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# Passivity-Based Control with Guaranteed Safety via Interconnection and Damping Assignment

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**Abstract:** In this paper, we study a Passivity-Based Control (PBC) design that solves asymptotic stability with guaranteed safety problem via Interconnection and Damping Assignment (IDA) approach. Akin to the classical IDA-PBC method, the original system is transformed via a state-feedback to a port-Hamiltonian system where the corresponding interconnection and damping matrices and the energy function are shaped according to the given set of unsafe states and to the desired equilibrium point. By embedding it in a hybrid control framework, we show how the global results can also be obtained. We illustrate the efficacy of our proposed method on a nonlinear second-order system.

*Keywords:* Passivity-based control, stabilization with guaranteed safety, hybrid control, interconnection and damping assignment.

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## 1. INTRODUCTION

Energy-based modeling and control design framework has become an indispensable tool for analyzing and controlling complex multi-domain physical systems. It enables one to gain insight and to control such complex systems through the use of the classical concept of energy and the exchange thereof between different physical entities. For example, the analysis and control of systems described by Euler-Lagrange equation have been investigated and discussed thoroughly in Ortega et al. (1998). The concept has found many control applications in electro-mechanical systems, such as, robotics, and power systems (see e.g., Jayawardhana & Weiss (2008); Garcia-Canseco et al. (2010); Ortega et al. (2001); Ortega & Garcia-Canseco (2004); Kotyczka & Lohmann (2009)).

Another well-known energy-based modeling and control design framework is the port-Hamiltonian framework which is closely related to the Euler-Lagrange framework (through the use of Legendre transformation) and has a nice structure in the state equations. The energy exchange between physical elements and the dissipated energy are encapsulated in the interconnection and damping matrices in the vector field. We refer interested readers on the port-Hamiltonian framework to the textbook of van der Schaft (1999) and to the articles in Ortega et al. (2002); Ortega & Garcia-Canseco (2004); Ortega et al. (2008). Control design methods that are based on port-Hamiltonian framework have recently been proposed, such as, the Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) which will be the main focus of this paper, and the Energy-Balancing Passivity-Based Control in Jeltsema et al. (2004).

Generally speaking, the IDA-PBC method concerns with the design of a state feedback control law such that the closed-loop system has a desirable port-Hamiltonian structure (i.e., it has desired interconnection and damping matrices, as well as, a desired energy function). By an appropriate design of these

interconnection and damping matrices and of the energy function, the stabilization of a desired equilibrium can be achieved. A generalization of IDA-PBC method has appeared in Battle et al. (2008) where the interconnection and damping matrices are lumped.

In this paper, we investigate the generalization of IDA-PBC to solve the problem of stabilization with guaranteed safety. Here, safety means that all admissible state trajectories do not violate system constraints or enter a set of unsafe states. In practical applications, especially in advanced instrumentations, robotics and complex systems, it is common that the system has state constraints or set of unsafe states, i.e. the subset of state domain that must be avoided. In this regards, the notion of safety must be also considered as an integral part in the control design process in addition to stability and robustness consideration.

The incorporation of safety aspect into the stabilization of the closed-loop system has been considered before in Ngo et al. (2005); Tee et al. (2009); Romdlony & Jayawardhana (2014a,b); Ames et al. (2014). In Romdlony & Jayawardhana (2014a,b); Ames et al. (2014), the well-known Control Lyapunov Function-based control method is combined with the Control Barrier Function-based control method which is proposed in Wieland & Allgöwer (2007) to solve the problem. The proposed control method does not impose unboundedness of energy function on the boundary of the set of unsafe states as imposed in Ngo et al. (2005); Tee et al. (2009).

As an alternative to the aforementioned methods for solving stabilization with guaranteed safety problem, we propose in this paper an energy-based method for solving this problem that offers a nice energy interpretation. The main approach behind our proposed method (as presented later in Proposition 2) is to assign a desired energy function such that it has a minimum at the desired equilibrium point and has local maxima in the set of unsafe states. Thus with an appropriate interconnection

and damping matrices, the closed-loop system will converge to the minima (that includes the desired one) while avoiding the region of concavity where the unsafe state belongs to.

Although the proposed method can ensure that all admissible trajectories are safe, the method may not give a global stability result. This is due to the existence of multiple minima in the desired energy function.

In our second result (as given later in Proposition 4), we propose a hybrid control strategy that combines the global stability result of IDA-PBC with respect to the set of equilibria and another state-feedback controller that can steer the system from the set of undesired equilibria to the desired one. Hence, global stability with guaranteed safety is achieved.

In Section II, we briefly review the stabilization via IDA-PBC which is based on Ortega et al. (2002); Ortega & Garcia-Canseco (2004); Ortega et al. (2008) and the notion of stabilization with guaranteed safety as introduced in Romdlony & Jayawardhana (2014a,b). In Section III, we propose methods for (local) stabilization with guaranteed safety via IDA-PBC. In Section IV, a hybrid control method for achieving global result is discussed.

