

# Pattern Generation of Biped Walking Constrained on Parametric Surface

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**Abstract**—This paper describes a generation method for spatially natural biped walking. By limiting the COG (Center of Gravity) motion space to a sculptured surface, the degree of freedom of the COG matches to the number of the ZMP (Zero Moment Point) equations. The COG motion can be uniquely generated along a specified surface satisfying the ZMP constraint with low calculation cost. Spatial and time parts are separable by representing motion surface of the COG as parametric variables. The motion surface defines the relative height of the COG from the landing foot position. Thus, the proposed method reflects geometric information directly to the motion planning without considering walk stability.

In this paper, we show two actual examples, walking pattern including mostly stretched knee and going up stairs. The validity of the proposed method is confirmed by simulation. Walking with mostly stretched knee is also shown in experiment using HRP-2.

**Index Terms**—Biped robot, Waking pattern generation, Dynamic walking, Parametric surface

## I. INTRODUCTION

Recently so-called human-friendly machines have begun to spread in society, for example, pet robot, human-like robot, high performance home robot and so on. Industrial countries will need such machines due to the fast aging society. Because humanoid robots have the advantage of working in environments which is designed for human being, they are expected to assist our society in future. From the practical point of view, it is necessary for humanoid robot to have the capability of walking where humans can. The vertical motion of the COG contributes to the improvement of performance of humanoid robot such as stability, joint load torque and so on. For example, human walk contains mostly stretched knee with low load torque. In contrast, you will squat down when the scaffold is unstable. Thus, the aim of our research is to realize an efficient and stable walk as well as human in real-time.

The generation methods of the vertical COG motion based on the ZMP can be classified into three categories, (a) Time base, (b) Constraint plane base, and (c) Constraint surface base, as shown in Fig. 1. Here, time base approach (a) provides a vertical COG motion as a time function. Constraint plane approach (b) shows the vertical velocity of the COG is constant which is derived by analytical solution of the linear inverted pendulum. Constraint surface

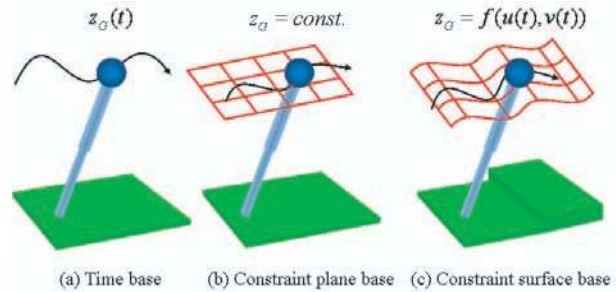


Fig. 1. Classification of the vertical COG motion

approach (c) is our new approach which means that the COG motion follows desired surface. In this approach, the spatially natural walking pattern is generable on the motion surface which is designed by considering a landing position and a movable space of legs.

The rest of this paper is organized as follows. The previous works are discussed in Section II. In section III, a dynamic model of the COG of humanoid robot which is constrained on parametric surface is shown. Section IV presents how to generate the COG motion constraint on parametric surface. In section V, general form of constraint surface is shown. Then, some applications and its results are shown in VI. Finally, we conclude in Section VII.

## II. RELATED WORKS

Many researches have proposed a variety of walking pattern generation methods[1]-[3], [5]-[12].

An analytical solution of the linear inverted pendulum has been utilized to generate a walking pattern in real-time on the assumption with the constant [1]-[3]. Kajita [1], [2] proposed the pattern generation method that is able to specify arbitrary landing time and position by applying preview control. Harada [3] also realized real-time gait planning using analytical solution and the ZMP [4] trajectory which is expressed as spline function. In this model, since it has to assume the COG height to be constant, it can not deal with the up-and-down COG motion exactly.

