}_m^2,$$

where $D_m > 0$ is the rotational inertia of the rotor.

Neglecting the capacitive effects in the windings of the motor and considering a rigid shaft, we have that the potential energy $\mathcal{V}$ of the system is only due to the interactions between the magnetic materials in stator and rotor[5], i.e. $\mathcal{V} = \mathcal{V}(q_m)$. We get the *Lagrangian*

$$\mathcal{L}(\dot{q}_e, \dot{q}_m, q_m) = \frac{1}{2}\dot{q}_e^T D_e \dot{q}_e + \mu^T \dot{q}_e + \frac{1}{2}D_m \dot{q}_m^2 - \mathcal{V}(q_m) \tag{3.2}$$

To model the *external forces* we will assume that the dissipative effects are linear time invariant and only due to the resistances in the windings $r_i \geq 0$, $i = 1, ..., n_e$, and the mechanical friction $R_m \geq 0$. The control forces are the voltages applied to the windings $u \in \mathcal{R}^{n_s}$, $n_s \leq n_e$. We consider here fully actuated as well as underactuated machines, that is, machines where the voltages can be applied only to stator windings (e.g., induction motor), or to both stator and rotor windings (e.g., synchronous motor with field windings). Hence, we find convenient to partition the vector of generalized electrical coordinates as $q_e = [q_s^T, q_r^T]^T \in \mathcal{R}^{n_e}$, $q_s \in \mathcal{R}^{n_s}$, $q_r \in \mathcal{R}^{n_r}$, $n_r = n_e - n_s$, where the subscripts s, r are used to denote variables related to windings with and without actuation respectively. In the case of underactuated machines the partition coincides with stator and rotor variables as well. Notice however, that there are also machines, like the PM synchronous, PM stepper and variable reluctance motors, where q_e

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[4]See [14] for relaxations of the latter restriction.

[5]This energy contribution is zero if there are magnetic materials in only one part (stator or rotor), and the reluctance properties of the other part is uniform.

consists only of stator variables which are directly actuated by the stator voltages, see the examples in Section 3.3. Finally, we assume[6] that the load torque y_L in the mechanical subsystem is of the form

$$y_L(q_m, \dot{q}_m) = [k_1 + k_3 \dot{q}_m^2] \tanh\left(\frac{\dot{q}_m}{\rho}\right) + k_2 \dot{q}_m \tag{3.3}$$

with a scaling parameter $\rho > 0$.

With the considerations above, and applying the Euler-Lagrange equations to (3.2), we obtain the *equations of motion* of the generalized machine as

$$D_e \ddot{q}_e + W_1(q_m) \dot{q}_m \dot{q}_e + W_2(q_m) \dot{q}_m + R_e \dot{q}_e = M_e u \tag{3.4}$$

$$D_m \ddot{q}_m - y(\dot{q}_e, q_m) + R_m \dot{q}_m = -y_L \tag{3.5}$$

where

$$W_1 := \frac{dD_e(q_m)}{dq_m}, \ W_2 := \frac{d\mu(q_m)}{dq_m}, \ R_e := \text{diag}\{r_s \mathscr{I}_{n_s}, r_r \mathscr{I}_{n_r}\}, \ M_e := \begin{bmatrix} \mathscr{I}_{n_s} \\ 0 \end{bmatrix}$$

with y the generated *torque*

$$y = h(\dot{q}_e, q_m) := \frac{1}{2} \dot{q}_e^T W_1 \dot{q}_e + W_2^T \dot{q}_e + \eta(q_m) \tag{3.6}$$

and

$$\eta(q_m) = -\frac{dV}{dq_m} \tag{3.7}$$

which is also bounded and periodic in q_m.

Notice that the machine is *fully characterized* by its dynamic parameters $D := \text{diag}\{D_e, D_m\}$, M_e, μ, η and the dissipation parameters $R := \text{diag}\{R_e, R_m\}$. Therefore, in the sequel we will refer to the system described by (3.4), (3.5), (3.6) as $\Sigma(D, R, M_e, \mu, \eta)$. Similarly, the electrical subsystem (3.4), (3.6) will be denoted by $\Sigma_e(D_e, R_e, M_e, \mu, \eta)$, and the mechanical subsystem (3.5) by $\Sigma_m(D_m, R_m)$.

3.2. Remarks

1. We have derived here the model of the electrical machine using the Lagrangian formalism. It is clear that the same model can be obtained from the application of the basic force laws. Besides its esthetic appeal and generality, variational modeling is the natural framework for passivity-based designs, where energy functions and workless forces must be identified. Furtheremore, as pointed out in [71], we tend to believe that "the long-term value of variational techniques precludes their dismissal merely on the ground of conceptual difficulties or lack of physical insight when employed by the novice".

2. If we introduce the flux[7] λ as defined by (3.1) into (3.4), we get the voltage

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[6]The presence of a load torque y_L of this form ensures that to every bounded y there exists a bounded $\dot{q}_m$. Except from this, as will be shown below, the load torque y_L plays no role in the torque tracking problem. Also, as shown in [16] it can be treated as an external disturbance for the speed tracking problem.

[7]We recall that fluxes and currents are the generalized electrical momenta and velocities of the Hamiltonian formalism [71].

balance equation $\dot{\lambda} + R_e \dot{q}_e = M_e u$. Particularly useful for further developments is the following relationship between motor fluxes and rotor currents for motors where the rotor windings are short circuited (induction motors)

$$\dot{\lambda}_r + R_r \dot{q}_r = 0 \tag{3.8}$$

where $\lambda := [\lambda_s^T, \quad \lambda_r^T]^T$ and $R_e = \text{diag}\{R_s, \quad R_r\}$ with $R_s = r_s \mathscr{I}_{n_s} \in \mathscr{R}^{n_s \times n_s}$, $R_r = r_r \mathscr{I}_{n_r} \in \mathscr{R}^{n_r \times n_r}$. The importance of (3.8) is that it defines a dynamic relationship between $\dot{q}_r$ and λ_r which is unaffected by the control action, hence we will have to take this into account when defining a "desired behaviour" for the machine.

3. An interesting property of $\Sigma(D, R, M_e, \mu, \eta)$ and other magnetic field devices in which currents are of main interest, is that the electric charges are *ignorable* [71], (also known as cyclic in mechanics [2]). That is, the Lagrangian of the system does not contain q_e (although it contains the corresponding currents $\dot{q}_e$). It must be pointed out that when choosing the form (2.1) of the Euler-Lagrange equations, it is crucial that the currents are expressed in their natural frames (where electric charges can be obtained by integration of currents) to avoid introduction of quasi coordinates. See also [71].

4. For machines with mechanical commutation, the relation between port and rotor currents introduces *non-holonomic constraints* [71], [50] and the dynamic equations cannot be obtained directly from (2.1) with the given Lagrangian. Instead quasi coordinates could be introduced, and the dynamic equations derived by using the Boltzmann-Hamel [71] or Gaponov [50] form of the Euler-Lagrange equations. These procedures are however quite involved, and the dynamic equation for this class of machines is therefore usually not derived from variational principles, but by the use of basic laws as Faraday's law, Ohm's law and Euler's law. Also, since the Euler-Lagrange procedure is based on energy properties, the dynamic equations for machines with nonlinear magnetics can also be derived by this procedure. The switched reluctance machine is an example of a machine in which the magnetic nonlinearities must be taken into account. We will *not consider* machines with mechanical commutation or nonlinear magnetics in this paper, but refer the reader to [20] where the latter issue is addressed.

3.3. *Examples*

In [41] several examples of electric machines described by $\Sigma(D, R, \mu, \eta)$ are given. We present here two examples of *fully actuated* machines, i.e., where $n_s = n_e$ and $M_e = \mathscr{I}_{n_e}$.

(i) For the 3ϕ *PM synchronous motor* [33] we have $n_e = 3$ and the parameters
$$D_e(q_m) =$$

$$\begin{bmatrix} L_{ls} + L_A - L_B\cos 2n_p q_m & -\frac{1}{2}L_A - L_B\cos 2\left(n_p q_m - \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B\cos 2\left(n_p q_m + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_A - L_B\cos 2\left(n_p q_m - \frac{\pi}{3}\right) & L_{ls} + L_A - L_B\cos 2\left(n_p q_m - \frac{2\pi}{3}\right) & -\frac{1}{2}L_A - L_B\cos 2(n_p q_m + \pi) \\ -\frac{1}{2}L_A - L_B\cos 2\left(n_p q_m + \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B\cos 2(n_p q_m + \pi) & L_{ls} + L_A - L_B\cos 2\left(n_p q_m + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$\tag{3.9}$$

$$\mu(q_m) = \lambda_m \begin{bmatrix} \sin n_p q_m \\ \sin\left(n_p q_m - \dfrac{2\pi}{3}\right) \\ \sin\left(n_p q_m + \dfrac{2\pi}{3}\right) \end{bmatrix} \qquad (3.10)$$

where L_{ls}, L_A, L_B are inductance parameters, n_p is the number of pole pairs and λ_m is the amplitude of the flux linkage established by the permanent magnet.

(ii) The 2ϕ *PM stepper motor* has $n_e = 2$ and the following parameters [75]

$$D_e = \begin{bmatrix} L & 0 \\ 0 & L \end{bmatrix} \qquad (3.11)$$

$$\mu(q_m) = \frac{K_m}{N_r} \begin{bmatrix} \cos(N_r q_m) \\ \sin(N_r q_m) \end{bmatrix} \qquad (3.12)$$

L is the self-inductance of each winding, and K_m is the torque constant. N_r is the number of rotor teeth of same polarity. In this case the torque has a term due to the interaction between the permanent magnet and the magnetic material in the stator (detent torque), and $\eta(q_m) = -K_D \sin(4N_r q_m)$, $K_D \approx 5 - 10\%$ of $K_m i_0$, where i_0 is the rated current.

4. Problem formulation and Design Procedure

4.1. *Problem formulation*

We will assume here that the currents of the actuated windings $\dot{q}_s$ and rotor position q_m and velocity $\dot{q}_m$ are *available for measurement*. Also, we will take as basic *regulated variable* the generated torque y, which is however *unmeasurable* since it depends on the variables $\dot{q}_r$. Notice that the motor speed is related to the latter via a simple linear *passive* operator (3.5). Thus by regulating $y - y_L$ and ensuring passivity in closed loop, as can be done in passivity-based control, we can expect to have a good behaviour in the mechanical subsystem with a simple (PI) speed outer loop.