## 2. PRELIMINARIES

Throughout this paper, we consider a non-linear affine system described by

$$\dot{x} = f(x) + g(x)u \quad (1a)$$

$$y = h(x) \quad (1b)$$

where  $x \in \mathbb{R}^n$  denotes the state vector,  $u, y \in \mathbb{R}^m$  denote the control input and the output of the system, respectively. The functions  $f(x)$ ,  $g(x)$  and  $h(x)$  are  $\mathcal{C}^1$ , and  $g(x)$  and its left annihilator  $g^\perp(x) \in \mathbb{R}^{(n-m) \times n}$  are full rank for all  $x \in \mathbb{R}^n$ . For  $a \in \mathbb{R}^n$ , we define  $\mathbb{B}_\varepsilon(a) := \{x \in \mathbb{R}^n \mid \|x - a\| < \varepsilon\}$ .

### 2.1 Stabilization via IDA-PBC

Let us now recall the results on the IDA-PBC design method as discussed in Ortega & Garcia-Canseco (2004).

The IDA-PBC method aims at stabilizing the system (1) at a desired equilibrium  $x^*$  by designing a feedback law  $u = \beta(x)$  that transforms (1) into a port-Hamiltonian structure and has a desirable damping component ensuring the asymptotic stability of  $x^*$  (which is the minimum of the desired energy function). More precisely, suppose that we can design an energy function  $H_d: \mathbb{R}^n \rightarrow \mathbb{R}$  and interconnection and damping matrices  $J_d, R_d: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$  such that

$$g^\perp(x)f(x) = g(x)^\perp(J_d(x) - R_d(x))\nabla H_d \quad (2a)$$

$$\nabla^2 H_d(x^*) > 0 \quad (2b)$$

$$J_d(x) = -J_d^\top(x) \quad (2c)$$

$$R_d(x) = R_d^\top(x) \geq 0. \quad (2d)$$

where  $x^* = \arg \min H_d(x)$  is the desired equilibrium. Then, the stabilizing feedback law  $u = \beta(x)$  via IDA-PBC is given by

$$\beta(x) = (g^\top(x)g(x))^{-1}g^\top(x)((J_d(x) - R_d(x))\nabla H_d(x) - f(x)). \quad (3)$$

Using this control law, the closed-loop system can be represented as a port-Hamiltonian system in the form of

$$\dot{x} = (J_d(x) - R_d(x))\nabla H_d(x) \quad (4)$$

where  $x^*$  is (locally) stable equilibrium point. Furthermore,  $x^*$  is asymptotically stable if it is an isolated minimum, and is

globally stable if  $H_d$  is proper and  $x^*$  is the largest invariant set of (4) in  $\{x \in \mathbb{R}^n \mid -\nabla^\top H_d(x)R_d(x)\nabla H_d(x) = 0\}$ .

We define  $\mathcal{E} := \{x \mid \nabla H_d(x) = 0\}$  as a set of equilibria which contains also the desired equilibrium point  $x^*$ . As will be shown later in Section III, our construction of  $H_d$  using IDA-PBC for solving the stabilization with guaranteed safety problem (which will be defined shortly) may result in  $\mathcal{E}$  that is not a singleton. Thus, the sole use of IDA-PBC may only stabilize  $x^*$  locally although the closed-loop system is globally safe. In Section IV, we show how to modify the IDA-PBC approach for solving the global stabilization case. In this regards, we denote  $\mathcal{E}_u := \mathcal{E} \setminus x^*$  as the set of undesired equilibria.

A straightforward generalization of IDA-PBC has recently been proposed in Battle et al. (2008) where, instead of restricting the closed-loop system to a particular structure with the interconnection and damping matrices  $J_d(x)$  and  $R_d(x)$ , we can lumped both matrices into a single matrix  $F_d(x)$  which satisfies

$$F_d(x) + F_d^\top(x) \leq 0. \quad (5)$$

The new PDE that has to be solved is

$$g^\perp(x)f(x) = g^\perp(x)F_d(x)\nabla H_d(x) \quad (6)$$

and its corresponding control input is given by

$$u = \beta(x) = (g^\top(x)g(x))^{-1}g^\top(x)(F_d(x)\nabla H_d(x) - f(x)) \quad (7)$$

In this case, the resulting port-Hamiltonian closed-loop system is given by

$$\dot{x} = F_d(x)\nabla H_d(x) \quad (8)$$

and this control design is often referred to as the Simultaneous IDA-PBC approach.

### 2.2 Stabilization with guaranteed safety

As can be seen above, the IDA-PBC is mainly focused on the stabilization of a point without taking into account the safety of the closed-loop system.

Before we discuss the inclusion of the safety aspect into the IDA-PBC design, let us first recall the problem of stabilization with guaranteed safety which has been studied recently in Romdlony & Jayawardhana (2014a) and Romdlony & Jayawardhana (2014b).

We denote  $\mathcal{X}_0 \subset \mathbb{R}^n$  as the set of initial conditions,  $\mathcal{D} \subset \mathbb{R}^n$  as the set of unsafe states where  $\mathcal{D} \cap \mathcal{X}_0 = \emptyset$ . Moreover, we always assume that  $x^* \in \mathcal{X}_0$ .