Some generation methods of humanoid walking pattern considering multi rigid body dynamics were developed. Nagasaka [5] proposed that a generation method of walking pattern using optimal gradient method under some

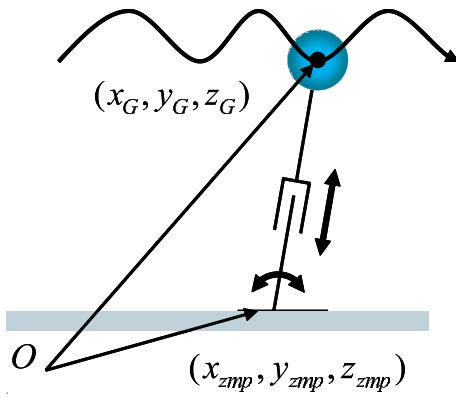


Fig. 2. Inverted pendulum model

objective functions and generates some optimal walking patterns in off-line. Kagami [6] achieved to fast generation of dynamically stable humanoid robot walking pattern. Lim [7] compensated the ZMP trajectory using upper-body motion off-line. Generally, because these methods are needed a large computation cost, therefore, it is difficult to quick change of walking pattern.

Our method follows the one proposed by Nishiwaki et al. [8]. Here, in their method, the ZMP error in generated walking pattern is compensated by the horizontal COG motion. On the contrary, in our method, the ZMP error is compensated so that the COG can follow the given motion surface. The feature of proposed method is that walking pattern is divided into a time and spatial parts respectively. The motion surface is given as relative height of the COG to the landing position. The geometric information is directly used in the motion surface without considering the ZMP constraint.

### III. DESCRIPTION OF THE COG MOTION CONSTRAINED ON PARAMETRIC SURFACE

Generally, humanoid robot can be expressed as the COG and the angular momentum around the COG as shown in Fig.2. Let  $\mathbf{x}_G = [x_G, y_G, z_G]^T$  be the trajectory of the COG corresponding the ZMP trajectory. Also, let  $\mathbf{L}_G = [L_{Gx}, L_{Gy}, L_{Gz}]^T$  be the angular momentum of the robot about the COG. When the external force does not act to humanoid robot, the ZMP position can be calculated as [3],

$$x_{zmp} = \frac{M(x_G(\ddot{z}_G + g) - (z_G - z_{zmp})\ddot{x}_G) - \dot{L}_{Gy}}{M(\ddot{z}_G + g)}, \quad (1)$$

$$y_{zmp} = \frac{M(y_G(\ddot{z}_G + g) - (z_G - z_{zmp})\ddot{y}_G) + \dot{L}_{Gx}}{M(\ddot{z}_G + g)}, \quad (2)$$

where,  $M$  is a total weight of humanoid robot,  $g$  is gravity acceleration. Here, since the influence of angular momentum during walking is small, let us assume the angular momentum can be neglected in the ZMP equations:

$$x_{zmp} = x_G - \frac{(z_G - z_{zmp})\ddot{x}_G}{\ddot{z}_G + g} \quad (3)$$

$$y_{zmp} = y_G - \frac{(z_G - z_{zmp})\ddot{y}_G}{\ddot{z}_G + g} \quad (4)$$

Looking at (3) and (4), it is observed that two ZMP equations include three translational variables. Therefore, the COG motion can not be specified arbitrarily. Let us express the COG motion as parametric surface.

$$x_G = a(u(t), v(t)), \quad (5)$$

$$y_G = b(u(t), v(t)), \quad (6)$$

$$z_G = c(u(t), v(t)), \quad (7)$$

where,  $a, b$  and  $c$  are functions which characterize the shape of surface.  $u(t)$  and  $v(t)$  are parameters which make a time response on the constraint surface. Substituting (5)-(7) into (1) and (2), the ZMP position are rewritten as,

$$x_{zmp} = a - \frac{(c - z_{zmp})(a_u \ddot{u} + a_v \ddot{v} + a_{uu} \dot{u}^2 + 2a_{uv} \dot{u} \dot{v} + a_{vv} \dot{v}^2)}{c_u \ddot{u} + c_v \ddot{v} + c_{uu} \dot{u}^2 + 2c_{uv} \dot{u} \dot{v} + c_{vv} \dot{v}^2 + g}, \quad (8)$$

