Geodesy on the Move, Forsberg, Feissel, Dietrich (eds), Springer-Verlag,Berlin New York, 58-78, 1998

PAST AND FUTURE DEVELOPMENTS IN GEOPOTENTIAL MODELING

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ABSTRACT

This paper reviews the development and estimation of geopotential models over the past 96 years, starting from simple ellipsoidal normal gravity models to complex high degree (360/500) spherical harmonic expansions. The paper is written to show the evolutionary changes that have taken place in the mathematical models and data used. The discussion considers geopotential models from surface gravity data, satellite tracking data, and combination solutions that incorporate numerous data types including satellite altimeter data. A number of questions are posed that relate to future modeling efforts.

INTRODUCTION

A geopotential model is one that represents the Earth's gravity or gravitational potential and/or gravity values. The estimation of a variety of different models, from the simple to the complex, has evolved significantly in this century. Vetter (1994) discusses various geopotential models used in astrodynamics starting from the very early times (1950's) of space programs. Nerem, Jekeli and Kaula (1995) provide a review of the gravity field determination process, including a review of global scale modeling. Bouman (1997) provides a comprehensive discussion and analyses of geopotential estimation procedures and results starting from the 1970's. This current paper starts earlier than the previous reviews, going back to the time when estimates of normal gravity formula parameters were being determined.

GRAVITY FORMULA PARAMETERS

Theoretical studies done by many investigators (Clairaut, deSitter, Helmert, Pizzetti, Somigliana, Lambert, etc.) showed that variations of gravity on the surface of a rotating surface would take the form:

$$\gamma = \gamma_E \left(1+\beta_1 \sin^2\phi+\beta_2 \sin^2 2\phi+\cdots\right) \tag{1}$$

where γ is the "normal" value of gravity, γ_E is equatorial gravity, β_1, β_2 are constants to be determined or calculated from assumptions on the figure upon which the gravity variations were to be described, and ϕ is geodetic latitude. Based on estimates of the mass of the Earth, its equational radius, and flattening, the parameters of the gravity formula could be determined. The inverse aspect was to determine the parameters of the gravity formula using gravity measurements distributed as widely as possible. One of the first determinations in this century was by Helmert in 1901, who used gravity measurements at 1603 stations. As additional gravity measurements were acquired, various estimates of the parameters were made by Bowie (1917), Heiskanen (1928, 1957), Jeffreys (1937, 1941, 1948), Zhongolovich (1952), Uotila (1957), Kaula (1958), etc. The improved estimates not only included surface gravity data, but also information on the lunar orbit.

The gravity variations described by (1) are latitude dependent only. An extension of the assumption to the case where the equator of the Earth could be an ellipsoid led to a gravity formula that had a longitude dependent term:

$$\gamma = \gamma_E\left(1+\beta_1\sin^2\phi+\beta_2\sin^2 2\phi+\beta_3\cos^2\phi\cos 2(\lambda-\lambda_0)\right) \tag{2}$$

where λ_0 is the longitude of the major axis of the equatorial ellipse. Parameters of the formula were determined by Helmert in 1915, Heiskanen in 1924 and 1928, Niskanen in 1945, Jeffreys in 1948, Uotila in 1957 and 1962, etc. The analyses evolved with the continuing improvement in the availability of surface gravity data.

The early parameter estimates were obtained from point gravity measurements where values were reduced to a zero elevation. As more data became available, gravity anomalies were used. To prevent the over weighting of areas in which dense data was available mean values in cells of various size (e.g. 1°x1°) were estimated. In the Heiskanen 1938 solution 1,802 1°x1° cells were used, while Uotila, in 1962, had 11,294 1° cells. The type of anomaly to use in the analyses was also pertinent to the estimates, with some groups favoring isostatic anomalies and others free-air anomalies. In addition, the magnitude of the adopted value of gravity at Potsdam, based on a measurement by Kühnen and Furtwanger in 1906, later turned out to be too large by about 14 mgal.

EARLY SPHERICAL HARMONIC EXPANSIONS OF TERRESTRIAL ANOMALIES

We next consider a spherical harmonic expansion of gravity anomalies, $\Delta g(\theta,\lambda)$, where θ is the co-geocentric latitude:

$$\Delta g(\theta,\lambda) = \sum_{n=0}^{M}\sum_{m=0}^{n}(a_{nm}\cos m\lambda + b_{nm}\sin m\lambda)P_{nm}(\cos\theta) \tag{3}$$

Dubovskii carried out two expansions to degree 6 in 1937. One expansion assumed zero value for the anomalies in unsurveyed areas while the second version assumed that Pratt isostatic anomalies were zero. In 1941-1943 Jeffreys used free-air gravity anomalies in 10° cells to determine—through a least squares adjustment—coefficients to degree 4. Based on error estimates, Jeffreys felt that no degree 4 terms were reliably determined nor were several degree 3 terms. Jeffreys emphasized the need for error analysis in the estimation of the coefficients. In a 1943 paper Jeffreys used the coefficients of the almost degree 3 expansions to calculate geoid undulations.

In 1952 Zhongolovich described expansions to degree 8 based on the analyses of 10°x10° anomalies estimated from approximately 26,000 point anomalies. A description of the two solutions (first variant, second variant) may also be found in Molodenskii, Eremeev, and Yurkina (1960). The data distribution was such that the data in the southern hemisphere was only 3% of that known in the northern hemisphere. The geoid undulations implied by one of the variants is given in Brovar, Magnitsky, and Shimbirer (1961). Uotila was to later note (1957) that the undulations implied by the Zhongolovich analyses were much larger (-160 to

180m) than seen in his analysis. Kaula (1959) described the estimation of 1°x1°, 5°x5°, 10°x10°, and 30°x30° anomalies. The 1° anomalies were estimated taking into account gravity and elevation data. A total of 8,000 1° values were estimated within 860 5°x5° cells. Ten degree anomalies were also formed from the 1°x1° cells, both extrapolated and interpolated. The 30° anomalies were adjusted to be consistent with one constraint based on satellite estimates of the 2, 4, and 6 zonal harmonies and five constraints from inadmissible harmonics. The corrections to the anomalies were used to obtain revised 10° cells which were then used to determine spherical harmonic coefficients to degree 8 using the orthogonality relationships. The coefficients were used to construct global geoid undulation maps. This solution was one of the first to introduce a constraint from satellite gravity field information into the analysis of terrestrial data.