*Definition 1. (Safety).* Consider an autonomous system

$$\dot{x} = f(x), \quad x(0) \in \mathcal{X}_0, \quad (9)$$

where  $x(t) \in \mathbb{R}^n$ , the system is called *safe* if for all  $x(0) \in \mathcal{X}_0$  and for all  $t \in \mathbb{R}_+$ ,  $x(t) \notin \mathcal{D}$ .

*Stabilization with guaranteed safety control problem:* Consider the system in (1) with a given set of initial conditions  $\mathcal{X}_0 \subset \mathbb{R}^n$  and set of unsafe state  $\mathcal{D} \subset \mathbb{R}^n$ , design a feedback law  $u = \beta(x)$  such that the closed loop system is safe and  $x^*$  is asymptotically stable, i.e. for all  $x(0) \in \mathcal{X}_0$ , we have that  $x(t) \notin \mathcal{D}$  for all  $t$  and  $\lim_{t \rightarrow \infty} \|x(t)\| = x^*$ . Moreover, when  $\mathcal{X}_0 = \mathbb{R}^n \setminus \mathcal{D}$  we call it the *global stabilization with guaranteed safety control problem*.

Note that in the latter definition, there is a slight modification to the one used in Romdlony & Jayawardhana (2014a,b). Instead of stabilizing the origin as considered in these papers, we consider here the stabilization of arbitrary admissible equilibria

$x^*$ . Here, the set of admissible equilibria is given by  $\mathcal{E} = \{x \in \mathbb{R}^n | g^\perp(x)f(x) = 0\}$ .

Let us now recall the result of stabilization of the origin with guaranteed safety as discussed in (Romdlony & Jayawardhana, 2014a, Proposition 1).

*Proposition 1.* Consider the autonomous system (9) with a given set of unsafe state  $\mathcal{D}$  which is assumed to be open. Suppose that there exists a proper and lower-bounded  $\mathcal{C}^1$  function  $W : \mathbb{R}^n \rightarrow \mathbb{R}$  such that

$$W(x) > 0 \quad \forall x \in \mathcal{D} \quad (10a)$$

$$L_f W(x) < 0 \quad \forall x \in \mathbb{R}^n \setminus (\mathcal{D} \cup \{0\}) \quad (10b)$$

$$\mathcal{U} := \{x \in \mathbb{R}^n | W(x) \leq 0\} \neq \emptyset \quad (10c)$$

$$\overline{\mathbb{R}^n \setminus (\mathcal{D} \cup \mathcal{U})} \cap \overline{\mathcal{D}} = \emptyset \quad (10d)$$

then the system is safe with  $\mathcal{X}_0 = \mathbb{R}^n \setminus \mathcal{D}$  and the origin is asymptotically stable.

The function  $W$  that satisfies the hypotheses in Proposition 1 is called Lyapunov-Barrier function. In comparison to the related barrier function as used in Ngo et al. (2005) and Tee et al. (2009), the Lyapunov-Barrier function is not necessarily unbounded on the boundary of the unsafe state set.

### 3. STABILIZATION WITH GUARANTEED SAFETY VIA IDA-PBC

As a first step towards the inclusion of safety aspect into the IDA-PBC design, we consider the problem of stabilization of a desired equilibrium  $x^*$  with guaranteed safety by combining the standard IDA-PBC with the result in Proposition 1 as follows.

*Proposition 2.* Given a set of unsafe state  $\mathcal{D}$  which is open, suppose that there exist  $H_d, J_d, R_d$  such that (2) holds and satisfy

$$H_d(x) > 0 \quad \forall x \in \mathcal{D} \quad (11a)$$

$$\mathcal{U} := \{x \in \mathbb{R}^n | H_d(x) \leq 0\} \neq \emptyset \quad (11b)$$

$$\overline{\mathbb{R}^n \setminus (\mathcal{D} \cup \mathcal{U})} \cap \overline{\mathcal{D}} = \emptyset. \quad (11c)$$

Then the control law  $u = \beta(x)$  where  $\beta$  as in (3) solves stabilization of  $x^*$  with guaranteed safety control problem. Moreover, if  $x^*$  is the unique minimum of  $H_d$  and  $H_d$  is proper, then the result holds globally (i.e.,  $\mathcal{X}_0 = \mathbb{R}^n \setminus \mathcal{D}$ ).

**Proof.** By the assumption of (2a), the substitution of control law (3) into the system (1) results in a closed-loop system that is in the port-Hamiltonian structure as in (4). For the sake of simplicity, we denote the right hand side of (4) by  $F(x)$ .

It is easy to verify that

$$\dot{H}_d = \nabla^\top H_d(x(t))(J_d(x(t)) - R_d(x(t)))\nabla H_d(x(t)) \leq 0. \quad (12)$$

for all  $x(t) \in \mathbb{R}^n \setminus \mathcal{D}$ .

First, we prove that the closed-loop system is globally safe, i.e., for all  $x(0) \in \mathbb{R}^n \setminus \mathcal{D}$ , the corresponding state trajectory  $x(t)$  never enters  $\mathcal{D}$ .