Our control problem can therefore be formulated as follows:

Output feedback torque tracking problem. Consider the $2n_e + 2$ dimensional machine model $\Sigma(D, R, M, \mu, \eta)$ with state vector $[q_e^T, \dot{q}_e^T, q_m, \dot{q}_m]^T$, inputs $u \in \mathcal{R}^{n_s}$, regulated output y, measurable outputs $\dot{q}_s$, q_m, $\dot{q}_m$ and smooth disturbance $y_L(t) \in \mathcal{L}_\infty$. Find conditions on D, R, μ, η such that, for all smooth desired output functions $y_d(t) \in \mathcal{L}_\infty$ with known derivative $\dot{y}_d(t) \in \mathcal{L}_\infty$, *global torque tracking* with internal stability is achieved, i.e. $\lim_{t \to \infty}(y - y_d) = 0$ with all internal signals bounded. Further, for *underactuated* machines, we will require asymptotic *flux amplitude tracking*, that is, for a given *bounded function* $\beta(t) \geq \delta > 0$, we must have that $\lim_{t \to \infty} |\,\|\lambda_r\| - \beta(t)\,| = 0$.

4.2. *Design procedure*

The rationale of our design, takes off from the passive subsystems decomposition of Section 2 and, "disregarding" the mechanical dynamics, attempts to regulate the generated torque y by imposing a desired value to the currents $\dot{q}_e$. There are therefore three natural steps to follow:

1. Apply the passive subsystems decomposition of Section 2 to the motor, to view Σ_e as the "system to be controlled", and Σ_m as a *passive disturbance*. To ensure
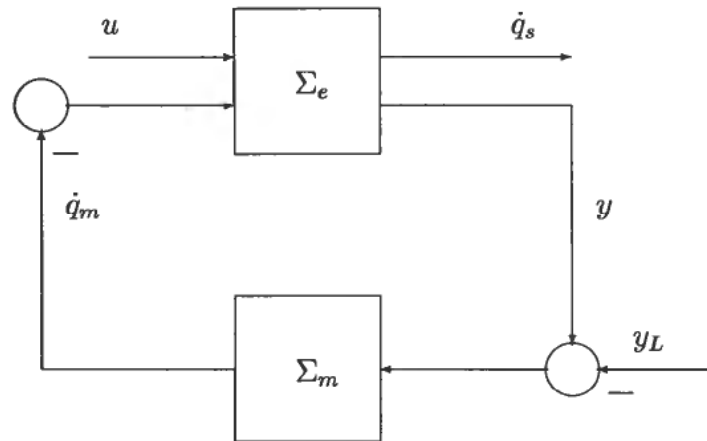
Figure 1.   Passive subsystem decomposition.

the latter does not "destroy" the stability of the loop we must *inject damping* to Σ_e so as to strengthen its passivity property to *strict* passivity.

2. Define a set of "*attainable*" currents $\dot{q}_{ed}$, i.e., those for which we can find a control law that ensures $\lim_{t\to\infty}\dot{q}_e = \dot{q}_{ed}$. To this end we *shape the energy* of the closed loop to match a desired energy (storage) function, which we choose here as

$$H_{ed} := \frac{1}{2}\dot{\tilde{q}}_e^T D_e \dot{\tilde{q}}_e,$$

where the current error is defined as

$$\dot{\tilde{q}}_e := \dot{q}_e - \dot{q}_{ed} \tag{4.1}$$

3. Among the attainable currents choose $\dot{q}_{ed}$ to deliver the desired torque y_d, that is, such that if $\dot{q}_e \equiv \dot{q}_{ed}$ then $y \equiv y_d$. Finally, give conditions under which $\lim_{t\to\infty} y = y_d$ with internal stability.

5.   Strict passifiability via damping injection

In this section we will carry out the first step of our design procedure, namely, decomposition of the model into passive subsystems Σ_e, Σ_m, and strict passifiability of Σ_e.

5.1.   Feedback decomposition

Proposition 5.1.   The system $\Sigma(D, R, M_e, \mu, \eta)$ can be represented as the negative feedback interconnection of two *passive subsystems* (see Fig. 1)

$$\Sigma_e: \mathscr{L}_{2e}^{n_s+1} \to \mathscr{L}_{2e}^{n_s+1} : \begin{bmatrix} u \\ -\dot{q}_m \end{bmatrix} \mapsto \begin{bmatrix} \dot{q}_s \\ y \end{bmatrix}$$

$$\Sigma_m: \mathscr{L}_{2e} \to \mathscr{L}_{2e} : (y - y_L) \mapsto \dot{q}_m$$

Proof

The proof is a corollary of Lemma 2.1 noting that the Lagrangian of the electric machine (3.2) can be decomposed into

$$\mathscr{L}_e(\dot{q}_e, q_m) = \frac{1}{2}\dot{q}_e^T D_e \dot{q}_e + \mu^T \dot{q}_e, \quad \mathscr{L}_m(q_m, \dot{q}_m) = \frac{1}{2}D_m \dot{q}_m^2 - \mathscr{V}(q_m)$$

5.2. *Conditions for damping injection*

We now identify a class of machines for which—*injecting damping to* Σ_e—we can ensure that the map from control input to measurable output is *output strictly passive*. The physical interpretation of the assumptions is given after the proposition.

Proposition 5.2.   Consider the subsystem $\Sigma_e(D_e, R_e, \mu, \eta)$. Assume

A.1. $R_e := \text{diag}\{R_s, R_r\}$ with $R_s \in \mathscr{R}^{n_s \times n_s}$, $R_r \in \mathscr{R}^{n_r \times n_r}$ diagonal matrices and $R_r > 0$

A.2. The $n_r \times n_r$-dimensional (2, 2) block of the matrix

$$W_1 = \frac{dD_e}{dq_m}$$

is zero, i.e.

$$W_1 = \begin{bmatrix} (W_1)_{11} & (W_1)_{12} \\ (W_1)_{21} & 0_{n_r \times n_r} \end{bmatrix}$$

A.3. The non-actuated rotor components of the vector μ are independent of q_m, that is

$$W_2 = \frac{d\mu}{dq_m} := \begin{bmatrix} W_{2s} \\ 0 \end{bmatrix}, \; W_{2s} \in \mathscr{R}^{n_s}$$

Under these conditions there is a nonlinear *output feedback* of the form

$$u = v + W_{2s}\dot{q}_m - K_1(q_m, \dot{q}_m)\dot{q}_s \tag{5.1}$$

such that the map $v \mapsto \dot{q}_s$ is *output strictly passive* for all $q_m, \dot{q}_m \in \mathscr{L}_{2e}$.

Proof

The dynamics of Σ_e is described by (3.4) which we repeat here for ease of reference

$$D_e\ddot{q}_e + W_1\dot{q}_m\dot{q}_e + W_2\dot{q}_m + R_e\dot{q}_e = M_e u$$

Closing the loop with (5.1) we get

$$D_e\ddot{q}_e + C_e(q_m, \dot{q}_m)\dot{q}_e + R_{es}(q_m, \dot{q}_m)\dot{q}_e = M_e v \tag{5.2}$$

where we have defined

$$C_e := \frac{1}{2}W_1\dot{q}_m, \; R_{es} := R_e + \frac{1}{2}W_1\dot{q}_m + \begin{bmatrix} K_1 & 0 \\ 0 & 0 \end{bmatrix} \tag{5.3}$$

Taking the time derivative of the total energy of Σ_e, that is

$$H_e := \frac{1}{2}\dot{q}_e^T D_e \dot{q}_e,$$

along the trajectories of Σ_e we get

$$\dot{H}_e = \dot{q}_s^T v - \dot{q}_e^T R_{es}\dot{q}_e$$

Suppose that

$$K_1 = K_1^T > \sup_{q_m, \dot{q}_m}\left\{\frac{\dot{q}_m^2}{4}(W_1)_{12}R_r^{-1}(W_1)_{12}^T - \frac{1}{2}(W_1)_{11}\dot{q}_m\right\} \tag{5.4}$$

Then, by using standard matrix results, the symmetric matrix R_{es} can be shown to be

uniformly positive definite in the sense that $\inf_{q_m, \dot{q}_m} \lambda_{min}\{R_{es}\} \geq \alpha > 0$, where $(\cdot)_{ij}$ denotes the (i, j)-th submatrix of a block matrix and $\lambda_{min}\{\cdot\}$ is the minimum eigenvalue. Integration of $\dot{H}_e$ completes the proof.

5.3. *Remarks*

1. Notice that strict passivity is achieved, via the *nonlinear damping term $K_1 \dot{q}_s$*, (with K_1 explicitly depending on q_m and $\dot{q}_m$) which recovers the positivity of the "damping" matrix R_{es}.

2. In the case of full actuation, i.e. $n_s = n_e$, the required positivity of R_{es} is guaranteed if

$$K_1 = K_1^T > -R_e - \frac{1}{2} W_1 \dot{q}_m \qquad (5.5)$$

3. Assumption *A.1.* is a reasonable condition of *damping of the nonactuated dynamics* which is satisfied in all electric machines. The problem is with condition (5.4), which shows that to overcome the imprecise knowledge of the rotor resistances we will have to inject *high gains* into the loop.

4. *A.2.* is a *decoupled dynamics condition* equivalent to requiring that in the magnetic coenergy the contribution of the terms quadratic in $\dot{q}_r$ be independent of q_m. Physically, this translates into the condition that if there are rotor windings, then the rotor flux induced by the rotor currents must be independent of the rotor position. This means that the stator must have uniform reluctance properties (non-salient and of uniform magnetic material). This assumption is satisfied by many machines, e.g. classical Park [74] and poly-phase [72] machines.

5. Since the torque (3.6) consists of one component due to the currents, and the other of purely magnetical origin, and since we have no control on the magnetic fields from the permanent magnets, it is reasonable to expect that we must eliminate the effect on Σ_e of the flux linkages due to the permanent magnets. This explains the need for assumption *A.3.* Physically, this assumption also means that if the machine has rotor windings, then the stator must have uniform reluctance properties i.e., if the machine has permanent magnets, then they can only be placed on the rotor. As can be seen from (5.1), the term from the permanent magnets must be cancelled out. The need for this cancellation is a drawback of the scheme. However, the term is generally a vector with periodic functions in a measurable quantity (position), and proportional to a constant which can be precisely identified.

6.   Current tracking via energy shaping

We now define the "attainable" currents $\dot{q}_{ed}$ of the second step of the design procedure.

Proposition 6.1.   *If* in (5.1) v and $\dot{q}_{ed} \in \mathscr{L}_\infty^{n_e}$ satisfy

$$M_e v = D_e \ddot{q}_{ed} + C_e \dot{q}_{ed} + R_{es} \dot{q}_{ed} \qquad (6.1)$$

then (see (4.1) for the definition of the current error) $\tilde{\dot{q}}_e \rightarrow 0$ as $t \rightarrow \infty$ independently of q_m, $\dot{q}_m$ and the choice of $\dot{q}_{ed}$. Furthermore, when $\dot{q}_{ed}$ is bounded then $\dot{q}_m$, $\dot{q}_e$ and y also bounded. In addition, boundedness of $\ddot{q}_{ed}$ ensures that $\ddot{q}_e$ and v are also bounded.