In 1962 Uotila described the analysis of 1°x1° anomalies to determine, using a least squares adjustment, spherical harmonic coefficients to degree 4. Two solutions were made, one fixing the second, third, and fourth degree zonal harmonics from satellite data. Geoid undulation maps were also produced. Rapp (1969a) described the analyses of a global set of 5° anomalies (55% based on terrestrial gravity estimates) for harmonic coefficients to degree 8. Solutions for the coefficients were made using a least squares adjustment and a quadrature procedure. Undulation differences between the two solutions reached 20m, emphasizing the difference between the two types of solutions.

Except for special situations, the era of calculating harmonic coefficients from terrestrial data alone became less important because the terrestrial data coverage remained poor and information from the analyses of satellite data was increasing—both in accuracy and in maximum expansion degree possible.

EARLY SATELLITE AND COMBINATION GEOPOTENTIAL MODELS

With the launch of Sputnik in 1958, and the start of the Vanguard satellites, procedures for the analysis of the tracking data rapidly evolved. Initial results gave a few zonal harmonic coefficients and then a few tesseral coefficients (e.g. Cook, Izsak, Jacchia, King-Hele, O'Keefe, Kozai, Smith, D.E, etc.). For example, Kaula (1963) describes the use of Baker-Nunn camera observations of three satellites to obtain estimates of 35 selected coefficients with a maximum complete degree to 4 and selected coefficients to degree 7. In 1966 the Smithsonian Astrophysical Observatory documented (Lundquist and Veis, 1966) the first of three SAO Standard Earth models. The SE I was based solely on satellite observations and was complete to degree 8 with selected additional resonance and zonal terms. The SAO SE II was described by Gaposchkin and Lambeck in 1970 and SAO SE III in 1973 (Gaposchkin). Both SE II and III determined potential coefficient models that combined satellite information with terrestrial gravity data. SE II was complete to degree 16, while SE III was complete to degree 18.

The Applied Physics Laboratory (APL) of the Johns Hopkins University produced a number of geopotential models using Doppler data from the Transit navigation satellites. The APL 1.0 model (1963) was complete to degree 4, while the APL 3.5 model (1965) was complete to degree 8. Other models (e.g. APL 4.5 and 5.0) were also produced in this series. Guier and Newton (1965) describe a solution where estimates of odd degree zonal harmonies were combined with estimates of non-zonal harmonies and even degree zonal harmonies derived by King-Hele to produce a geopotential model complete to degree 8.

The Naval Weapons Laboratory (NWL) also developed geopotential models using the Doppler data from the Transit system. Anderle (1996) described the NWL-5E-6 solution that was complete to degree 7. The NWL-8 geodetic parameter sets (8B, 8C, 8D) (Anderle and

Smith, 1967) included geopotential models that were complete to degree 12 with additional resonance terms. The preferred model was NWL-8D.

During the 1960's interest in combining satellite and terrestrial data in a joint estimation process increased. Kaula (1966) implemented a procedure that used harmonic coefficient estimation from gravity anomalies using the orthogonality relationships and those coefficients (estimated from satellite data) to obtain an adjusted set of potential coefficients and gravity anomalies. The procedure described by Kaula for this joint adjustment has been used for more than 30 years (with revisions) to obtain some of the high degree models we consider today (1997). In the Kaula 1966 study, 5° equal area anomalies were used with the mean of several sets of satellite-derived potential coefficients to estimate a geopotential model, complete to degree 12.

Results from an alternative way to combine terrestrial gravity data with satellite determinations of the geopotential were presented at the IUGG meeting in Lucerne, Switzerland in 1967. Rapp (1969b) described the estimation of 2,261 5° anomalies from terrestrial data and their combination in a least squares adjustment, with the degree 8 SAO SE I model, to obtain a set of potential coefficients to degree 14. In this solution, normal equations were formed using equation (3) as a basis where $\Delta\bar{g}$ (and its accuracy) were used to form normal equations in which the potential coefficients were parameters to be estimated. A similar solution was carried out by Köhnlein (1967) to degree 15.

At this point there were two different techniques that could be used for the combination of terrestrial and satellite data: one procedure required a global set of anomalies and involved an adjustment of satellite derived from the terrestrial data through orthogonality relationships; the other combined the satellite and terrestrial data through normal equations with the potential coefficients as parameters. Rapp (1969c) compared the two model approaches theoretically and carried out solutions with both methods using common data sets.