$$y_{zmp} = b - \frac{(c - z_{zmp})(b_u \ddot{u} + b_v \ddot{v} + b_{uu} \dot{u}^2 + 2b_{uv} \dot{u} \dot{v} + b_{vv} \dot{v}^2)}{c_u \ddot{u} + c_v \ddot{v} + c_{uu} \dot{u}^2 + 2c_{uv} \dot{u} \dot{v} + c_{vv} \dot{v}^2 + g}, \quad (9)$$

where,  $(\cdot)_u = \frac{\partial(\cdot)}{\partial u}$ ,  $(\cdot)_v = \frac{\partial(\cdot)}{\partial v}$ ,  $(\cdot)_{uu} = \frac{\partial^2(\cdot)}{\partial u^2}$ ,  $(\cdot)_{uv} = \frac{\partial^2(\cdot)}{\partial u \partial v}$ ,  $(\cdot)_{vv} = \frac{\partial^2(\cdot)}{\partial v^2}$ . Two ZMP equations can be represented by two parametric variables. Since the COG motion is kinematically related with two parametric variables, a time response of the COG motion can be generated by the ZMP position by preparing constraint surface of the COG motion in world coordinates in advance.

### IV. GENERATION OF THE COG MOTION CONSTRAINT ON PARAMETRIC SURFACE

There are two kind of methods of walking pattern generation. One is designed so that generated the ZMP may follow reference ZMP[2]. Another approach can be solved as the ZMP inverse problem to satisfy reference ZMP as a time function. Because the first approach which is based on linear tracking control is difficult to apply the ZMP equations which include non-linear terms. Therefore, the second approach is suitable to generate the COG motion.

#### A. Calculation of the COG motion

Digitalizing  $u(t)$  and  $v(t)$  at sampling period  $\Delta t$ , velocity and acceleration at each step during considering motion are obtained as follows[6], [8]:

$$\dot{u}(t) = \frac{u_{k+1} - u_k}{\Delta t}, \quad \ddot{u} = \frac{u_{k+1} - 2u_k + u_{k-1}}{\Delta t^2} \quad (10)$$

$$\dot{v}(t) = \frac{v_{k+1} - v_k}{\Delta t}, \quad \ddot{v} = \frac{v_{k+1} - 2v_k + v_{k-1}}{\Delta t^2} \quad (11)$$

$$k = 1 \dots N$$

Then, the ZMP error at  $k$ -th step which is defined from difference between reference ZMP position  $(x_{zmp}^{ref}, y_{zmp}^{ref})$  and generated the ZMP position, is given by

$$e_{x,k} = x_{zmp,k}^{ref} - x_{zmp,k}, \quad (12)$$

$$e_{y,k} = y_{zmp,k}^{ref} - y_{zmp,k}. \quad (13)$$

To generate the COG motion satisfying the ZMP reference, the ZMP error at each step should be zero. Substituting (8)-(11) into (12) and (13), the ZMP error can be equivalently rewritten as

$$f_{x,k}(u_{k+1}, u_k, u_{k-1}, v_{k+1}, v_k, v_{k-1}) = 0, \quad (14)$$

$$f_{y,k}(u_{k+1}, u_k, u_{k-1}, v_{k+1}, v_k, v_{k-1}) = 0, \quad (15)$$

where, function  $f_{x,k}$  and  $f_{y,k}$  are composed of  $u_k, v_k$  and those forward and back one step. the ZMP error vector  $\mathbf{F}$  during considering walking motion is defined as

$$\mathbf{F} = [\mathbf{f}_x^T, \mathbf{f}_y^T]^T \in \mathbb{R}^{2N} \quad (16)$$

$$\mathbf{f}_x = [f_{x,1}, \dots, f_{x,N}]^T \in \mathbb{R}^N$$

$$\mathbf{f}_y = [f_{y,1}, \dots, f_{y,N}]^T \in \mathbb{R}^N$$

Then, the generation problem of the COG motion can be formulated as follows.