Proof

Rewriting (5.2) in terms of the error signals gives

$$D_e\ddot{\tilde{q}}_e + C_e\dot{\tilde{q}}_e + R_{es}\dot{\tilde{q}}_e + \psi \tag{6.2}$$

with

$$\psi := M_e v - (D_e\ddot{q}_{ed} + C_e\ddot{q}_{ed} + R_{es}\dot{q}_{ed}) \tag{6.3}$$

Then (6.1) implies $\psi \equiv 0$, and the dynamics of the system is fully described by

$$D_e\ddot{\tilde{q}}_e + C_e\dot{\tilde{q}}_e + R_{es}\dot{\tilde{q}}_e = 0 \tag{6.4}$$

$$D_m\ddot{q}_m + R_m\dot{q}_m = y - y_L \tag{6.5}$$

These equations are locally Lipschitz in state, and under the assumptions on the desired torque and load torque, they are continuous in t, so there exists $t_1 > 0$ such that in the time interval $[0, t_1)$ the solutions exists and are unique.

Taking the time derivative of the desired energy function

$$H_{ed} = \frac{1}{2}\dot{\tilde{q}}_e^T D_e \dot{\tilde{q}}_e$$

along the trajectories of (6.2) we get

$$\dot{H}_{ed} = -\dot{\tilde{q}}_e^T R_{es}\dot{\tilde{q}}_e, \ \forall t \in [0, t_1) \tag{6.6}$$

It follows from (6.6) and the proof of Proposition 5.2 that

$$\dot{H}_{ed} = -\dot{\tilde{q}}_e^T R_{es}\dot{\tilde{q}}_e \le -\alpha\|\dot{\tilde{q}}_e\|^2, \ \forall t \in [0, t_1) \tag{6.7}$$

and we can conclude that

$$\|\dot{\tilde{q}}_e(t)\| \le m_e\|\dot{\tilde{q}}_e(0)\|e^{-\rho_e t}, \ \forall t \in [0, t_1) \tag{6.8}$$

holds with

$$m_e = \sqrt{\frac{\lambda_{max}(D_e)}{\lambda_{min}(D_e)}}$$

and

$$\rho_e = \frac{\alpha}{\lambda_{max}(D_e)},$$

which are independent of t_1.

From this, and since $\dot{q}_{ed} \in \mathscr{L}_\infty^{n_e}$, we deduce that $\dot{q}_e$ is bounded on the open interval $[0, t_1)$. Now, we are going to prove that it remains bounded also on the *closed* interval $[0, t_1]$. To this end, notice that the right hand side of (6.5) is also bounded on $[0, t_1)$, thus its solution cannot grow faster than an exponential, and consequently $\dot{q}_m, q_m$ remain bounded $\forall t \in [0, t_1]$. This in its turn ensures the boundedness of R_{es}, and consequently $\|\dot{\tilde{q}}_e\|$ cannot escape to infinity in this time interval.

Since m_e, ρ_e and α are independent of t_1, it is possible to repeat this argument for a new initial condition, to define solutions on the time interval $[t_1, 2t_1]$. This shows that it is possible to extend this procedure to prove existence of solutions for the whole real axis, and the system cannot have finite escape time. It follows from (6.8) that

$$\lim_{t\to\infty} \dot{\tilde{q}}_e = 0 \tag{6.9}$$

and we can conclude that $\dot{q}_e$ is bounded, which implies that y is also bounded. From this, and the definition of the load torque (3.3), it follows from $\dot{q}_m$ remains bounded. Now, it follows from (6.4) and (6.3) that $\ddot{q}_e$ and v will be bounded if $\ddot{q}_{ed}$ is also bounded.

7.   From current tracking to torque tracking

It is now convenient to take a brief respite and recapitulate our derivations. The first step of Subsection 4.2 was carried out in Section 5 where we designed an inner control loop to ensure that (5.2) defines a strictly passive mapping $v \mapsto \dot{q}_s$. In Section 6 we carried out the second step, that is, we established a relationship between the control signal v and $\dot{q}_{ed}$ (6.1) which implies $\lim_{t \to \infty} \dot{q}_e = \dot{q}_{ed}$. The third step, to which this section is devoted, demands the definition of $\dot{q}_{ed}$ from the "attainable" set that delivers the desired torque and the establishment of conditions under which current tracking implies torque tracking. Notice that these steps are not straightforward since D_e, C_e and R_{es} (6.1), and y (3.6), depend on q_m and $\dot{q}_m$, thus some additional conditions on the couplings between the subsystems must be satisfied. These conditions are expressed in terms of restrictions on the parameters D_e, R_e, μ, η of the general electric machine model.

7.1.   *Desired current behaviour*

Motivated by (3.6) we proposed to define $\dot{q}_{ed}$ such that, for a given desired torque y_d the equation

$$y_d = \frac{1}{2}\dot{q}_{ed}^T W_1 \dot{q}_{ed} + W_2 \dot{q}_{ed} + \eta \tag{7.1}$$

holds. This gives

$$y - y_d = \frac{1}{2}\dot{\tilde{q}}_e^T W_1 \dot{\tilde{q}}_e + \dot{\tilde{q}}_e^T W_1 \dot{q}_{ed} + W_2^T \dot{\tilde{q}}_e$$

with the error signal $\dot{\tilde{q}}_e := \dot{q}_e - \dot{q}_{ed}$. Since W_1 and W_2 are bounded, it follows that asymptotic *torque tracking* will be achieved if we can ensure $\lim_{t \to \infty} \dot{q}_{ed}$ with $\dot{q}_{ed} \in \mathscr{L}_\infty^{n_e}$.

- It is clear then that to attain the torque tracking objective we must define bounded $\dot{q}_{ed}$ and v such that (7.1) and (6.1) both hold.

Towards this end, first notice that in the case of *fully actuated* machines $M_e = \mathscr{I}_{ne}$ and there are no restrictions on the set of "attainable" currents. That is, in this case, *for any* given $\ddot{q}_{ed}$, $\dot{q}_{ed}$, we can make $\psi \equiv 0$ by a suitable selection of v.

Now, to treat the case of *underactuated* machines, where we do not have enough control actions to directly set $\psi \equiv 0$ for any given $\dot{q}_{ed}$, we find it convenient to partition $\psi = [\psi_s^T, \psi_r^T]^T$ with $\psi_s \in \mathscr{R}^{n_s}$. Since $\psi_s \equiv 0$ can always be solved with a suitable choice of v, (see the definition of M_e), we concentrate our attention on the solution of $\psi_r \equiv 0$. We will show in the sequel that, although (6.1) and (7.1) are intimately related, additional restrictions on the machine model are required for a simultaneous solution. Interestingly enough, it turns out that these "decoupling" conditions are (a stronger version of) the well known *Blondel-Park* (BP) transformation conditions which are fundamental in the analysis of rotating machines [41], [47].

7.2.   *Decoupling conditions*

The following definition is in order:

Definition 7.1.   We say the machine $\Sigma(D, R, M_e, \mu, \eta)$ is *BP transformable* if there exists a current transformation

$$\dot{z}_e = P(q_m)\dot{q}_e = P_1 \mathbf{e}^{-Uq_m}\dot{q}_e \tag{7.2}$$

such that the dynamics of Σ_e in these coordinates are *independent* of q_m (but still depend on $\dot{q}_m$). P_1 is any nonsingular constant matrix. If furthermore the matrix U is of the form

$$U = \begin{bmatrix} 0 & 0 \\ 0 & U_{22} \end{bmatrix} = -U^T = \begin{bmatrix} 0 & 0 \\ 0 & -U_{22}^{-1} \end{bmatrix}$$

then we say the machine is *strongly BP* transformable.

From the structure of the matrix U above we can see the strong BP transformability means that the decoupling is achieved rotating *only the rotor variables*. As will become clear later, this condition is needed when the rotor circuits are not actuated, as in the induction motor case.

In the fundamental paper [41] necessary and sufficient conditions for BP transformability are given. Since our definition of the BP transformation slightly differs from the one given in [41], and for the sake of self-containment, we give below a simplified version of their theorem, whose proof is given in Appendix A.

Proposition 7.1. If there exists a *constant* matrix $U \in \mathcal{R}^{n_e \times n_e}$ solution of

$$UD_e - D_e U = W_1 \tag{7.3}$$

$$R_e U = UR_e \tag{7.4}$$

$$UW_2 = \frac{dW_2}{dq_m} \tag{7.5}$$

then $\Sigma(D, R, m_e, \mu, \eta)$ *is BP transformable.* In this case the dynamics of Σ_e (see 3.4)) in the coordinates $\dot{z}_e$ is described by

$$D_e(0)P_1^{-1}\ddot{z}_e + UD_e(0)P_1^{-1}\dot{q}_m\dot{z}_e + W_2(0)\dot{q}_m + R_eP_1^{-1}\dot{z}_e = \mathbf{e}^{-Uq_m}M_eu = M_eu' \tag{7.6}$$

while the dynamics of Σ_m (see (3.5)) is described by

$$D_m\ddot{q}_m + R_m\dot{q}_m = y - y_L \tag{7.7}$$

$$y = \dot{z}_e^TP_1^{-T}UD_e(0)P_1^{-1}\dot{z}_e + W_2^T(0)P_1^{-1}\dot{z}_e + \eta \tag{7.8}$$

Example 7.1. Park's transformation

For a class of synchronous machines with inductance matrix as in (3.9), the BP transformation to the $dq0$-frame is given as [33]

$$P(q_m) = \frac{2}{3} \begin{bmatrix} \cos(n_pq_m) & \cos\left(n_pq_m - \frac{2\pi}{3}\right) & \cos\left(n_pq_m + \frac{2\pi}{3}\right) \\ \sin(n_pq_m) & \sin\left(n_pq_m - \frac{2\pi}{3}\right) & \sin\left(n_pq_m + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{7.9}$$

This transformation can also be written as

$$P(q_m) = P_1\mathbf{e}^{-Uq_m}$$

where

$$U = \frac{n_p}{\sqrt{3}} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \qquad P_1 = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ 0 & -\sqrt{3} & \sqrt{3} \\ 1 & 1 & 1 \end{bmatrix}$$

and U satisfies (7.3)–(7.5). The inverse transformation is given as

$$P^{-1}(q_m) = \mathbf{e}^{Uq_m}P_1^{-1} = \begin{bmatrix} \cos(n_p q_m) & \sin(n_p q_m) & 1 \\ \cos\left(n_p q_m - \dfrac{2\pi}{3}\right) & \sin\left(n_p q_m - \dfrac{2\pi}{3}\right) & 1 \\ \cos\left(n_p q_m + \dfrac{2\pi}{3}\right) & \sin\left(n_p q_m + \dfrac{2\pi}{3}\right) & 1 \end{bmatrix} \quad (7.10)$$

Several slightly different forms are encountered in the literature, [19], [23]. This stems from the choice of the matrix P_1, which is sometimes chosen from a power preserving objective, or with the objective of making P_1 orthogonal.

