Passivity-based integral control of a boost converter for large-signal stability

R. Leyva, A. Cid-Pastor, C. Alonso, I. Queinnec, S. Tarbouriech and L. Martinez-Salamero

Abstract: This paper presents a new integral control law for a boost converter that ensures the stability of large-signal operation. The integral controller is derived in two steps by means of passivity-based control theory. First, a static law, which ensures global stability, is obtained. Secondly, a combination of the storage function of this static law and a new positive semidefinite storage function results in a new integral control law. The proposed regulator satisfies the usual transient specifications and behaves robustly for parameter uncertainty. Global stability is guaranteed even if the duty cycle saturation is taken into account. Simulation and experimental results verify the theoretical predictions.

1 Introduction

DC-to-DC switching converters exhibit a nonlinear dynamic behaviour and the control to output transfer function of their linearised models is in many cases of nonminimum phase type. The use of linear controllers in such converters only ensures the local stability of the switching regulator.

The nonlinear nature of switching converters has prompted some authors to use nonlinear control for regulation purposes. Feedback linearisation, sliding-mode control and passivity-based control are some of the nonlinear strategies that have been used in recent years in the field of switching converters. In [1] and [2], feedback linearisation is applied in converters with no uncertainty in the parameters. These papers do not consider the saturation of the duty cycle and the implementation of the control law requires some analog divisions which constrain the operation region where control exists. The use of the sliding-mode has been analysed in [3, 4] where only local stability is guaranteed in spite of the nonlinear nature of the control. More recently, passivity-based control techniques have also been studied in [5], although the resulting control assumes perfect knowledge of all converter parameters, including input voltage and load resistance. Another interesting approach is reported in [6] where passivity-based controllers are derived using the concept of incremental energy.

Also, experimental prototypes of passivity control law in converters have been reported in literature [7] and [8]. A digital control is reported in [7], although the solution is too expensive for commercial purposes. In [8], the analog control law proposed is linear but does not solve the uncertainties in the parameters.

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Although the application of passivity techniques to control switching converters is a relatively new subject in the field of Control Theory, passivity concepts have long been used [9–11] to analyse systems represented by state equations.

The idea of passivity stems from Circuit Theory's definition of the energetic behaviour of n-port linear networks, which were later extended to the interconnection of nonlinear passive circuits [12].

A passivity-based control of switching converters should not assume linear dynamic behaviour of the power cell nor neglect the control saturation problem when a large-signal perturbation appears.

This paper analyses control laws that consider a possible saturation of the duty cycle. These laws guarantee that there will be no steady-state error in the output voltage tracking of step-type references even if there is uncertainty in the converter parameters or switching ripple influence.

2 Boost converter dynamics—review

Figure 1 shows a boost converter whose dynamic behaviour during T_{ON} and T_{OFF} can be expressed as follows

$$\dot{x}_a = A_i x_a + b_i \quad i = 1, 2 \tag{1}$$

where

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1/RC \end{bmatrix} \quad b_1 = \begin{bmatrix} Vg/L \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad b_2 = \begin{bmatrix} Vg/L \\ 0 \end{bmatrix}$$

and

$$x_a = \begin{bmatrix} i_a \\ v_a \end{bmatrix}$$

The state vector components are i_a, which represents the inductor current, and v_a, which represents the capacitor voltage. These variables are measurable or available for feedback purposes.

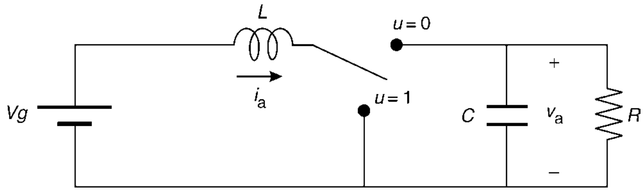

Fig. 1 *Equivalent circuit of a boost convertor*

Equation (1) can be expressed in compact form as follows

$$\dot{x}_a = (A_1 x_a + b_1)u + (A_2 x_a + b_2)(1 - u) \tag{2}$$

or, equivalently:

$$\dot{x}_a = A_2 x_a + b_2 + (A_1 - A_2)x_a u + (b_1 - b_2)u \tag{3}$$

where $u = 1$ during T_{ON} and $u = 0$ during T_{OFF}.

If the switching frequency is significantly higher than the converter's natural frequencies, this discontinuous model can be approximated by a continuous averaged model, which contains a new variable d_a. In the [0,1] subinterval, d_a is a continuous function and constitutes the converter duty cycle.

In the boost converter $b_1 = b_2$, which leads to

$$\dot{x}_a = A_2 x_a + b_2 + (A_1 - A_2)x_a d_a \tag{4}$$

Considering that the system variables consist of two components:

$$\begin{aligned} x_a &= x_e + x \\ d_a &= d_e + d \end{aligned} \tag{5}$$

where x_e and d_e represent the equilibrium values and x and d are the perturbed values of the state and duty cycle.

