

Patterns of strong coupling for LHC searches

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ABSTRACT: Even though the Standard Model (SM) is weakly coupled at the Fermi scale, a new strong dynamics involving its degrees of freedom may conceivably lurk at slightly higher energies, in the multi TeV range. Approximate symmetries provide a structurally robust context where, within the low energy description, the dimensionless SM couplings are weak, while the new strong dynamics manifests itself exclusively through higher-derivative interactions. We present an exhaustive classification of such scenarios in the form of effective field theories, paying special attention to new classes of models where the strong dynamics involves, along with the Higgs boson, the SM gauge bosons and/or the fermions. The IR softness of the new dynamics suppresses its effects at LEP energies, but deviations are in principle detectable at the LHC, even at energies below the threshold for production of new states. We believe our construction provides the so far unique structurally robust context where to motivate several LHC searches in Higgs physics, diboson production, or WW scattering. Perhaps surprisingly, the interplay between weak coupling, strong coupling and derivatives, which is controlled by symmetries, can override the naive expansion in operator dimension, providing instances where dimension-8 dominates dimension-6, well within the domain of validity of the low energy effective theory. This result reveals the limitations of an analysis that is both ambitiously general and restricted to dimension-6 operators.

KEYWORDS: Beyond Standard Model, Effective field theories, Global Symmetries, Technicolor and Composite Models

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1 Introduction

The primary mode of exploration of the energy frontier at the Large Hadron Collider (LHC) is through the search for on-shell produced new states. A secondary but nonetheless important mode is through the study of the high-energy behavior of SM processes, as they can be affected by the presence of off-shell heavy states. This second mode is arguably more important the stronger the new dynamics (a detailed discussion of this point will be given in a forthcoming paper by [1], see also the presentation [2]; this was also illustrated in ref. [3]). The most motivated instance of such a new strong dynamics at around the weak scale is given by the scenario of Higgs compositeness [4–11], which represents one of the very few solutions of the hierarchy problem. In that scenario the new dynamics necessarily

concerns the Higgs boson, the longitudinal components of electroweak vector bosons and possibly the third quark family.

The direct involvement in the strong dynamics of the other degrees of freedom, the transverse polarizations of the vector bosons in particular, is more like an option that does not even seem well motivated both from a phenomenological and a model building perspective. In particular, as concerns vector compositeness, the literature seems to lack a structurally robust scenario, based on symmetries and dynamics, which implements it. It is perhaps for that reason that, while the implications of Higgs (and top) compositeness at the LHC have been widely and carefully studied, those of vector compositeness have not. That is certainly a pity given the great amount of data the LHC will be harvesting on these particles.

The main goal of this paper is to construct, from an effective field theory (EFT) perspective, structurally robust scenarios of strong coupling where the weakly coupled SM emerges in the low-energy limit. We will classify a number of different scenarios where the SM fermions, vectors or scalars become involved in the strong dynamics, and discuss their implications. An interesting corollary of our analysis is that structurally robust “accidents”, due to symmetry and dynamics, can overcome the naive suppression of the energy expansion and boost the effects of dimension-8 operators above that of dimension-6 operators, and that within the domain of validity of the EFT description.

This paper is organized as follows. In section 2 we review the case of a Strongly-Interacting Light Higgs (SILH) and use it as a playground to introduce our logic and to apply it to different hypotheses, such as that of an accidentally light Higgs (ALH), detailed in appendix B. In section 3 we present a picture for vector compositeness, where only higher derivative interactions are associated with a strong coupling. Similar ideas are behind the discussions in section 3.2, where an approximate extended supersymmetry favors higher-derivative strong interactions involving fermions. In section 4 we combine these ideas to depict the possible patterns of strong coupling at around the weak scale. We also leave for the appendices detailed discussions of the Minimal Coupling (MC) assumption and of the specific dimension-8 operators that are relevant to our analysis.

2 The SILH, its operators and its power-counting

Before introducing new proposals for strongly coupled dynamics at the weak scale, we would like to recall the Strongly-Interacting Light Higgs [12] (SILH) scenario — the reader familiar with these concepts can skip to section 3. In our view the SILH represents a benchmark in the domain of weak-scale effective Lagrangians, and that for various reasons: its motivation by the hierarchy problem, its simplicity and its robustness. In particular, the SILH construction is made robust by the existence of at least one class of explicit UV completions [10, 11], based on warped extra-dimensions [13]. A brief review of the SILH will also allow us to introduce its operator basis, which we shall employ in the rest of the paper, as well as to synthetically explain some points, which perhaps lend themselves to misunderstandings (in appendix A we further digress on the notion of MC).

The SILH relies on the basic assumption that the microscopic theory basically consists of two sectors. On one hand there are the SM fermions and gauge bosons, and on the other there is an approximately conformally invariant strongly coupled sector, which we simply dub the CFT and from which the Higgs sector originates. The CFT is weakly coupled to the SM and develops a mass-gap at around the TeV scale. The Higgs doublet emerges as a light composite degree of freedom, and can be naturally light, provided it is a pseudo-Nambu-Goldstone boson (PNGB) [4–9] living in a coset \mathcal{G}/\mathcal{H} , of some approximate global symmetry \mathcal{G} of the CFT. A nice property of this scenario is that the gauge and proto-Yukawa couplings can conceivably be the only source of explicit breaking of \mathcal{G} , and consequently be fully responsible for the electroweak vacuum dynamics. With the constraint of custodial symmetry and group compactness,¹ there is a unique option for the minimal coset fitting just the Higgs doublet and no other scalar: $\text{SO}(5)/\text{SO}(4)$ [10, 11]. The simplest next options would be $\text{SO}(6)/\text{SO}(5)$ [14, 15] and $\text{SO}(6)/\text{SO}(4) \times \text{SO}(2)$ [16] involving respectively an additional singlet and an additional doublet.

It is also legitimate to entertain the possibility of a composite scalar that happens to be light because of some unexplained accident. As we shall discuss below and explain in more detail in appendix B, such an Accidentally Light Higgs (ALH) can serve as an instructive term of comparison. Yet another possibility is to have a light Higgs boson arising from an accidental global supersymmetry of the CFT [19, 20]. Nevertheless, we will not discuss this last possibility here any further.

2.1 The one-coupling-one-scale assumption

The simplest possible assumption for the CFT dynamics at the TeV scale is that it be broadly characterized by just one overall mass scale m_* , describing the mass of the resonances, and one overall coupling strength g_* , describing their interactions.² One can certainly consider scenarios with more structure than that. Nonetheless it is reassuring to know that some of the simplest calculable models obey that structure. In holographic composite Higgs model [10, 11], the role of m_* and g_* is played respectively by the Kaluza-Klein (KK) mass m_{KK} and the coupling arising from 5D gauge interactions:

$$m_* \sim \frac{1}{R} \equiv m_{KK}, \quad g_* \sim \frac{g_5}{\sqrt{\pi R}} \equiv g_5 \sqrt{\frac{m_{KK}}{\pi}}, \quad (2.1)$$

where πR is the compactification length and g_5 the 5D gauge coupling. A slightly more general situation would arise in string theory or in generic large- N gauge theories. For instance in type I string theory, the role of m_* would be played by the string scale, but there would be two different couplings, for open and for closed string states, respectively scaling like $\sqrt{g_S}$ and g_S , where g_S is the string coupling. Similarly, in the case of large- N gauge theories, m_* identifies the lightest hadron mass and two couplings, $g_* \sim 4\pi/\sqrt{N}$

¹Group compactness follows from the request of unitarity, if one wants to interpret \mathcal{G} as a global symmetry of the CFT. We shall come back to this point in the next section.

²To be perhaps more concrete, this state of things may arise because the ultraviolet theory, the CFT, is indeed controlled by just one coupling and a few parameters, like the number of colors. But it may also arise because of the presence of several comparable couplings and structural parameters.

and $g'_* = 4\pi/N$, describe respectively the interactions of mesons and glueballs, but one is easily convinced that, as long as the SM fields couple to just mesonic operators, only one coupling matters in practice. Finally, in the limit $g_* \sim 4\pi$, our counting reduces to Naive Dimensional Analysis (NDA) [21]. It would of course be interesting to also consider more general situations.

Now, below the scale m_* , we can write an effective Lagrangian for the composite resonances (scalars, vectors or spinors), denoted collectively by Φ , which, by a symmetry or just by accident, happen to be lighter than m_* . This Lagrangian can be written as a “loop” expansion³ In the case of phenomenological models based on 5D constructions the expansion is in KK-loops. The same structure arises also in string theory and large- N gauge theories from an expansion in topologies, of the worldsheet and of Feynman diagrams respectively.:

$$\mathcal{L}_{\text{eff}} = \frac{m_*^4}{g_*^2} L\left(\frac{\Phi}{m_*}, \frac{\partial_\mu}{m_*}\right) = \frac{m_*^4}{g_*^2} \left[\sum_{n=0}^{\infty} \left(\frac{g_*^2}{16\pi^2}\right)^n L_n\left(\frac{\Phi}{m_*}, \frac{\partial_\mu}{m_*}\right) \right], \quad (2.2)$$

where we are working with non-canonically normalized fields, and denoting by L_n generic functions. Effective Lagrangians so structured will be at the basis of all our discussions. From them it will be straightforward to power count the coefficients of the higher-dimensional operators deforming the SM Lagrangian. In particular, one finds that scattering amplitudes involving n -quanta scale like $\mathcal{A}_n \propto g_*^{n-2}$.

2.2 Partial compositeness

In composite Higgs models it is usually assumed that the SM fermions and gauge bosons are not part of the CFT, and that their involvement can be dialed by the choice of their mixings with the strongly coupled states [10, 11, 22]. Those mixing parameters can also be viewed as a measure of the (partial) compositeness of the corresponding SM state. In models based on warped extra-dimensions these parameters codify the shape of the wave function of the SM states: the more the wave function is peaked towards the throat of AdS, the more composite the state is. This state of affairs can be neatly described in the operator language by the use of the AdS/CFT correspondence [23]. In all models considered in the literature so far, the following operator mixings played a leading role

$$\mathcal{L}_{\text{mix}} = \epsilon_A A_\mu J^\mu + \epsilon_\psi \psi \mathcal{O}_\psi + h.c., \quad (2.3)$$

where A_μ and ψ indicate generically gauge fields and fermions of the SM, while J^μ and \mathcal{O}_ψ are respectively gauge currents and composite fermion operators of the strong sector. The coupling to A_μ just corresponds to the weak gauging of a $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ subgroup of the global symmetry \mathcal{G} of the CFT. We have dropped all internal indices to simplify notation. To properly interpret the ϵ 's, we must also specify the normalization of the fields, which amounts to specifying the strength of the amplitude with which they interpolate between the vacuum and particle states. Canonical A_μ and ψ interpolate with

³Throughout the paper, L is meant to be a power series in $g_*^2/16\pi^2$, but its dependence in the coupling is left implicit.

strength 1 for SM one-particle states, while canonical J^μ and \mathcal{O}_ψ interpolate with amplitude $\sim g_*^{n-1}$ for n particle states from the strong sector. Considering the case $n = 1$, we can thus interpret the ϵ 's in eq. (2.3) as the mixing between elementary and composite quanta with corresponding quantum numbers. One is easily convinced that the microscopic mixing Lagrangian in eq. (2.3) corresponds to an effective low-energy Lagrangian of the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{g_*^2} \left\{ m_*^4 L \left(\frac{\Phi}{m_*}, \frac{D_\mu}{m_*}, \frac{\epsilon_A \hat{F}_{\mu\nu}^i}{m_*^2}, \frac{\epsilon_\psi \hat{\psi}}{m_*^{3/2}} \right) - \frac{1}{4} (\hat{F}_{\mu\nu}^i)^2 + i \bar{\hat{\psi}} \gamma^\mu D_\mu \hat{\psi} \right\}, \quad (2.4)$$

where,

$$D_\mu \equiv \partial_\mu + i \epsilon_A T_i \hat{A}_\mu^i, \quad (2.5)$$

$$\hat{F}_{\mu\nu}^i \equiv \partial_\mu \hat{A}_\nu^i - \partial_\nu \hat{A}_\mu^i - \epsilon_A f^{ijk} \hat{A}_\mu^j \hat{A}_\nu^k, \quad (2.6)$$

with T_i and f^{ijk} respectively the generators and structure constants of the gauge group.

Several comments on the above equation are in order. Given $[D_\mu, D_\nu] = i \epsilon_A \hat{F}_{\mu\nu}$, the exhibited explicit dependence of L on the field-strength is redundant: we exhibit it only in order to better stress the difference with other scenarios we shall discuss later. Furthermore, as it was the case for eq. (2.2), and as we shall better discuss in the next section, L is given by of a loop expansion. Notice also the non-canonical normalization of all the fields, including the elementary ones: $\hat{A}_\mu \equiv g_* A_\mu$ and $\hat{\psi} \equiv g_* \psi$. When $\epsilon_{A,\psi} \rightarrow 0$, the SM fields A_μ and ψ decouple. Accordingly, the SM gauge couplings are given by $g_A \equiv \epsilon_A g_*$: the universality of gauge interactions fixes the mixing ϵ_A . We stress that if it weren't for the explicitly exhibited ϵ independent elementary field kinetic terms, the ϵ 's could be absorbed into the definition of A_μ and ψ , and these would be coupled as strongly as the Φ 's; the combination of small ϵ 's and our notation makes it instead clear that A_μ and ψ are elementary fields.

Eq. (2.4) gives rise to contributions to the scattering amplitude among n_e elementary and n_c composite states satisfying the scaling

$$\mathcal{A}_{n_e, n_c} \propto \epsilon^{n_e} g_*^{n_e + n_c - 2}, \quad (2.7)$$

where ϵ indicates collectively ϵ_A and ϵ_ψ , and we omitted the energy dependence. As already said, the ordinary gauge couplings are $g_A \equiv \epsilon_A g_*$. On the other hand, in the fermion sector there is more freedom. As the Higgs multiplet is issued from the strong dynamics, the Yukawa coupling will have the form (with an obvious notation)

$$y_\psi \sim \epsilon_{\psi_L} \epsilon_{\psi_R} g_*, \quad (2.8)$$

so that only the product $\epsilon_{\psi_L} \epsilon_{\psi_R}$ is fixed. In principle, it is possible, and sometimes phenomenologically convenient, to have either ϵ_{ψ_L} or ϵ_{ψ_R} to be large. In particular, models with $\epsilon_{t_R} \sim 1$ are especially attractive [16]. Notice that this situation can arise naturally if the operator \mathcal{O}_{t_R} mixing to t_R , has dimension satisfying $3/2 < \dim[\mathcal{O}_{t_R}] < 5/2$, such that the interaction of t_R with the CFT is relevant and becomes strong at low-energies, turning *de facto* t_R into a strongly-interacting composite fermion. In any case we assume the flavor structure associated with the ϵ_ψ parallels that of the SM, minimizing violation of flavor symmetry [17, 18].