$$\mathbf{F}(\lambda) = \mathbf{0}, \quad (17)$$

$$\lambda = [\mathbf{u}^T, \mathbf{v}^T]^T \in \mathbb{R}^{2N},$$

$$\mathbf{u} = [u_1, \dots, u_N]^T \in \mathbb{R}^N,$$

$$\mathbf{v} = [v_1, \dots, v_N]^T \in \mathbb{R}^N.$$

The objective function in (17) which consists non-linear terms can not be solved analytically. Thus, the numerical solution by using Newton's method is calculated as

$$\lambda_i = \lambda_{i-1} + \left( \frac{\partial \mathbf{F}(\lambda_{i-1})}{\partial \lambda_{i-1}} \right)^{-1} \mathbf{F}(\lambda_{i-1}), \quad (18)$$

$$\text{if } \|\lambda_i - \lambda_{i-1}\| \leq \epsilon \text{ then } \lambda = \lambda_i$$

$$\text{else } i = i + 1$$

where,

$$\frac{\partial \mathbf{F}(\lambda_i)}{\partial \lambda_i} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}, \quad (19)$$

$$\mathbf{A} = \frac{\partial \mathbf{F}_x}{\partial \mathbf{u}}, \mathbf{B} = \frac{\partial \mathbf{F}_x}{\partial \mathbf{v}}, \mathbf{C} = \frac{\partial \mathbf{F}_y}{\partial \mathbf{u}}, \mathbf{D} = \frac{\partial \mathbf{F}_y}{\partial \mathbf{v}}.$$

Although  $2N \times 2N$  inverse matrix is needed in the process of convergent calculation,  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  and  $\mathbf{D}$  which are tridiagonal matrices respectively, can be reduced computational cost. Initial value of  $\lambda_0$  can be used the ZMP reference. Then, substituting calculated parameters  $u(t)$  and  $v(t)$  into (5)-(9), the COG motion and the ZMP reference corresponding to the COG motion can be obtained.

The proposed method is essentially formulated as well as fast generation method[6]. the ZMP error of fast generation method is modified by horizontal motion of the COG. In contrast, the COG motion is generated to follow the given constraint surface. Especially, this method makes it easy to take kinematics in world coordinates into consideration.

## B. Evaluation

To evaluate proposed method, the example of biped walking pattern generation is shown by using a simple

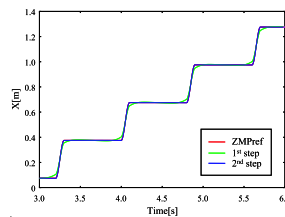


Fig. 3. ZMP trajectory in sagittal plane

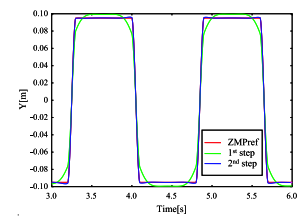


Fig. 4. ZMP trajectory in lateral plane

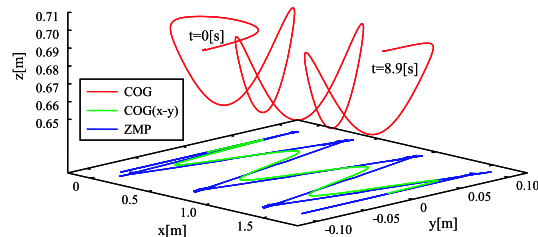


Fig. 5. COG and ZMP trajectory

inverted pendulum model. Simple constraint surface is given as

$$a(u, v) = u(t)$$

$$b(u, v) = v(t)$$

$$c(u, v) = 0.05 \cos\left(\frac{2\pi u(t)}{d_x}\right) + 0.005 \sin\left(\frac{2\pi u(t)}{d_y}\right)$$