If  $x(0) \in \mathcal{U}$  (i.e.  $H_d(x(0)) \leq 0$ ) by the definition of  $\mathcal{U}$  then it follows from (12), that  $H_d$  is non-increasing along the trajectory  $x(t)$  satisfying  $\dot{x} = F(x)$ , thus  $H_d(x(t)) - H_d(x(0)) \leq 0$  for all  $t \in \mathbb{R}_+$ . Hence, it implies that  $H_d(x(t)) \leq 0$  for all  $t \in \mathbb{R}_+$ . In other words, the set  $\mathcal{U}$  is forward invariant and  $\lim_{t \rightarrow \infty} x(t) \in \mathcal{U}$ .

Moreover by (11a) and the fact that  $\mathcal{D} \cap \mathcal{U} = \emptyset$ , the state trajectory  $x(t) \notin \mathcal{D}$  for all  $t \in \mathbb{R}_+$ .

It remains now to show that for all  $x(0) \in \mathbb{R}^n \setminus (\mathcal{D} \cup \mathcal{U})$ , we also have the property that  $x(t) \notin \mathcal{D}$  for all  $t \in \mathbb{R}_+$ . In this case, we note that  $H_d(x(0)) > 0$  and, as before,  $H_d$  is non-increasing along the trajectory of  $x$  for all  $t$ .

Since the set  $\overline{\mathbb{R}^n \setminus (\mathcal{D} \cup \mathcal{U})}$  does not intersect with the set  $\overline{\mathcal{D}}$ , it implies that the trajectory  $x(t)$  will not enter  $\mathcal{D}$  before it first reach the boundary of  $\mathbb{R}^n \setminus (\mathcal{D} \cup \mathcal{U})$ , in which case,  $H_d(x) = 0$ . Once the trajectory  $x(t)$  is on the boundary of  $\mathbb{R}^n \setminus (\mathcal{D} \cup \mathcal{U})$ , by the fact that  $H_d(x(t)) - H_d(x(0)) \leq 0$ , the state trajectory  $x(t)$  will remain in  $\mathcal{U}$  for the remaining  $t$ . Thus the closed-loop system is globally safe with the admissible set of initial conditions  $\mathcal{X}_0 = \mathbb{R}^n \setminus \mathcal{D}$ .

We will now prove the asymptotic stability of  $x^*$ . By the local convexity of  $H_d$  in the neighborhood of  $x^*$  (c.f. the assumption (2b)) and by (12) we can use  $H_d$  as a Lyapunov function to show the stability of  $x^*$ .

In this case, we define  $\mathcal{X}_0$  as the largest domain of convexity of  $H_d$  around  $x^*$  excluding  $\mathcal{D}$ . By the convexity of  $H_d$  in  $\mathcal{X}_0$  and by (12), it follows that  $\mathcal{X}_0$  is forward invariant.

In particular, for all  $x(0) \in \mathcal{X}_0$ ,  $x(t)$  is bounded for all  $t$  and by the application of La-Salle invariance principle,  $x(t)$  converges to the largest invariance set contained in  $\mathcal{M} := \{x \in \mathcal{X}_0 | \nabla^\top H_d(x)R_d(x)\nabla H_d(x) = 0\}$ . By the strict convexity of  $H_d$  in  $\mathcal{X}_0$ , such an invariant set is given by  $\{x^*\}$ . In combination with the global safety property as proven above, we achieve the (local) stability of  $x^*$  with guaranteed safety.

Finally, if  $x^*$  is the unique minimum of  $H_d$  and  $H_d$  is proper then the global results holds by the use of La-Salle invariance principle.  $\square$

It is easy to observe that instead of finding  $J_d$  and  $R_d$  separately as in Proposition 2, we can simultaneously design them as pursued in Battle et al. (2008) where we need to find  $F_d(x)$  such that (5) and (6) hold. Note that this relaxed condition does not change our previous result on the stability, neither on the safety of the closed-loop system. It only relaxes the solvability of the PDE in the expense of port-Hamiltonian structure. More precisely, we state it in the following corollary.

*Corollary 3.* Given a set of unsafe state  $\mathcal{D}$  which is open, suppose that there exist  $H_d$  and  $F_d$  such that (2b), (5), (6) hold and satisfy (11). Then the control law  $u = \beta(x)$  as in (7) solves stabilization of  $x^*$  with guaranteed safety control problem.  $\triangle$

*Example 1.* In order to illustrate the main result in Proposition 2, let us consider the following system.

$$\begin{aligned} \dot{x}_1 &= -x_1^3 + 2.25x_1x_2^2 + 3.5x_2^3 - 1500x_2 \\ \dot{x}_2 &= u. \end{aligned} \quad (13)$$

It can be shown that the origin can be made globally-asymptotically stable (GAS) using a simple control law  $u = -kx_2$  with  $k > 0$ . First, we note that the  $x_1$ -subsystem is input-to-state stable (ISS) with respect to  $x_2$  (for example, using  $V(x_1) = \frac{1}{2}x_1^2$  as the ISS Lyapunov function). Hence, if we let  $u = -kx_2$ , the  $x_2$ -subsystem converges exponentially to zero, and this implies that, by the ISS property of  $x_1$ -subsystem,  $x_1(t)$  converges also to zero.