7.3. *Remarks*

1. For the purpose of the present paper, the key feature of BP transformable machines is that the components of torque that also involves currents become *independent of rotor position* (see (7.8)) when expressed in suitable coordinates. Notice also that, for constant speed, the electrical subsystem in (7.6) is linear time invariant when u' is taken as the new input. This fundamental property has been exploited in the literature to determine stability properties in stationary operation [70].

2. The underlying fundamental assumption for the machine to be BP transformable, is that the windings are sinusoidally distributed [74], giving a sinusoidal air-gap magnetomotive force (MMF) and sinusoidally varying elements in the inductance matrix D_e. For a practical machine, this means that the first harmonic in a Fourier approximation of the MMF must give a sufficiently close approximation of the real MMF. Examples of machines in which higher order harmonics must be taken into account, are the squarewave brushless DC motors in [48], and machines with significant saliency in the air gap [65].

3. It is interesting to remark that the BP transformation cannot be derived from a *canonical transformation* [21] $z = z(q)$ of the generalized coordinates and momenta. This fact is presented in Appendix B.

4. Since the matrix U is real and skew-symmetric, it follows that $\mathbf{e}^{-Uq_m}$ is an orthogonal transformation, and the transformation $P(q_m)$ is bounded.

Main results

We now proceed to relate the property of BP transformability with our problem of definition of desired currents $\dot{q}_{ed}$. This is carried out in the following subsections, where we treat underactuated $(n_s < n_e)$ and fully actuated machines $(n_s = n_e)$ separately.

8.1. *Underactuated machines, $n_s < n_e$*

Proposition 8.1. Assume the machine $\Sigma(D, R, M_e, \mu, \eta)$ is *strongly BP transformable*, satisfies $\mu = \eta \equiv 0$, the (2, 1) block of D_e is nonsingular, and that $\beta(t)$ is a bounded strictly positive twice differentiable function with bounded first and second order derivatives. Under these conditions, the following definition of $\dot{q}_{ed}$ satisfies (6.1) and (7.1) for any given y_d

$$\dot{q}_{ed} = \begin{bmatrix} \dot{q}_{sd} \\ \dot{q}_{rd} \end{bmatrix} = \begin{bmatrix} (D_e)_{21}^{-1}\left[\mathscr{I}_{n_r} + (D_e)_{22}\left\{ \frac{1}{\beta^2}y_d U_{22}^{-1} + \frac{\dot{\beta}}{\beta}R_r^{-1} \right\} \right]\lambda_{rd} \\ -\left[\frac{1}{\beta^2}y_d U_{22}^{-1} + \frac{\dot{\beta}}{\beta}R_r^{-1} \right]\lambda_{rd} \end{bmatrix} \quad (8.1)$$

where λ_{rd} is the solution of the differential equation

$$\dot{\lambda}_{rd} = \frac{1}{\beta^2(t)}y_d R_r U_{22}^{-1}\lambda_{rd} + \frac{\dot{\beta}(t)}{\beta(t)}\lambda_{rd} \quad (8.2)$$

with initial conditions such that $\|\lambda_{rd}(0)\| = \beta(0)$. Furthermore,

$$\|\lambda_{rd}(t)\| = \beta(t), \ \forall t \geq 0$$

Proof

The last statement of the proposition follows immediately by taking the time derivative of

$$\frac{1}{2}\|\lambda_{rd}\|^2,$$

substituting (8.2), and using the fact that strong BP transformability implies $R_r U_{22}^{-1} + (R_r U_{22}^{-1})^T = 0$.

To simplify the notation of the rest of the proof we find it convenient to introduce the *desired flux* $\lambda_d = [\lambda_{sd}^T, \lambda_{rd}^T]^T$ as

$$\lambda_d := D_e \dot{q}_{ed} \quad (8.3)$$

Some simple calculations using (6.3), (5.3) and (8.3) show that[8]

$$\psi_r \equiv 0 \Leftrightarrow \dot{\lambda}_{rd} + R_r \dot{q}_{rd} = 0 \quad (8.4)$$

Now, notice that *BP transformability* of the machine allows us to write (7.1) as (see (7.3))

$$y_d = \dot{q}_{ed}^T U D_e \dot{q}_{ed}$$

which, in terms of the desired fluxes and currents looks like

$$y_d = \dot{q}_{ed}^T U \lambda_d$$

If further the machine is *strongly BP* transformable then

$$y_d = \dot{q}_{rd}^T U_{22} \lambda_{rd}$$
$$= -\dot{\lambda}_{rd}^T R_r^{-1} U_{22}\lambda_{rd} \quad (8.5)$$

where we have set $\psi_r \equiv 0$ and used (8.4) in the last equation. From this we see that for (7.1) to hold, we must define λ_{rd} such that (8.5) always holds. It is straightforward to verify that this is the case when $\dot{\lambda}_{rd}$ is defined as in (8.2), using $U_{22}^{-T} = -U_{22}^{-1}$ (see Definition 7.1), the symmetry of R_r and the fact that $\|\lambda_{rd}(t)\| = \beta(t)$.

The proof is completed using (8.4) to obtain $\dot{q}_{rd}$ and the definition of λ_d to calculate $\dot{q}_{sd}$.

Our main result for underactuated machines is contained in the theorem below.

Theorem 8.1. Consider the machine model (3.4), (3.5), (3.6). Assume that the machine

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[8]See (3.1) and the first remark in Subsection 3.2 for the physical motivation behind this choice of relations between desired fluxes and currents.

is strongly BP transformable (Definition 7.1), $\mu = 0$, $\eta = 0$, $(D_e)_{21}$ is nonsingular and *A.1.–A.2.* of Proposition 5.2. hold. Under these conditions, there exists a *dynamic output feedback* controller that ensures global asymptotic torque tracking with internal stability. Furthermore, for all $\beta(t)$ (bounded strictly positive twice differentiable with bounded first and second order derivatives), we have $\lim_{t\to\infty} |\|\lambda_r\| - \beta(t)| = 0$.

Proof

The control law is obtained from (5.1) and (6.3), setting $\psi_s \equiv 0$. To this end, we use the definition of $\dot{q}_{ed}$ of Proposition 8.1. Notice that $\ddot{q}_{ed}$ is bounded, and can be computed from the available measurements provided $\dot{y}_d$ is known.

Convergence of $\tilde{q} \to 0$ follows from the arguments of Section 6. Boundedness of $\dot{q}_{ed}$ follows from (8.1) and the boundedness of λ_{rd} and y_d. This establishes asymptotic torque tracking.

Electrical rotor flux tracking is a consequence of the convergence of the currents to their desired values and the chosen norm of λ_{rd}, since

$$\lambda_{rd} - \lambda_r = (D_e)_{21}(\dot{q}_{sd} - \dot{q}_s) + (D_e)_{22}(\dot{q}_{rd} - \dot{q}_r)$$

and $D_e(q_m)$ is bounded with respect to q_m.

8.2. *Fully actuated machines, $n_s = n_e$*

For fully actuated machines $M_e = \mathcal{I}_{n_e}$, and as previously explained, we can make $\psi = 0$ by a suitable selection of ν for given $\dot{q}_{ed}$ and $\ddot{q}_{ed}$. The main difficulty is to find $\dot{q}_{ed}$ such that (7.1) is satisfied. This is done by choosing the desired currents from the BP transformed torque equation, since the matrices relating the transformed currents and the torque are not dependent on q_m, which considerably simplifies the choice.

Theorem 8.1. Consider the machine model (3.4), (3.5), (3.6). Assume the machine is BP transformable (Definition 7.1), and *A.1.–A.3.* of Proposition 5.2. hold. Under these conditions, if the desired currents and their derivatives are defined as

$$\dot{q}_{ed} = \mathbf{e}^{Uq_m}P_1^{-1}\dot{z}_{ed} \tag{8.6}$$

$$\ddot{q}_{ed} = U\mathbf{e}^{Uq_m}P_1^{-1}\dot{q}_m\dot{z}_{ed} + \mathbf{e}^{Uq_m}P_1^{-1}\ddot{z}_{ed} \tag{8.7}$$

where $\dot{z}_{ed}$ is chosen to satisfy

$$y_d - \eta = \dot{z}_{ed}^T P_1^{-T} U D_e(0) P_1^{-1}\dot{z}_{ed} + W_2^T(0)P_1^{-1}\dot{z}_{ed} \tag{8.8}$$

with $\dot{z}_{ed}, \ddot{z}_{ed} \in \mathcal{L}_\infty^{n_e}$, and the *dynamic output feedback* controller defined in (5.1), with

$$\nu = D_e\ddot{q}_{ed} + C_e\dot{q}_{ed} + R_{es}\dot{q}_{ed} \tag{8.9}$$

ensures global asymptotic torque tracking with internal stability.

Proof

The expression for the torque in the transformed system is, according to (7.7)

$$y - \eta = \dot{z}_e^T P_1^{-T} U D_e(0) P_1^{-1}\dot{z}_e + W_2^T(0)P_1^{-1}\dot{z}_e$$

Setting $\dot{\tilde{z}}_e = \dot{z}_e - \dot{z}_{ed}$ and using (8.8) gives

$$y - y_d = \dot{\tilde{z}}_e^T P_1^{-T} U D_e(0) P_1^{-1}\dot{\tilde{z}}_e + 2\dot{\tilde{z}}_e^T P_1^{-T} U D_e(0) P^{-1}\dot{z}_{ed} + W_2^T(0)P_1^{-1}\dot{\tilde{z}}_e \tag{8.10}$$

Since $\dot{\tilde{z}}_e = P(q_m)\dot{\tilde{q}}_e$, and $P(q_m)$ is a bounded transformation, it follows that

$$\lim_{t\to\infty}\dot{\tilde{q}}_e = 0 \qquad \Leftrightarrow \qquad \lim_{t\to\infty}\dot{\tilde{z}}_e = 0$$

$$\Downarrow$$

$$\lim_{t\to\infty} y = y_d$$

It is clear that $\dot{z}_{ed} \in \mathcal{L}_\infty^{n_e} \Leftrightarrow \dot{q}_{ed} \in \mathcal{L}_\infty^{n_e}$, and $\ddot{z}_{ed} \in \mathcal{L}_\infty^{n_e}$, $\dot{q}_m \in \mathcal{L}_\infty \Rightarrow \ddot{q}_{ed} \in \mathcal{L}_\infty^{n_e}$. Defining ν as in (8.9) gives $\psi = 0$, and the arguments of Section 6 hold.

8.3. *Remarks*

1. Notice that the assumption in Proposition 8.1. that the $(2, 1)$ block of D_e is nonsingular implies that the number of actuated and nonactuated windings must be equal. This is the case for typical induction motors, where this matrix is a nonsingular rotation matrix.