2.3 The effective lagrangian at the electroweak scale

Putting together the concepts presented above, it is straightforward to derive the general structure of the effective Lagrangian for a strongly-interacting light Higgs. This corresponds to reduce the general scenario of the previous section to the simplest case in which the set of light states from the CFT contains just the Higgs doublet. In this case, the L in eq. (2.4) is given by

$$L = \sum_n \left(\frac{g_*^2}{16\pi^2} \right)^n L_n, \quad (2.9)$$

$$L_n \equiv L_n \left(\frac{\hat{H}}{m_*}, \frac{D_\mu}{m_*}, \frac{\epsilon_A \hat{F}_{\mu\nu}^i}{m_*}, \frac{\epsilon_\psi \hat{\psi}}{m_*^{3/2}} \right), \quad (2.10)$$

with \hat{H} indicating the non-canonically normalized Higgs doublet field. In the leading term L_0 the SM (fermion and gauge) fields appear only as external spectators. Instead, the one-loop term L_1 can be written as the sum of a pure CFT contributions, $L_1^{(0)}$, and contributions arising from the exchange of one virtual SM state $\epsilon^2 L_1^{(1)}$ that depends implicitly on ϵ :

$$L_1 = L_1^{(0)} + \epsilon^2 L_1^{(1)}. \quad (2.11)$$

The above structure generalizes in the obvious way at the higher loop order.

In the simplest and well motivated situation where the Higgs H spans the $\text{SO}(5)/\text{SO}(4)$ coset, the above Lagrangian will respect the symmetry and the selection rules it imposes. The H dependence will arise via the Goldstone matrix $U = e^{i\Pi}$ and will be determined by the CCWZ construction [24, 25], while the dependence on the symmetry-breaking mixings ϵ 's will be constrained by the $\text{SO}(5)$ selection rules (see ref. [26] for a thorough discussion).

Within our assumptions, the estimate of the operator coefficients in the effective Lagrangian is a simple matter of power counting. Using the definition of the dimension-6 operators \mathcal{O}_i in table 1, we illustrate the leading results for three different scenarios in table 2. The coefficients c_i are defined by

$$\mathcal{L}_6 = \frac{1}{m_*^2} \sum_i c_i \mathcal{O}_i. \quad (2.12)$$

The first scenario in table 2 corresponds to an ALH that happens to be lighter than the other resonances just because of some accidental cancellation, and not because of a symmetry –see appendix B. The second scenario we consider is that of a general PNGB strongly-interacting light Higgs (GSILH), defined by the most general L satisfying the $\text{SO}(5)$ selection rules. The third scenario is the slightly more specific case of the SILH considered in [12], where L_0 is not completely generic because of restricted properties of the dynamics at the scale m_* . This third class describes, for instance, Little Higgs models [27] and Holographic composite Higgs models [10, 11].

A few explanations of the results of table 2 are in order. First of all, we should give a motivation for our choice of operators. Our choice singles out $\mathcal{O}_{W,B}$ and $\mathcal{O}_{2W,2B,2G}$ as the only operators involving vectors that can be generated at tree level by the exchange of massive vectors, in a theory where the two derivative terms involve the minimal power of the

$\mathcal{O}_H = \frac{1}{2}(\partial^\mu H ^2)^2$	$\mathcal{O}_{y\psi} = H ^2 \bar{\psi}_L H \psi_R$
$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$	$\mathcal{O}_{2B} = -\frac{1}{2}(\partial_\rho B_{\mu\nu})^2$
$\mathcal{O}_6 = H ^6$	$\mathcal{O}_{2W} = -\frac{1}{2}(D_\rho W_{\mu\nu}^a)^2$
$\mathcal{O}_W = \frac{i}{2}(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D^\nu W_{\mu\nu}^a$	$\mathcal{O}_{2G} = -\frac{1}{2}(D_\rho G_{\mu\nu}^A)^2$
$\mathcal{O}_B = \frac{i}{2}(H^\dagger \overleftrightarrow{D}_\mu H) \partial^\nu B_{\mu\nu}$	$\mathcal{O}_{3W} = \frac{1}{3!} \epsilon_{abc} W_\mu^{a\nu} W_{\nu\rho}^b W^{c\rho\mu}$
$\mathcal{O}_{HW} = i(D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a$	$\mathcal{O}_{3G} = \frac{1}{3!} f_{ABC} G_\mu^{A\nu} G_{\nu\rho}^B G^{C\rho\mu}$
$\mathcal{O}_{HB} = i(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	$\mathcal{O}_{L,R}^\psi = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{\psi}_{L,R} \gamma^\mu \psi_{L,R})$
$\mathcal{O}_{BB} = H ^2 B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_L^{(3)\psi} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{\psi}_L \sigma^a \gamma^\mu \psi_L)$
$\mathcal{O}_{GG} = H ^2 G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{4\psi} = \bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma^\mu \psi$

Table 1. Dimension-6 operators used in our analysis. Notice that our normalization differs from previous literature.

	$ H ^2$	$ H ^4$	\mathcal{O}_H	\mathcal{O}_6	\mathcal{O}_V	\mathcal{O}_{2V}	\mathcal{O}_{3V}	\mathcal{O}_{HV}	\mathcal{O}_{VV}	$\mathcal{O}_{y\psi}$
ALH	m_*^2	g_*^2	g_*^2	g_*^4	g_V	$\frac{g_V^2}{g_*^2}$	$\frac{g_V^2}{g_*^2} g_V$	g_V	g_V^2	$y_\psi g_*^2$
GSILH	$\frac{y_t^2}{16\pi^2} m_*^2$	$\frac{y_t^2}{16\pi^2} g_*^2$	g_*^2	$\frac{y_t^2}{16\pi^2} g_*^4$	g_V	$\frac{g_V^2}{g_*^2}$	$\frac{g_V^2}{g_*^2} g_V$	g_V	$\frac{y_t^2}{16\pi^2} g_V^2$	$y_\psi g_*^2$
SILH	$\frac{y_t^2}{16\pi^2} m_*^2$	$\frac{y_t^2}{16\pi^2} g_*^2$	g_*^2	$\frac{y_t^2}{16\pi^2} g_*^4$	g_V	$\frac{g_V^2}{g_*^2}$	$\frac{g_V^2}{16\pi^2} g_V$	$\frac{g_*^2}{16\pi^2} g_V$	$\frac{y_t^2}{16\pi^2} g_V^2$	$y_\psi g_*^2$

Table 2. Estimated coefficients (c_i) of different operators appearing in the effective Lagrangian for a strongly interacting Higgs, under different hypotheses: an accidentally small electroweak scale and accidentally light Higgs (ALH), a general SILH (GSILH) scenario, and the proper SILH of [12] where the additional assumption of MC is considered (see appendix A). The subscript V can denote W, B, G according to the basis defined in table 1. For the ALH scenario the entries in the first two columns emphasize the need for tuning, w.r.t. the NDA estimate (see appendix B).

fields which is compatible with symmetry. In particular that is the case for a renormalizable theory. Now, as it turns out, the simplest incarnations of the Little Higgs and holographic Higgs models, are to a good approximation described at the scale m_* by theories in the above mentioned class.⁴ That property is essentially a corollary of the mechanism of collective breaking [27] enforced in these models: the scale of the resonances m_* is parametrically smaller than the genuine cut-off Λ of the theory, at which no weakly coupled description is tenable. In such models, apart from $\mathcal{O}_{W,B}$ and $\mathcal{O}_{2W,2B,2G}$, all operators involving vectors can only be generated via loops. Such a structure was dubbed in ref. [12], for somewhat obvious reasons, Minimal Coupling (MC). In appendix A we give a detailed discussion of

⁴Notice that these models are not precisely renormalizable at the scale m_* as the breakdown $\mathcal{G} \rightarrow \mathcal{H}$ at a higher scale, implies the existence of scalars in a non-linear σ -model.

MC in the context of extra-dimensional models. A second important property of our choice of basis is that we only need two operators of the form $H^\dagger H F_{\mu\nu} F^{\mu\nu}$, which we choose to be \mathcal{O}_{GG} and \mathcal{O}_{BB} . Now, it turns out that in the case where the Higgs is a PNGB, the coefficient of these two operators is subject to the same selection rules, and thus to the same protection, that control the Higgs potential. The reason is quite simple, and can be understood even without delving into the details of SO(5) and of its breaking: if we were to turn-off all interactions of the SM apart from color and electric charge,⁵ the neutral Higgs h would remain an exact Goldstone boson. Therefore, in such a limit, h would have to appear always coupled derivatively. Since \mathcal{O}_{GG} and \mathcal{O}_{BB} contain non-derivative Higgs interactions in the presence of gluonic and photonic backgrounds, their coefficients should be suppressed. In practice that means that in the case of a PNGB Higgs, \mathcal{O}_{GG} and \mathcal{O}_{BB} are absent in L_0 and appear first in $\epsilon^2 L_1^{(1)}$ in eq. (2.11), together with the leading contribution to the Higgs potential. Both for \mathcal{O}_{GG} , \mathcal{O}_{BB} and for the potential the leading contributions come from the top sector at 1-loop. These effects carry some model dependence, associated with the choice of the SO(5) quantum numbers for the CFT operators that mix with the top as in eq. (2.3). The estimates in table 2 correspond to the scenario where the need for tuning in the Higgs potential is minimized. That is the scenario where either $\epsilon_{t_L} \sim y_t/g_*$, $\epsilon_{t_R} \sim 1$ or $\epsilon_{t_L} \sim 1$, $\epsilon_{t_R} \sim y_t/g_*$ [26], in which case the coefficients of \mathcal{O}_{GG} , \mathcal{O}_{BB} feature an extra suppression $\propto y_t^2/g_*^2$ [12].

3 Effective theories of strong multipolar interactions

We are interested in exploring patterns of strong coupling beyond the SILH scenario described above. In particular, we would like to consider the case in which the SM vector bosons also arise from some strong dynamics at the TeV. At first sight, however, that seems difficult to reconcile with the observed weakness of the SM gauge interactions (see, for instance, ref. [28] to get a better taste of the difficulties). Indeed, one could conceive a situation where the SM vector bosons are composite in the multi-TeV range and the weakness of their effective coupling results from the large number of degrees of freedom in the underlying dynamics, like it happens in large- N gauge theories. However, in that situation the coupling of the SM vector bosons to the heavy resonances would also be suppressed, so that the new dynamics would not appear as genuinely strong, or, more precisely, it would not appear stronger than the SM dynamics. In order to counter that, the underlying dynamics should be more structured and involve at least two couplings, a weak one g , describing the low-energy strength of the gauge interactions, and a stronger one g_* , describing the interactions of gauge bosons with massive resonances. It would also be desirable to have such state of affairs be made structurally robust by symmetry. That could in principle provide, even in the absence of an explicit UV complete realization of the scenario, a set of rules to consistently power count the coefficients in the effective Lagrangian, as in the SILH case.

A technically natural situation where the vector bosons are involved in a dynamics with coupling $g_* > g$ just above the electroweak scale can be pictured as follows. Notice

⁵Notice that this would amount to just gauging $U(1)_{\text{EM}} \subset SU(2)_L \times U(1)_Y$.

first of all that the gauge coupling g , the one appearing in the covariant derivative, controls the *monopole* charge (*electric*, *chromo-electric* etc. for the various gauge factors of the SM) of the light degrees of freedom. The effective Lagrangian, however, also describes all *higher-multipole* interactions, which are unavoidably associated with the existence of structure at the fundamental scale m_* . It is then intuitive that there could exist situations in which the resonances (at least the light ones) have small charges, controlled by g , but large multipoles, controlled by g_* . The simplest limiting situation would be a strong dynamics producing a composite photon A_μ without any charged light degrees of freedom.⁶ As a matter of fact the effective theory is similar to the one resulting in the SM below the electron mass scale, with the photon and the neutrinos as the only remaining degrees of freedom.

The effective theory for a photon A_μ with multipolar interactions depends only on the field-strength $F_{\mu\nu}$ and on its derivatives:

$$\mathcal{L}_{\text{eff}} = \frac{m_*^4}{g_*^2} L \left(\frac{\hat{F}_{\mu\nu}}{m_*^2}, \frac{\partial_\mu}{m_*}, \frac{\hat{\Phi}}{m_*} \right), \quad (3.1)$$

where $\hat{\Phi}$ denotes other, neutral, degrees of freedom of the theory (again hatted fields are non-canonically normalized), and we have relaxed the minimal coupling assumption.⁷ While A_μ behaves like a free field at low-energies, its interactions grow with energy. For instance, the amplitude for light by light scattering is proportional to $g_*^2 E^4 / m_*^4$, and reaches a strength g_*^2 at $E \sim m_*$. Starting from such an obviously technically natural situation, one can imagine deforming the effective theory by endowing the light resonances with small charges. That amounts to deforming the ordinary derivatives into covariant derivatives:

$$\partial_\mu \Phi \rightarrow (\partial_\mu + i\epsilon q_\Phi A_\mu) \Phi, \quad (3.2)$$

where $q_\Phi \sim O(1)$ while $\epsilon \equiv g/g_* \ll 1$. Again, as the covariant derivatives are not renormalized, the smallness of ϵ is technically natural. It realizes the intuitive situation where the light composites have small charges but large multipoles.

It is interesting to generalize the above situation to the case of N_A vectors A_μ^i of a non-abelian gauge group \mathcal{G} . Neglecting for simplicity matter fields, the strong dynamics must have the following features:

- There must exist N_A composite photons associated with the gauge group $U(1)^{N_A}$.
- The non-abelian symmetry \mathcal{G} must be a global symmetry of the strong sector under which the photons transform in the adjoint representation of dimension N_A .

⁶We do not need to worry about the weak gravity conjecture [29]: in this limiting case, conflict is avoided provided heavy charged resonances at the scale m_* exist, while in a more realistic case the light states have small but non-zero charges.

⁷For an explicit example of this power-counting without minimal coupling, consider the contribution to the $F_{\mu\nu}^4$ operator from confining large N gauge theory where some of its constituents have coupling g_* to some external photon field A_μ . In this situation the coefficient of $F_{\mu\nu}^4$ in the low energy theory is easily power counted to be $g_*^4 N / (16\pi^2 m_*^2)$ where m_* is the hadron mass scale. Now, the backreaction of the strong gauge theory to the photon propagation is controlled by the parameter $\kappa \equiv g_*^2 N / 16\pi^2$, so that for $\kappa < 1$ one can speak of an elementary photon, separated from the confining sector. For $\kappa \sim 1$ the photon becomes part of the strong sector: in this limit the coefficient of $F_{\mu\nu}^4$ is precisely dictated by eq. (3.1).