We will now consider the problem of stabilization of (13) with guaranteed safety via IDA-PBC. Assume that the set of unsafe state is defined by  $\mathcal{D} = \{x \in \mathbb{R}^n | (x_1 - 2)^2 + (x_1 - 2)x_2 + x_2^2 <$

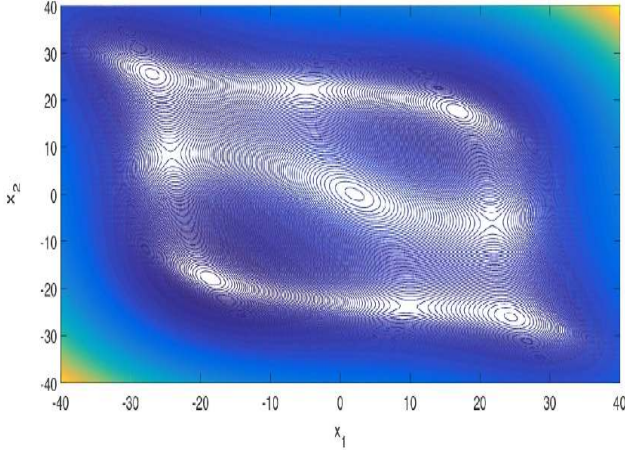


Fig. 1. The contour of the desired energy function  $H_d$  as used in Example 1. The function  $H_d$  has four minima, one maximum, and four saddle-points.

10}. For simplicity, we consider the following  $H_d : \mathbb{R}^2 \rightarrow \mathbb{R}$

$$H_d = (x_1^2 \ x_1 x_2 \ x_2^2) \begin{pmatrix} 1 & 0.5 & -0.125 \\ 0.5 & 1 & 0.5 \\ -0.125 & 0.5 & 1 \end{pmatrix} \begin{pmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \end{pmatrix} - 1000(x_1 - 2 \ x_2) \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \begin{pmatrix} x_1 - 2 \\ x_2 \end{pmatrix} + 10000$$

which is proper and has minima at the desired equilibrium  $x^* = (-18.6467, -17.8454)^\top$  and at other equilibria  $x_{u1} = (-26.948, 25.532)^\top$ ,  $x_{u2} = (16.7688, 17.7117)^\top$ ,  $x_{u3} = (24.3953, -26.0258)^\top$ . The contour plot of this  $H_d$  is shown in Figure 1.

Now, in order to design the controller as in Corollary 3, we need to solve the PDE (6) where in this case,  $g^\perp(x) = (g_1(x) \ 0)$ , with  $g_1 : \mathbb{R}^2 \rightarrow \mathbb{R}$  and  $F_d(x) = \begin{pmatrix} a(x) & b(x) \\ c(x) & d(x) \end{pmatrix}$  that must be designed and also satisfy (5). It follows directly from (6) that we need to satisfy

$$-x_1^3 + 2.25x_1x_2^2 + 3.5x_2^3 - 1500x_2 = a(x)\nabla_{x_1}H_d(x) + b(x)\nabla_{x_2}H_d(x). \quad (14)$$

A possible solution to this equation is to let  $a(x) = -0.5$  and  $b(x) = 1$ . In order to fulfill (5), we can take  $c(x) = -1$  and  $d(x) = c_1$  with  $c_1 \leq 0$ . Using these numerical values, the simulation results of the closed-loop system with several different initial conditions are shown in Figure 2. It can be seen from this figure that we achieve the (local) stabilization with guaranteed safety at the desired equilibrium point  $x^*$ . One can also notice from the simulation that there exists other attractive equilibrium points  $x_{u1}, x_{u2}, x_{u3}$ . Moreover, we achieve global stabilization with guaranteed safety with respect to  $\mathcal{E} = \{x^*, x_{u1}, x_{u2}, x_{u3}\}$ .  $\triangle$

As shown in Example 1, the region-of-attraction of the desired equilibrium point can rather be restrictive. For this example, we plot in Figure 3 the numerically-estimated region-of-attraction (RoA) for every equilibria in  $\mathcal{E}$ . In this plot, the RoA for  $x^*$  is shown in yellow, while that for the other equilibria  $x_{u1}, x_{u2}$  and  $x_{u3}$  are shown in red, blue, and green, respectively.