Equation (4) can be written as follows

$$(\dot{x}_e + \dot{x}) = A_2(x_e + x) + b_2 + (A_1 - A_2)(x_e + x)(d_e + d) \tag{6}$$

which results in

$$\dot{x} = Ax + Bxd + bd \tag{7}$$

where

$$A = A_2 + (A_1 - A_2)d_e \quad B = (A_1 - A_2)$$

and

$$b = (A_1 - A_2)x_e$$

and matrices A, B and b are given by

$$A = \begin{bmatrix} 0 & -d'_e/L \\ -d'_e/C & -1/RC \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1/L \\ -1/C & 0 \end{bmatrix} \tag{8}$$

and

$$b = \begin{bmatrix} v_e/L \\ -i_e/C \end{bmatrix}$$

where $d'_e = 1 - d_e$.

The dynamics of the perturbed system are summarised in the following equations

$$L\frac{di}{dt} = -d'_e v + vd + v_e d \tag{9}$$

$$C\frac{dv}{dt} = d'_e i - \frac{1}{R}v - id - i_e d \tag{10}$$

The purpose of the sections below is to find a control law that guarantees stability even if perturbations are large: that is to say, the system guarantees a return to the equilibrium point taking into account that the control variable d belongs to the $[-d_e, 1 - d_e]$ interval. On the other hand, once nominal stability in the large is assured, the regulator should fulfil the robustness and the usual transient specifications in small-signal operation.

3 Linear control laws for large-signal stability in boost converter

This Section derives a static control law for the boost converter that guarantees global stability. As is stated in Appendix 9.1, the passivity-based control of a system requires a storage function and a passive output that satisfy condition (29). In the case of a boost converter, multiplying (9) by i and (10) by v, we obtain:

$$Li\frac{di}{dt} = -d'_e vi + vid + v_e di \tag{11}$$

$$Cv\frac{dv}{dt} = d'_e vi - \frac{1}{R}v^2 - vid - i_e dv \tag{12}$$

Adding (11) and (12) yields

$$Li\frac{di}{dt} + Cv\frac{dv}{dt} = -\frac{1}{R}v^2 + (v_e i - i_e v)d \tag{13}$$

Choosing as passive output

$$y = h(x) = (v_e i - i_e v) \tag{14}$$

leads to

$$\dot{V}(x) + \frac{1}{R}v^2 = yd \tag{15}$$

where

$$V(x) = L\frac{i^2}{2} + C\frac{v^2}{2}$$

We can conclude that the system has a passive input-output characteristic and, also, that term $(1/R)v^2$ adds dissipativity to the system [13–15].

From theorem 2 in Appendix 9.1, the feedback connection of a memoryless function will modify the regulator dynamics and will guarantee the global asymptotic stability.

Therefore, choosing

$$d(t) = -\phi y(t) = -\phi(v_e i - i_e v) \tag{16}$$

as the control law results in asymptotic stability in the large.

The term $\phi()$ makes it possible to assume that gain is constant around the equilibrium point and that the duty cycle is saturated far from the equilibrium point. It can be observed that, since $-d_e \leq d \leq 1 - d_e$, then ϕ will be:

$$\phi(y) = \begin{cases} \phi_{max} & -\dfrac{d_e}{\phi_{max}} \leq y \leq \dfrac{1-d_e}{\phi_{max}} \\ \dfrac{1-d_e}{y} & y > \dfrac{1-d_e}{\phi_{max}} \\ \dfrac{d_e}{y} & y < \dfrac{d_e}{\phi_{max}} \end{cases} \tag{17}$$

Once global asymptotic stability has been guaranteed, an a posteriori small-signal analysis can be made to choose the correct value for constant ϕ_{max}.

The linearised control law is $d = -\phi_{max}(v_e i - i_e v)$. In addition, according to (9)–(10), the small-signal model of

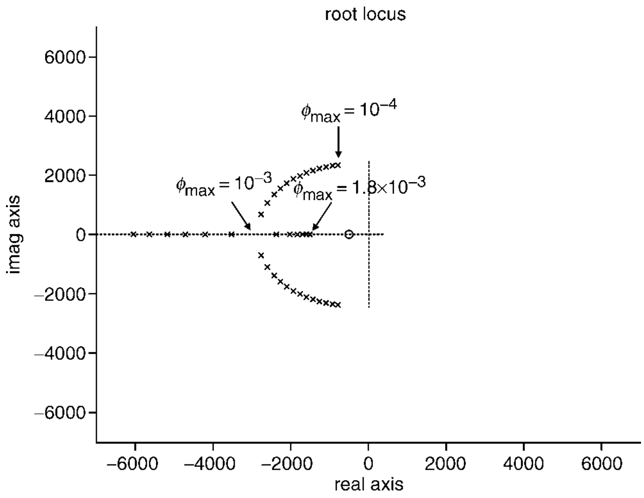

Fig. 2 *Root loci for a passivity-based static control*

a boost converter will be expressed as

$$\begin{bmatrix} \dot{i} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & -d'_e/L \\ d'_e/C & -1/RC \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} v_e/L \\ i_e/C \end{bmatrix} d \tag{18}$$

Hence, the corresponding loop gain will be given by

$$\begin{aligned} T(s) &= -\phi_{max}\left(v_e\frac{I(s)}{D(s)} - i_e\frac{V(s)}{D(s)}\right) \\ &= -\phi_{max}\frac{\left(\frac{v_e^2}{L}+\frac{i_e^2}{C}\right)s + \frac{1}{LC}\left(\frac{v_e^2}{R}\right)}{s^2+\frac{1}{RC}s+\frac{(d'_e)^2}{LC}} \end{aligned} \tag{19}$$

Although the model can consider inductor losses, the corresponding parasitics do not affect the previous derivations.