The $U(1)^{N_A}$ gauge symmetry guarantees that we have N_A massless vectors, each coming with two helicities, while the global symmetry \mathcal{G} is needed to render the gauging of \mathcal{G} a small deformation, as we will describe below. Notice that the $U(1)^{N_A}$ generators sit in the adjoint of \mathcal{G} , so that the symmetry group is actually the semidirect product $[\mathcal{G}]_{\text{global}} \rtimes [U(1)^{N_A}]_{\text{local}}$. Again, as before, all fields Φ must be neutral under the gauge group, so that all interactions, in principle strong, are dominated by irrelevant higher-derivative interactions. All this can be encoded in a general one-coupling-one-scale effective Lagrangian as in eq. (3.1). Now, starting from this theory, we can consider the smooth deformation of its symmetry according to

$$[\mathcal{G}]_{\text{global}} \rtimes [U(1)^{N_A}]_{\text{local}} \rightarrow [\mathcal{G}]_{\text{local}}. \quad (3.3)$$

Notice that although the gauge group is modified, the deformation is still smooth. In particular, the number of local generators is unaffected, so that the number of degrees of freedom is unchanged. As in the abelian case, the deformation will simply amount to replacing derivatives and field-strengths with their covariant form in eq. (2.5) and eq. (2.6), with $\epsilon_A \rightarrow \epsilon$.⁸ The corresponding effective Lagrangian will now have the form

$$\mathcal{L}_{\text{eff}} = \frac{m_*^4}{g_*^2} L \left(\frac{\hat{F}_{\mu\nu}^i}{m_*^2}, \frac{D_\mu}{m_*} \right), \quad (3.4)$$

where, contrary to eq. (2.4), the field-strength $\hat{F}_{\mu\nu}^i$ appears without a suppression factor. Higher-derivative interactions are thus controlled by the strong coupling g_* and by the scale m_* , while IR physics is controlled by the (classically) dimensionless weak coupling $g = \epsilon g_*$. The structure of our Lagrangian is natural, in the sense that it is the most general compatible with the symmetries, both exact and approximate. In particular, in the limit $\epsilon = 0$, where the deformation is turned off, our Lagrangian is the most general one respecting $[\mathcal{G}]_{\text{global}} \rtimes [U(1)^{N_A}]_{\text{local}}$ and described by a scale m_* and a strong coupling g_* , which in principle could be as large as $\sim 4\pi$. Once ϵ is turned on, we have the most general Lagrangian respecting an exact $[\mathcal{G}]_{\text{local}}$ and an approximate $[\mathcal{G}]_{\text{global}} \rtimes [U(1)^{N_A}]_{\text{local}}$. The important difference with respect to the usual discussion of naturalness of small parameters is that in our case ϵ deforms the symmetry into a group which is not a subgroup of the original one. In other words, $\epsilon = 0$ is not a point of enhanced symmetry, but a point of deformed symmetry. The consequences for naturalness are however the same: symmetry singles out $\epsilon = 0$ as a stable point.

As a matter of fact the situation encountered in our construction is fully analogous to the one encountered when going from the Galilei to the Poincaré group: the number of generators is unchanged but the group structure (and the corresponding Lie algebra) is modified. In that case the parameter playing the role of ϵ is represented by the inverse of the speed of light $1/c$: when $c \rightarrow \infty$, with everything else fixed, the Poincaré group is deformed into the Galilei group. According to that limiting procedure, the Galilei group is said to be an Inonu-Wigner (IW) contraction of the Poincaré group [30]. It is here worth recalling that the IW contraction of a Lie group \mathcal{G} with respect to a subgroup \mathcal{H} corresponds to a

⁸Notice that we use the normalization implied by eq. (3.1): the undeformed kinetic term is $-1/4g_*^2 F_{\mu\nu} F^{\mu\nu}$.

group with the same Lie algebra as \mathcal{G} , except for the commutators among the generators in \mathcal{G}/\mathcal{H} , which are *contracted* to zero. The contracted group is thus simply $\mathcal{H} \rtimes [\mathcal{G}/\mathcal{H}]_{\text{abelian}}$. The IW contraction precisely describes the relation between the two symmetry groups in eq. (3.3): $[\mathcal{G}]_{\text{global}} \rtimes [\text{U}(1)^{N_A}]_{\text{local}}$ is the contraction of the gauge group $[\mathcal{G}]_{\text{local}}$ with respect to its global subgroup $[\mathcal{G}]_{\text{global}}$. In practice all local generators are abelianized.

In section 4 we shall apply the above construction to the gauge interactions of the SM and study its compatibility with cases where the Higgs and/or some of the fermions participate in the new strong dynamics. In the next section we will instead add some remarks about the possibility of realizing the above scenario using the mechanism of partial compositeness.

3.1 Strong multipolar interactions and partial compositeness

Following the idea of partial compositeness, one could generalize eq. (2.3) to incorporate the coupling of the field-strength $F_{\mu\nu}$ to the strong sector by adding the term

$$\Delta\mathcal{L}_{\text{mix}} = \epsilon_F F_{\mu\nu} \mathcal{O}^{\mu\nu}, \quad (3.5)$$

where $\mathcal{O}^{\mu\nu}$ is some composite anti-symmetric two index tensor, normalized like in section 2.2. The presence of this extra mixing term can be accounted for by modifying L in eq. (2.4) according to

$$L \rightarrow L \left(\frac{\hat{\Phi}}{m_*}, \frac{D_\mu}{m_*}, \frac{\epsilon_F \hat{F}_{\mu\nu}}{m_*^2}, \frac{\epsilon_\psi \hat{\psi}}{m_*^{3/2}} \right). \quad (3.6)$$

In the effective theory below m_* , there are now two sources of couplings for A_μ . In the presence of light charged states ϵ_A (see eq. (2.3)) leads to the standard gauge coupling $g = \epsilon_A g_*$. On the other hand, there are also higher-derivative interactions generated by ϵ_F . For such terms, the addition of a vector leg will cost an effective coupling

$$g_{\text{eff}} \sim \epsilon_F g_* \frac{E}{m_*}. \quad (3.7)$$

This is certainly suppressed at low-energies, but it grows with energy, and, for maximal mixing $\epsilon_F = O(1)$, it becomes $O(g_*)$ at $E \sim m_*$. Indeed, the limiting case $\epsilon_F = O(1)$ schematically describes the scenario of strongly-coupled dipole interactions we outlined in the previous section. Furthermore, this operator mixing picture makes also clear that there is no contradiction in the choice $\epsilon_A \ll \epsilon_F \sim 1$: quantum effects from ϵ_F always involve at least one derivative acting on A_μ , that is to say they represent multipole interactions that do not affect the $A_\mu J^\mu$ term.

It must however be remarked that the terms in eq. (2.3) are crucially distinct from all other possible operator mixings, including eq. (3.5): in theories where there exists a separation of scales and the strong sector (possibly harboring the Higgs sector) is approximately conformally invariant above the weak scale, eq. (2.3) represents all the possible relevant or marginal mixing terms.⁹ The gauge coupling is obviously marginally relevant or irrelevant

⁹In models in which the Higgs is elementary we could also have a marginal/relevant coupling to the strong sector as discussed, for example, in ref. [31–34].

as the conserved vector current is constrained by unitarity to have exactly dimension 3 in the unperturbed CFT limit. As for fermions, when the dimension of \mathcal{O}_ψ ranges from its minimal value of $3/2$, as dictated by unitarity, and $5/2$, the couplings range from relevant to marginal. On the other hand, if we interpret $\mathcal{O}^{\mu\nu}$ in eq. (3.5) as belonging to a CFT, then the theory of unitary representation of the conformal group constrains its dimensionality to be ≥ 2 . Therefore the interaction is always irrelevant, apart for the limiting case where the dimension of $\mathcal{O}^{\mu\nu}$ is exactly equal to 2. Nevertheless, in this limit $\mathcal{O}^{\mu\nu}$ must correspond to a free field [35, 36]. Indeed, it must be the field-strength of another free massless gauge field. Anyway it does not describe states in the putative strongly interacting sector. For instance, the correlators of $\mathcal{O}^{\mu\nu}$ trivially factorize into disconnected 2-point functions and thus do not mediate any scattering among the SM vectors that are coupled to it. As $F_{\mu\nu}\mathcal{O}^{\mu\nu}$ is always irrelevant, whatever value its coefficient has in the far UV, becomes very small by RG evolution down to the TeV scale. We thus conclude that it is not possible to realize this idea and construct the scenarios described in the previous section using partial compositeness in models based on CFTs. By the AdS/CFT correspondence, this also tells us the impossibility to construct these scenarios using a warped extra-dimension with asymptotic AdS₅ geometry, like in the Randall-Sundrum model [13]. In other words, there seems to be no way to obtain the scenario of strong multipolar gauge interactions in a system which we can both easily control, via partial compositeness, and extrapolate to high-energies. On the other hand, we cannot exclude that such scenario might arise in a QFT where the vectors are fully composite, but which cannot be continuously deformed to a QFT where they are elementary.

To conclude this discussion, and to make sure we are not missing any opportunity, it is worth commenting on all other possible mixings between the SM vectors and fermions and a CFT. Notice that the terms in eqs. (2.3), (3.5) exhaust all the possibilities for the mixing of a single SM field with a CFT primary operator. In that sense they are leading in the derivative expansion: any other operators involving one fermion or one gauge boson of the SM will have more derivatives acting on it, and thus represent less relevant mixing to descendants of the CFT operators in eqs. (2.3), (3.5). One could then consider interactions involving SM composite operators. But also in that case one finds no options compatible with naturalness. Indeed one is quickly convinced that the resulting interactions are always strictly irrelevant apart for the limiting case where SM fermion bilinears are coupled to a CFT scalar with dimension 1. However by unitarity this scalar field must be an approximately free field so that its mass term will suffer from a hierarchy problem. This state of things encapsulates the essence of the Flavor problem of (conformal)-technicolor [37]. Amusingly, the obstruction to the construction of strong multipolar gauge interactions using partial compositeness and the difficulties of flavor, arise for a similar reason: the absence of relevant operators.

3.2 Fermions as composite pseudo-goldstini

The results of the previous section, and the known properties of NG-bosons, imply we can think of strongly coupled scenarios where the SM vectors and the Higgs are composites of a new strong dynamics which manifests itself only through higher derivative (multipolar)

interactions. In this section we want to characterize scenarios where the fermions, as well, are composites with higher-derivative strong interactions. By the discussion in the previous section, and similarly to the case of a derivatively coupled scalar or vector, we cannot implement this scenario in a CFT using partial compositeness, as the corresponding interactions would always be irrelevant. Therefore derivatively coupled fermions must arise directly as composites from the strong sector and cannot be deformed into elementary states.

Given a fermion field ψ , the simplest thinkable symmetry enforcing purely derivative interactions is just given by the transformation $\psi \rightarrow \psi + \xi$. Of course the Lie parameter ξ must be a Grassmann number, but notice this is not a supersymmetry, as spacetime coordinates are not affected. Now, aside the kinetic term $i\bar{\psi}\not{\partial}\psi$, which happens to be invariant up to a total derivative, higher order invariants must be functions of $\partial_\mu\psi$. The lowest order interaction has thus the schematic form $(\partial\psi)^4$ and arises at dimension 10. The resulting scattering amplitudes are extremely soft, behaving at low energy like $s^3 \propto E^6$. However, very much like in the case of bosons, under reasonable assumptions on the UV behaviour of the cross-section, unitarity and analyticity bound the low energy amplitude to be *not softer* than s^2 [38]. It thus seems the pure shift symmetry is disfavored by basic principles. The same conclusion of course applies to the case of multiple fermions ψ_i , each with its own shift symmetry $\psi_i \rightarrow \psi_i + \xi_i$.

A more plausible alternative scenario is to consider a non-linearly realized supersymmetry which, sticking to a single chiral fermion ψ (identified with the $\mathcal{N} = 1$ Goldstino) in Dirac notation, acts as [39]

$$\delta\psi = \xi + \frac{i}{2F^2}\partial_\mu\psi(\bar{\psi}\gamma^\mu\xi - \bar{\xi}\gamma^\mu\psi), \quad (3.8)$$

with F the Goldstino decay constant. In a one-coupling-one-scale scenario one has the scaling $F \sim m_*^2/g_*$, which generalizes the relation $f \sim m_*/g_*$ for the decay constant of a Goldstone boson. The effective operators describing the coupling of a Goldstino to itself and to other massless light fields (such as a complex scalar ϕ , a field-strength $F_{\mu\nu}$ or a generic matter fermion ψ_q) start at dimension 8, and always involve some extra derivative acting on the Goldstino field. At the lowest dimension, by use of the equations of motion, these reduce to [40–43]

$$\frac{i}{F^2}\bar{\psi}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi F_{\mu\rho}F_\nu{}^\rho, \quad \frac{i}{F^2}\partial_\mu\phi^\dagger\partial_\nu\phi\bar{\psi}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi, \quad (3.9)$$

$$\frac{1}{F^2}\bar{\psi}^2\partial_\mu^2\psi^2, \quad \frac{1}{F^2}\partial_\nu\bar{\psi}\gamma^\mu\psi\bar{\psi}_q\gamma_\mu\partial^\nu\psi_q, \quad \frac{1}{F^2}\partial_\nu\bar{\psi}_q\gamma^\mu\psi\bar{\psi}\gamma_\mu\partial^\nu\psi_q. \quad (3.10)$$

The idea here is to consider the more general case of $\mathcal{N} > 1$ supersymmetries, where \mathcal{N} of the SM fermions are identified with pseudo-Goldstini from some strong dynamics (virtually all of them for $\mathcal{N} = 45$). That would be a generalization of the Volkov-Akulov model for the Goldstino-neutrino [44]. Notice that having $\mathcal{N} > 8$ does not force us to include massless fields of high spin: since supersymmetry is non-linearly realized, multiplets do not have to be complete. While the strong dynamics is characterized by effective interactions generalizing eq. (3.10), the SM gauge and Yukawa interactions necessarily

break supersymmetry, as they correspond to non-derivative interactions for the pseudo-Goldstini. However, since these other interactions are weak, the corresponding breaking can be treated as a small perturbation of the strong dynamics controlled by the more sizeable coupling g_* . As a matter of fact, the situation where just the fermions, not the vectors nor the Higgs, belong to the new dynamics was already considered long ago in an interesting paper by Bardeen and Visnjic [45]. In the case of maximal R -symmetry, $SU(\mathcal{N})$, the resulting interactions at dimension 8 are

$$\begin{aligned}
 F^2 \det \left[\delta_{\mu\nu} + \frac{i}{2F^2} (\partial_\mu \bar{\psi}^a \gamma_\nu \psi^a - \bar{\psi}^a \gamma_\nu \partial_\mu \psi^a) \right] \\
 = i \bar{\psi}^a \gamma^\mu \partial_\mu \psi^a + \frac{1}{8F^2} \left[(\partial_\mu \bar{\psi}^a \gamma^\mu \psi^a)^2 + (\bar{\psi}^a \not{\partial} \psi^a)^2 - 2 \bar{\psi}^a \not{\partial} \psi^a \partial_\nu \psi^b \gamma^\nu \bar{\psi}^b \right. \\
 \left. - \bar{\psi}^a \gamma^\mu \partial_\nu \psi^a \bar{\psi}^b \gamma^\nu \partial_\mu \psi^b - \partial_\nu \bar{\psi}^a \gamma^\mu \psi^a \partial_\mu \bar{\psi}^b \gamma^\nu \psi^b + 2 \bar{\psi}^a \gamma^\mu \partial_\nu \psi^a \partial_\mu \bar{\psi}^b \gamma^\nu \psi^b \right] + \dots
 \end{aligned} \tag{3.11}$$

where $a, b = 1, \dots, \mathcal{N}$ represent the flavor indices.