In fact, the region-of-attraction is influenced by the choice of  $F_d$ , particularly, the damping part. In Figure 4 we show the

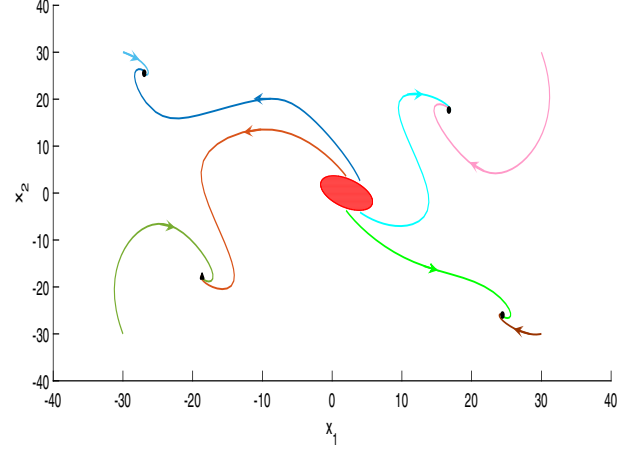


Fig. 2. The numerical simulation result of the closed-loop system using IDA-PBC method in Example 1 from eight different initial conditions. The desired equilibrium is shown in triangle while the other equilibria are shown in circle. The set of unsafe states  $\mathcal{S}$  is shown in the red elliptic-parabolic. All trajectories converge to the equilibria and avoid  $\mathcal{S}$

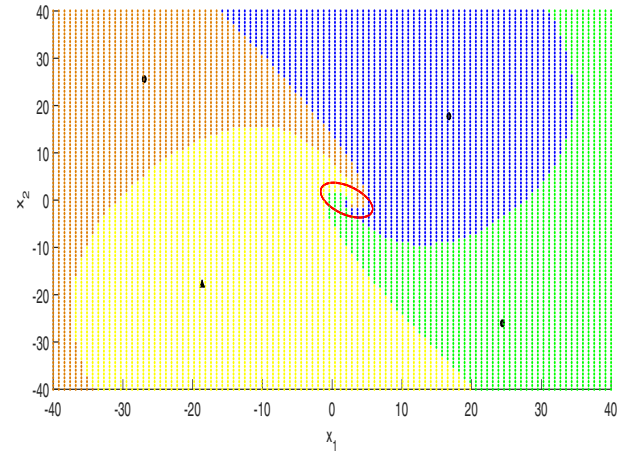


Fig. 3. The numerical estimation of region-of-attraction (RoA) of the closed-loop system in Example 1 for every equilibria with  $c_1 = -1$ . The RoA of the desired equilibrium point  $x^*$  is shown in yellow, while that for the other equilibria  $x_{u1}, x_{u2}$  and  $x_{u3}$  are shown in red, blue, and green, respectively. The boundary of  $\mathcal{S}$  is shown in red line.

different region-of-attraction for different damping element by varying the value of  $c_1$ . In this figure, the RoA of  $x^*$  has gained additional area on the upper side, as well as on the lower-right side. However, the RoA near the set of unsafe state is reduced.

In the following section we will discuss a hybrid strategy for achieving global stabilization with guaranteed safety.

#### 4. GLOBAL STABILIZATION WITH GUARANTEED SAFETY

As has been shown before in Section III, the IDA-PBC approach has allowed us to achieve local stabilization of a desired equilibrium with guaranteed safety. At the same time, it may also introduce undesired equilibrium points that prevent us from achieving a global stabilization with guaranteed safety. Despite this, if one is interested only in the safety aspect, the aforementioned proposed control can, in fact, guarantee the

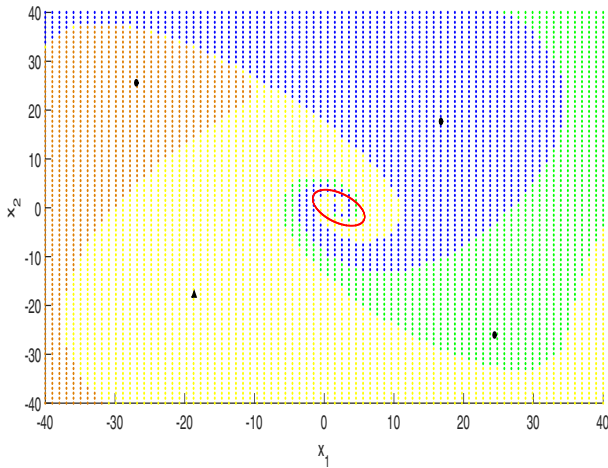


Fig. 4. The numerical estimation of the region-of-attraction (RoA) of the closed-loop system in Example 1 for every equilibria with  $c_1 = 0$ . The rest of the information is the same as that in Fig.3.

global safety, i.e., for all admissible initial condition  $\mathbb{R}^n \setminus \mathcal{D}$ , the state trajectory will never enter the set of unsafe state  $\mathcal{D}$ . Indeed, in our previous example, we have shown that the state trajectory from any initial condition converges to the set of equilibrium points  $\mathcal{E}$  without entering  $\mathcal{D}$ .

In this section, we propose a simple hybrid control strategy where we combine the IDA-PBC based state feedback that achieves set asymptotic stabilization with guaranteed global safety and other feedback controllers that can steer the system trajectories from the neighborhood of  $\mathcal{E}_u$  to the desired equilibrium point  $x^*$ . As will be shown later, this hybrid strategy provides a simple solution to the global stabilization with guaranteed safety.

Prior to describing our proposed hybrid controller, let us recall the following definitions on hybrid automaton as discussed in Lygeros et al. (2003).