Figure 2 shows the root loci as a function of ϕ_{max} for the linearisation of control law (16) for a converter whose set of parameters is $L = 200\,\mu H$, $C = 200\,\mu F$, $R = 10\,\Omega$, $Vg = 12$ V and $d_e = 0.5$. The coordinates of the equilibrium point are $v_e = 24$ V and $i_e = 5$ A. Notice that linear control techniques are applied once the global asymptotic stability of the switching regulator has been established.

Figures 3 and 4 show the simulation results which depict the converter start-up when $\phi_{max} = 10^{-3}$, where the switching period is $T_S = 20$ ms and $R_L = 220\,m\Omega$. The parameter ϕ_{max} has been chosen for settling time minimisation.

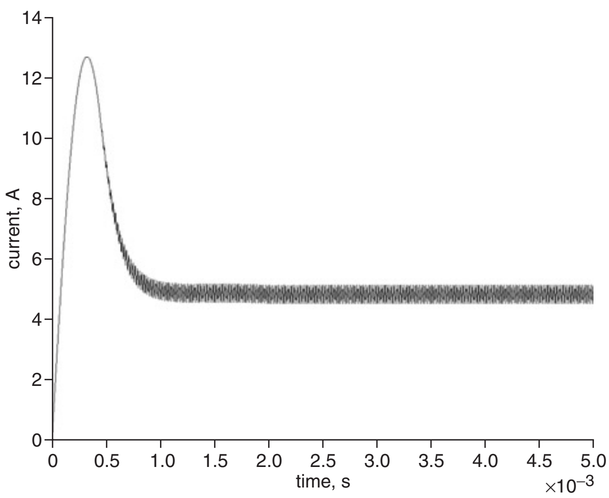

Fig. 3 *Convertor start-up current*

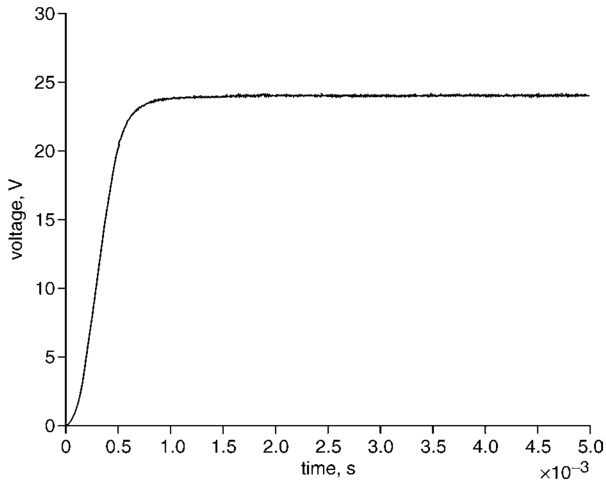

Fig. 4 *Converter start-up voltage*

However, the control law analysed in this section requires precise knowledge of the equilibrium point: that is to say, the values of parameters Vg, R. If these parameter values are not known, there will be a steady-state error in the capacitor voltage. To avoid this problem, in the next Section we present passivity-based controllers that eliminate the steady-state error of the output voltage.

4 Passivity-based integral control of the boost converter

In the Section above, the storage function was derived by means of incremental energy concepts [6] and it has been demonstrated that this function is of the Lyapunov type. In this Section, we move away from the physical concept of incremental energy to analyse a storage function that includes an integral function of the capacitor voltage error; i.e. the variable to be regulated.

The dynamics of the incremental values for boost converters are given by

$$\begin{aligned} \frac{di}{dt} &= -\frac{d'_e}{L}v + \frac{1}{L}vd + \frac{v_e}{L}d \\ \frac{dv}{dt} &= \frac{d'_e}{C}i - \frac{1}{RC}v - \frac{1}{C}id - \frac{i_e}{C}d \\ \frac{dz}{dt} &= v \end{aligned} \tag{20}$$

where z is the integral of the voltage incremental error.

On the basis of the previous passivity results, combined storage functions will be built for system (20) so that a new integral control can be derived that guarantees asymptotic stability for large signal perturbations. For this purpose, the storage function defined above will be used and combined with a new one, which includes an integral term.