DEVELOPMENTS IN THE 1970's

With the basic techniques for determination of the spherical harmonic expansion of the gravitational field established, various groups turned to the processing of additional data and new data types (e.g. satellite laser ranging). The first model (GEM-1) in the GEM series was developed at NASA/Goddard Space Flight Center (Lerch *et al.*, 1972). This model was based on satellite tracking data and was complete to degree 12. The NASA analyses used the numerical integration of the satellite equations of motion, and variational equations, in the model estimation process, in contrast to the SAO studies that used analytic orbit theory. A companion model, GEM-2, was estimated to degree 16 using available 5° terrestrial gravity anomalies. Other models from Goddard quickly evolved as more data became available and computer resources increased. The combination of the satellite and terrestrial data was done through the merger of normal equations where equation (3) was the basis of the terrestrial data observation equation. A list of the GEM models produced is given in Table 1. The GEM-9 and 10 models (Lerch *et al.*, 1979) were complete to 20 (GEM-9) and to 22 (GEM-10), with selected terms to degree 30. The surface gravity data were 1,654 5° mean anomalies used by Rapp (1977) in an analysis where potential coefficients were estimated from the terrestrial data to degree 52 using a quadrature procedure. Two additional models, GEM-10B and GEM-10C, were estimated (Lerch *et al.*, 1981) (the models were presented at meetings in 1978 and formally published in 1981) using the same tracking data as used for the GEM-9 estimation. For GEM-10B (complete to degree 36) 1,654 5° equal area terrestrial anomalies were used as well as estimates of ocean geoid undulations obtained from 200 passes of GEOS-3 data. The GEM-10C model was complete to degree 180. The model was developed by first calculating a global set of 1°x1° geoid undulations using the terrestrial anomalies and anomalies derived (Rapp, 1979a) from GEOS-3 altimeter data and the GEM-10B model. In the oceans, the derived undulations were replaced by estimates from the GEOS-3 altimeter. The global set of undulations was then used in a quadrature procedure to determine potential coefficients to degree 180. The GEM-10C model was then formed from the GEM-10B model (2 to 36) with the coefficients 37 to 180 from the quadrature results.

The World Geodetic System 1972 (WGS72) was described by Sepplin (1974). Part of this system was a geopotential model which was based on the combination of the NWL10E geopotential model (based on Doppler tracking data) with other satellite tracking data (e.g. optical and secor) with terrestrial and astro-geodetic data. The adopted model was complete to degree 19.

Estimation of geopotential models in Europe was now also underway, with the development of the GRIM-1 and GRIM-2 models, the latter being a combination model (satellite and terrestrial data) (Balmino, Reigber, Maynot, 1978).

Rapp (1978) described the formation of a global 1°x1° anomaly field that was used with the GEM-9 potential coefficient model to determine a set of adjusted coefficients to degree 12 and an adjusted anomaly file that was used to determine, through quadrature procedures, a model to degree 60. Higher degree adjustments and model estimations were not possible because of the extensive computer resources needed in the formation and inversion of several matrices. In 1979 software developed by Rizos (1979) became available that was extremely efficient in the calculation of harmonic coefficients given a global data set (analysis) and the calculation of functional values (e.g. anomalies or geoid undulations) from a set of potential coefficients. What could not be done in 1978 because of inefficient code became possible in 1979 with the procedures developed by Rizos. It was then a natural step to take the adjusted 1° anomalies from the Rapp (1978) combination solution and determine the potential coefficients to degree 180 with the procedures and results described in Rapp (1979b).

During this time period several issues were considered dealing with the estimation of potential coefficients from terrestrial data. One item had been identified by Pellinen (1966), who described a smoothing parameter, β_n, that would yield mean cap values of a function defined by a set of harmonic coefficients. Since harmonic coefficients of the anomalies are derived from mean anomalies, the implied smoothing was to be considered. If $\Delta\bar{g}$ represents mean anomalies in a cell $d\sigma$, the quadrature procedure for estimating potential coefficients would be

$$\begin{Bmatrix} C_{nm} \\ S_{nm} \end{Bmatrix} = \frac{1}{4\pi\gamma\beta_n(n-1)} \iint_\sigma \Delta\bar{g}\, P_{nm}(\cos\theta) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} d\sigma \tag{4}$$

The calculation of β_n depends on the cap size of a circular cap that has an area equivalent to the cell size in which the anomaly is given. Closed expressions for β_n were given by Pellinen, as described by Rapp (1977) and studied further by Katsambalos (1979). The use of equations such as (4) enabled the determination of expansions to a degree higher than that implied by the Nyquist guideline that the highest frequency to be measured from a cell of size $\theta°$ would be $180°/\theta°$. For example, solutions were made to degree 52 from 5° anomalies (Rapp, 1977).

A second area of concern related to the reduction of the gravity anomalies to be used in the combination solution to the surface (Is it the geoid ?) on which the quadrature procedure was to be applied. Tests were carried out (Rapp, 1977) approximating a gradient reduction term by terrain corrections. Magnitudes were calculated for the correction to potential coefficients if terrain corrections were applied to the data. The results pointed out the need for such corrections. Similar computations were carried out for the effect of atmospheric corrections on anomaly data to potential coefficient estimations. Again, it became clear that such correction terms were needed.

Another area related to the proper formation of the boundary value condition in relating anomalies to potential coefficients. Most combination solutions (quadrature or direct) used spherical approximations. However, as higher degree solutions were being sought, a more rigorous approach was needed. Lelgemann (1973) provided equations (to eccentricity squared terms) to correct coefficients estimated with spherical approximation equation to those that would be found with a more rigorous formulation. The correction terms were evaluated in Rapp (1997) and found to have a magnitude on the order of 0.3% of the coefficients. Although small, such terms could cause systematic effects and it was thought desirable to consider such effects in future combination solutions.

The 1970's also saw the development of techniques to combine satellite and terrestrial gravity data where the gravitational field was modeled by surface density values (Koch, 1971, 1974; Koch and Witte, 1970; and Chovitz and Koch, 1979) or by discrete gravity anomaly values (Rapp, 1974). The advantage of such methods was the simple incorporation of terrestrial data in the solution. Once the discrete values (surface density or gravity anomalies) were estimated, they could be converted to a geopotential model whose maximum degree was consistent with the size of the cell used in the solution. The surface density solution used 104/192 density values leading to geopotential models of degree 11/15. The Rapp (1974) solution used 184 15° equal area cells leading to potential coefficient models of degree 12. Although models developed from discrete estimates of gravity field signal were conceptually of interest, the actual application in orbit analyses for high resolution geopotential models was not an efficient process (as compared to the use of potential coefficients) and interest in such procedures declined.