In principle there is no obstruction to generalize this to the case where the R -symmetry reduces to a subgroup of $SU(\mathcal{N})$ and to the case where the vectors and the scalars (the Higgs multiplet) are also part of the approximately supersymmetric dynamics. Probably the most efficient way to proceed is by employing the proper generalization of the CCWZ [24, 25] coset construction to the case of a non-linearly realized space-time symmetry (supersymmetry), which was laid down by Volkov and Ogievetsky [46, 47] in the 70's, and for which a recent reappraisal can be found in ref. [48]. A proper investigation of that construction is however beyond the scope of our present discussion, and we leave it for future work. Here we will content ourselves by illustrating qualitatively which interaction structures can possibly arise when \mathcal{N} of the SM fermions are pseudo-Goldstini. That can be done by inspecting eq. (3.9), which suggests which classes of terms to expect when $\mathcal{N} > 1$, while obviously the underlying R -symmetry will constrain the contraction of the a indices of the Goldstini. As a matter of fact, we have also done a more quantitative analysis, by finding, through a Noether procedure, the most general Lagrangian and transformation laws at order $1/F^2$. In other words, we have explicitly constructed a Lagrangian and transformation laws that satisfy \mathcal{N} non-linear supersymmetries up to terms of order higher than the first in the $1/F^2$ expansion. In particular, we find interactions of the form

$$i \frac{\kappa_{ab}^{AB}}{F^2} F^{A\rho\mu} F_{\mu\nu}^B \bar{\psi}_a (\gamma_\rho \partial^\nu + \gamma^\nu \partial_\rho) \psi_b, \tag{3.12}$$

whose action is invariant under $\delta\psi^a = \xi^a + \dots$ and $\delta F_{\mu\nu}^A = i \frac{\kappa_{ab}^{AB}}{F^2} \partial_\mu (F_{\nu\tau}^B \bar{\psi}_a) \gamma^\tau \xi_b + \text{h.c.} + \dots$ up to terms at least $O(1/F^4)$.¹⁰ A relevant case arises when $\kappa_{ij}^{AB} F^{A\rho\mu} F_{\mu\nu}^B = W^{a\rho\mu} B_{\mu\nu} \sigma_{ij}^a$. In that case (a factor of) the underlying R -symmetry group is identified with the global subgroup of the SM gauge group factor $SU(2)_L$ and the field-strength couple to an isospin current, as shown in eq. (C.7) of appendix C. That represents a first step towards the

¹⁰Of course, unless the supersymmetry is exact, these higher order terms will generate, via quantum effects, symmetry breaking interactions at all orders, including $\mathcal{O}_{4\psi}$ at dimension-6. Nevertheless, the leading contribution to $\mathcal{O}_{4\psi}$ comes from loops involving for instance a non-supersymmetric operator of dimension 12 and will be suppressed by $\sim (g_*/4\pi)^4$ from closing the loops to reduce the number of legs.

construction of an action invariant under the non-linear transformation of \mathcal{N} Goldstini, a task that we leave for future work.

To conclude, we have sketched a situation where an approximate supersymmetry suppresses interactions of dimension-6 involving fermions and scalars/gauge bosons, in favor of interactions of dimension-8 with more derivatives, which we summarize in appendix C, eqs. (C.6)–(C.7), (C.13).

4 Applications to weak-scale effective lagrangians

Based on the ideas of the previous section we shall here present scenarios where all the SM degrees of freedom (fermions, gauge fields and the Higgs) can take part in a novel strong dynamics around the TeV scale. In those models where the gauge bosons are composite with strong multipolar interactions, the symmetry of the strong sector will generally be

$$[\mathcal{G}]_{\text{global}} \times [\text{U}(1)^N]_{\text{local}}, \quad (4.1)$$

which will be deformed by turning on the gauge couplings into

$$[\mathcal{H}_1]_{\text{global}} \times [\mathcal{H}_2]_{\text{local}}, \quad (4.2)$$

where $\mathcal{H}_1 \times \mathcal{H}_2$ is a subgroup of \mathcal{G} and $\dim[\mathcal{H}_2] = N$. Such a deformation, which slightly generalizes eq. (3.3), will allow us to implement the situation where the Higgs is a PNGB.

For a number of reasons we ended up labeling *Remedios* the scenarios with composite vectors: the vague esthetic analogy with a character of a famous novel, its rôle as a remedy to provide a physical interpretation of some LHC searches, and finally for the impossibility to provide an explicit UV realization.¹¹

4.1 Pure Remedios

The simplest scenario, though perhaps not the most motivated, is one where only the gauge fields (or part of them) participate in a strong multipolar dynamics. In such a pure *Remedios* only the operators associated with the SM field-strengths $W_{\mu\nu}^a$, $B_{\mu\nu}$ and $G_{\mu\nu}^A$ can appear enhanced by powers of the strong coupling g_* in the effective Lagrangian. Therefore the largest effects will be given by operators that are purely built from field-strengths and (covariant) derivatives. At the dimension-6 level there are just \mathcal{O}_{3W} , \mathcal{O}_{3G} and \mathcal{O}_{2V} ($V = W, B, G$). The coefficients of these operators are expected to be of order

$$c_{3W}, c_{3G} \sim g_*, \quad c_{2W}, c_{2B}, c_{2G} \sim 1. \quad (4.3)$$

In the electroweak sector the main effects of these dimension-6 operators are anomalous triple gauge couplings (TGC), which affect diboson production, and modifications of the gauge boson propagators, which affect $\psi\bar{\psi} \rightarrow \psi\bar{\psi}$:

$$c_{3W} \sim g_* \quad \Rightarrow \quad \delta\mathcal{A}(\bar{\psi}\psi \rightarrow V_T V_T) \sim gg_* \frac{E^2}{m_*^2}, \quad (V_T = W_T, Z_T) \quad (4.4)$$

$$\delta\mathcal{A}(V_T V_T \rightarrow V_T V_T) \sim gg_* \frac{E^2}{m_*^2}, g_*^2 \frac{E^4}{m_*^4}, \quad (4.5)$$

$$c_{2W}, c_{2B} \sim 1 \quad \Rightarrow \quad \delta\mathcal{A}(\psi\bar{\psi} \rightarrow V_T^* \rightarrow \psi\bar{\psi}) \sim g^2 \frac{E^2}{m_*^2}. \quad (4.6)$$

¹¹ *Remedios the Beauty was not a creature of this world* - Gabriel Garcia Marquez.

For $g_* \gg g$ the effects of $c_{2W,2B}$ are subleading to those of c_{3W} . However the high precision with which $e^+e^- \rightarrow V^* \rightarrow \psi\bar{\psi}$ was measured at LEP1/SLC and LEP2 makes the former effects more relevant at the present moment. These effects are encapsulated by the W, Y parameters defined in [49]:

$$W, Y \equiv c_{2W,2B} \frac{m_W^2}{m_*^2} \sim \frac{m_W^2}{m_*^2}. \quad (4.7)$$

Electron-positron data imply $W, Y \lesssim 10^{-3}$ [49], limiting this scenario to $m_* \gtrsim 3$ TeV. On the other hand, c_{3W} contribute mainly to TGC, which are presently less well measured. The effect of c_{3W} has the same structure as λ_γ defined in ref. [50]:

$$\lambda_\gamma \equiv c_{3W} \frac{m_W^2}{m_*^2} \sim g_* \frac{m_W^2}{m_*^2}, \quad (4.8)$$

whose present experimental constraint $\lambda_\gamma \lesssim \text{few} \times 10^{-2}$ [51, 52] leads to a weaker bound on these scenarios: $m_* \gtrsim 1.5 \sqrt{g_*/(4\pi)}$ TeV. However the sensitivity will improve with the advancement of the LHC program [1, 2, 53].¹²

Finally, effects in $\mathcal{A}(V_T V_T \rightarrow V_T V_T)$ have just started to be under LHC scrutiny using different channels [54–56]. Nevertheless, a self-consistent analysis including effects from higher-dimensional operators is still missing. In this respect, it is interesting to notice that our estimates in eq. (4.5) reveals a second contribution to $\mathcal{A}(V_T V_T \rightarrow V_T V_T)$ proportional to $g_*^2(E/m_*)^4$. Besides coming from two insertions of c_{3W} , a similar contribution is given by dimension-8 operators of the schematic form $(g_*^2/m_*^4)F_{\mu\nu}^4$ (see appendix C for details). These contributions cannot be neglected and, in fact, dominate over one single insertion of c_{3W} when $E^2/m_*^2 \gtrsim g/g_*$. Phrased differently, as soon as the dimension-6 contribution to $2 \rightarrow 2$ scattering is bigger than the SM one, the whole cross-section becomes sensitive to dimension-8 effects.¹³ Throughout the whole article, we will encounter different instances like this one where, as a consequence of approximate symmetries, a small set of dimension-8 effects $\propto g_*^2$ dominates specific $2 \rightarrow 2$ scattering processes. The power counting rule of our models is such that the field expansion is controlled by the coupling g_* , while the expansion in dimensions (fields and derivatives) is controlled by $1/m_*$. This guarantees that, as long as the analysis is limited to amplitudes with 4 legs, operators of dimension 10 or higher are suppressed by additional powers of $\sim E^2/m_*^2 \ll 1$ or $\sim g_*^2 v^2/m_*^2 \ll 1$. Within the domain of validity of the EFT, these effects are thus subdominant. Similarly, for exclusive processes with n legs, the leading contribution comes from the n -field operators of lowest dimensionality that respect the relevant symmetries. Depending on n , the dimension of these operators may well be ≥ 10 .

Analogous considerations apply when gluons G_μ^A are involved in the strong dynamics.

¹²Notice that in the *pure Remedios* scenario discussed here, the leading contributions to $\bar{\psi}\psi \rightarrow V_T V_T$ processes come from the dimension-6 operator \mathcal{O}_{3W} . In section 4.3.2, eq. (4.35), we will present a scenario where also fermions are composite in such a way that the analysis in terms of dimension-6 operators (such as in refs. [51, 52]) is insufficient and a specific study involving dimension-8 operators is necessary.

¹³Surprisingly, the peculiar helicity structure of these scattering amplitudes forbids interference between dimension-6 operators and the SM in $2 \rightarrow 2$ high-energy processes involving at least one transversely polarized gauge boson [57]. For amplitudes with only transverse polarizations, this implies that dimension-6 and dimension-8 operators will give comparable corrections $\propto g_*^2 E^4/m_*^4$ to the squared amplitude.

4.2 Remedios with a composite Higgs

Since the need for a strong dynamics at the TeV is mainly motivated by the hierarchy problem, it is worth considering strong sectors in which also the Higgs arises as a composite state. Furthermore, the little hierarchy problem forces us to assume the Higgs field is a PNGB parametrized by a coset \mathcal{G}/\mathcal{H} . That assumption directly translates into the request that the strong couplings of $W_{\mu\nu}^a$ and $B_{\mu\nu}$ preserve \mathcal{G} . Absent that request the Higgs mass would be destabilised by the strong dynamics. We can see two broad options to achieve that, as we illustrate in the next two sections.

4.2.1 Minimal composite Higgs model

Let us consider the simplest scenario where H is a PNGB multiplet describing the coset $\text{SO}(5)/\text{SO}(4)$ [10, 11]. In the standard construction, $\text{SU}(2)_L \times \text{U}(1)_Y$ gauges a subgroup of $\text{SO}(5)$, thus explicitly breaking the symmetry that ensures the masslessness of the Goldstones. As long as the weak gauge group is weak, the breaking of the symmetry is small and consequently the Higgs mass is smaller than the fundamental scale m_* , a necessary condition for the theory to be not badly tuned. However, if, in the limit where $g = g' = 0$, the vector bosons interact strongly via multipolar terms, they should do so respecting $\text{SO}(5)$. In particular, that implies W_μ^a must necessarily be a triplet under some additional $\widetilde{\text{SU}}(2) \not\subset \text{SO}(5)$.

Starting from the Minimal Composite Higgs Model (MCHM), the simplest option is then to consider a strong sector which, in the $g = g' = 0$ limit, has the symmetry group

$$\mathcal{G} = [\text{SO}(5) \times \widetilde{\text{SU}}(2) \times \text{U}(1)_X]_{\text{global}} \times [\text{U}(1)^4]_{\text{local}}, \quad (4.9)$$

the four local $\text{U}(1)$'s being associated with the three W_μ^a and B_μ . Under the global group, W_μ^a transforms as a triplet $(\mathbf{1}, \mathbf{3}, \mathbf{0})$, while B_μ is a total singlet. When g and g' are turned on, the group eq. (4.9) is *deformed and reduced*, following the inverse of the IW contraction of section 3. In particular, given $\text{SU}(2)_L \times \text{SU}(2)_R \cong \text{SO}(4) \subset \text{SO}(5)$, g gauges the diagonal $\text{SU}(2)$ inside $\text{SU}(2)_L \times \widetilde{\text{SU}}(2)$. On the other hand, g' gauges, as in standard constructions, the linear combination $T_{3R} + X$. The resulting effective Lagrangian has then the form

$$\mathcal{L}_{\text{eff}} = \frac{m_*^4}{g_*^2} L \left(U, \frac{\hat{F}_{\mu\nu}^i}{m_*^2}, \frac{D_\mu}{m_*} \right), \quad (4.10)$$

where $U = e^{i\Pi}$ is the Goldstone matrix that contains the Higgs and $\hat{F}_{\mu\nu}$ and D_μ are given by eqs. (2.5), (2.6), with $\epsilon = g/g_*$ and $\epsilon' = g'/g_*$ for respectively $\text{SU}(2)_L$ and $\text{U}(1)_Y$. The effective Lagrangian will also be written as an expansion in loops, controlled by $(g_*/4\pi)^2$, and in powers of the symmetry breaking parameters ϵ and ϵ' , as previously illustrated in eq. (2.9) and eq. (2.11). The Lagrangian in eq. (4.10) respects the symmetry eq. (4.9), apart from the terms associated with g and g' . According to the discussion in section 3.1, our Lagrangian can also be formally thought as arising in a scenario where weakly coupled vectors are mixed to the strong sector via the terms

$$\epsilon_W W_{\mu\nu}^a \mathcal{O}^{a\mu\nu} + \epsilon_B B_{\mu\nu} \mathcal{O}^{\mu\nu}, \quad (4.11)$$

and then the limit $\epsilon_{W,B} \rightarrow 1$ is taken.

Model	\mathcal{O}_{2V}	\mathcal{O}_{3V}	\mathcal{O}_{HW}	\mathcal{O}_{HB}	\mathcal{O}_V	\mathcal{O}_{VV}	\mathcal{O}_H	\mathcal{O}_{y_ψ}
Remedios (section 4.1)	1	g_*						
Remedios+MCHM (section 4.2.1)	1	g_*	g	g'	g_V	g_V^2	g_*^2	$y_\psi g_*^2$
Remedios+ISO(4) (section 4.2.2)	1	g_*	g_*	g'	g_V	g_V^2	λ_h	$y_\psi \lambda_h$

Table 3. Estimated coefficients of the dimension-6 operators for the different scenarios considered in the main text, neglecting loop effects. The subscript V can denote W, B, G according to the basis defined in table 1.