Thus, consider the following storage function

$$V(x) = V_1(x) + V_2(x) \tag{21}$$

where $V_1(x) = L(i^2/2) + C(v^2/2)$ and $V_2(x) = 1/2\gamma^2((L/d'_e)i + z)^2$. This storage function is positive definite and radially unbounded for the extended boost dynamics (20), and its time-derivative is

$$\dot{V}(x) = \dot{V}_1(x) + \dot{V}_2(x) \tag{22}$$

whose components, from (13) and (42)–(43) respectively,

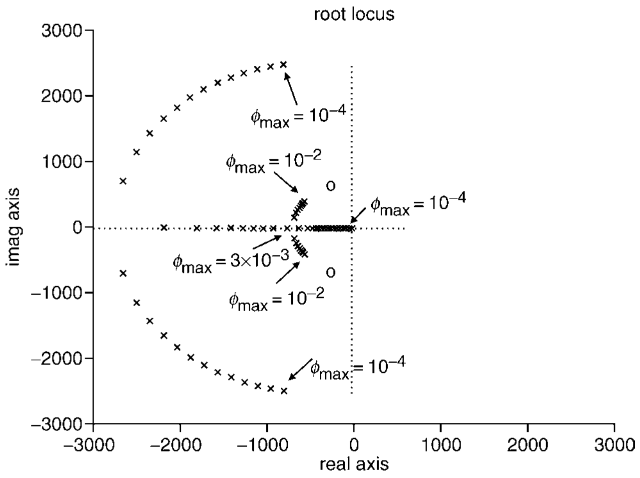

Fig. 5 *Root loci for gain loop (30)*

are

$$\dot{V}_1(x) = -\frac{1}{R}v^2 + (v_e i - i_e v)d \tag{23}$$

$$\dot{V}_2(x) = \frac{\gamma^2}{d_e'^3}(Vg + d_e'v)(d_e'z + Li)d \tag{24}$$

Hence,

$$\dot{V}(x) = -\frac{1}{R}v^2 + (v_e i - i_e v)d + \frac{\gamma^2}{d_e'^3}(Vg + d_e'v)(d_e'z + Li)d \tag{25}$$

With the following output

$$y = v_e i - i_e v + \frac{\gamma^2}{d_e'^3}(Vg + d_e'v)(d_e'z + Li) \tag{26}$$

(26) becomes

$$\dot{V}(x) \le yd \tag{27}$$

and the passive behaviour of the input-output characteristic is proven.

We will introduce a memoryless function $\phi()$, which represents a feedback gain that takes saturation into account,

$$d = -\phi(y) \tag{28}$$

and we shall define $k = \gamma^2/d_e'^2$, where γ and d_e' are constant values.

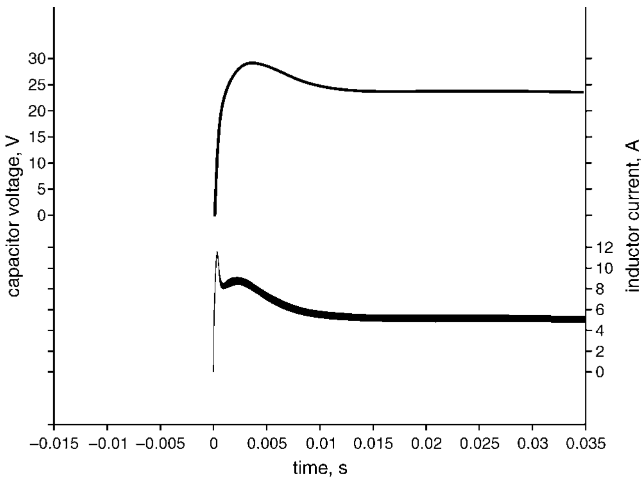

Fig. 6 *Inductor current and capacitor voltage during start-up*

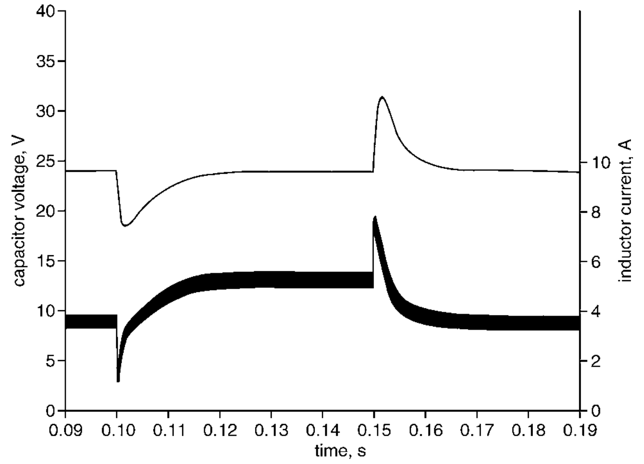

Fig. 7 *Inductor current and capacitor voltage responses for step perturbations in the input voltage*

Therefore, the control law:

$$d = -\phi y = -\phi(v_e i - i_e v + k(Vg + d_e'v)(d_e'z + Li)) \tag{29}$$

guarantees the global asymptotic stability of the switching regulator.

Note that if only $V_2(x)$ is taken as the storage function, then according to Appendix B, the control law becomes $d = -\phi_1(Vg + d_e'v)(d_e'z + Li)$. Nevertheless, this simpler version of the control law cannot provide any additional damping, since $\dot{V}_2(x) = yd$, and, so it is not suitable for most DC-to-DC converter applications.