2. Eq. (8.2) has a solution of the form

$$\lambda_{rd}(t) = \frac{\beta(t)}{\beta(0)} e(U_{22})^{-1} \rho_d(t) \lambda_{rd}(0), \; \dot{\rho}_d(t) = \frac{r_r}{\beta^2(t)} y_d(t), \; \rho_d(0) = 0 \qquad (8.11)$$

This gives an interpretation of the desired flux in terms of its rotation angle, whose speed is related to the desired torque.

3. During the derivation of the model and controller, we have assumed the use of a voltage source inverter for the input to the actuated windings. If the inverter used is a current source inverter or a fast current control is used, it follows that the inputs to the actuated windings will be the currents $\dot{q}_s \equiv \dot{q}_{sd}$, where $\dot{q}_{sd}$ is the vector of desired currents for the actuated windings, as defined in the previous sections.

4. The quadratic form in (8.8) is in general not easy to solve for the components of $\dot{z}_{ed}$. Examples of solutions for certain motors are given in Section 9.

5. For 3ϕ machines the currents of the transformed system are usually denoted with subscripts d, q and 0. For machines in which the symmetrical windings has an isolated neutral, the zero sequence of the transformed currents is exactly zero, which defines a natural choice for desired value of this current.

6. The desired transformed currents could be chosen from a similar objective as in the field oriented approach. The transformed torque equation is generally given as

$$y = c\{\lambda_d \dot{z}_q - \lambda_q \dot{z}_d\} \qquad (8.12)$$

where c is a constant, and λ_d, λ_q are d and q components of the transformed flux vector. If we can make $\lambda_q \dot{z}_d$ equal to zero and λ_d constant, it will be possible to control the torque by specifying $\dot{z}_q$.

9. Examples

9.1. *Two-phase model of uniform air gap squirrel cage induction motors*

In this section the controller (5.1), (6.1) with desired currents from (8.1)–(8.2), will be applied to the two-phase $\alpha\beta$ model of an n_p pole pair squirrel cage machine with uniform air gap. This model could be interpreted as an equivalent model of the usual 3ϕ machine, or stemming from a reduction of phases in the more general polyphase machine through transformations like those presented in [71]. It is common practice to substitute the (single)-cage rotor which has a uniform conductor distribution with a fictitious sinusoidally wound rotor (poles and phases corresponding to stator windings) for analytical purposes. This implies that only the first harmonic of the rotor MMF is modelled, but this is not detrimental for the subsequent analysis and controller design. However, care should be taken in cases of double-cage or deep bar rotors [71], [69].

The model has $n_e = 4$, $n_s = n_r = 2$, and its parameters are

$$D_e(q_m) = \begin{bmatrix} L_s \mathscr{I}_2 & L_{sr} e^{\mathscr{I} n_p q_m} \\ L_{sr} e^{-\mathscr{I} n_p q_m} & L_r \mathscr{I}_2 \end{bmatrix}, \; \mu = 0, \; \eta = 0 \qquad (9.1)$$

$$R_e = \begin{bmatrix} R_s \mathscr{I}_2 & 0 \\ 0 & R_r \mathscr{I}_2 \end{bmatrix}, \; M_e = \mathscr{I}_2 \qquad (9.2)$$

where L_s, L_{sr}, L_r are inductance parameters, R_r, R_s are resistances, and

$$\mathscr{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \; e^{\mathscr{J} n_p q_m} = \begin{bmatrix} \cos(n_p q_m) & -\sin(n_p q_m) \\ \sin(n_p q_m) & \cos(n_p q_m) \end{bmatrix} \quad (9.3)$$

Equations (7.3)–(7.5) are satisfied for

$$U_{22} = -n_p \mathscr{J} = n_p \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

and $e^{\mathscr{J} n_p q_m}$ is the BP transformation of the rotor quantities from the rotor reference system to the stator reference system. This gives the desired rotor flux (see (8.2) and (8.11))

$$\lambda_{rd} = e^{\rho_d \mathscr{J}} \begin{bmatrix} \beta(t) \\ 0 \end{bmatrix}, \; \dot{\rho}_d = \frac{R_r}{n_p \beta^2(t)} y_d(t), \; \rho_d(0) = 0$$

where the rotation angle (relative to the rotor system) is defined by the familiar relation between desired *slip*, $\dot{\rho}_d$ and desired torque y_d.

The choice of desired currents then follows from (8.1), and the input to the stator windings is given by

$$u = v - K_1(\dot{q}_m)\dot{q}_s$$
$$v = L_s \ddot{q}_{sd} + L_{sr} e^{\mathscr{J} n_p q_m} \ddot{q}_{rd} + L_{sr} n_p \mathscr{J} e^{\mathscr{J} n_p q_m} \dot{q}_m \dot{q}_{rd} + R_s \dot{q}_{sd} + K_1(\dot{q}_m)\dot{q}_{sd}$$

where

$$K_1(\dot{q}_m) = \left(\frac{n_p^2 L_{sr}^2}{4\varepsilon} \dot{q}_m^2 + k_1 \right) \mathscr{J}_2, \; 0 < \varepsilon < \min\{R_s, R_r\}, \; k_1 \geq 0$$

has been chosen to satisfy (5.4).

The application of this observerless control scheme was first proposed in [16], and its extension to speed and position control is presented in [56].

It is important to underscore that in [56] it has been shown that under the simplifying assumptions of constant speed and that the inverter can be modelled as a current source, the passivity-based control *reduces exactly* to the well known indirect field oriented control (FOC). For a voltage source inverter, the difference from indirect FOC is the additional nonlinear damping term introduced in the input. Furthermore, in [73] it is shown that the global stability properties are preserved for all estimates $\hat{T}_r$ of rotor time constant

$$T_r = \frac{L_r}{R_r},$$

such that

$$\frac{1}{2} T_r < \hat{T}_r < \infty,$$

which is a condition that will be satisfied in most practical applications.

9.2. *Synchronous motors*

In the last years, synchronous motors, and especially permanent magnet motors have become an attractive alternative to induction motors in the low to medium power range [6]. These machines are generally more expensive than induction motors[9], but

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[9]The difference in price will be reduced as low price high-energy permanent magnets become available.

have higher efficiency due to the fact that the rotor losses are negligible. They also have reduced size compared to the induction motors.

The controller as given by (5.1), (8.9) with currents satisfying (8.8), can be applied to this type of motor as follows.

Using the transformation given in (7.9)–(7.10), and the model given in (3.9)–(3.10), the torque can be expressed in new coordinates $\dot{z}_e = [\dot{z}_d, \dot{z}_q, \dot{z}_0]^T$ as

$$y = \frac{3n_p}{2}\{(L_d - L_q)\dot{z}_d\dot{z}_q + \lambda_m\dot{z}_q\} \tag{9.4}$$

where

$$L_d = L_{ls} + \frac{3}{2}(L_A + L_B), \; L_q = L_{ls} + \frac{3}{2}(L_A - L_B).$$

The desired currents are chosen as

$$\dot{z}_{ed} = \begin{bmatrix} 0 \\ \dfrac{2y_d}{3n_p\lambda_m} \\ 0 \end{bmatrix}$$

from which it follows that $\dot{z}_{ed}, \ddot{z}_{ed} \in \mathscr{L}_\infty^3$, whenever $y_d, \dot{y}_d \in \mathscr{L}_\infty$.

To satisfy (5.5), taking the uncertainty of the resistances into account, we choose K_1 as

$$K_1 = -\frac{1}{2}W_1\dot{q}_m + k\mathscr{I}_3, \; k > 0$$

The input is then given from (8.9) and (5.1).

It must be pointed out that if $L_B \neq 0$, the airgap is nonuniform. According to Remark 5.3. our method can be used for such machines as long as there are no rotor windings, which is the case here.

The above approach can also be extended to synchronous reluctance motors[10] with the same inductance matrix as in (3.9). In these machines there are no permanent magnets or windings in the rotor, hence $\lambda_m = 0$ and the torque is given as [22]

$$y = \frac{3n_p}{2}(L_d - L_q)\dot{z}_d\dot{z}_q$$

from which it follows that one of the desired currents should be constant, and the other proportional to desired torque.

Also, if the synchronous machine has a field winding on the rotor instead of permanent magnets, λ_m will be proportional to the current in the field winding, which is usually chosen to be constant or varying according to a field weakening objective. The choice of the other desired currents could be done as previously explained.

9.3. *PM stepper motor*

As a last example, we show in this section how to apply the proposed controller (5.1), (8.9) with currents satisfying (8.8), to a PM stepper motor.

With the model given in (3.11)–(3.12), the transformation to the dq-frame is [41]

$$P(q_m) = e^{-Uq_m} = \begin{bmatrix} \cos(N_rq_m) & \sin(N_rq_m) \\ -\sin(N_rq_m) & \cos(N_rq_m) \end{bmatrix}, \; U = \begin{bmatrix} 0 & -N_r \\ N_r & 0 \end{bmatrix}$$

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[10]This motor has been proposed as an alternative to other AC-machines, see [40].

where U satisfies (7.3)–(7.5). This transformation is orthogonal, and $P^{-1}(q_m) = P^T(q_m)$.

The torque expressed in new coordinates $\dot{z}_e = [\dot{z}_d, \dot{z}_q]^T$ is

$$y = K_m \dot{z}_q - K_D \sin(4N_r q_m)$$

and we choose the desired currents as

$$\dot{z}_{ed} = \begin{bmatrix} 0 \\ \dfrac{1}{K_m} \{y_d + K_D \sin(4N_r q_m)\} \end{bmatrix}$$

Notice that (5.5) is satisfied for the choice $K_1 = k\mathscr{I}_2 > 0$, where $k > 0$, and $y_d \in \mathscr{L}_\infty$ implies $\dot{z}_{ed} \in \mathscr{L}^2_\infty$, and boundedness of $\ddot{q}_{ed}$ follows from the boundedness of $\dot{y}_d$ and $\dot{q}_m$. The input is then given from (8.9) and (5.1).

As previously pointed out, the underlying assumption of BP-transferability is the sinusoidally distribution of the MMF. It can be discussed whether this is a good approximation in the case of stepper motors, with concentrated windings, significant air gap saliency and often hybrid rotor construction. The BP-transform above has however been used in several applications, and we have adopted this convention here.

Its worth to point out that the controller in [4] can be obtained from the passivity based approach if it is applied to the full system, without dividing the system into electrical and mechanical parts as done here.

10. Experimental results

To illustrate the performance of the proposed method, we implemented the scheme in Sub-section 9.1. to control speed/position of a 4-pole 3ϕ squirrel cage induction motor with $\varDelta$-connected stator phases from the German company Lust GmbH. We give here only a brief description of the equipment and the experiments and refer the reader to [51] for further details.