where,  $d_x = 0.30$  and  $d_y = 0.25$  are given according to a step length and width respectively. Reference ZMP is given as step cycle is 0.8 s (single support phase 0.7 s / double support phase 0.1 s), step length is 0.3m, and walking velocity is 1.35 km/h. Motion period is set to 8.9 s,  $u(t)$  and  $v(t)$  are digitalized at 5 ms ( $N = 1780$ ). For faster computation of (18), we regarded as  $\mathbf{B} \approx \mathbf{0}$  and  $\mathbf{C} \approx \mathbf{0}$  in (19). From this point of view, matrix of (19) can be tridiagonal matrix. Hence, (18) can be calculated as  $O(N)$ . The situation of convergence is shown in Fig.3 and Fig.4. There is no first step with a large numerical error. The generated COG trajectory is shown in Fig.5.

In this case, Maximum ZMP error converges less than 1mm at 3 steps in spite of using the approximation matrix in (19). Computation time at 3 steps needs 4.4ms(Xeon 3.0GHz dual, Linux 2.4.22, gcc 2.95). Maximum ZMP error ( $\max(e_{x,k}, e_{y,k})$ ) became  $\{12.9 \times 10^3[\text{mm}], 102.3 \times 10^3[\text{mm}]\}$ ,  $\{31.5[\text{mm}], 19.4[\text{mm}]\}$ ,  $\{1.9[\text{mm}], 1.7[\text{mm}]\}$ ,  $\{0.3[\text{mm}], 0.3[\text{mm}]\}$  at initial, first, second and third steps respectively. Although the surface made from simple trigonometric function and it can be derived analytically in (19), this result shows the algorithm is essentially quick.

## V. CONSTRAINT SURFACE BY SPLINE FUNCTION

If a surface which can be solved the third partial differentiation explicitly in (19) is used, (18) can be calculated quickly. In order to determine constraint surface freely,

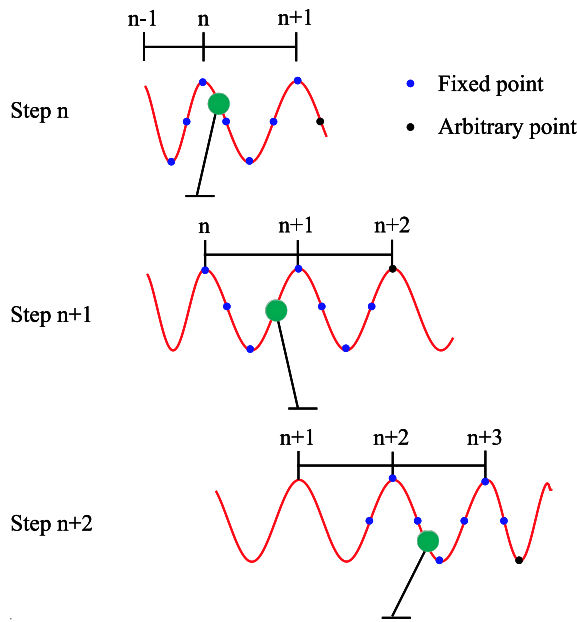


Fig. 6. Sequential surface generation

interpolation function as parametric surface can be used. The constraint function (5)-(7) is needed as

- Insert data has no influence on whole shape of surface.
- Third order differentiation can be defined.
- Low computation

Thus, Riesenfeld spline surface which is based on B-spline is suitable to represent constraint surface of the COG motion under these conditions. In Resenfelt spline function, the surface is generated by partial data which depends on desired differentiable order. Therefore, constraint surface can be generated sequentially as shown in Fig.6. This feature is similar in that the one foot place is set at least in advance.