Let a hybrid automaton be described by the tuple  $(Q, X, F, Q_0 \times X_0, Dom, E, \mathcal{G}, \mathcal{R})$  where  $Q \subset \mathbb{Z}_+$  is a finite set of discrete variables,  $X \in \mathbb{R}^n$  is the set of continuous variables,  $F : Q \times X \rightarrow X$  defines the vector field of the continuous variables,  $Q_0 \times X_0$  is the set of initial conditions,  $Dom : Q \rightarrow X$  defines the domain of each discrete variable  $q \in Q$ ,  $E \subset Q \times Q$  denotes the set of edges that describe different transitions/jumps between different discrete state. The set  $\mathcal{G} : E \rightarrow X$  defines the guard conditions that can initiate the transition or jump to another discrete state. The maps  $\mathcal{R} : E \times X \rightarrow X$  defines the resetting of the continuous variables following a transition/jump.

Using the above notion of hybrid automaton, we consider hybrid automaton as shown in Figure 5 as our proposed hybrid strategy. In this setting,  $Q = \{1, 2\}$ ,  $X = \mathbb{R}^n$ , the set of initial condition is given by  $Q_0 \times X_0 = \{1\} \times \mathbb{R}^n \setminus \mathcal{D}$ . For  $q = 1$ ,  $F(1, x)$  is a vector field of the closed-loop system using the IDA-PBC method, i.e.,  $F(1, x) = (J_d(x) - R_d(x)) \nabla H_d(x)$ . On the other hand,  $F(2, x)$  is a vector field of the closed-loop system using another state-feedback control law  $u = k(x)$  that can steer the system trajectories from the neighborhood of  $\mathcal{E}_u$  to the desired one  $x^*$  without entering  $\mathcal{D}$ . If the latter state-feedback controller exists then the global stabilization with guaranteed safety problem is solvable by combining it with the IDA-PBC control via hybrid automaton as in Figure 5. In this case, the

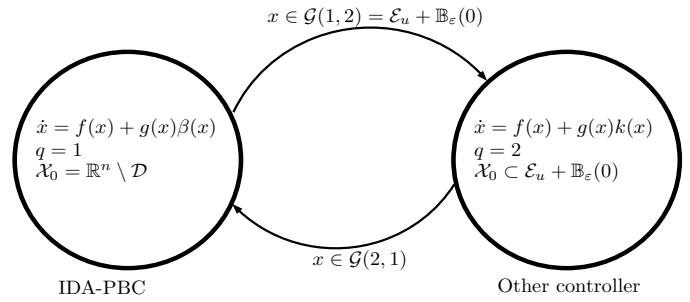


Fig. 5. Hybrid automaton used in Proposition 4 for solving global stabilization with guaranteed safety by using IDA-PBC and another local stabilizing feedback controller.

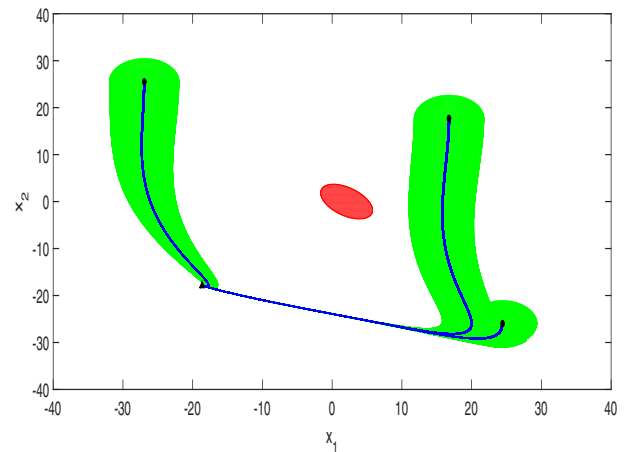


Fig. 6. The plot of positively invariant set  $\Omega(x(0))$  for the system in Example 1 using a state feedback  $u = -Kx$ , with  $K = [1339.0 \quad 4673.4]$ . The plot of  $\Omega(\mathcal{E}_u + \mathbb{B}_5(0))$  is shown in green while the plot of  $\Omega(\mathcal{E}_u + \mathbb{B}_{0.1}(0))$  is shown in blue.

guards  $\mathcal{G}(1, 2)$  and  $\mathcal{G}(2, 1)$  are defined by the neighborhood of  $\mathcal{E}_u$  and the boundary of the positive invariant set due to the application of  $u = k(x)$  that contains the neighborhood of  $\mathcal{E}_u$ , respectively. The jump map  $\mathcal{R}$  is simply given by an identity.

We note that the existence of the second state-feedback control law  $u = k(x)$  is a mild assumption. For this controller to exist, we need only to assume that  $x^*$  is reachable from any point in the neighborhood of  $\mathcal{E}_u$  without entering  $\mathcal{D}$ .