DEVELOPMENTS IN THE 1980's

The geopotential modeling effort actively continued in the 1980's, driven by more precise tracking data (primarily satellite laser tracking), a larger number of satellites with a variety of different inclinations and heights, satellite altimeter data (GEOS-3 and SEASAT), and improving terrestrial gravity anomaly data sets. In addition, the new data was making it possible to study new areas (e.g. plate motion and dynamic ocean topography) of research leading to a need for increasingly accurate geopotential models.

The efforts at NASA/GSFC now incorporated SLR data from Lageos and satellites that had laser reflectors. The GEM-L2 model (Lerch, Klosko, Patel, 1982) utilized GEM-9 data with 2 1/2 years of Lageos (launched in 1976) data. This model was complete to degree 20, although improvements over the GEM-9 model (due to the Lageos data) were primarily below degree 8. Data (laser, S band, and altimeter) from SEASAT was used with the GEM-9 model to produce several models tailored to SEASAT. The PGS-S4 model included the normal equations from GEM-10B, surface gravity, normal equations based on 5° data, and SEASAT altimeter data. This model was complete to degree 22 with additional terms.

With the approval of the TOPEX/POSEIDON (T/P) project, an effort to improve the Earth's geopotential was started with the goal of providing improved orbit determination for use with the T/P altimeter data. One of the first pre-launch models was developed at NASA/GSFC and designated GEM-T1 (Marsh *et al.*, 1988). This model was complete to degree 36 and based solely on satellite tracking data from 27 satellites.

The development of the GRIM models continued in Germany (DGFI) and France (GRGS). The GRIM-3 model (Reigber *et al.*, 1983) used optical, laser, and Doppler measurements and

1°x1° gravity anomalies from terrestrial and altimeter (GEOS-3) derived sources. The solution was complete to degree 36. An updated version (GRIM3-L1) of this model incorporated 16 months of Lageos SLR data, 1°x1° anomalies from the analysis of SEASAT data (Rapp, 1983a), an updated set of 1°x1° terrestrial anomalies (Rapp, 1983b), and several constraint equations from resonance effects. The anomaly data entered into the solution through normal equations where the anomalies where the observed values and the potential coefficients were the parameters. The maximum degree of the GRIM3-L1 model was 36.

The group at the Center for Space Research at the University of Texas at Austin started the estimation of geopotential models with the TEG-1S and TEG-1 model (Tapley *et al.*, 1988/1989). The TEG-1 model incorporated a variety of tracking data from 14 satellites, 1° surface gravity data, and direct SEASAT and GEOSAT altimeter data, and was complete to degree 50. The TEG-1S model excluded the direct altimeter data from the solution, although crossover information was included.

Numerous high degree expansions using different techniques and the improved terrestrial and altimeter derived anomalies were made during the 1980's. An important aspect in some solutions was the research described by Colombo (1981), who developed techniques to analyze and synthesize values on a grid on a sphere using FFT techniques. The quantities of interest could be given/computed as point or mean values. Colombo also showed how a least squares collocation procedure could be used to obtain high degree solution that would minimize the standard deviations of the adjusted coefficients. He also studied the Pellinen smoothing factors to determine improved values that would take into account noise in the anomaly data as well as cell size in determining optimal de-smoothing ($1/\beta_n$) factors. His suggestion was to use new β_n factors (q_n) in the discrete form of (2) using mean anomalies. The revised q_n values were: β_n^2 for $0 \le n \le N/3; \beta_n, N/3 < n \le N;$ and 1 for n>N, where $N=180°/\theta°$, and θ is the cell size in which the anomalies were given. A problem with the factors was the discontinuity that would take place when n=N/3 and at n=N. Numerical tests with these factors are described in Colombo (1981) and in Rapp (1986).

Rapp (1981) described a geopotential model (OSU81) that was complete to degree 180. This model used an updated 1° anomaly file (42,585 terrestrial-based values, 37,905 values derived from SEASAT altimeter data), and a set of a priori potential coefficients to degree 36 that were based on several models including some coefficients (e.g. zonal, resonant) determined in coefficient specific solutions. The 1° anomaly values lacking an estimate were computed from the a priori model and assigned an accuracy of ±30 mgals. The OSU81 model used optimum quadrature weights suggested by Colombo. The standard deviations of the non-adjusted coefficients were estimated using a propagated noise effect, with uniform anomaly accuracy and a component for the sampling error.

In 1985 Wenzel started procedures to carry out a sub-optimum least squares solution for geopotential coefficients combining an a priori geopotential model, terrestrial 1° anomalies, and 1° geoid undulations in the ocean areas derived from satellite altimeter data. A sub-optimum procedure was needed because of the large number of coefficients to be estimated (e.g. 32,761 for a degree 180 solution) in a rigorous least squares adjustment. The solution carried out by Wenzel assumed that the off diagonal elements of the normal equation of the combination solution were zero so that the inversion of the very large matrix became simply the inversion of a diagonal element. The advantage of the procedure (and others like it) was that a global anomaly field was not needed; different types of data (e.g. anomalies and undulations) could be combined and accuracy estimates for the adjusted coefficients could be determined. The model (GPM-2) developed by Wenzel was complete to degree 200.

Hajela (1985) described a solution to degree 250 using a least squares collaboration procedure suggested by Colombo (1981). The solution was made from 1° anomalies and was not a combination model as prior solutions. The power contained at the higher degrees was considerably smaller than other solutions, indicating too much smoothing could be taking place.