Resenfeld spline surface can be calculated as

$$\mathbf{x}_G(u, v) = \sum_{i=1}^M \sum_{j=1}^N \mathbf{P}_{ij} B_i(u) B_j(v), \quad (20)$$

where,  $M$  and  $N$  are a number of data specified parameter  $u$  and  $v$  respectively.  $\mathbf{P}_{ij}$  and data position.  $B_i(u)$  and  $B_j(v)$  are B-spline coefficient which can be calculated by de Boor-CoX's recurrence formula according to desired differentiable order. A partial differentiation of surface can be also obtained as

$$\begin{aligned} & \frac{\partial^{k+l} \mathbf{x}_G(u, v)}{\partial u^k \partial v^l} \\ &= \sum_{i=1}^M \sum_{j=1}^N \mathbf{P}_{ij} \frac{\partial^k B_i(u)}{\partial u^k} \frac{\partial^l B_j(v)}{\partial v^l}. \end{aligned} \quad (21)$$

In case of surface, the coefficient of B-spline according to parameter  $u(u)$  and  $v(t)$  can be calculated separately. Thus, the constraint surface by using Resenfelt spline can be also calculated quickly.

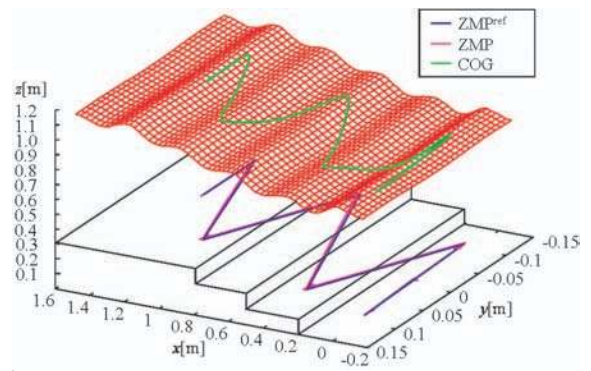


Fig. 7. COG motion

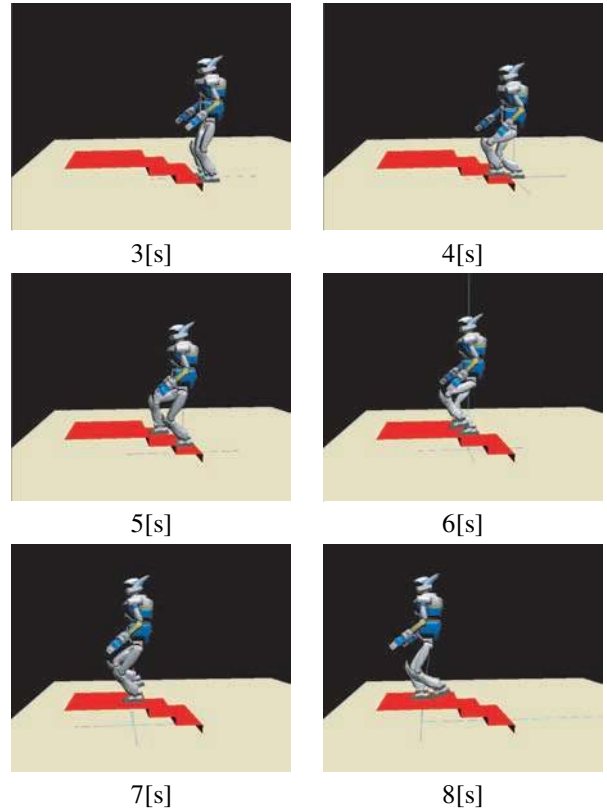


Fig. 8. Snapshot of climb up stair

## VI. APPLICATIONS

In this section, we demonstrate two kinds of gait generation for a humanoid robot. We use a detailed model of humanoid robot HRP-2 [13], which is a multi rigid body system. We define a fixed point on the waist link as a virtual the COG, whose motion is determined by our method. In other words, the whole robot is treated as a simple 3D inverted pendulum for pattern generation. Adding information of foot trajectories, we can calculate the leg joint angles, velocities and accelerations by inverse kinematics. Finally, by using 1 and 2, we obtain the ZMP.