It is now straightforward to estimate the size of the different contributions to the effective Lagrangian: only operators that are invariant under \mathcal{G} can arise from the strong dynamics. In particular, at dimension-6 the only such operators are \mathcal{O}_H and \mathcal{O}_{3W} for which we thus expect

$$c_H \sim g_*^2, \quad c_{3W} \sim g_* . \quad (4.12)$$

On the other hand \mathcal{O}_W and \mathcal{O}_{HW} are not invariant under $\widetilde{SU}(2)$, while \mathcal{O}_B and \mathcal{O}_{HB} are forbidden by $SO(4)$ (since $H^\dagger \overleftrightarrow{D}_\mu H$ and $D_{[\mu} H^\dagger D_{\nu]} H$ belong to $(\mathbf{1}, \mathbf{3})$ of $SU(2)_L \times SU(2)_R$). The coefficients of those other operators will therefore be suppressed by powers of the weak couplings, like in ordinary composite Higgs scenarios [12]. It perhaps does not make much sense to assume here MC (see appendix A), as, unlike for the case of the standard composite Higgs scenario, we do not possess here any explicit, if partial, UV completion, and even less one that satisfies MC. Finally, the coefficients c_{2W} and c_{2B} are of $O(1)$, as in eq. (4.3), contrary to the SILH where they are suppressed by $O(g^2/g_*^2)$. A summary of these results is given in table 3.

On the other hand, at the dimension-8 level, in addition to the $F_{\mu\nu}^4$ terms discussed above, further operators with $O(g_*^2)$ coefficient will arise. In particular, we shall have operators involving two H and two field-strengths, e.g., $\epsilon^{abc} D^\mu H^\dagger \sigma^a D_\nu H W_{\mu\rho}^b W^{\nu\rho c}$ (see appendix C). These operators, however, only contribute to the scattering of four bosons, through $O(g_*^2)$ corrections to $VVVV$, $VVhh$ and V^3h vertices.

4.2.2 Remedios with ISO(4)

If we are willing to possibly give up UV-completions within quantum field theory (as explained, this might even have to be the case for *Remedios* models), we can consider scenarios in which the Higgs is a PNGB issued from the spontaneous breakdown of a non-compact group. The simplest such option is given by $SO(4,1)/SO(4)$, see for instance the discussions in ref. [58, 59].¹⁴ However, its combination with *Remedios* does not introduce novelties with respect to $SO(5)/SO(4)$, as we still need to add an additional $\widetilde{SU}(2)$ to include the W_μ^a in the strong dynamics. A more interesting example is to consider $ISO(4)$, the isometric group of the 4D Euclidean space, consisting of the semidirect product of 4D rotations and translations: $SO(4) \rtimes T_4$. The Higgs field can here be identified with the

¹⁴We can write a perfectly consistent effective Lagrangian based on $SO(4,1)/SO(4)$, given the unitarity of the residual symmetry $SO(4)$. However a unitary CFT cannot respect a non-compact symmetry such as $SO(4,1)$: within ordinary QFT, there cannot exist a UV-completion with linearly realized $SO(4,1)$.

coset $\text{ISO}(4)/\text{SO}(4)$ corresponding to the transformation

$$\begin{aligned} H &\rightarrow H + c, \\ H &\rightarrow RH, \end{aligned} \tag{4.13}$$

where c parametrizes T_4 translations while R is an $\text{SO}(4)$ rotation of the 4-component Higgs.¹⁵

Now, with respect to $\text{SO}(5)$, the important novelty is that $\text{ISO}(4) = \text{SO}(4) \rtimes T_4$ possesses irreducible representations of dimension 3. These are the $(\mathbf{3}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{3})$ of the $\text{SU}(2)_L \times \text{SU}(2)_R$. We can thus fit the triplet of electroweak bosons W_μ^a in the $(\mathbf{3}, \mathbf{1})$ while respecting the full $\text{ISO}(4)$.

Concerning instead the hypercharge field B_μ , the simplest option is to take it as a singlet, as in the previous example. That choice obviously preserves $\text{ISO}(4)$, in particular the Higgs would remain a Goldstone. Alternatively, B_μ could be fit into an incomplete $(\mathbf{1}, \mathbf{3})$ of $\text{SO}(4)$. That would break $\text{SO}(4)$ down to $\text{SU}(2)_L \times \text{U}(1)_R$, but would leave T_4 untouched. In practice the coset would become $[\text{SU}(2) \times \text{U}(1) \rtimes T_4]/\text{SU}(2) \times \text{U}(1)$: the Higgs would remain a Goldstone, but the custodial symmetry would be maximally broken. In what follows we shall focus on the first option, for which the strong dynamics, in the pure Remedios limit, has symmetry group

$$\mathcal{G} = [\text{ISO}(4)]_{\text{global}} \rtimes [\text{U}(1)^4]_{\text{local}}, \tag{4.14}$$

with the gauge bosons of $\text{U}(1)^4$ transforming like a $(\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under $\text{SO}(4)$. Moreover, $\text{ISO}(4)$ is assumed to be spontaneously broken to $\text{SO}(4)$.

Before discussing the effective Lagrangian and its implications, it may be worth recalling that $\text{ISO}(4)$ is a IW contraction of both $\text{SO}(5)$ and $\text{SO}(4, 1)$. Geometrically this is quite evident: the isometries of the plane ($\text{ISO}(N)$) coincide with the local limit of the isometries of the sphere ($\text{SO}(N)$), or of any hyperboloid ($\text{SO}(N-m, m)$). As a consequence of that, the effective Lagrangian for $\text{ISO}(4)/\text{SO}(4)$ can be viewed as a particular limit of that for $\text{SO}(5)/\text{SO}(4)$. In the CCWZ language [24, 25] (and in the notation of ref. [60]), that amounts to replacing everywhere d_μ and E_μ according to

$$d_\mu^\alpha \hat{T}_\alpha + E_\mu^A T_A = -ie^{-i\pi^\alpha \hat{T}_\alpha} \partial_\mu e^{i\pi^\alpha \hat{T}_\alpha} \longrightarrow \frac{-i}{\epsilon} e^{-i\epsilon\pi^\alpha \hat{T}_\alpha} \partial_\mu e^{i\epsilon\pi^\alpha \hat{T}_\alpha} \equiv d_\mu^\alpha(\epsilon) \hat{T}_\alpha + E_\mu^A(\epsilon) T_A, \tag{4.15}$$

and then letting $\epsilon \rightarrow 0$, so that $d_\mu^\alpha \rightarrow \partial_\mu \pi^\alpha$ and $E_\mu^A \rightarrow 0$. This specific result for d_μ and E_μ simply corresponds to the metric flatness of the $\text{ISO}(4)/\text{SO}(4)$ coset. One important consequence is that in the \mathcal{G} symmetric limit, that is for $g = g' = 0$, the effective Lagrangian only depends on derivatives of the Higgs field: $L \equiv L(\partial H)$.

Invariance under \mathcal{G} strongly restricts the interactions generated by the strong dynamics. Among the dimension-6 operators of table 1, only \mathcal{O}_{3W} , \mathcal{O}_{HW} , \mathcal{O}_{2W} , \mathcal{O}_{2B} have coefficients enhanced according to the pattern

$$c_{3W}, c_{HW} \sim g_*, \quad c_{2W}, c_{2B} \sim 1. \tag{4.16}$$

¹⁵The same reasoning of the previous footnote applies: there cannot exist any ordinary unitary QFT with linearly realized $\text{ISO}(4)$ acting as UV completion.

In particular, the shift symmetry T_4 implies the constraint $c_H = 0$. Indeed c_H controls the curvature of the coset manifold: it is positive for $\text{SO}(5)/\text{SO}(4)$, negative for $\text{SO}(4,1)/\text{SO}(4)$ and vanishing for $\text{ISO}(4)/\text{SO}(4)$. Similarly, T_4 forbids direct strong-sector contributions to c_W , c_B , c_T , which nicely helps control corrections to electroweak observables, while $\text{SO}(4)$ forces $c_{HB} = 0$. The operator $|\Box H|^2$, as it respects \mathcal{G} , can appear with $O(1)$ coefficient. However, by use of the equations of motion it can be easily seen to only affect \mathcal{G} -invariant interactions of dimension greater than 8. Its fate is however different when the small deformations of \mathcal{G} are taken into account, as we shall discuss later. Finally, as in the previous examples, at dimension-8 level we encounter new operators with $O(g_*^2)$ coefficients, which contribute to $VV \rightarrow VV$ at $O(g_*^2 E^4/m_*^4)$, see appendix C, eq. (C.8) in particular.

\mathcal{G} -breaking effects and phenomenology. In order to obtain a low energy description coinciding with the SM, we must necessarily deform \mathcal{G} . In particular $\text{ISO}(4)$ must be explicitly broken. There are in principle multiple model building options for the structure of that breaking, at each order in the derivative and field expansion in the effective Lagrangian. Moreover an important structural factor concerns the possible involvement of the fermions in the strong dynamics. Here we shall make the conservative and consistent assumption that all the sources of \mathcal{G} -breaking are associated with the $\text{SU}(2)_L \times \text{U}(1)_Y$ gauge couplings, the Yukawa couplings and the Higgs potential. Moreover we shall take the fermions to be elementary, that is we shall assume their interactions with the strong sector are all associated with the (weak) couplings of the SM. On the other hand, we will remain agnostic onto the source of these couplings. In particular we shall not rely on partial compositeness, which provides a direct way to estimate the coefficients of symmetry breaking operators of arbitrary dimension. Nonetheless, we find it more than reasonable to assume that, whatever the underlying theory, each and every SM interactions is accompanied by a tower of higher derivative interactions involving precisely the same fields and suppressed just by the suitable powers of $1/m_*$ with respect to the lower derivative terms. As it will turn out, this tower of higher derivative operators is the leading source of \mathcal{G} -breaking higher dimension operators. The other source is given by quantum corrections saturated at the scale m_* . These effects are unescapable unless some fine tuning is allowed. They thus provide a lower bound on the size of the operator coefficients.

According to the above discussion, aside the gauge couplings $g \equiv \epsilon g_*$ and $g' \equiv \epsilon' g_*$, we shall consider as leading \mathcal{G} -breaking effects the top Yukawa and the Higgs potential

$$\mathcal{L}_{\text{break}} = \epsilon_t g_* [\bar{Q}_L H t_R + \dots] + \epsilon_2 m_*^2 [|H|^2 + \dots] - \epsilon_4 \frac{g_*^2}{2} [|H|^4 + \dots] , \quad (4.17)$$

where for each of the three expression in square brackets, the dots represent higher derivative operators with the same field content and purely suppressed by powers of $1/m_*$. Here we should identify $\epsilon_t g_* \equiv y_t$, $\epsilon_2 m_*^2 \equiv m_H^2$ and $\epsilon_4 g_*^2 \equiv \lambda_h$ with the corresponding SM coupling renormalized at the scale m_* . Notice that, given our power counting rules for operator coefficients at the scale m_* , in the interesting case where m_* is not much larger than a few TeV, renormalization effects are typically not very important. A relevant exception is given

by the effects breaking the Higgs shift symmetry that are induced by the top Yukawa. In particular the Higgs quartic receives a sizable IR contribution from top quark loops:

$$\Delta\lambda_h = \frac{3y_t^4}{4\pi^2} \ln \frac{m_*}{m_t}. \quad (4.18)$$

As is well known from the case of the MSSM, for m_* of order a few TeV, and given the observed value of the Higgs mass, the IR and UV contributions are comparable: $\Delta\lambda_h \sim \lambda_h$. We shall thus make this rough identification throughout our estimates.

Aside calculable IR effects, the breaking of ISO(4) in eq. (4.17) will propagate through quantum corrections at the cut-off scale m_* , of which we can only offer an order of magnitude estimate, and whose detailed value depends on the full theory. First and foremost we have the top sector correction to the Higgs mass parameter, expected to scale like

$$\Delta m_H^2 \sim \frac{3y_t^2}{4\pi^2} m_*^2. \quad (4.19)$$

That implies the usual estimate of the tuning

$$\frac{(125 \text{ GeV})^2}{\Delta m_H^2} \sim \left(\frac{400 \text{ GeV}}{m_*} \right)^2, \quad (4.20)$$

or equivalently, given $\lambda_h \sim \Delta\lambda_h$ and eq. (4.18),

$$\frac{\lambda_h v^2}{\Delta m_H^2} \sim 2 \frac{m_t^2}{m_*^2} \ln \frac{m_*}{m_t} \equiv \frac{v^2}{\tilde{f}^2} \ln \frac{m_*}{m_t}. \quad (4.21)$$

In our scenario, $\tilde{f} \equiv m_*/y_t$ thus represents the natural scale for v , the Higgs vacuum expectation value (VEV).¹⁶

We can now estimate the size of the coefficients of dimension-6 operators that arise when \mathcal{G} is deformed by the SM couplings:

- The first class of operators involves four derivatives and two powers of the Higgs field, schematically of the form $H^\dagger \partial^4 H / m_*^2$. When the partial derivatives ∂_μ are deformed into covariant derivatives D_μ , the resulting structures are classified according to the number of commutators, $[D_\mu, D_\nu] = -igW_{\mu\nu}^a \sigma^a/2 - ig'B_{\mu\nu}/2$, they involve. These resulting operators will thus involve 0, 1 or 2 field-strengths, each weighted by the corresponding gauge coupling. There is just one operator not involving any field-strength, and that is $|\Box H|^2/m_*^2$, which respects \mathcal{G} and which we already encountered in the discussion of the \mathcal{G} -symmetric limit. In the presence of the SM couplings (eq. (4.17)) its consequences are however different. By applying a field redefinition this operator can be rewritten in terms of the operators in table 1. In particular,

¹⁶It is interesting to compare this results with ordinary composite Higgs models based on a compact coset, like SO(5)/SO(4). In that case the natural scale for non linearities in the Higgs field and thus for the Higgs VEV is $f \equiv m_*/g_*$. Instead, in a flat non-compact coset like ISO(4)/SO(4) the quantity m_*/g_* does not represent the natural scale for the VEV: in the unbroken theory all points in the infinite H -plane are equivalent. Only when ϵ_i are turned on, does there arise a natural scale for the VEV, and given by \tilde{f} .

one finds $c_6 \sim \lambda_h^2$, $c_{4\psi} \sim y_\psi^2$ and more importantly a contribution to $c_{y\psi}$ that gives a universal correction to the Higgs couplings to fermions:

$$c_{y\psi} \sim y_\psi \lambda_h \quad \Rightarrow \quad \delta g_{h\psi\psi} \sim \frac{m_h^2}{m_*^2}, \quad (4.22)$$

where $\Delta\mathcal{L}_{\psi\psi}^h = (h/v)(\delta g_{h\psi\psi} m_\psi \bar{\psi}\psi + \text{h.c.})$. Concerning operators involving field-strengths, by a straightforward analysis one can prove they give rise to independent contributions to \mathcal{O}_{HB} , \mathcal{O}_W , \mathcal{O}_B , \mathcal{O}_{BB} . In particular that implies $c_{HB} \sim g'$ and, more interestingly,

$$c_B \sim g', \quad c_W \sim g \quad \Rightarrow \quad \delta\hat{S} \sim \frac{m_W^2}{m_*^2}, \quad (4.23)$$

and

$$c_{BB} \sim g'^2 \quad \Rightarrow \quad \delta g_{h\gamma\gamma} \sim \frac{e^2 v^2}{m_*^2}, \quad (4.24)$$

where \hat{S} (and for later \hat{T}) are defined in [49] and are proportional to the Peskin-Takeuchi parameters [61], while modifications to the Higgs coupling to photons are normalized as $\Delta\mathcal{L}_{\gamma\gamma}^h = (h/v)\delta g_{h\gamma\gamma} F_{\mu\nu} F^{\mu\nu}$. Notice that both c_B and c_{HB} break the custodial $\text{SO}(4)$, while c_B also breaks T_4 . Nevertheless, one insertion of g' saturates the necessary selection rule. As a result the relative size of $c_B \sim c_{HB} \sim (g'/g)c_W$ from eq. (4.23) is the same as in the SILH (without MC), while c_{HW} is enhanced by $\sim g_*/g$ (see eq. (4.16)).