Once the nonlinear control law has been established, ϕ and k will be selected to ensure that the regulator system behaves satisfactorily in small-signal operation. By linearising the control law (29), we obtain:

$$d_{lin} = -\phi_{max}\left(\left(v_e + k\frac{Vg}{d_e'}L\right)i - i_e v + k\,Vg\,z\right) \tag{30}$$

The corresponding loop gain is given by

$$T(s) = -\phi_{max}\left(\left(v_e + k\frac{VgL}{d_e'}\right)\frac{I(s)}{D_{lin}(s)} - i_e\frac{V(s)}{D_{lin}(s)} + kVg\frac{Z(s)}{D_{lin}(s)}\right) \tag{31}$$

where $I(s)/D_{lin}(s)$, $V(s)/D_{lin}(s)$ and $Z(s)/D_{lin}(s)$ are derived from a linear model of system (20).

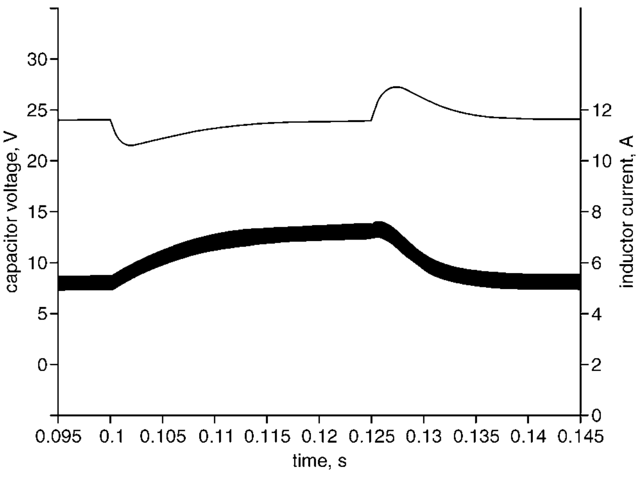

Fig. 8 *Inductor current and capacitor voltage responses for step perturbations in the load resistance*

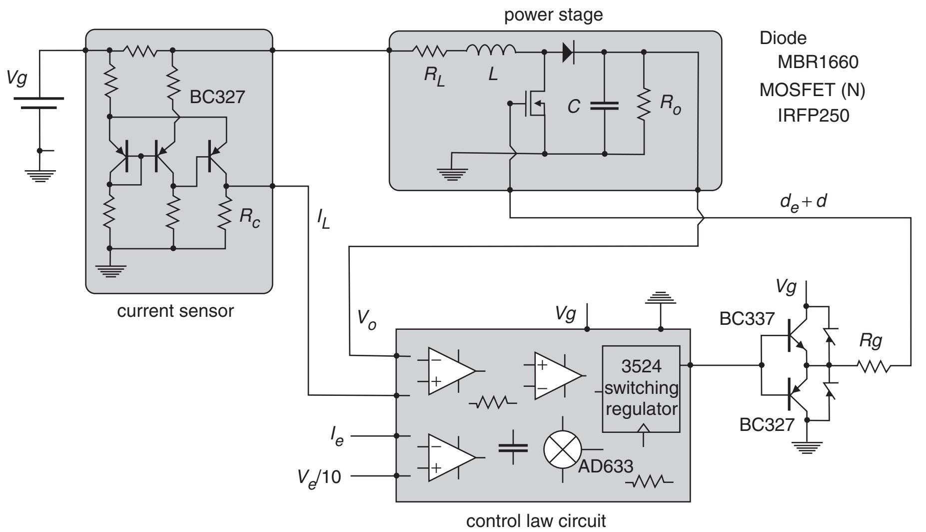

Fig. 9 *Curcuit scheme*

Figure 5 shows the root loci for the gain loop (30) and the converter parameters of the previous section. A value of $k = 500$ is used and the values of $\phi_{\max}$ are within the interval [0.0001–0.01].

Figure 6 illustrates an ACSL simulation of the control law (29) with $\phi_{\max} = 0.003$, which was chosen for settling time minimisation. The waveforms depicted in the Figure are for the inductor current and capacitor voltage during start-up which, in fact, represents a large-signal perturbation around the equilibrium point of the state variables. Fig. 7 shows the system response when the input voltage has step changes: first it decreases from 17 V to 12 V and then returns to 17 V. Notice that the system behaves robustly by returning to the desired output voltage with zero steady-state error after a short transient period. We should point out that the control was designed using a nominal input voltage of 12 V. Similarly, Fig. 8 illustrates the system responses for step changes in the resistance load from 10 Ω a 20 Ω and then a return to 10 Ω. Again we point out that the system, which was designed considering a nominal value of the load resistance of 10 Ω, behaved robustly.