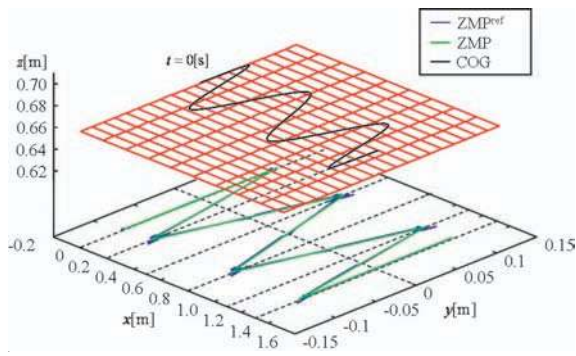


Fig. 9. COG motion (constant height)

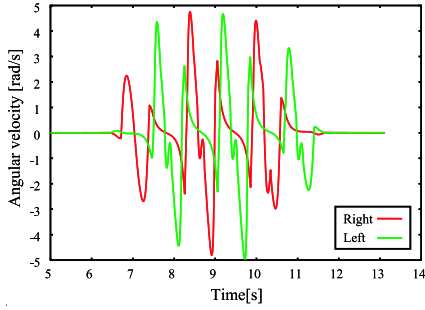


Fig. 10. Knee joint angular velocity(constant height)

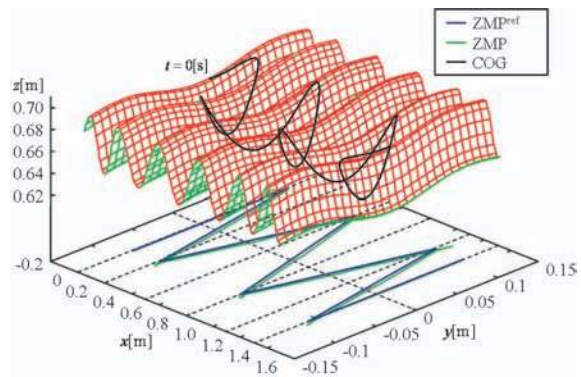


Fig. 11. COG motion (on parametric surface)

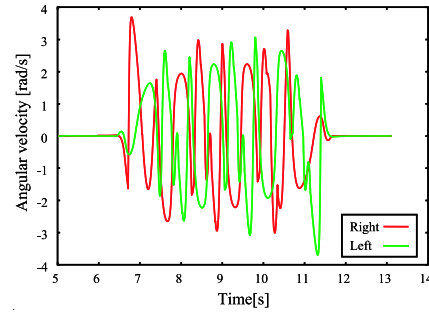


Fig. 12. Knee joint angular velocity(up down height)

It is used to evaluate the stability of the generated walking pattern.

#### A. Go up stairs

With known geometry of stairs, we can determine the constraint surface for stair climbing. The plenty of degrees of freedom of parametric surface allows us to improve kinematic limitation of knee extension by introducing up and down motion of the waist. Fig.7 shows the ZMP trajectory and the parametric surface determined by Riesenfeld spline with 11 points in lateral direction and with 15 points in sagittal direction for each step. Fig.8 shows the snapshots of the simulation. By this simulation, we confirmed a stable stair climbing.

#### B. Walking with mostly stretched knee

A walking pattern generated by 3D Linear Inverted Pendulum (3D-LIPM)[1] tends to have singularity and excessive knee joint speed problem since it requires planer constraint of waist (COG). Fig.9 shows the constraint surface of 3D-LIPM and Fig.10 shows the resulted speed of knee joints. The joint speed exceeds the capability of the knee joint of HRP-2 up to about 4[rad/s], and we cannot realize this walking pattern. We can avoid this problem by lowering hip height, but it results awkward walking and larger power consumption.

By introducing our parametric constraint, we can generate more natural and more efficient walking pattern using mostly stretched knees. Fig.11 show our parametric surface for our "human-like" walking. Since it is designed to lower hip at every support exchange, the robot can avoid over extension of knees, thus it can avoid excessive joint speed.