The equivalent parameters[11] are:

	Nominal parameters of motor system	
R_s	Stator resistance	1·9 Ω
R_r	Rotor resistance	3·0 Ω
L_s	Stator inductance	0·130 H
L_r	Rotor inductance	0·130 H
L_{sr}	Mutual inductance	0·120 H
n_p	Number of pole pairs	2
$\dot{q}_{mN}$	Rated speed	3000 rpm
β_N	Rated rotor flux amplitude	0·2 Wb
D_m	Inertia (rotor and load)	5·53 kgcm^2
y^N	Nominal torque	1·5 Nm
P_{rated}	Rated power	400 W
u_{DC}	DC-link voltage of inverter	300 V
I_{MAX}	Maximum line currents	4·5 A

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[11]Identification of all 2ϕ motor parameters was carried out by the manufacturer on sample motors in the same production series. They were given in the motors data sheet with $\pm (5 - 10)\%$ accuracy.

Position was measured with an incremental encoder and a quadruple counter, giving a position resolution of

$$\frac{2\pi}{4\cdot4096} \approx 4\cdot0 \cdot 10^{-4} \text{ rad.}$$

Speed was estimated from position measurements using a backward difference approximation, and computed at one third of the main sampling frequency to increase resolution. A dSPACE controller board with a TMS320C31 main processor and a TMS320P14 slave processor were used for controller implementation. Two of the line currents were measured using LEM transducers, and converted to digital signals using 12 bit A/D converters. The controller was first implemented in SIMULINK, and then converted to C-code, compiled and downloaded to the board using dSPACE software. Symmetric carrier-based PWM was implemented on the slave processor. The switching pattern is calculated at a frequency of 10 kHz for the three phases, from voltage references transferred from the main processor at the end of each sampling interval. For the discretization a simple first order Euler approximation was used, with a sampling time of $T_{sampl} = 300$ μs. The combination of this sampling time with the number of encoder lines and backward difference estimation, gave a speed resolution of

$$\frac{1}{4\cdot4096}/(3T_{sampl}) \cdot 60 \approx 4\cdot1 \text{ rpm}$$

(See [42]) for speed or position tracking, the desired torque was chosen as in [56], with an extra integral term to compensate for the unknown load torque (friction in bearings of motor and inertial load), giving

$$y_d = D_m\ddot{q}_{md} - z - f(q_m - q_{md}) + \hat{y}_L$$
$$\dot{z} = -az + b(\dot{q}_m - \dot{q}_{md}),\ a, b > 0,\ z(0) = \dot{q}_m(0) - \dot{q}_{md}(0)$$
$$\dot{\hat{y}}_L = -\gamma e,\ \gamma \geq 0$$

where $q_{md}, \dot{q}_{md}$ are the rotor position and speed reference, and $f \equiv 0$ for speed tracking. The error term in the integral action was set to $e = q_m - q_{md}$ for position tracking, and $e = \dot{q}_m - \dot{q}_{md}$ in the case of speed tracking. The references were generated by linear first order filtering of step/square-wave signals from implementations of the SIMULINK signal generator. Higher order derivatives of references for position/speed and flux amplitude $\beta(t)$ were obtained from state space representations of linear filters.

The only true way of verifying flux tracking is to compare measured flux with its reference. Unfortunately, flux measurement is far from trivial, and hence not implemented in the experimental setup. We have therefore used the current model observer from [5] to estimate flux amplitude $\hat{\lambda}_d$

$$\hat{\lambda}_d = \frac{L_{sr}}{T_r p + 1}\hat{i}_d,\quad p := \frac{d}{dt}$$
$$\hat{i}_d = \cos(\theta_a)i_\alpha + \sin(\theta_a)i_\beta$$
$$\hat{i}_q = -\sin(\theta_a)i_\alpha + \cos(\theta_a)i\beta$$
$$\theta_a = n_p q_m + \int_0^t \frac{L_{sr}}{T_r}\frac{\hat{i}_q}{\hat{\lambda}_d}dt$$

where $T_r = L_r/R_r$ and i_α, i_β are stator currents of the equivalent 2ϕ model. These were calculated from the 3ϕ line current measurements I_A and I_B. Due to the singularity in the flux speed calculation, it was necessary to substitute $\hat{\lambda}_d$ in the division above by a small constant $c = 0\cdot001$ whenever $\hat{\lambda}_d \leq c$. This estimator was run in parallel with the controller, and not used for feedback purposes.
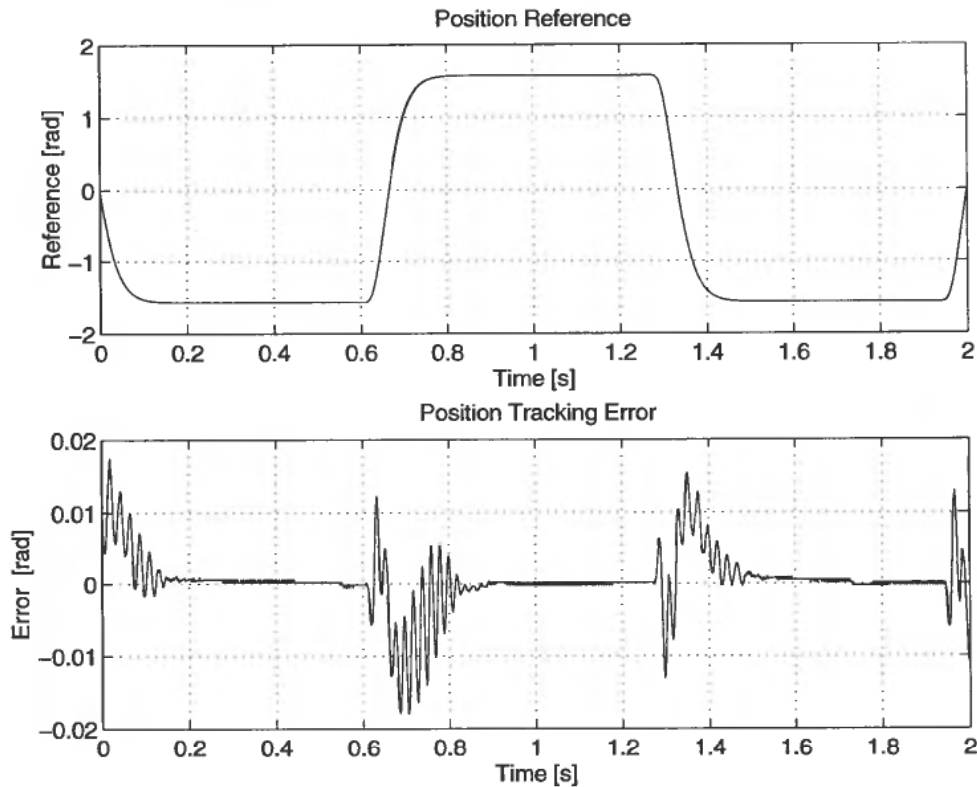
Figure 2.   Position reference $q_{md}(t)$ (upper figure) and tracking error $(q_m - q_{md})$.

A sampling time of $300\,\mu s$ was high enough to allow for simultaneously logging of several signals, without significant response deterioration. By running only the code strictly necessary for the implementation (no observer, only logging of execution time), the sampling time could be set to $T = 147\,\mu s$.

The following parameters were used for position tracking: $a = 1000, b = 95, f = 41, \gamma = 70$. For speed and flux tracking we used $a = 1000, b = 320, f = 0, \gamma = 3 \cdot 85$.

Figure 2 shows position reference and tracking error for a 180° turn of the motor. As can be seen from the lower plot the maximum error is approximately 1°. The next figure shows that precision of position control was only restricted by the resolution of the position measurement. The digital jittering could be removed by using a deadzone in the position controller. Due to position dependent friction, controller parameters had to be set low enough to avoid oscillatory behaviour at certain "worst-case" positions. In the lower plot of Fig. 3 the flux error during position tracking for a constant reference of $\beta = 0 \cdot 2$ Wb is shown. Perfect control can hardly be expected with parameters taken directly from the data sheet and a simple discretization, and there is a maximum error of $0 \cdot 009$ Wb during transients. Reference voltages for PWM and measured line currents for two of the three phases are shown in Fig. 4. Voltage references were below the theoretical limits for the PWM (± 150 V), and the line currents did not saturate. This was also the case for all of the experiments reported here, but for space limitations we do not include more than one plot of currents and voltages.

In Figs. 5 and 6 we have shown examples of flux tracking and speed regulation with
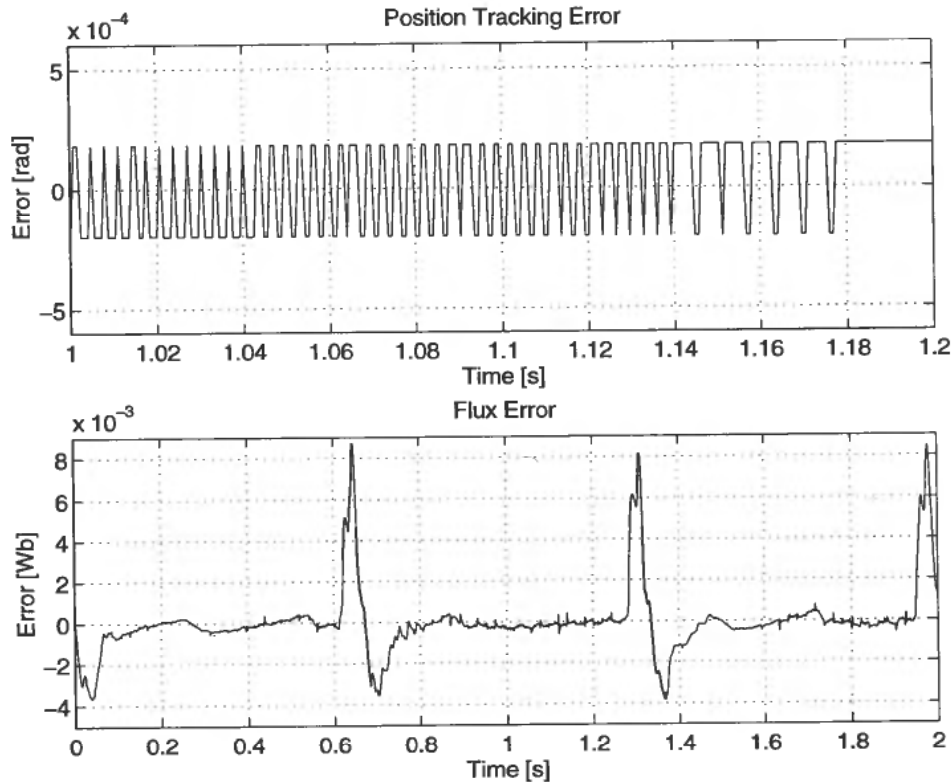
Figure 3. Window of position tracking error and flux error $(\hat{\lambda}_d - \beta)$.

and without the term $\dot{\beta}(t)$ in the controller. Comparison shows that this term is necessary for high quality flux tracking. Finally, in Fig. 7 an example of speed tracking with a reference close to a step, and constant $\beta = 0{\cdot}2$ Wb is shown. The speed follows its reference closely, and results in only minor peaks in flux during transients, due to unmodelled effects.