- A second class of effects results from the dressing of \mathcal{G} -breaking interactions with powers of ∂_μ/m_* , as captured by the dots in eq. (4.17). In particular, \mathcal{O}_H can be viewed as just a 2-derivative iteration of the Higgs quartic term, protected by the same selection rules, and thus controlled by the same small parameter $\epsilon_4 g_*^2 \equiv \lambda_h$:

$$c_H \sim \lambda_h \quad \Rightarrow \quad \delta g_{hVV} \sim \frac{m_h^2}{m_*^2}, \quad (4.25)$$

where $\Delta\mathcal{L}_{VV}^h = (h/v)\delta g_{hVV} m_W^2 \left(W^{+\mu} W_\mu^- + \frac{Z^\mu Z_\mu}{2\cos^2\theta_W} \right)$ parametrizes deviations of the Higgs couplings to vectors. Similarly, higher derivative dressing of the Yukawa couplings, $y_\psi \bar{\psi}_L \psi_R \square H$, can be shown to lead, upon use of the H equation of motion, to $c_{y\psi} \sim y_\psi \lambda_h$. Although these other effects are comparable to eq. (4.22), they are distinguished, as they are in general not universal across fermion species.

- A third class of \mathcal{G} -breaking operators is generated by loop-effects. These are typically sub-leading, except for c_T , which might not receive any tree-level contribution as it corresponds to a violation of two units of custodial isospin, $\Delta I_c = 2$, that is not present in eq. (4.17). At the one-loop level, however, this custodial breaking can be achieved either by two insertions of g' (g' can be assigned to a $(\mathbf{1}, \mathbf{3})$ of $\text{SO}(4)$) or by four insertions of y_t (y_t can be assigned to a $(\mathbf{1}, \mathbf{2})$). A contribution in the first class is given by diagram figure 1(a):

$$c_T \sim \left(\frac{g_*}{4\pi} \right)^2 \times g'^2 \quad \Rightarrow \quad \delta\hat{T} \sim \left(\frac{g_*}{4\pi} \right)^2 \times \tan^2\theta_W \frac{m_W^2}{m_*^2}, \quad (4.26)$$

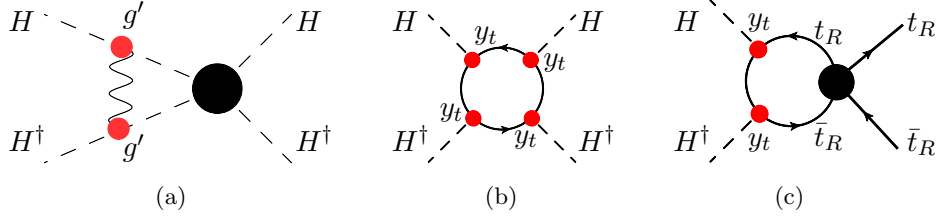


Figure 1. Diagrams a and b: leading contributions to the operator \mathcal{O}_T . Diagram c: leading effects in the case of t_R compositeness (for t_L compositeness with the obvious replacement $R \rightarrow L$). The black blob denotes $O(g_*^2)$ vertices from the $\mathcal{G} \equiv [\text{ISO}(4)]_{\text{global}} \times [\text{U}(1)^4]_{\text{local}}$ preserving strong dynamics at loop momentum virtuality $\sim m_*$. The small red blobs denote instead vertices associated with the weak deformation and breakdown of \mathcal{G} by the SM couplings (g, g', y_t, λ_h).

while a contribution in the second class is instead given by diagram figure 1(b), and simply corresponds to a deformation at virtuality of order m_* of the SM top loop contribution to the ρ -parameter:

$$c_T \sim \frac{y_t^4}{16\pi^2} \Rightarrow \delta\hat{T} \sim \left(\frac{y_t}{4\pi}\right)^2 \times \frac{m_t^2}{m_*^2}. \quad (4.27)$$

This second contribution, even if not enhanced by the strong dynamics could be as important as eq. (4.26), especially for g_* smaller than 4π .

In summary, the main phenomenological effects in ISO(4) models are captured by eqs. (4.16), (4.22)–(4.27). We find that the Higgs coupling to photons, $\delta g_{h\gamma\gamma}$ from eq. (4.24), is a factor $\sim (4\pi/y_t)^2$ larger than in the SILH [12], and gives a rather strong constraint in this type of models: $m_* \gtrsim 2 \text{ TeV}$ from a 2‰ measurement of $\delta g_{h\gamma\gamma}$ [62]. A comparable constraints is also given by the \hat{S} parameter from eq. (4.23), while \hat{T} seems slightly less important because of the $\tan^2 \theta_W$ and loop suppressions. On the other hand, corrections to the Higgs coupling to vectors and fermions, δg_{hVV} and $\delta g_{h\psi\psi}$, are suppressed compared to the case of a generic composite Higgs scenario, and provide, at the moment, weaker constraints.

Nevertheless, while contributions to the observables of eqs. (4.22)–(4.27) are ubiquitous in new physics scenarios, the g_* -enhanced effects eq. (4.16) are unique to our model. Apart from $c_{3W} \sim g_*$, which is present in all Remedios scenarios, we now also have $c_{HW} \sim g_*$, which mainly affects TGC and the $hZ\gamma$ vertex:

$$\delta g_1^Z = \frac{\delta \kappa_\gamma}{\cos^2 \theta_W} = \frac{\delta g_{hZ\gamma}}{\sin \theta_W \cos \theta_W} = -\frac{m_Z^2}{m_*^2} \frac{c_{HW}}{g} \sim \frac{m_Z^2}{m_*^2} \frac{g_*}{g}, \quad (4.28)$$

$$\lambda_\gamma = \frac{m_W^2}{m_*^2} \frac{c_{3W}}{g} \sim \frac{m_W^2}{m_*^2} \frac{g_*}{g}, \quad (4.29)$$

where $\delta g_1^Z, \delta \kappa_\gamma, \lambda_\gamma$ correspond to anomalous TGC, normalized according to ref. [50],¹⁷ while $\delta g_{hZ\gamma}$ describes the anomalous $HZ\gamma$ vertex according to $\Delta \mathcal{L}_{hZ\gamma} = \delta g_{hZ\gamma} (h/v) F_{\mu\nu} Z^{\mu\nu}$.

¹⁷We recall that, at the dimension-6 level, the other parameters are fixed: $\lambda_Z = \lambda_\gamma$ and $\delta \kappa_Z = \delta g_1^Z - \tan^2 \theta_W \delta \kappa_\gamma$.

Notice the interesting correlation between δg_1^Z , $\delta \kappa_\gamma$ and $\delta g_{hZ\gamma}$, which could single out these scenarios if deviations from the SM predictions were to appear in the future. Taking into account the present constraint from $h \rightarrow \gamma\gamma$ and EWPT, the allowed size of these other effects could still be as large as a few per-cent which is within the reach of the ongoing LHC run, or of future electron-positron colliders. It is important to point out these new contributions to $hZ\gamma$ are potentially even larger than the SM one, as that arises at one-loop. Perhaps more importantly, the relative size of the new physics effects in $hZ\gamma$ and in $h\gamma\gamma$ is

$$\frac{\delta g_{h\gamma Z}}{\delta g_{h\gamma\gamma}} \sim \frac{g_*}{g}. \quad (4.30)$$

Thus, deviations larger than $O(1)$ in $BR(h \rightarrow Z\gamma)$ are possible at the moment.

Like in all models with composite Higgs, it is also worth considering the situation where one, or both, of the chiralities of the top quark is part of the strong dynamics. Consider first the case where t_R is composite. One class of important effects is again given by ∂_μ/m_* dressing of SM operators. The leading effect at the dimension-6 level is given by $\bar{t}_R \not{D}^3 t_R/m_*^2$, which upon use of the equations of motion leads to $c_{y_t} \sim y_t^3$ and to $c_L^t \sim c_L^{(3)t} \sim y_t^2$ corresponding to a specific linear combination of \mathcal{O}_L^t and $\mathcal{O}_L^{(3)t}$. The first effect corresponds to an $(m_t/m_h)^2$ enhancement w.r.t. eq. (4.22). The second one gives a relative correction to the $Z t_L \bar{t}_L$ vertex of order m_t^2/m_*^2 , leaving the $Z b_L \bar{b}_L$ vertex unaffected. The other class of contributions arise from loops, and the leading one is shown in figure 1(c). This gives rise, among other subleading effects, to $c_R^t \sim (g_*/4\pi)^2 y_t^2$, implying a relative correction to the $Z t_R \bar{t}_R$ vertex of order $(g_*/4\pi)^2 (m_t/m_*)^2$. Analogous effects are generated in the case of a composite t_L , (figure 1(c), with $t_R \rightarrow t_L$). Now the novelty is that also the $Z b_L \bar{b}_L$ vertex is modified by a relative amount $(g_*/4\pi)^2 (m_t/m_*)^2$, implying the rather strong constraint $m_* \gtrsim 5 (g_*/4\pi) \text{ TeV}$.

4.3 Composite fermions

In this section we shall discuss the implications of fermion compositeness, in its different incarnations of partial compositeness, described in section 2.2, and approximate supersymmetry, described in section 3.2.

4.3.1 Partially composite fermions

In section 2.2 we discussed how to incorporate composite fermions in a strongly coupled sector by taking the limit in which the effective mixing parameter ϵ_ψ in eq. (2.3) becomes $O(1)$ at the IR scale where the CFT develops a mass-gap. If the SM fermions are composite, the main beyond the SM (BSM) effects are associated with 4-fermion contact-interactions, described by $\mathcal{O}_{4\psi}$. In order to avoid severe constraints from flavor-violating processes, these interactions should preserve an approximate family symmetry. If that is the case, one of the most important process to unravel composite fermions is $\psi\psi \rightarrow \psi\psi$ at high energy, as its amplitude receives an energy-growing contribution from $\mathcal{O}_{4\psi}$:

$$\delta \mathcal{A}(\psi\psi \rightarrow \psi\psi) \simeq \epsilon_\psi^4 g_*^2 \frac{E^2}{m_*^2}. \quad (4.31)$$

At the LHC these effects are being searched in the high-energy angular distributions of dijets, for quark compositeness, and in the spectrum of Drell-Yan processes when also the leptons are composite. In the former case, the leading contribution is from qq initial states, as these have larger PDFs than $q\bar{q}$. In both cases, the data from LHC run 1 imply $m_* \gtrsim (g_*\epsilon_\psi^2/4\pi) \times 60 \text{ TeV}$ (see table 5 of ref. [63], and also [64–67]). The lower bound on m_* seems quite stringent, but it is quickly reduced if fermions are not fully composite, that is for $\epsilon_\psi < 1$. Indeed, provided the Higgs field is fully composite, stronger constraints can arise, for sufficiently small $\epsilon_\psi < 1$, by considering processes involving bosons along with fermions. In particular, the new physics contribution to $\psi\bar{\psi} \rightarrow V_L V_L/hV_L$, scales like $\sim \epsilon_\psi^2 g_*^2 E^2/m_*^2$, which, compared to eq. (4.31), features fewer powers of ϵ_ψ .

Here we want to elaborate more on such scenarios, focussing for simplicity on the case where the Higgs is composite and the fermions are partially composite, while the transverse polarizations of gauge bosons are elementary (that is, we here do without the Remedios construction). To analyze which are the most relevant observables to test these scenarios, we adopt a power-counting following eq. (2.3). One is easily convinced that, as long as $\epsilon_\psi > g/g_*$, the operators with the largest coefficients are those with the largest number of Higgs or fermion fields (as opposed to field-strengths and derivatives), as they directly probe the strong dynamics. We can divide these operators into 3 classes (see table 1 for notation):¹⁸ a) $\mathcal{O}_{4\psi}$, whose main impact was discussed above, b) operators that only affect Higgs physics, \mathcal{O}_H , \mathcal{O}_6 , $\mathcal{O}_{y\psi}$, and c) the 8 independent operators

$$\mathcal{O}_L^q, \mathcal{O}_L^{(3)q}, \mathcal{O}_R^u, \mathcal{O}_R^d, \mathcal{O}_L^l, \mathcal{O}_L^{(3)l}, \mathcal{O}_R^e, \mathcal{O}_T, \quad (4.32)$$

which, in the vacuum $H = (0, v/\sqrt{2})$, give modifications to electroweak observables and are already constrained by pre-LHC measurements. Parametrically, all coefficients of the operators of eq. (4.32) are of order $\sim \epsilon_\psi^2 g_*^2$, with the exception of \mathcal{O}_T whose coefficient depends on the details of custodial symmetry breaking (here we are not assuming any particular composite Higgs model). The bounds on these coefficients come mainly from precision Z -physics at LEP. In particular, the seven partial widths $\Gamma(Z \rightarrow \bar{\psi}\psi)$, with $\psi = e_{L,R}, \nu_L, u_{L,R}, d_{L,R}$, roughly place a constraint $m_* \gtrsim (g_*\epsilon_\psi/4\pi) \times 40 \text{ TeV}$ on seven linear combinations of the coefficients of the 8 operators in eq. (4.32) [52, 64]. Nevertheless, in the presence of a sizeable breaking of the custodial symmetry, the effects of \mathcal{O}_T on the Z propagator could partially compensate the $Z\bar{\psi}\psi$ vertex corrections, in such a way that one linear combination of the operators in eq. (4.32) remains invisible to processes on the Z -pole, thus escaping LEP1 constraints [68]. That particular combination modifies $\bar{\psi}\psi \rightarrow W^+W^-$, but the constraints placed by LEP2 data (in the form of the parameter g_1^Z discussed above) are one order of magnitude weaker [51, 52] than those placed by Z -pole data. Future measurements at the LHC, either in $\bar{\psi}\psi \rightarrow V_L V_L$ [1, 2] or $\bar{\psi}\psi \rightarrow V_L h$ [3], will instead be able to provide a more sensitive probe. It is amusing that in this case, the

¹⁸We focus on operators invariant under the full family symmetry $\text{SU}(3)^5$. This is the set of operators that will definitely be present whatever our assumptions about the origin of flavor, in particular, whatever our assumption about the underlying flavor symmetry. Moreover, this implies that operators that change the chirality of the fermions ($\propto \bar{\psi}_L \psi_R$) are proportional to the Yukawa couplings and can then be neglected, with the exception of $\mathcal{O}_{y\psi}$ whose impact on Higgs physics can still be important.

best observables to test fermion and Higgs compositeness are those traditionally associated with anomalous TGC [50].