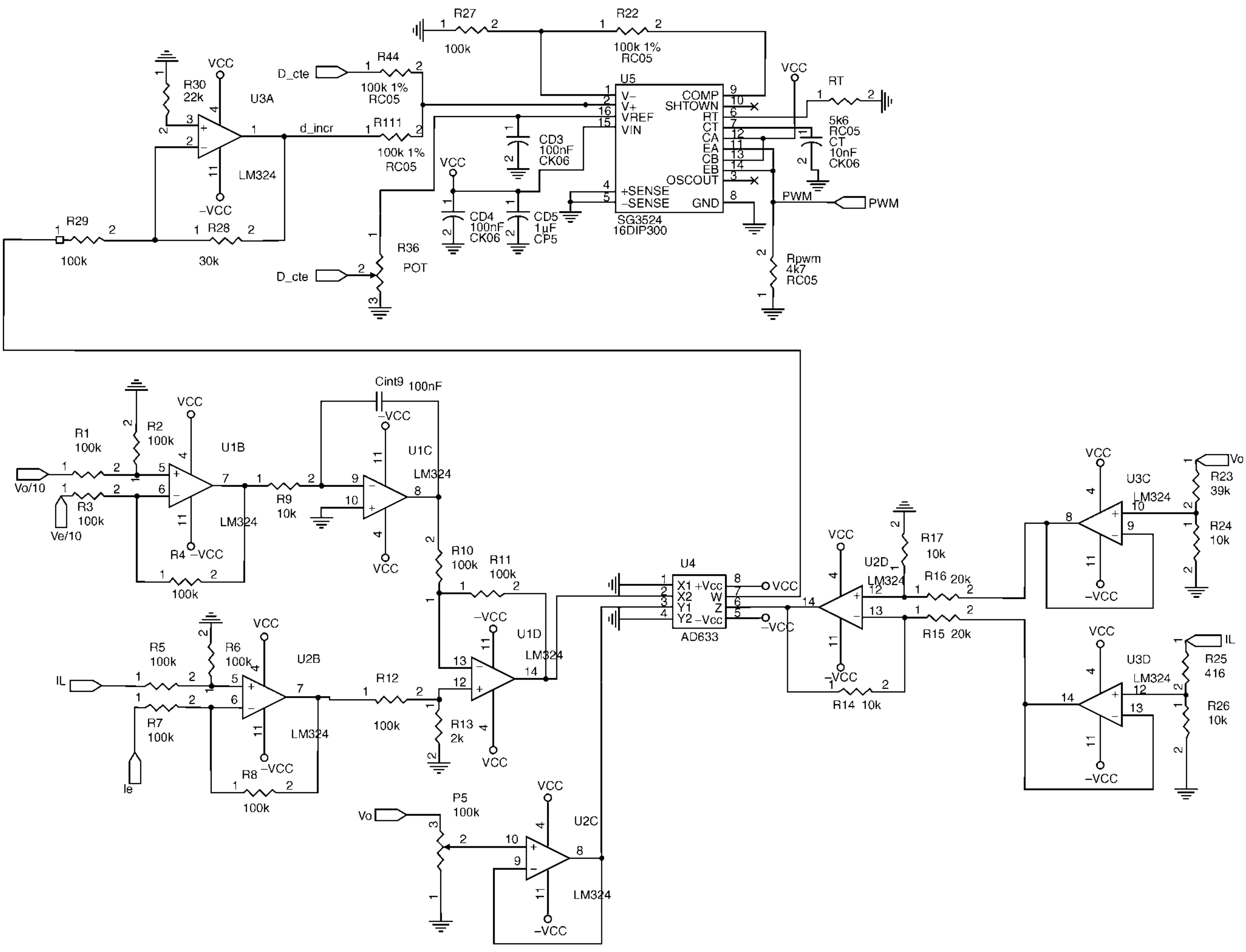

Fig. 10 *Detail of control law circuit*

5 Experimental results

Figures 11, 12 and 13 show the experimental waveforms for the set of parameters used in the sections above and Figs. 9 and 10 show the regulator implementation scheme. Figure 11 shows the measured converter start-up, which is in perfect agreement with the simulated response of Fig. 6. Similarly, measured system responses for input variations (Fig. 12) and load perturbations are in good agreement with the respective simulations of Figs. 7 and 8.