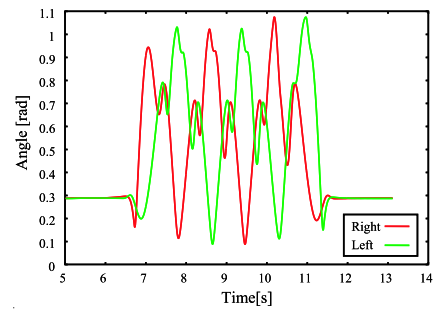


Fig. 13. Knee joint angle

Fig.12 shows the resulted speed of knee joints, which is within the speed limit. As shown in Fig.13, the knee joints are mostly stretched for every step in this walking pattern. Fig.14 and 15 shows sagittal and lateral ZMP trajectories for the generated pattern. The maximum error between the final ZMP (dotted lines) and the reference ZMP (thin lines) is about 2[cm], which is small enough compared with foot size of HRP-2. Since it guarantees the stable dynamic walk, we tested the walking pattern on the actual humanoid robot HRP-2. Fig.16 shows snapshots of walking using parametric surface of Fig.11.

## VII. CONCLUSION

This paper presents walking pattern generation method of the COG motion constraint on parametric surface. Introducing a constraint plane, the COG motion with vertical motion during walking can be generate naturally.

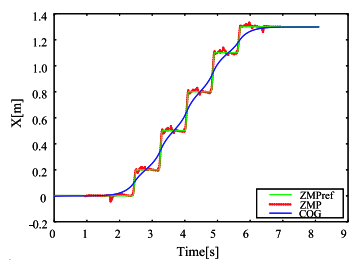


Fig. 14. ZMP trajectory in sagittal plane

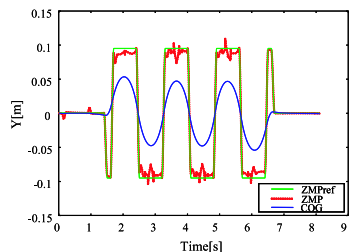


Fig. 15. ZMP trajectory in lateral plane

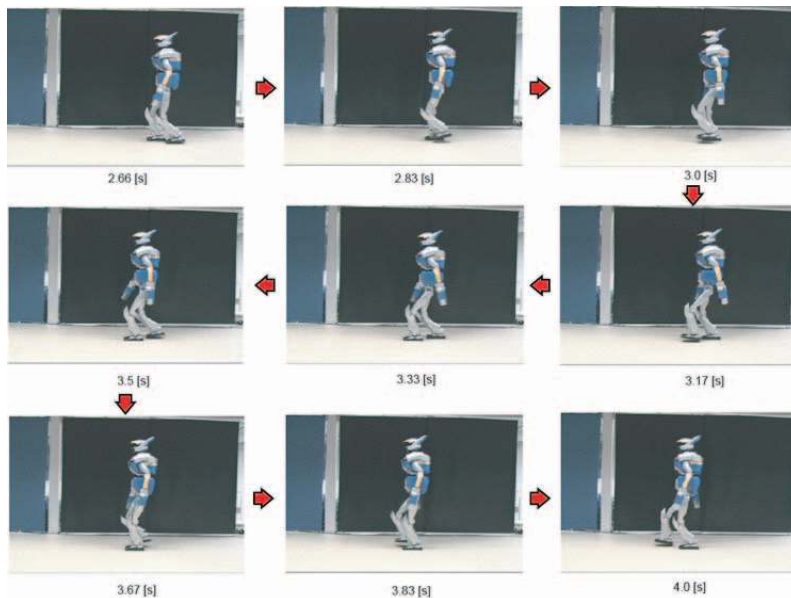


Fig. 16. Snapshot of walk

Digitalizing surface parameter, the COG motion is generatable essential quickly. The effectiveness of proposed method is shown by two applications, going up a stair in numerical simulation and walking with mostly stretched knee in experimental result.

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