Rapp and Cruz (1986a) described several solutions using 1° data and GEM-L2 as the a priori model This solution used a downward continuation procedure to reduce surface anomalies to the ellipsoid and the ellipsoidal correction terms (to anomalies and to potential coefficients) developed by Cruz (1986). A usual quadrature type combination using the Colombo modified weights was used to determine an adjusted coefficient set to degree 20 and an adjusted 1° anomaly set. This global set of anomalies was expanded to degree 250 using the optimal estimation procedure implemented by Hajela. To reduce the power reductions at high degrees all 1° anomaly standard deviations were set to ±1 mgal. Two models (OSU86C,D) were released that merged the adjusted coefficients with the coefficients from the optimum estimation procedure. In these models the anomaly standard deviations in the combination solutions were restricted to a certain range to reduce the impact of large anomaly residuals in regions of high standard deviation.

Rapp and Cruz (1986b) described the first potential coefficient solution to degree 360. The first steps in the creation of the model were to create a nearly global 30'x30' anomaly file. This required the formation of a terrestrial 30' anomaly file that was merged with an altimeter derived from a GEOS-3/SEASAT data anomaly set. Of the 259,200 possible 30' values, 38% were based on adjusted 1° data with the rest coming from the terrestrial and altimeter data. The global 30' data was used in the quadrature procedure to determine the potential coefficients to degree 360. The ellipsoidal corrections and anomaly reduction to the ellipsoid were used. Two solutions (OSU86E,F) were generated using two different anomaly data sets. The final models were a merger of the coefficients from the adjustment of the GEM-L2 model and 1°x1° data with the coefficients from the quadrature procedure.

One should also note the development of the WGS84 geopotential model (White, 1986; DMA, 1987), which is complete to degree 180. This model was initially developed using satellite tracking data (Doppler, laser, GPS), surface gravity data, and geoid height information derived from SEASAT altimeter data. The merger of the normal equations and their solution gave a model complete to degree 41. This model was removed from a global 1°x1° anomaly set and the resulting residuals developed, with a quadrature procedure, to degree 180. The final WGS84 model was a merger of the degree 41 combination solution and the coefficients from degree 42 to 180 from the quadrature solution. The coefficients were initially available only to degree 18 and then later to degree 180.

In 1988 Weber and Zomorrodian suggested a procedure to create a tailored high degree geopotential model. The basic idea was to improve an existing model by the incorporation of new anomaly data in a region where improved geoid undulation computations (e.g.) were desired. The initial work described an improved model valid in Iran that was complete to degree 180 with GPM2 as the starting model. Based on the success of this work, others were developed for different areas. The IFE 88E2 model, based on the OSU86F model, was complete to degree 360 and tailored to the improved gravity data available in Europe (Basic *et al*, 1989). A discussion on the advantages and disadvantages of tailored geopotential models is found in Kearsley and Forsberg (1990).

As part of the pre-launch studies for the Topex/Poseidon mission, Tapley *et al.* (1989) described the estimation of a combination model to degree 50 (TEG-1) and a corresponding model (TEG-1S) , where direct altimeter data was withheld from the solution. Shum *et al.* (1989) described the estimation of the PTGF-4 and PTGF-4A geopotential models using data from 15 satellite and normal equations from 1°x1° terrestrial data created by Pavlis (1988). The two models were complete to degree 50 with the PTGF-4A including SEASAT and GEOSAT sea surface height data, which was not included in the PTGF-4 model. The PTGF-4A solution also estimated harmonic coefficients of the expansion, to degree 10, of dynamic ocean topography (the separation between the ocean surface and the geoid).

Improved models for the high degree combination solutions were made possible by the introduction of ellipsoidal harmonics and the use of conversion equations to transform ellipsoidal harmonics coefficients to spherical harmonic coefficients. The discussion of the transformation can be found in Gleason (1988) and Jekeli (1988). The introduction of these techniques into the quadrature based combination solution is described in Rapp and Pavlis (1990). The use of the almost closed expressions eliminated the need for the less than satisfactory series expressions used for correction terms. In addition, rigorous procedures were described in Pavlis (1988) to properly define the boundary value problem that was being carried and the development of appropriate correction terms to the mean gravity anomalies being used in the combination solution.

Rapp and Pavlis (1990) describe the development of two degree 360 geopotential models (OSU89A,B) that differ only in the treatment of anomaly estimates in which no directly observed gravity anomaly data was available. The OSU89B model incorporated anomalies for "empty" cells calculated from elevation data based on spherical harmonic models incorporating a topographic/isostatic hypothesis (Pavlis and Rapp, 1990). The combination model was based on the GEM-T2 model (Marsh *et al.*, 1989), which was complete to degree and order 36, with an additional 616 coefficients up to degree 50 and order 43. Improved 30' anomaly data sets were available for both terrestrial data and altimeter derived anomalies from GEOS-3/SEASAT data (Hwang, 1989) and a limited set of Geosat data in the oceans near Antarctica.

The developments in the 1980's were made possible, in part, by the introduction of vector processing computers, the increased speed of the computers, and the increased memory and storage capabilities. The new supercomputers required, for efficient use of the system, the re-writing of much software that had been previously used by sequential processors. The supercomputer provided the computer resources to process large amounts of new tracking data and enabled the extension of spherical harmonic models to higher degrees. The need for faster computers continues in the 1990's, where significant computer resources are required for the estimation of global geopotential models and the parameters associated with it.

DEVELOPMENTS IN THE 1990's

Geopotential models evolved rapidly starting early in the 1990's. The GEM-T3 model from NASA/GSFC was described by Lerch *et al.* (1992). This model was complete to degree 50, using tracking data from 31 satellites plus satellite altimeter data from GEOS-3, SEASAT, and GEOSAT, with improved surface gravity data. (A model based only on satellite tracking data was designated GEM-T3S and was also complete to degree 50.) The model estimation included three models (to degree 15) of the dynamic ocean topography for each of the altimeter missions as described by Nerem *et al* (1994b). The GEM-T3 combination model was found to give orbits more accurate than the GEM-T3S model. In the development of these and earlier models (e.g. GEM-T2) the optimal data weighting technique described by Lerch (1991) was used.