4.3.2 Fermions as composite pseudo-goldstini

In section 3.2 we discussed the scenario in which fermions arise as composite pseudo-Goldstini of a strong sector with $\mathcal{N} > 1$ spontaneously broken supersymmetries. The phenomenological interesting feature of these models is that the higher-dimensional fermion interactions must involve derivatives, inducing softer effects at low energy. Indeed, from eq. (3.11) we have

$$\delta\mathcal{A}(\psi\psi \rightarrow \psi\psi) \simeq g_*^2 \frac{E^4}{m_*^4}, \quad (4.33)$$

and, hence, we expect milder constraints coming from, e.g., dijet or Drell-Yan collider searches.

The above suppression in $\psi\psi \rightarrow \psi\psi$ makes other observables equally relevant. An example is given by the case in which the Higgs is also composite. Here, beside the effects of Higgs compositeness captured by the SILH scenario, we expect additional genuinely strongly interacting contributions from the dimension-8 operators in eq. (3.9), that modify the amplitudes for longitudinal diboson pair production

$$\delta\mathcal{A}(\bar{\psi}\psi \rightarrow V_L V_L) \simeq g_*^2 \frac{E^4}{m_*^4}. \quad (4.34)$$

As in the SILH, there are also dimension-6 contributions to $\mathcal{A}(\bar{\psi}\psi \rightarrow W_L W_L)$ coming from \mathcal{O}_W and \mathcal{O}_B , but these are suppressed by weak couplings: $\delta\mathcal{A}(\bar{\psi}\psi \rightarrow W_L W_L) \sim g^2 E^2/m_*^2$, and are subleading w.r.t. eq. (4.34) for $E \gtrsim (g/g_*)m_*$. Therefore, in this scenario, apart from the obvious effects on Higgs observables due to the Higgs compositeness, the most important effects are modifications in the high-energy production of dibosons (including neutral $Z_L Z_L$ and $Z_L h$ final states) and quark/lepton pairs, according to eq. (4.34) and eq. (4.33) respectively.

For the case of composite gauge bosons *à la* Remedios we also expect the operators in eqs. (3.9), (3.12) (see also appendix C) to be induced by the strong dynamics. This leads to

$$\delta\mathcal{A}(\bar{\psi}\psi \rightarrow V_T V_T) \simeq g_*^2 \frac{E^4}{m_*^4}. \quad (4.35)$$

From vector compositeness *per-se* we also have the sizeable dimension-6 operator coefficient $c_{3W} \sim g_*$, implying, from an insertion of an ordinary gauge $\bar{\psi}\psi V_T$ vertex, a contribution $\mathcal{A}(\bar{\psi}\psi \rightarrow V_T V_T) \sim g g_* E^2/m_*^2$ (eq. (4.4)). In this contribution the weakness of the ordinary gauge vertex is compensated by the lower power of $E/m_* < 1$. Nevertheless, the dimension-6 contribution exceeds the SM one, $\sim g^2$, only for $(g_*/g)E^2/m_*^2 \gtrsim 1$, precisely where the dimension-8 contribution eq. (4.35) becomes fully dominant. In other words, as soon as

contributions from these scenarios are larger than the SM, they are dominated by the dimension-8 operators.¹⁹

These scenarios are also interesting because they give rise to sizeable contributions, eq. (4.35), to neutral diboson pair production, including photons, without giving rise at the same time to large deviations in electroweak observables or to any other observable affected by dimension-6 operators. This motivates the possibility to extend the analysis of $q\bar{q} \rightarrow VV$ at the LHC in a consistent way, providing richer avenues for explorations and prospects. In particular, it could be possible to combine measurements in $q\bar{q} \rightarrow WW, WZ, W\gamma$ with those with neutral final-states $q\bar{q} \rightarrow ZZ, Z\gamma, \gamma\gamma$ to test this class of SM deformations [1, 2].

5 Conclusions

In this paper we have argued that, although the SM is weakly-coupled at the electroweak scale, it may still conceivably emerge from a strong dynamics just above the TeV scale. The weakness of the SM couplings at low-energies could be the consequence of approximate symmetries: similarly to the pions in QCD, the SM particles could correspond to the lightest degrees of freedom of the strong sector whose interactions, at very low energy $E < m_*(g/g_*)$, could be dominated by small symmetry breaking effects, and thus appear weak.

The hierarchy problem strongly motivates to consider the Higgs a composite state directly participating in a strong dynamics above the weak scale. For this reason composite Higgs scenarios have been extensively studied in recent years. Nevertheless, if a strong sector is supposed to be around the TeV, it is also worth entertaining the possibility that other SM states may be part of it. The LHC at 13 – 14 TeV offers for the first time a powerful probe of the nature of the SM particles in that crucial energy range. The study of the high-energy behaviour $\psi\psi \rightarrow \psi\psi$ and $\psi\bar{\psi} \rightarrow VV$ offers at present the best sensitivity, but in the future also the study of $VV \rightarrow VV$ could play a role.

We have here characterized several new patterns of strong dynamics, encompassing not just the Higgs boson, but also the SM gauge bosons and fermions. The importance of our constructions should be clear. They provide structurally robust parametrizations, based on symmetry and dynamics, of possible SM deformations. In turn each of these parametrizations offers a specific ordering of the observables according to their sensitivity to new physics. An overall guideline for the design of LHC research strategies is thus devised. From a more theoretical perspective, the simple and the robust principles underlying our constructions provide well definite power-counting rules for the derivative and field expansions within the effective Lagrangian, as well as a direct assessment of its domain of applicability. The value of these neat rules is best appreciated by considering the situations where they lead to results countering naive expectations. To that end, we should stress the recurrent emergence of scenarios where, well within the domain of validity of the

¹⁹Notice also that in the SM the largest contribution to the $\psi\bar{\psi} \rightarrow WW$ cross-section comes from the $+-$ vector helicity structure, which in the high energy limit is not modified by dimension-6 effects, but by dimension-8 ones [1, 2, 57]. Therefore, the contribution of dimension-8 versus dimension-6 operators to the cross-section is not only enhanced by the g_*/g factor, but also by this interference term.

effective field theory description, the leading effects in $2 \rightarrow 2$ scatterings are captured by dimension-8 operators rather than by dimension-6 ones.

The main new idea considered here has been to have (some of) the SM gauge bosons arise from a strong sector, in such a way that the strong dynamics manifests itself purely through higher-derivative (multipolar) interactions. The extreme IR softness of these strong interactions accounts for their having escaped detection in collider experiments thus far. Their structure is enforced by a specific symmetry, while the ordinary gauge interactions, which dominate the low energy dynamics, arise as a small deformation of that symmetry. In that respect our construction is technically natural. Indeed, the physical situation depicted by our scenario is fully analogous to the one encountered in the low-energy effective Lagrangian for neutral atoms and photons, where all interactions are necessarily higher derivatives (multipoles). In that case, the resulting IR softness of the light-matter scattering amplitudes is the well known cause for the color of the sky. Our construction and its consequences, exotic as they may appear, are in fact as natural as that color.

Based on the above idea, we constructed explicit scenarios for new physics, which we dubbed *Remedios*, and outlined their phenomenology. Their crucial property is to produce deformations of the SM amplitudes suppressed by powers of E/m_* but enhanced by a strong coupling g_* : at sufficiently high-energy, but still below the fundamental cut-off scale m_* , these deformations can become sizeable and even overcome the SM contributions. The LHC at 13 – 14 TeV is thus well suited to explore these scenarios by looking at the differential cross-sections for diboson production. Indeed, we have shown that Remedios models always feature large effects in TGC. In particular, they always have a sizable λ_γ , which could be seen in $\psi\bar{\psi} \rightarrow V_T V_T$. As matter of fact, we should stress that we are not aware of other self-consistent and robust constructions where the study of TCG offers a better option than the search for the underlying new resonances: in order for new physics to show up loud and clear in deformations of the SM amplitudes, below the threshold for new physics, the new dynamics should be rather strong. The strength of the new dynamics is precisely the novelty offered by our construction. It should be reminded that in the scenarios of composite Higgs the corrections to TCG are not that important, given the SM gauge bosons are weakly coupled (elementary) at all energies. Finally, another important class of effects is given by deformations of the vector propagators, in particular those described by the W, Y, Z parameters of ref. [49], which could be sizable enough to saturate the experimental bounds coming from LEP. The exploration of these other effects is more for (far) future high-luminosity e^+e^- colliders.

We have also explored the possibility to combine the Remedios construction for gauge bosons with a composite PNGB Higgs. If the latter arises from a non-compact coset, in particular $\text{ISO}(4)/\text{SO}(4)$, additional sizable deviations (beside those expected either in the pure composite Higgs scenario or in the pure Remedios scenario) are expected in $h \rightarrow Z\gamma$ and TGC (in particular, κ_γ), following the interesting correlation shown in eq. (4.28).

Finally, we have extended the analysis to scenarios in which the SM fermions are also strongly coupled at m_* . Four-fermion interactions represent here the main new physics effect of fermion compositeness, and constraints from studies of the angular distributions

of high-energy dijets are very strong. Nevertheless, we have argued that softer fermion interactions (and hence milder constraints) can arise in models based on approximate supersymmetry, where part of, or all of, the SM fermions have the interpretation of pseudo-Goldstini. This idea is not new, as it was already put forward in the 80's [45]. However we believe it acquires new vitality offering new perspectives in the general context of strongly coupled scenarios just above the weak scale. For instance, the combination of Remedios models with soft composite fermions implies that the leading effects to $\psi\bar{\psi} \rightarrow V_T V_T$ come from dimension-8 operators, which, with respect to the SM, scale as $g_*^2 E^4 / (g^2 m_*^4)$. A detailed phenomenological analysis of these effects will be given in ref. [1, 2].

Our main results are summarized in tables 2 and 3, where an estimate of the coefficients of the main induced operators are presented for the different strongly coupled scenarios. That can be useful to motivate certain high-energy searches in Higgs physics, diboson production, or WW scattering, which were not theoretically well justified before, because of the absence of BSM scenarios that predicted large indirect effects in these differential distributions, without the appearance of new resonances in direct searches. In the end, our constructions offer a map of the possible geography of new physics at the weak scale. Even if the resulting geography looks exotic, the map is constructed according to well definite structural assumptions that reflect broad properties of the underlying dynamics. It seems to us that, notwithstanding all its limitations, the charting of new physics that we outlined is conceptually solid and offers a well-motivated alternative to the standard approach based on the fully general dimension-6 effective Lagrangian. The latter, in fact, is based on the hypothesis that the scale separation represents the unique parameter controlling the hierarchy between different terms in the EFT expansion; such an assumption is often at odds with explicit model-building, and certainly at odds with the phenomenological requirement of strong coupling that is needed to justify indirect, rather than direct, searches. The occurrence of situations where the dimension-8 operators dominate makes the relevance of our construction comparatively evident. In the end, the study of the new experimental data will offer a proof or, to the very least, a partial disproof of our assumptions.

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A Minimal coupling from extra-dimensions

In this appendix we would like to elaborate on the notion of Minimal Coupling (MC), as employed in the main text. The notion was introduced in ref. [12], in a somewhat empirical way, to describe the coefficient structure of the effective operators that arises in specific,

and interesting, classes of models. As such it is a robust and well defined notion. It arises, as we shall better illustrate in this section, when the low-energy effective Lagrangian is dominantly fathered by a Lagrangian at the scale m_* involving the lowest possible number of fields and derivatives. In that sense it is probably an accident of this perhaps not too general class. In ref. [12] it was however remarked how the same features intriguingly emerge in certain classes of strings theories [70], which seemingly cannot be described at the string scale by an effective Lagrangian with the smallest power of fields. In any case, true or false, the MC can be imposed or not imposed to the analysis of ref. [12], without a change of the conceptual foundations of that work. Moreover for the case of a strongly coupled sector, with $g_* \sim 4\pi$, all suppressions from MC disappear, and all models satisfy the same power counting. Ref. [69] pointed out that the notion cannot be formalized in a general effective Lagrangian, as a statement concerning the appearance of gauge fields only through covariant derivatives. The basic point, with which we agree, is that by power counting, there is no distinction between two powers of a covariant derivatives and a field strength. As the readers of ref. [69] may be under the impression that the results of ref. [12] are incorrect, we thought it would be worth to show in detail how the structure of operator suppression arises in a simple, yet non-trivial, example. That is the goal of the present appendix.

We will illustrate how the arguably simplest 5D construction leads to a low-energy effective Lagrangian effectively satisfying MC. Let us consider, for the sake of the argument, a 5D Yang-Mills theory compactified on a circle S_1 of radius R . At the tree level the light degrees of freedom are represented by the vector zero modes A_μ^a and by the Wilson lines $W^a \equiv \frac{1}{2\pi R} \oint A_5^a$. The low-energy effective Lagrangian is determined by the parameters of the microscopic 5D Lagrangian describing physics at distances $\ll R$, and by the effects at the KK threshold. The simplest option is to assume only one microscopic scale at which the 5D theory is strongly coupled. Equivalently, that amounts to assuming the coefficients of the 5D Lagrangian satisfy naive dimensional analysis (NDA). Given the 5D coupling g_5 , a simple analysis of loop effects allows to conclude that, in the absence of intervening new physics at lower scales, the theory becomes strong at around the scale

$$\Lambda_5 \equiv \frac{16\pi^2}{g_5^2}, \quad (\text{A.1})$$

and therefore using the NDA ansatz:

$$\mathcal{L}_5 = \frac{\Lambda_5}{16\pi^2} L(D_M/\Lambda_5) = \frac{\Lambda_5}{16\pi^2} \left\{ -\frac{1}{4} F_{MN}^2 + \frac{c_1}{\Lambda_5^2} F_{MN} D_P^2 F^{MN} + \frac{c_2}{\Lambda_5^2} F_{MN}^3 + \dots \right\}. \quad (\text{A.2})$$

Notice that the NDA ansatz is compatible with naturalness, in that, starting from the leading F_{MN}^2 term, all others would be generated from loops with coefficients of precisely the NDA size. For instance, the terms associated with $c_{1,2}$ are expected to arise from log divergences at 2-loops. One could even slightly generalize the NDA assumption by assuming that the 5D physics is itself characterized by one scale M_5 and one dimensionless coupling \tilde{g}_* :

$$\mathcal{L}_5 = \frac{M_5}{\tilde{g}_*^2} L(D_M/M_5), \quad (\text{A.3})$$

and with the identification

$$\frac{M_5}{\tilde{g}_*^2} \equiv \frac{1}{g_5^2} \quad \Longrightarrow \quad M_5 = \Lambda_5 \frac{\tilde{g}_*^2}{16\pi^2} < \Lambda_5. \quad (\text{A.4})$$

The scale M_5 could be interpreted as the scale where new 5D resonances are expected. The NDA case eq. (A.2) simply corresponds to the strongly coupled limit $\tilde{g}_* \sim 4\pi$. Considering now the general ansatz in eq. (A.3), it is easy to deduce the general structure of the effective Lagrangian below the compactification scale. It will be organized as a triple expansion in loops and in inverse powers of m_{KK} and M_5 . The loop expansion parameter g_* and resonance scale m_* are identified according to eq. (2.1), but here, for a more precise identification, we take $m_* = 1/(\pi R)$, as the physical scale of a compactified extra dimension is determined by its length πR . The expansion in $1/M_5$ can be expressed in terms of the dimensionless parameter $m_*/M_5 \ll 1$. The result is then

$$\mathcal{L}_{\text{eff}} = \frac{m_*^4}{g_*^2} \sum_{n,m} \left(\frac{g_*^2}{16\pi^2} \right)^n \left(\frac{m_*^2}{M_5^2} \right)^m L_{n,m} \left(\frac{D_\mu}{m_*}, \frac{F_{\mu\nu}^a}{m_*}, \frac{W^a}{m_*} \right). \quad (\text{A.5})$$

The leading order term $L_{0,0}$ simply arises by integrating out the KK resonances at tree level in a 5D theory with the minimal Yang-Mills Lagrangian, that is the first term in eq. (A.2). This Lagrangian enjoys some accidental properties, which went by the name *minimal coupling* in ref. [12]. In particular, operators in the class of \mathcal{O}_{HW} , \mathcal{O}_{BB} and \mathcal{O}_{3W} do not appear in $L_{0,0}$. These terms however appear in general at the next order in the expansion and are thus suppressed by either a loop factor or by

$$\frac{m_*^2}{M_5^2} \sim \left(\frac{g_*^2}{\tilde{g}_*^2} \right)^2. \quad (\text{A.6})$$

Notice that in the case of a 5D theory based on the ordinary NDA, we have $\tilde{g}_* \sim 4\pi$, so that the above factor is equivalent to a 2-loop suppression. From our construction it is clear that the only way to fully eliminate the suppression dictated by MC is to assume $M_5 \sim m_*$, corresponding to the full lack of validity of the 5D description.