The parameter tuning was done online while observing responses, and the scheme was easy to tune, as compared to implementations of other observer-based FOC schemes.

Further experimental results from the application of this controller can be found in [15] (speed tracking and flux regulation), [29] (comparison to feedback linearizing controllers) and [51] (position, speed and flux tracking with comparison to FOC).

11. Concluding remarks

We have studied in this paper the output feedback global tracking problem for the generalized electric machine's model. A passivity-based method is used to design the controller which proceeded in three steps. First, the machine is decomposed as the feedback interconnection of two passive subsystems—electrical and mechanical. Then, a nonlinear damping is injected to make the electrical subsystem strictly passive. Finally, an energy-shaping controller is designed to make the currents converge exponentially to a desired value that delivers the required torque. Our main contribution is the establishment of physically interpretable conditions on the model such that the method can be successfully applied. To further relax these conditions, it is our belief that we must combine passivity ideas with the powerful new dynamic extension
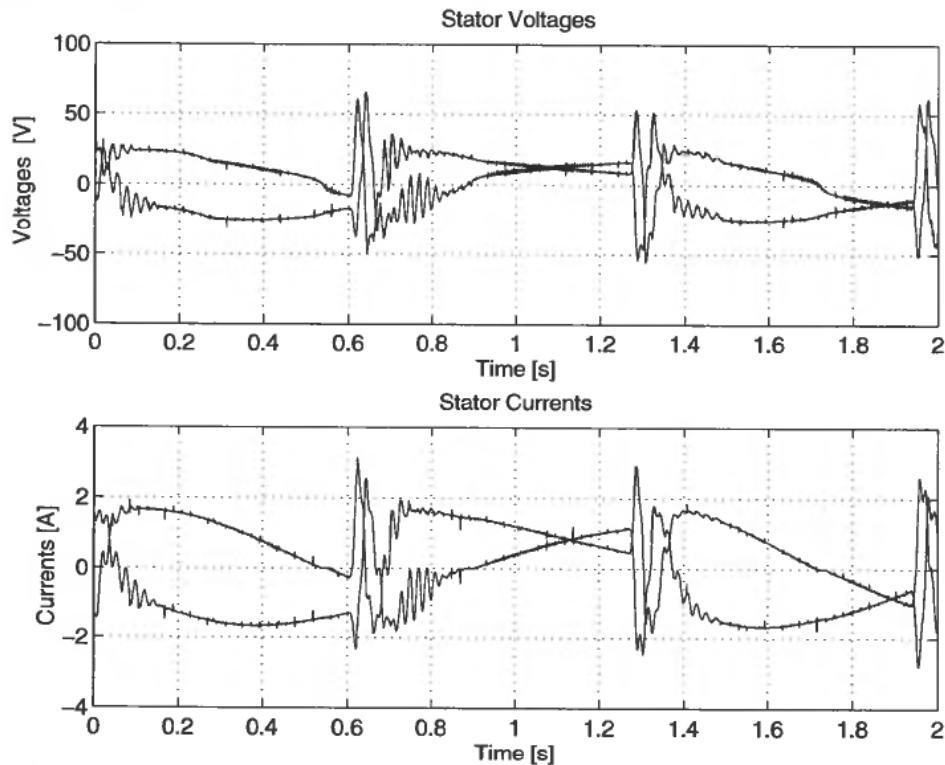
Figure 4. 3ϕ stator phase voltage references u_a, u_b and measured line currents I_A, I_B.

techniques for stabilization of nonlinear systems. Some research along these lines for the robotics problem has been reported in [8].

The passivity-based approach gives control schemes which provides global stability results for the resulting system. Further, there is no need for observers since unmeasurable states are not used, and there are no singularities, hence obviating the need for special precautions to be taken at for example start-up. The performance of the scheme, as measured with the exponential convergence rate of desired currents (and consequently outputs) to their desired values, can be explicitly derived for each machine using the results in Sections 5 and 6. It follows that the rate of convergence is restricted by the time constant of the unactuated windings. This is a consequence of the fact that additional damping cannot be injected into this dynamics, since the involved states are unmeasurable. As discussed in [29] the same limitation applies to feedback linearizing schemes, where the restriction is now imposed on the observer convergence.

To establish the relationship of our controller to existing schemes we notice that our control signal consists of a nonlinear damping term added to the reference dynamics. Henceforth, following [53], it can be classified as an indirect vector control scheme, which is the most widely used implementation of field oriented control, (especially well suited for operation close to zero speed [42]). In particular, for the speed control of the induction motor, it is shown in [56] that the passivity-based controller exactly reduces to indirect field oriented control under some simplifying assumptions, namely constant speed and current fed inverter, (for which the additional problem of stator dynamics is not present). Also, this connection to indirect FOC shows that under the same

Estimated Speed and its Reference

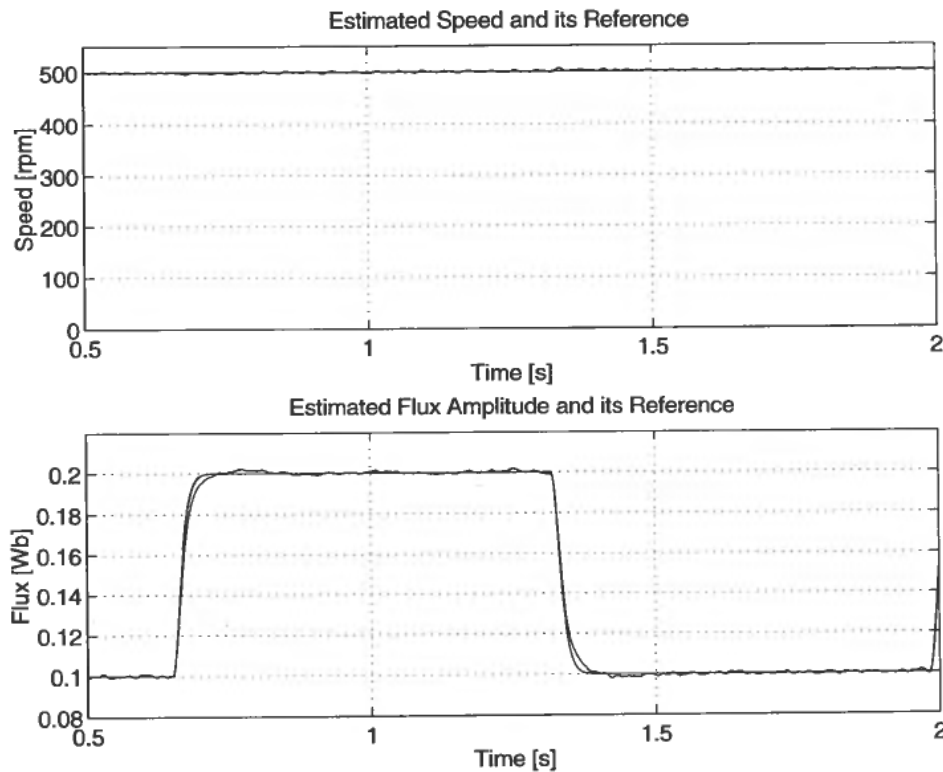**Estimated Flux Amplitude and its Reference**

Figure 5. Example of flux tracking. Speed and its reference in upper figure. Flux amplitude and its reference in lower figure.

assumptions, the computational burden of the passivity based approach is no greater than in this implementation of FOC.

In practice, the assumptions of constant and known parameters will not hold. For instance resistances will vary due to temperature changes and the skin effect at high frequencies, and inductances will change when magnetic saturation occurs. To assess the sensitivity of the stability of indirect field oriented control *vis a vis* these assumptions we carried out in [73] a robustness analysis. In that paper we proved that stability is preserved despite large variations in rotor resistance and inductance. In view of the downward compatibility mentioned above, these robustness properties are inherited by the passivity based controller. It may be possible to enhance performance by identifying parameters on-line. Globally stable adaptive controllers have been reported in [14], unfortunately requiring full state measurement. The relaxation of this assumption, together with extension to the case of nonlinear magnetics, use of speed observers, and friction compensation are the main topics for our future research in this field. Some interesting work on rotor resistance adaptation may be found in [12].

Since the passivity based approach exploits the systems energy properties, which are invariant under a change of coordinates, this gives the possibility of controller implementation in a general *dq*-frame, chosen from the objectives of minimizing computational burden and increasing numerical robustness.

Experimental results from the application of the proposed controller to an induction motor have been included. Even if an *ad hoc* discretization and a speed estimation
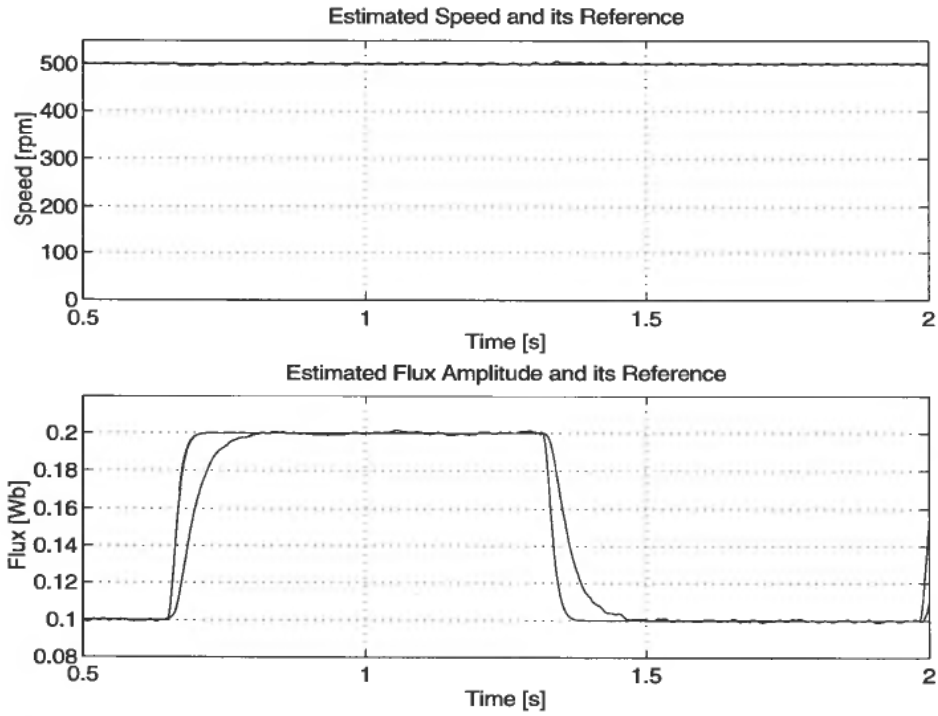
Per J. Nicklasson, Romeo Ortega and Gerardo Espinosa-Pérez

Estimated Speed and its Reference

Estimated Flux Amplitude and its Reference

Figure 6.   Example of flux tracking. Same as previous figure, but without $\dot{\beta}(t)$ in controller.