The TEG-2B model (Tapley *et al.*, 1991) was a combination model based on satellite tracking data, surface gravity data, and altimeter data and was complete to degree 50. An update of the GRIM models was started with the cooperative effort of DGFI/GRGS. The first in the

GRIM4 series were the satellite-alone model (S1) and the combination model (C1) as described by Schwintzer *et al.* (1991). The C1 model added normal equations formed from 1° data sets with high frequency components (degree 51 to 360) removed using the OSU86F degree 360 model to reduce the aliening effects when the degree 50 model was estimated. The GRIM-4 model development continued with the S2/C2 model and the S3/C3 model described by Schwintzer *et al*. (1992). The S3 model was complete to degree 50, with coefficients to degree 20, plus additional zonal and resonant coefficients, reliably determined. The C3 solution (complete to degree 60) incorporated surface gravity data and direct altimeter data with a dynamic ocean topography model (to degree 10) also solved for. The most recent GRIM models are designated GRIM4-S4 and GRIM4-C4 (Schwintzer *et al.*, 1997). The satellite model is complete to degree 60 plus several resonance terms while the combination model is complete to degree 72 and included additional surface gravity data not previously used in GRIM model estimation. Mean (1° cells) geoid undulation values in ocean areas, based on a mean sea surface corrected by the Levitus dynamic ocean topography model, were introduced in the solution. The new tracking data included the GPS tracking of T/P.

Geopotential model development for the Topex/Poseidon project continued with the estimation of the three JGM models. These models were developed by NASA/GSFC and the University of Texas at Austin. The JGM-1 model was a pre-launch model while JGM -2 was a post-launch model that included T/P laser and DORIS tracking data (Nerem *et al.*, 1994a). Both models are combination models using OSU (1991 vintage) 1° surface data and are complete to degree 70. Direct altimeter data from GEOS-3, SEASAT and GEOSAT were also used in the solution. The JGM -3 model (Tapley *et al.*, 1996), released in 1994, was estimated starting from the JGM -1 model with the addition of SLR data to several satellites, DORIS tracking of T/P and SPOT2, and GPS tracking of T/P. The model, complete to degree 70, includes the surface gravity data used in previous JGM models.

The above discussion has been restricted to the low degree satellite and combination models where normal equations are formed to solve the coefficients of the maximum feasible degree, which has been degree 70 for the models described. Significant progress has been made in the development of high degree geopotential models and this will now be discussed.

Rapp, Wang, and Pavlis (1991) described the OSU91A model, which is complete to degree 360. This was a blended model where the coefficients to degree 50 were based on a combination solution starting from the GEM-T2 model, incorporating 1° surface data and GEOSAT altimeter data, with coefficients from degree 51 to 360 from a quadrature type of combination solution as used in the OSU89 model estimation. The 30' anomalies used in the quadrature solution were based on a merger of terrestrial data, altimeter derived anomalies, and the topographic/isostatic anomalies for cells in which no other estimate was available.

During this time period, alternative techniques to the quadrature combination solution were developed that attempted to estimate a high degree model using a least squares adjustment technique with certain approximations that would allow high degree (360) models to be estimated. Bosch (1987) described an adjustment method that ordered the unknown coefficients in such a way that the normal equation became quite patterned, enabling a high degree model to be estimated with the neglect of certain off-diagonal terms. Gruber and Bosch (1992a) describe the degree 360 DGFI92A model that started from the normal equations of the GRIM4-S2 model adding 30' and 1° terrestrial anomaly information as well as a sea surface height model based on GEOSAT altimeter data. The least squares solution was carried out using the patterned structure approach. In 1992 Gruber and Bosch describe the OGE12 model which was complete to degree 360. The GRIM4-C2 coefficients and their standard deviation complete to degree 50 formed the a priori model. The patterned structure was used so that the largest matrix to be inverted was 361x361. A solution led to a preliminary 360 model which was then used to fill in gaps in the surface anomaly and sea surface height data sets. Gruber and Anzenhofer (1993) describe the GFZ93a and GFZ93b degree 360 geopotential model that are based on the GRIM4-C3 models and estimated using the block diagonal structure. The GFZ93b model used the Basic/Rapp (1992) mean sea surface while the GFZ93a model used a 9 month mean sea source derived from ERS-1 fast delivery data.

Gruber, Anzenhofer and Rentsch (1995) report on the development of the degree 360 GFZ95A model whose a priori basis was GRIM4-C4B. Improved terrestrial anomaly files and geoid height data (derived from a 2-year ERS-1 mean sea surface with dynamic ocean topography correction) were used in an iterative block-diagonal solution. The authors suggest an improved implementation of the procedure detailed by Bosch (1993), where by the full satellite only normal equations can be used with normal equations from the non-global anomaly and undulation data sets. A re-ordering leaves a full normal system in the upper left part of the normals with block-diagonal parts and sparse matrices for off diagonal components at higher degrees. This procedure was implemented (Gruber *et al.*, 1996) using new terrestrial data, a 3-year ERS-1 mean sea surface, altimeter derived gravity anomalies, with the normal equations and potential coefficients of the GRIM4-S4 model as the a priori model. Five different solutions (GFZ96) to degree 350 were computed testing different weighting procedures and data set combinations.