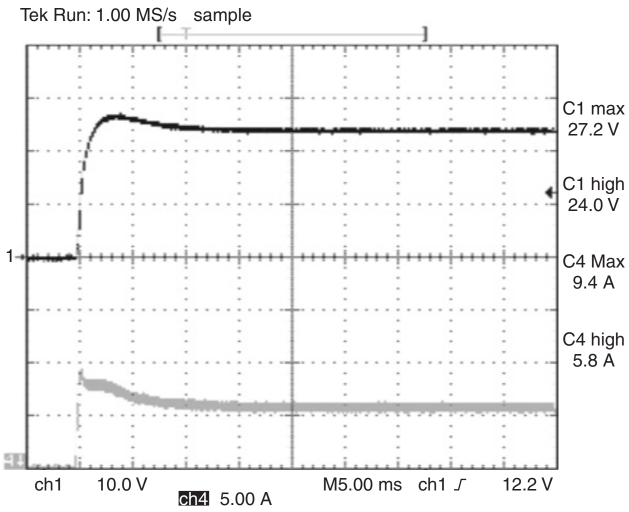

Fig. 11 *Experimental start-up*

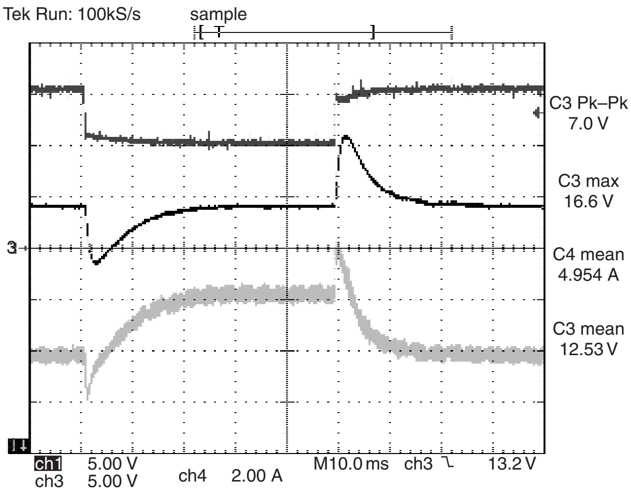

Fig. 12 *System response to input voltage variations*

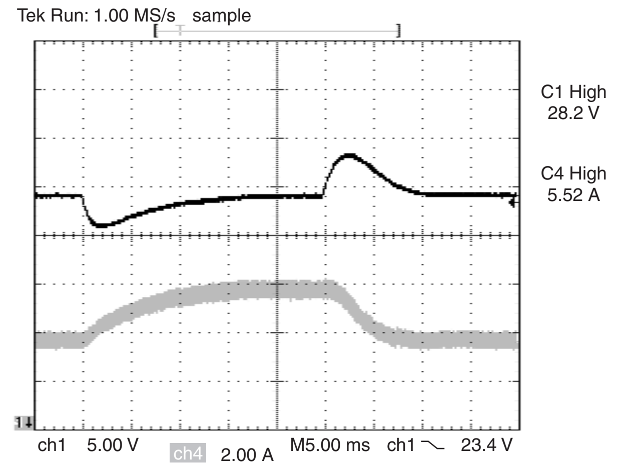

Fig. 13 *System response to load resistance perturbations*

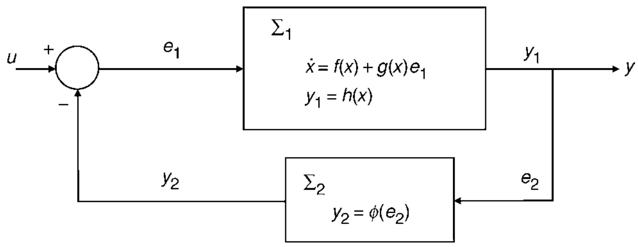

Fig. 14 *Feedback scheme*

6 Conclusions

This paper has shown that passivity-based integral control is an effective solution for regulating DC-to-DC nonminimum phase switching converters under large-signal operation. The control law can be easily implemented by means of an analog multiplier, standard operational amplifiers and a pulse width modulator. In addition, as global stability is assured, it does not require start-up aid circuits.

It is a simple task to apply it to canonical converters such as buck or buck-, and research is currently in progress to determine whether it can be extended to higher order converters.

7 Acknowledgment

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9 Appendices

9.1 Basic passivity concepts

Consider a single-input-single-output (SISO) nonlinear system represented by

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x) \quad (32)$$

where $x \in R^n$, $u \in R$ and $y \in R$. The function f is locally Lipschitz, and the functions g and h are continuous.

Definition 1: System (32) will be passive if there exists a differentiable continuous positive semidefinite function $V(x(t))$ such that

$$uy \geq \frac{\partial V}{\partial x}\dot{x} \quad (33)$$

where $V(x(t))$ is known as the storage function.

Definition 2 [14]: System (32) will be strictly passive with respect to the output if there exists a continuous differentiable positive semidefinite function $V(x(t))$ such that

$$uy \geq \frac{\partial V}{\partial x}\dot{x} + \delta y^2 \quad (34)$$

where δ is a positive constant.

If a system is strictly passive with respect to the output, we often need to know if the following property is satisfied

$$y(t) \equiv 0 \implies x(t) = 0 \quad \text{when } u(t) = 0 \quad (35)$$

Condition (35) is known as zero-state observability.

Theorem 1 [14]: If system (32) is passive with a positive definite storage function $V(x)$, then the system origin for $u = 0$ is stable.

Proof: Since $\dot{V} \leq uy$ then $\dot{V} \leq 0$. In addition, $V(x)$ is positive definite and hence $V(x)$ is a Lyapunov function.

If the system is strictly passive with respect to the output with a positive definite storage function $V(x)$ and is also zero-state observable, then the system origin is asymptotically stable.

Proof: Since $\dot{V} \leq uy$ then $\dot{V} \leq 0$. On the other hand, $V(x)$ is positive definite and therefore $V(x)$ is a Lyapunov function. In addition, the observability condition establishes that the only solution that can identically remain in $S = \{x \in R^n / h(x) = 0\}$ is the trivial solution $x(t) = 0$. Hence, from La Salle's invariance principle the origin must be asymptotically stable. Moreover, if $V(x)$ is not radially bounded, then the origin is globally asymptotically stable.

Definition 3: Considering the time-invariant nonlinear static system or memoryless function

$$y = \phi(u) \quad (36)$$

where ϕ: $R \to R$ is locally Lipschitz.

The static system will be strictly passive with respect to the input if

$$uy \geq \delta u^2 \quad (37)$$

where δ is a positive constant.