**Proposition 4.** Assume the system as in Proposition 2 with the given control law  $u = \beta(x)$  and a proper  $H_d$ . Suppose that there exist a constant  $\delta > 0$  and a control law  $u = k(x)$  such that for all  $x_0 \in \mathcal{E}_u + \mathbb{B}_\delta(0)$  the corresponding state trajectory converges to  $x^*$  and is safe, i.e., the positive invariant set  $\Omega(\mathcal{E}_u + \mathbb{B}_\delta(0)) =: \Phi$  does not intersect  $\mathcal{D}$ . Then the global stabilization with guaranteed safety problem is solvable using hybrid control as in Figure 5 with  $\mathcal{G}(1, 2) = \mathcal{E}_u + \mathbb{B}_\epsilon(0)$ ,  $0 < \epsilon < \delta$ ,  $\mathcal{G}(2, 1) = \partial\Phi$  and  $\mathcal{R} = \text{Id}$ .

**Proof.** As assumed in the proposition, the hybrid automaton is initialized with the first mode  $q = 1$ .

Following the same proof as in Proposition 2, the properness of  $H_d$  along with inequality (12) implies that the state trajectories  $x$  asymptotically converges to  $\mathcal{E}$ . It has also been proven in Proposition 2 that the control law  $u = \beta(x)$  with a proper  $H_d$  guarantees global safety property of the closed-loop system. It remains to show that  $x(t) \rightarrow x^*$  for the hybrid system.

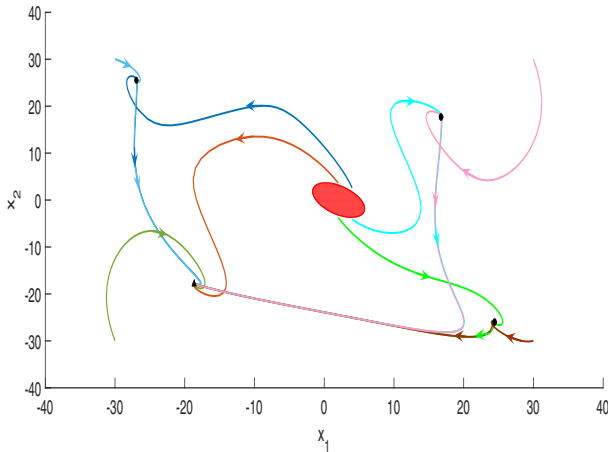


Fig. 7. The numerical simulation result of the closed-loop system in Example 1 using hybrid control method as in Proposition 4. The closed-loop trajectories are based on the same initial conditions as those used in Fig. 2 and all trajectories converge to  $x^*$  without entering  $\mathcal{D}$  (shown in red).

By the global attractivity of  $\mathcal{E}$ ,  $x$  converges to  $x^*$  or to  $\mathcal{E}_u$ . If for some  $x(0)$ ,  $x$  converges to  $x^*$  then the transition to  $q = 2$  will never happen and we obtain our result. Otherwise, there exists  $T > 0$  such that  $x(T) \in \partial(\mathcal{E}_u + \mathbb{B}_\varepsilon(0))$  which will initiate the jump to  $q = 2$ . During the jump, we have  $x^+(T) = x(T) =: x_T$  by our assumption and the closed-loop system will be described by

$$\dot{x} = f(x) + g(x)k(x), \quad x(T) = x_T \in \mathcal{E}_u + \mathbb{B}_\varepsilon(0).$$

By our assumption on  $k(x)$ , the state trajectory  $x$  will remain in the positively invariant set  $\Omega(\mathcal{E}_u + \mathbb{B}_\varepsilon(0))$  and in particular, will never jump to  $q = 1$ . Thus  $x$  converges to  $x^*$  as desired. This proves our claim.  $\square$

The proposed approach provides a practical solution to the global stabilization with guaranteed safety. In this case, in addition to the IDA-PBC conditions, we need to find stabilizing controllers for only a finite and arbitrary small set of initial conditions. Hence, we may not need to design a large number of switched controllers defined on different polytope/manifold which can be numerically intractable for higher-order systems.

Let us now consider again the same system as in Example 1 where the IDA-PBC based controller is designed with  $c_1 = -1$ . One can evaluate directly that by applying  $u = -Kx$  where  $K = [1339.0 \quad 4673.4]$ , it can steer the system trajectories from any initial condition in  $\mathcal{E}_u + \mathbb{B}_5(0)$ . Indeed, Figure 6 shows the positively invariant set of the closed-loop system for initial condition in  $\mathcal{E}_u + \mathbb{B}_5(0)$  (shown in green) and in  $\mathcal{E}_u + \mathbb{B}_{0.1}(0)$  (shown in blue). Equip with this simple controller, we implement the hybrid control strategy as described in Proposition 4 and the simulation results are shown in Figure 7 where we use the same initial conditions as those used in Figure 2. In comparison to the results in Figure 2, we have now the global convergence of  $x$  to  $x^*$  using the hybrid control.

## 5. CONCLUSIONS

The use of energy-based control design has been shown to be applicable for solving the problem of stabilization with guaranteed safety. The avoidance of unsafe state is achieved by an appropriate design of the energy function which may result

into the existence of attractive undesired equilibria. By adopting a hybrid control framework, we can obtain the global result with less restrictive conditions on the other mode.

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