Several tailored geopotential models have also been developed in the 1990's. Li and Sideris (1994) describe the calculation of a degree 500 model using 5'x5' anomalies in Canada using the tailored model approach (Weber and Zomorrodian, 1988) starting from the OSU91A model. The new model (OSU91AT) was tested using geoid undulation information from GPS measurements as benchmarks. The tailored model to degree 360 gave slightly better results than the tailored model to degree 500. Motao *et al.* (1996) discuss the formation of a tailored (to degree 360) model (DQM94) using an improved set of gravity anomalies for China. The starting model was OSU91A. The tailored approach was implemented using ellipsoidal corrections for the anomalies, downward continuation and ellipsoidal harmonics. The residual coefficients were added to the original OSU91A model that left the OSU91A model unchanged to degree 19 and modified the residual by a weighting factor varying from zero at degree 20 to 1 at degree 180 and above. Anomaly and undulation tests were carried out to test the improvement of the DQM94 model over the OSU91A model.

Another type of tailored geopotential model is the kind designed to provide optimum orbit accuracy for a specific satellite. One example is the tuned model (PGS-S4) for SEASAT. A recent example is the DGM-E04 model (Scharroo, Visser, and Mets, 1998) which is a tuned version of JGM -3 to provide improved orbits for the ERS satellites. The tuning is carried out by adjusting a selected set (1100) of JGM-3 coefficients using crossover differences as the observations and including a scaled JGM-3 normal matrix to help stabilize the solution.

The final two geopotential models to be described for this paper were presented at the Gravity, Geoid, and Marine Geodesy meeting held in October 1996 in Tokyo. The TEG-3 model is described by Tapley *et al.* (1997). This model is a combination model to degree 70 with significant amounts of new, over JGM-3, satellite tracking data incorporated into the solution. The surface gravity data used for TEG-3 was the same as used in the JGM-3 model development.

Lemoine *et al.* (1997) describes the development of the EGM96 model which is complete to degree 360. This model incorporates new (over that used in JGM-3) satellite tracking data and terrestrial gravity data (1° and 30' cells) as well as 30' anomalies derived from GEOSAT (primarily) and ERS-1 altimeter data. The coefficients are a blend of three computational procedures. From degree 2 to 70 the coefficients are based on a least squares adjustment involving satellite tracking data, 1° terrestrial data, direct altimeter data, and fill-in anomalies in areas lacking data. The coefficients from degree 71 to 359 are taken from a block diagonal combination solution (Pavlis *et al.*, 1996) using normal equations derived from the satellite tracking data as a priori values. The coefficients at degree 360 are taken from a quadrature combination solution using the a priori satellite model and a global 30' anomaly set. (The adjusted anomalies from this solution were expanded to degree 500 but found useful to degree 460.)

The 1990's to date have been very productive in the estimation of improved geopotential models. The models have become increasingly accurate due, primarily, to more and new (e.g. GPS, TDRSS, and DORIS) tracking data, significant improvements in our terrestrial gravity data base, and in creating the mathematical model to unite the observations and parameters. The model improvement has been a significant challenge in the era of increased data accuracy. Areas in which the model development was critical included: Tides (both ocean and satellite effects); atmospheric drag; direct solar radiation pressure; Earth radiation pressure; spacecraft attitude effects; reference frame definitions and realization, etc. In addition, we have seen a significant effort to provide realistic error estimates for geopotential models (and related parameters). Such error estimates are needed so that propagated error estimates for quantities depending of the geopotential (e.g. satellite orbit error, geoid undulation, gravity anomaly, etc.) can be reliably determined.

TEMPORAL VARIATIONS OF THE GEOPOTENTIAL

The previous discussion has considered the geopotential to be constant in time. In fact, evidence has accumulated over the past ten years that demonstrates time changes can be estimated for selected zonal coefficients (Eanes and Bettadpur, 1996, Cazenave *et al.*, 1996, and Nerem and Klosko, 1996). Estimates of zonal rates to degree 5 have been made with the most reliable rate being for the degree 2 zonal coefficient. For the EGM96 geopotential model, the time rate of the zonal degree 2 term was fixed from a previous analysis (Nerem and Klosko, 1996) with the epoch for this term defined as 1986.0. All other coefficients (except for degree 2,1 terms which depend on polar motion) refer to a mean time span of the data used in the solution.

Geopotential changes are to be expected because of the changing mass distribution in the land, oceans, and atmosphere (NRC, 1997). The detection of the very small changes will be possible through the analysis of data from selected satellite geopotential mapping missions, as noted in the next section.

A LOOK TO THE FUTURE

In the near term (2-3 years) we can expect additional geopotential models that build on the increasing information available to the model community. New types of information could be incorporated into geopotential estimation. Suggestions have previously been made that dynamic ocean topography (DOT) models based on ocean circulation models might be used to correct altimeter derived sea surface heights to geoid undulations that are incorporated in the least squares estimation instead of solving for a DOT model. Questions exist as to the accuracy of a DOT model and the use of a static or time dependent model. The resolution, both in geographic sense and in a temporal sense, must be clearly defined.

Models higher than degree 360 models have been developed for special tailored models as well as developed from mean anomaly sets whose cell size would suggest that the maximum reasonable degree is $180°/\theta°$, although actual tests reveal expansions carried out above this maximum degree contain useful information. One would expect that more reliable high degree solutions could be obtained if cell sizes smaller than 30’ could be estimated. The next logical size to work with would be 15’ cells. Estimates of 15’x15’ sea surface heights and gravity anomalies for much of the world are now possible. Combination solutions using the quadrature adjustment approach would not be a problem with the block diagonal approach being somewhat more complicated. With some additional effort, potential coefficient models to degree 720 are possible using 15’ cell size for the anomaly estimates.

An alternative to higher degree geopotential models may be a combination of a high degree spherical harmonic model with harmonic wavelets. The wavelets can be introduced in a joint model in a way to be consistent with the data density available for a solution. The combined model can be estimated from both satellite data and terrestrial data. The resultant model can be used to calculate various gravimetric quantities such as geoid undulations, deflections of the vertical, etc. The theory for the development of the combined model procedure can be found in Freeden and Windheuser (1997) and Freeden and Schneider (1997).