Theorem 2: Considering the feedback system of Fig. 14, where Σ_1 is a dynamical system of the form:

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)e_1$$
$$y_1 = h_1(x_1) \quad (38)$$

and Σ_2 is a memoryless function of the type

$$y_2 = \phi(e_2)$$

where the functions f and g are continuous in t and locally Lipschitz in u, ϕ is continuous and $f(0) = 0$ and $h(0) = 0$. Inputs to Σ_1 and Σ_2 are given respectively by

$$e_1 = u_1 - y_2$$
$$e_2 = y_1$$

If Σ_1 is a passive system and Σ_2 is a memoryless system that is strictly passive with respect to the input, then the feedback system is strictly passive with respect to the output.

Proof: Observe that

$$e_1 y_1 \geq \frac{\partial V_1}{\partial x_1}\dot{x}_1$$
$$e_2 y_2 \geq \delta_2 y_1^2 \quad (39)$$

Then

$$u_1 y_1 \geq \frac{\partial V_1}{\partial x_1}\dot{x}_1 + \delta_2 y_1^2 \quad (40)$$

9.2 Storage function derivations in boost converters

Consider the storage function for a given system

$$V(x) = \frac{1}{2}(\alpha i + \beta v + \gamma z)^2 \geq 0 \quad (41)$$

To apply a passive-based control, we have to choose an output such that the input-output characteristic is passive, i.e.:

$$d(t)y(t) \geq \dot{V}(x(t))$$

Observe that

$$\begin{aligned}\dot{V}(x) &= (\alpha i + \beta v + \gamma z)\left(\alpha\frac{di}{dt} + \beta\frac{dv}{dt} + \gamma\frac{dz}{dt}\right)\\ &= \alpha^2 i\frac{di}{dt} + \alpha\beta i\frac{dv}{dt} + \alpha\gamma i\frac{dz}{dt}\\ &\quad + \beta\alpha v\frac{di}{dt} + \beta^2 v\frac{dv}{dt} + \beta\gamma v\frac{dz}{dt}\\ &\quad + \gamma\alpha z\frac{di}{dt} + \gamma\beta z\frac{dv}{dt} + \gamma^2 z\frac{dz}{dt}\end{aligned} \quad (42)$$

Substituting the values of the time-derivatives for the boost converter case (20) into expression (42) yields

$$\begin{aligned}\dot{V}(x) &= -\alpha^2\frac{d_e' iv}{L} + \gamma^2 zv + \alpha\beta\frac{i(d_e' iR - v)}{RC} + \beta^2\frac{d_e' iv}{C} - \beta^2\frac{v^2}{RC}\\ &\quad + \alpha\gamma iv - \alpha\beta\frac{v^2 d_e'}{L} - \gamma\alpha\frac{zvd_e'}{L} + \gamma\beta\frac{z(d_e' iR - v)}{RC}\\ &\quad + \beta\gamma v^2 + l(x)\cdot d\end{aligned} \quad (43)$$

where the terms $l(x)$ depending on the duty cycle are irrelevant and have not been explicitly shown.

Some terms, such as $-\beta^2(v^2/RC)$, have a defined sign in the whole domain; however, terms like $\beta^2(d_e' iv/C)$ do not have a defined sign and, in order to verify the passive nature of the input-output characteristic, we force them to be zero. Therefore, we constrain the analysis to the case $\beta = 0$ and expression (43) becomes

$$\dot{V}(x) = -\alpha^2 \frac{id_e' v}{L} + \gamma^2 zv + \alpha\gamma iv - \gamma\alpha \frac{zd_e' v}{L} + l(x)|_{\beta=0} \cdot d \tag{44}$$

After some manipulations, expression (B.4) can be written as

$$\dot{V}(x) = \left(-\alpha^2 \frac{d_e'}{L} + \alpha\gamma\right) iv + \left(-\gamma\alpha \frac{d_e'}{L} + \gamma^2\right) zv + l(x)\big|_{\beta=0} \cdot d \tag{45}$$

We can observe that the first two terms do not have a defined sign. Therefore, we will make them zero. Solving the equations

$$\begin{cases} -\alpha^2 \dfrac{d_e'}{L} + \alpha\gamma = 0 \\ -\gamma\alpha \dfrac{d_e'}{L} + \gamma^2 = 0 \end{cases} \tag{46}$$

we obtain $\alpha = \gamma L/d_e'$ and hence expression (45) yields

$$\dot{V}(x) = l(x)\big|_{\beta=0,\, \alpha=\gamma(L/d_e')} \cdot d \tag{47}$$

where

$$l(x)\big|_{\beta=0,\, \alpha=\gamma(L/d_e')} = \gamma^2 \frac{(Vg + v\, d_e')(zd_e' + Li)}{d_e'^3} \tag{48}$$

This procedure can be applied in other canonical converters, as well as in high order converters. On the other hand, the procedure makes it possible to analyse a boost converter model, which includes inductance losses resistance R_L. In this case, values of β and α become

$$\beta = \frac{\alpha R_L C}{L d_e'} \tag{49}$$

and

$$\alpha = \frac{\gamma L d_e' R}{d_e'^2 R + R_L} \tag{50}$$