Speed and its Refereence

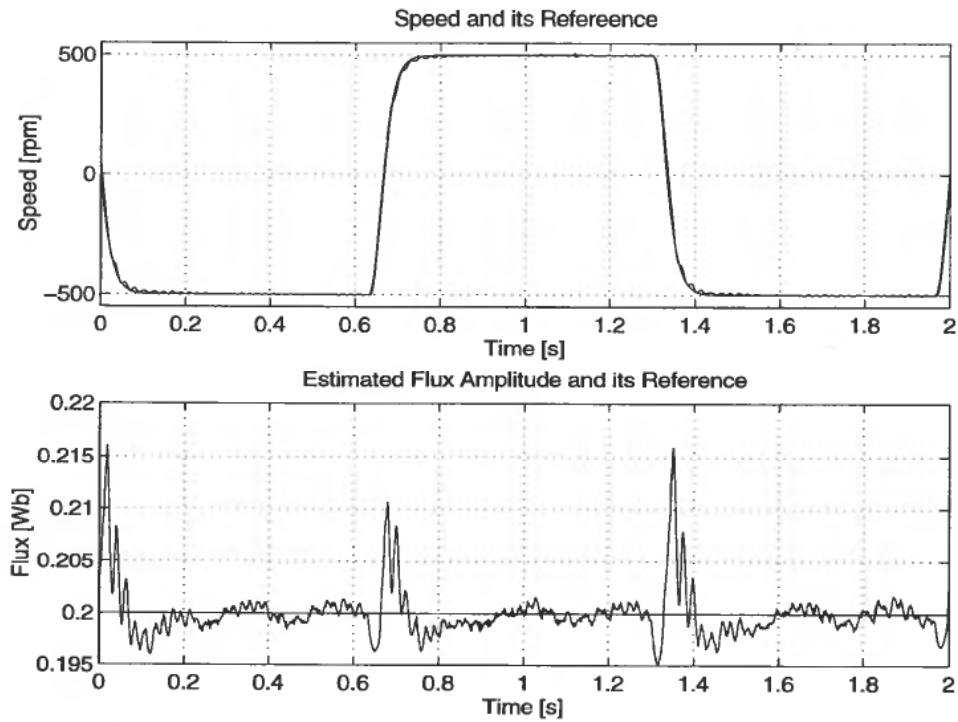Estimated Flux Amplitude and its Reference

Figure 7.   Example of speed tracking with constant flux reference.

scheme not considered in the theoretical analysis have been used, high-performance position/speed and flux tracking was experienced. As expected from a scheme with global stability results, controller tuning was an easy task.

As pointed out in [65], there is a potential advantage in working with a general model instead of specializing the model equations, and we hope that the results in this paper will motivate for additional research in the area of servo applications utilizing a general machine's model and its passivity properties. One such problem is tracking control of a robot manipulator with AC-drives. In a recent paper [57] we have provided an answer to this question by extending the results in [10] to the global case and the use of a general BP transformable machine.

Appendices

A. *Proof of Proposition 7.1.*

For the proof of Proposition 7.1, we need the following lemma:

Lemma A.1.

$$\frac{dD_e(q_m)}{dq_m} \neq 0 \Rightarrow U + U^T = 0 \tag{A.1}$$

Proof

Note that the differential equation (7.3) has the unique solution $D_e(q_m) = e^{Uq_m} D_e(0) e^{-Uq_m}$ [28].

$$D_e = D_e^T \Rightarrow e^{Uq_m} D_e(0) e^{-Uq_m} = e^{-U^T q_m} D_e(0) e^{U^T q_m}$$
$$\Rightarrow e^{U^T q_m} e^{Uq_m} D_e(0) = D_e(0) e^{U^T q_m} e^{Uq_m}$$
$$\Rightarrow e^{U^T q_m} e^{U} q_m = \mathscr{I}$$
$$\Rightarrow U^T + U = 0$$

The third implication follows from the fact that U and D_e do not commute, unless

$$\frac{dD_e(q_m)}{dq_m} = 0 \quad (7.3),$$

and this implies that $f(U) = e^{U^T q_m} e^{Uq_m}$ and D_e cannot commute, unless $f(U) \equiv \mathscr{I}$.

Proof of Proposition 7.1.

From (7.2) it follows that

$$\dot{q}_e = e^{Uq_m} P_1^{-1} \dot{z}_e \tag{A.2}$$
$$\ddot{q}_e = e^{Uq_m} P_1^{-1} \ddot{z}_e + U e^{Uq_m} P_1^{-1} \dot{q}_m \dot{z}_e \tag{A.3}$$

After inserting these two equations into (7.6), and multiplying from the left by e^{-Uq_m}, we get

$$e^{-Uq_m} D_e(q_m) e^{Uq_m} P_1^{-1} \ddot{z}_e + e^{-Uq_m} D_e(q_m) U e^{Uq_m} P_1^{-1} \dot{q}_m \dot{z}_e$$
$$+ e^{-Uq_m} W_1(q_m) \dot{q}_m e^{Uq_m} P_1^{-1} \dot{z}_e + e^{-Uq_m} W_2(q_m) \dot{q}_m$$
$$+ e^{-Uq_m} R_e e^{Uq_m} P_1^{-1} \dot{z}_e = e^{-Uq_m} M_e u \tag{A.4}$$

From $D_e(q_m) = \mathbf{e}^{Uq_m}D_e(0)\mathbf{e}^{-Uq_m}$, and since (7.4) implies that $\mathbf{e}^{Uq_m}R_e = R_e\mathbf{e}^{Uq_m}$ [38], we note that

$$\mathbf{e}^{-Uq_m}D_e(q_m)\mathbf{e}^{Uq_m} = D_e(0) \tag{A.5}$$

$$\mathbf{e}^{-Uq_m}D_e(q_m)U\mathbf{e}^{Uq_m} = D_e(0)U \tag{A.6}$$

$$\mathbf{e}^{-Uq_m}W_1(q_m)\mathbf{e}^{Uq_m} \overset{(7.3)}{=} \mathbf{e}^{-Uq_m}[U\mathbf{e}^{Uq_m}D_e(0)\mathbf{e}^{-Uq_m} - \mathbf{e}^{Uq_m}D_e(0)U\mathbf{e}^{-Uq_m}]$$
$$= UD_e(0) - D_e(0)U \tag{A.7}$$

$$\mathbf{e}^{-Uq_m}R_e\mathbf{e}^{Uq_m} = R_e \tag{A.8}$$

In addition, it follows from (7.5) that $W_2(q_m) = \mathbf{e}^{Uq_m}W_2(0)$. This implies that $\mathbf{e}^{-Uq_m}W_2(q_m) = W_2(0)$, which is constant with respect to q_m. Finally, inserting (A.5–A.8) into (A.4), gives

$$D_e(0)P_1^{-1}\ddot{z}_e + UD_e(0)P_1^{-1}\dot{q}_m\dot{z}_e + W_2(0)\dot{q}_m + R_eP_1^{-1}\dot{z}_e = \mathbf{e}^{-Uq_m}M_eu$$

For the transformed mechanical system Σ_m, we have the relations

$$\dot{q}_e^T W_1(q_m)\dot{q}_e = \dot{z}_e^T P_1^{-T}\mathbf{e}^{U^Tq_m}W_1(q_m)\mathbf{e}^{Uq_m}P_1^{-1}\dot{z}_e$$

$$\overset{(A.1)}{=} \dot{z}_e^T P_1^{-T}\mathbf{e}^{-Uq_m}W_1(q_m)\mathbf{e}^{Uq_m}P_1^{-1}\dot{z}_e$$

$$\overset{(A.7)}{=} \dot{z}_e^T P_1^{-T}[UD_e(0) - D_e(0)U]P_1^{-1}\dot{z}_e$$

$$= 2z_e^T P_1^{-T}UD_e(0)P_1^{-1}\dot{z}_e = 2\dot{z}_e^T P_1^{-T}D_e(0)U^TP_1^{-1}\dot{z}_e \tag{A.9}$$

$$W_2^T(q_m)\dot{q}_e = W_2^T(q_m)\mathbf{e}^{Uq_m}P_1^{-1}\dot{z}_e$$
$$= W_2^T(0)P_1^{-1}\dot{z}_e \tag{A.10}$$

B.   *Lemma on the BP transformation*

Lemma B.1.   Unless $U \equiv 0$, the velocities $\dot{z} = [\dot{z}_e^T, \dot{q}_m]^T$ introduced by the BP transformation cannot be derived from a transformation $z = Z(q)$ of the generalized coordinates $q = [q_e^T, q_m]^T$.

Proof

The transformation from the generalized electrical velocities $\dot{q}_e$ and the generalized mechanical velocity $\dot{q}_m$ to $\dot{z} = [\dot{z}_e^T, \dot{q}_m]T$ is

$$\begin{bmatrix} \dot{z}_e \\ \dot{q}_m \end{bmatrix} = \begin{bmatrix} P_1\mathbf{e}^{-Uq_m} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_e \\ \dot{q}_m \end{bmatrix} \tag{B.1}$$

If $z = Z(q)$, then we must have

$$\frac{\partial Z}{\partial q} = \begin{bmatrix} P_1\mathbf{e}^{-Uq_m} & 0 \\ 0 & 1 \end{bmatrix} \tag{B.2}$$

since

$$\dot{z} = \frac{\partial Z}{\partial q}\dot{q}.$$

From this, we find that z_e must be of the form

$$z_e = Z_e(q) = P_1\mathbf{e}^{-Uq_m}q_e + c, \quad c \in \mathscr{R}^{n_e}$$

Taking the total time derivative gives

$$\dot{z}_e = \frac{\partial Z_e}{\partial q}\dot{q} = -P_1\mathbf{e}^{-Uq_m}U\dot{q}_mq_e + P_1\mathbf{e}^{-Uq_m}\dot{q}_e$$

from which it follows that since the BP transformation (see (7.2)) is defined as $\dot{z}_e = P_1 \mathbf{e}^{-Uq_m} \dot{q}_e$ we must have

$$-P_1 \mathbf{e}^{-Uq_m} U \dot{q}_m q_e = 0, \ \forall q_e$$

For this to hold, U must be the zero matrix, since $P_1 \mathbf{e}^{-Uq_m}$ is nonsingular, and consequently for $U \neq 0$ there is no transformation $z_e = Z_e(q)$ such that

$$\dot{z}_e = \frac{\partial Z_e}{\partial q} \dot{q}.$$

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