Before developing such models, it may be appropriate to consider if a rectangular cell size should continue to be the way in which mean values are estimated. Jekeli (1996) notes the aliasing impact if the function averaging is done over constant angular blocks, as is the usual case. He suggests such errors can be reduced by forming spherical cap averages. To implement this procedure one would need to define the location of the spherical caps as well as carry out a new estimation of the mean values which is a substantial effort. Additional discussion on the recovery of harmonic coefficients from mean values on a sphere is given by Albertella and Sacerdote (1995).

As one goes to the higher degrees improved procedures will be needed to reduce the surface gravity anomalies to the ellipsoid. Downward contribution techniques need to be studied to determine the data needed and method to use for the reduction.

The most accurate high degree models are currently blended models where the low degree component comes from the combination of satellite tracking data, direct altimeter data, and terrestrial data. As part of the solution a model for DOT needs to be estimated. Could the block diagonal approach be expanded so that altimeter data and DOT estimates be incorporated in the model determination?

Improved data sets would also be of help in forming new models. Terrestrial gravity data in some selected land areas (parts of South and Central America, Asia, and Africa); in ocean areas (primarily southern oceans); and in the polar regions are still quite poor. Altimeter data is plentiful but not widely used in the shallow water areas. Improved shallow water tide models are needed so that data now edited from altimeter analysis is kept in the low degree combination model or in the recovery of gravity anomalies from altimeter data in shallow water or coastal water regions. Higher resolution and more consistent elevation data on a global basis would be helpful in the reduction of surface gravity data.

Much remains to be done to improve the data and analysis techniques now being used for the estimation of geopotential models. Steps can be taken as an evolutionary effort or one may wish to make significant changes in the way gravity anomaly data is represented and in the model to be estimated for the Earth’s gravitational potential. Reliable accuracy estimates for any geopotential model are an important aspect of any modeling effort.

In a few years data will become available from satellite missions designed to determine a more precise knowledge of the Earth’s gravitational potential. The missions currently underway are CHAMP (1999), GRACE (2001), and GOCE (2004?). The analysis of the data from each mission will provide a considerable challenge. Numerous questions need to be considered in such analyses. For example, what is the ideal representation of the geopotential for use in these missions? How can existing geopotential models be used in the data analysis? How can existing terrestrial gravity anomaly and satellite altimeter data be incorporated in the modeling effort? How will time changes in the gravity field be modeled? A variety of approaches to the analyses can be done prior to the acquisition of data from the new missions.

CONCLUSION

This paper has reviewed the estimation of the Earth's geopotential-from parameters of an ellipsoid gravity formula to geopotential models complete to spherical harmonic degree 500. Much can be done in the next few years to improve the accuracy of our existing models and to extend them to higher degrees is such high degree model will be useful. Alternative modeling techniques also provide additional opportunities for creative research. Much has been done in the past 96 years; much remains to be done.

ACKNOWLEDGMENT

The preparation of this report was supported, in part, by NASA (Contract NAS5-32352) through Hughes STX. Comments from the reviewers were helpful in improving the original text.

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Table 1. Geopotential Models Developed in the 1970's

Model	Type*	Date	Max. Complete Degree
GEM-1	S	1972	8
GEM-2	C	1972	16
GEM-3	S	1972	12
GEM-4	C	1972	16
WGS-72	C	1974	19
GEM-5	S	1974	12
GEM-6	C	1974	16
GEM-7	S	1976	16
GEM-8	C	1976	25
GEM-9	S	1979	20
GEM-10	C	1979	22
GEM-10B	C	1978	36
GEM-10C	C	1978	180
SAO SEIII	C	1973	18
GRIM1	S	1976	10
GRIM2	C	1978	23
OSU78	C	1978	60
OSU79	C	1979	180

* S = satellite tracking data; C = combination solution

Table 2. Geopotential Models Developed in the 1980's

Model	Type*	Date	Max. Complete Degree
GEM-L2	S	1982	20
PGS-S4	C	1982	22
GRIM3	C	1983	36
GRIM3B	C	1984	36
GRIM3-L1	C	1985	36
GEM-T1	S	1988	36
GEM-T2	S	1989	36
TEG-1	C	1988	50
PTGF-4/4A	C	1989	50
OSU81	C	1981	180
Hajela	T	1984	250
GPM2	C	1985	200
OSU86C,D	C	1986	250
OSU86E,F	C	1986	360
WGS84	C	1987	180
IFE88E2	t	1989	360
OSU89A/B	C	1989	360

* S = satellite tracking data; C = combination solution; T = terrestrial data only; t = tailored model

Table 3. Geopotential Models Developed in the 1990's

Model	Type*	Date	Max. Complete Degree
GEM-T3	C	1992	50
GEM-T3S	S	1992	50
TEG-2B	C	1991	50
GRIM4-S1	S	1991	50
GRIM4-C1	C	1991	50
GRIM4-S3	S	1992	50
GRIM4-C3	S	1992	50
JGM-1	C	1994	60
JGM-2	C	1994	70
JGM-3	C	1994	70
DGM-E04	t	1997	70
GRIM4-S4	S	1997	60
GRIM4-C4	C	1997	72
OSU91A	C	1991	360
DGFI92A	C	1992	360
OGE12	C	1992	360
GFZ93	C	1993	360
GFZ95A	C	1995	360
GFZ96	C	1996	359
Li/Sideris	t	1994	500
DQM94	t	1996	360
TEG-3	C	1997	70
EGM96	C	1997	360

* S = satellite tracking data; C = combination solution; t = tailored model