The above arguments indicate that a version of MC necessarily applies in theories that admit a range of lengths where they are described by a weakly coupled 5D theory. We cannot conclude that the same pattern of suppression must appear in other scenarios, like large- N gauge theories or in the effective descriptions of strings. Yet there are intriguing indications that a suppression of the same type also exists in those other contexts (see in particular ref. [70]). But we do not have a proof that must happen by necessity.

B An accidentally light Higgs

Here we shall illustrate the scenario where the Higgs boson, while arising from a strong dynamics, cannot be interpreted as a PNGB. Its small mass purely appears as the result of some unexplained tuning. We dub this scenario the Accidentally Light Higgs (ALH),

see also ref. [71]. For the ALH the Higgs potential is not dictated by selection rules, and one expects the generic function (working with the neutral Higgs component h):

$$V(h) = \frac{m_*^4}{g_*^2} F\left(\frac{h^2}{f^2}\right) = m_*^2 f^2 \left\{ a_2 \frac{h^2}{f^2} + a_4 \left(\frac{h^2}{f^2}\right)^2 + a_6 \left(\frac{h^2}{f^2}\right)^3 + \dots \right\}. \quad (\text{B.1})$$

To reproduce a VEV $\langle h \rangle = v$ and a mass m_h much below their natural expectations, $f \equiv m_*/g_*$ and $m_* \sim \text{TeV}$ respectively, one needs an (accidental) tuning of the parameters of the potential, away from the generic expectation $a_i \sim O(1)$. There are virtually as many ways to tune as there are parameters in the potential, that is infinitely many. Considering the Higgs VEV and mass, which are determined by $(\xi \equiv v^2/f^2)$

$$\xi \iff F'(\xi) = a_2 + 2a_4\xi + 3a_6\xi^2 + \dots = 0 \quad (\text{B.2})$$

$$\frac{m_h^2}{m_*^2} = 2F''(\xi)\xi = 2(2a_4 + 6a_6\xi + \dots)\xi, \quad (\text{B.3})$$

one can however qualitatively distinguish three main regions of parameter space, according to whether $\xi(m_*/m_h)$ is smaller than, comparable to, or larger than $O(1)$. The first region, $\xi m_h/m_* \ll 1$, can also be characterized by $a_2 \ll a_4^2 \ll a_{n \geq 6} \sim O(1)$. In this region, the non-renormalizable terms ($a_{n \geq 6}$) are just a small perturbation around the minimum controlled by the first two terms. One can for instance check that the relative size of the deviations from the SM in the Higgs self-couplings is controlled by $\sim a_2/a_4^2 \sim \xi m_*/m_h \ll 1$. The smallness of these corrections is controlled by the smallness of ξ , which in turns follows from the important tuning $a_2 \ll a_4^2 \ll 1$. A milder tuning is achieved for $a_2 \sim a_4^2 \ll a_{n \geq 6} \sim O(1)$ which also generically corresponds to the intermediate region $\xi m_*/m_h \sim O(1)$. Here we have $O(1)$ deviations from the SM in the Higgs self-couplings, even in the presence of a separation of scales. The reason for such a seemingly non-decoupling effect is the sizable coupling $g_*^2 \gg \lambda_h \sim a_4 g_*^2$ controlling the higher order terms, and the culprit is just the tuning. Finally, in the third region $\xi m_h/m_* \gg 1$, while remaining at $\xi \ll 1$, the deviations from the SM in Higgs self-interactions can be larger than $O(1)$. That result is nicely illustrated by focussing on the trilinear self-coupling which, from eq. (B.1), reads

$$\lambda_{3h} = \frac{6m_h^2}{v} \left(1 + \frac{2}{3} \frac{F'''(\xi)\xi}{F''(\xi)} \right), \quad (\text{B.4})$$

where we singled out the SM result normalized as $\lambda_{3h}^{SM} \equiv 6m_h^2/v$, corresponding to $F''' = 0$. The region $\xi m_h/m_* \gg 1$ corresponds to a situation where $a_{n \geq 6}$, and not a_2 , are used to tune F'' at the minimum, and thus m_h^2 , very small in such a way that $F''' \xi / F'' \gg 1$. This third region is thus characterized by an additional tuning of the physical Higgs mass m_h . An explicit example is obtained for instance by considering small perturbations around the tuned potential $F(\xi) \equiv (\xi - \xi_0)^3$ for which $m_h = 0$. Notice indeed that we can write the relative correction to the trilinear as

$$F'''(\xi)\xi/F''(\xi) \sim (\xi m_*/m_h)^2 F'''(\xi) \sim (\xi m_*/m_h)^2. \quad (\text{B.5})$$

showing that, in the absence of further cancellations in F''' , it is precisely controlled by $\xi m_*/m_h$.

When $\xi m_*/m_h \gg 1$ the Higgs self-coupling can in principle be as large as $\sim m_h g_*$, which implies a large scattering amplitude for $hh \rightarrow hh$:

$$\mathcal{A}(hh \rightarrow hh) \sim g_*^2, \quad (\text{B.6})$$

already at an energy of order m_h . For large $g_* \sim 4\pi$, Higgs self-interactions could be rather strongly coupled just around threshold. This scenario is already constrained by the LHC data on double Higgs production, but, to our knowledge, a detailed study is missing.

C Dimension-8 operators

In this appendix we list the CP-even, custodial preserving, dimension-8 operators that can give important $\sim g_*^2/m_*^4$ contact-interaction contributions to $2 \rightarrow 2$ scatterings at high-energy. Operators of higher-dimension always give contributions suppressed by powers of $\sim E^2/m_*^2 \ll 1$ to $2 \rightarrow 2$ scattering amplitudes. We limit ourselves to processes that involve at least a pair of bosons and use field redefinitions (equivalent to equations of motion) to rewrite terms with derivatives (e.g. $D_\mu B^{\mu\nu}$, $D_\mu W^{a\mu\nu}$, $\square H$, $\not{\partial}\psi$) as terms with fields (see also [72–74]). Operators of the form $|H|^2 \mathcal{O}_6$, with \mathcal{O}_6 a dim-6 operator, can be read directly from, e.g., ref. [75, 76] and generalization to operators with gluons is straightforward, so we omit them here.

$(\mathbf{X}_{\mu\nu})^4$. In models with the *Remedios* structure, we find

$$\text{SU}(2)_L : \quad {}_8\mathcal{O}_{4W} = W_{\mu\nu}^a W^{a\mu\nu} W_{\rho\sigma}^b W^{b\rho\sigma} \quad {}_8\mathcal{O}'_{4W} = W_{\mu\nu}^a W^{b\mu\nu} W_{\rho\sigma}^a W^{b\rho\sigma} \quad (\text{C.1})$$

$${}_8\mathcal{O}_{4\widetilde{W}} = W_{\mu\nu}^a W^{a\nu\rho} W_{\rho\sigma}^b W^{b\sigma\mu} \quad {}_8\mathcal{O}'_{4\widetilde{W}} = W_{\mu\nu}^a W^{b\nu\rho} W_{\rho\sigma}^a W^{b\sigma\mu} \quad (\text{C.2})$$

$$\text{U}(1)_Y : \quad {}_8\mathcal{O}_{4B} = B_{\mu\nu} B^{\mu\nu} B_{\rho\sigma} B^{\rho\sigma} \quad {}_8\mathcal{O}_{4\widetilde{B}} = B_{\mu\nu} B^{\nu\rho} B_{\rho\sigma} B^{\sigma\mu} \quad (\text{C.3})$$

$$\text{SU}(2)_L \times \text{U}(1)_Y : \quad {}_8\mathcal{O}_{2WB} = W_{\mu\nu}^a W^{a\mu\nu} B_{\rho\sigma} B^{\rho\sigma} \quad {}_8\mathcal{O}'_{2WB} = W_{\mu\nu}^a B^{\mu\nu} W_{\rho\sigma}^a B^{\rho\sigma} \quad (\text{C.4})$$

$${}_8\mathcal{O}_{2\widetilde{W}\widetilde{B}} = W_{\mu\nu}^a W^{a\nu\rho} B_{\rho\sigma} B^{\sigma\mu} \quad {}_8\mathcal{O}'_{2\widetilde{W}\widetilde{B}} = W_{\mu\nu}^a B^{\nu\rho} W_{\rho\sigma}^a B^{\sigma\mu}. \quad (\text{C.5})$$

Notice that $B_{\mu\nu} \widetilde{B}^{\mu\nu} B_{\rho\sigma} \widetilde{B}^{\rho\sigma}$ (and similar for W) can be eliminated in favor of the above using the properties of the Levi-Civita tensor.

$D\psi^2(\mathbf{X}_{\mu\nu})^2$. Strongly interacting fermions and vectors generate

$${}_8\mathcal{O}_{TWW} = \mathcal{T}^{\mu\nu} W_{\mu\rho}^a W_{\nu}^{a\rho} \quad {}_8\mathcal{O}_{TBB} = \mathcal{T}^{\mu\nu} B_{\mu\rho} B_{\nu}^{\rho} \quad (\text{C.6})$$

$${}_8\mathcal{O}_{TWB} = \mathcal{T}^{a\mu\nu} W_{\mu\rho}^a B_{\nu}^{\rho} \quad (\text{C.7})$$

where $\mathcal{T}^{\mu\nu} = \frac{i}{4} \bar{\psi}(\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu) \psi$ and $\mathcal{T}^{a,\mu\nu} = \frac{i}{4} \bar{\psi}(\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu) \sigma^a \psi$ for $\text{SU}(2)_L$ doublets. On the other hand ${}_8\mathcal{O}_{JWW} = \epsilon^{abc} J_\psi^{a\nu} W_{\rho\mu}^b \overleftrightarrow{D}_\nu \widetilde{W}^{c\rho\mu}$, ${}_8\mathcal{O}_{JWB} = J_\psi^{a\nu} W_{\rho\mu}^a \overleftrightarrow{D}_\nu \widetilde{B}^{\rho\mu}$ are odd under both C and P , and CP even ($J_\psi^\nu = \bar{\psi} \gamma^\nu \sigma^a \psi$, $J_\psi^\nu = \bar{\psi} \gamma^\nu \psi$ denote universal $\text{SU}(2)_L \times \text{U}(1)_Y$ currents, the extension to other cases being straightforward). Operators of the form $J_\psi^\nu B^{\mu\rho} D_\mu \widetilde{B}_{\rho\nu}$ (and similarly for $W_{\mu\nu}^a$), or operators involving $\bar{\psi}(\gamma^\mu \overleftrightarrow{D}^\nu - \gamma^\nu \overleftrightarrow{D}^\mu) \psi$ vanish due to Bianchi identities. The operators ${}_8\mathcal{O}_{JWW}$, ${}_8\mathcal{O}_{JWB}$ and ${}_8\mathcal{O}_{TWB}$ cannot arise in the model of section 4.2.1, as they are not singlets under \mathcal{G} in eq. (4.9) — the former are also suppressed for ψ pseudo-Goldstini.

$D^4 H^4$. In models where the Higgs is composite,

$${}_8\mathcal{O}_{\{D\}H} = (D_{\{\mu}H^\dagger D_{\nu\}}H)^2 \quad {}_8\mathcal{O}_{DH} = (D_\mu H^\dagger D^\mu H)^2 \quad (\text{C.8})$$

mediate interaction between four (longitudinal) vectors, that might be relevant in the model $\text{ISO}(4)/\text{SO}(4)$ of section 4.2.2, where the leading contribution from \mathcal{O}_H is suppressed. Operators that involve the $\mu \leftrightarrow \nu$ antisymmetric part of $D_\mu H^\dagger D^\nu H$ transform as a $(\mathbf{1}_L, \mathbf{3}_R)$ of $\text{SU}(2)_L \times \text{SU}(2)_R$ and those with the symmetric part of $D_\mu H^\dagger \sigma^a D^\nu H$ as $(\mathbf{3}_L, \mathbf{3}_R)$, so that both break custodial symmetry; while the custodial preserving $(D_{[\mu}H^\dagger \sigma^a D_{\nu]}H)^2$ can be rewritten as eq. (C.8) using the properties of Pauli matrices. For completeness, we list $(H^\dagger D_\mu D_\nu H + D_\mu D_\nu H^\dagger H)^2$ and $(H^\dagger \sigma^a D_\mu D_\nu H - D_\mu D_\nu H^\dagger \sigma^a H)^2$ which however vanish in ISO models.

$D^2 H^2 (X_{\mu\nu})^2$. On the other hand,

$${}_8\mathcal{O}_{HWW} = D_\mu H^\dagger D_\nu H W_\rho^{a\mu} W^{a\nu\rho}, \quad {}_8\mathcal{O}_{HBB} = D_\mu H^\dagger D_\nu H B_\rho^\mu B^{\nu\rho} \quad (\text{C.9})$$

$${}_8\mathcal{O}_{HWW}'' = D_\mu H^\dagger \sigma^a D_\nu H W_\rho^{b\mu} W^{c\nu\rho} \epsilon^{abc}, \quad {}_8\mathcal{O}_{HWB} = D_\mu H^\dagger \sigma^a D_\nu H W_\rho^{a\mu} B^{\nu\rho} \quad (\text{C.10})$$

$${}_8\mathcal{O}_{HWW}' = D_\mu H^\dagger D^\mu H W_{\nu\rho}^a W^{a\nu\rho}, \quad {}_8\mathcal{O}_{HBB}' = D_\mu H^\dagger D^\mu H B_{\nu\rho} B^{\nu\rho} \quad (\text{C.11})$$

$${}_8\mathcal{O}_{HWB}' = D_\mu H^\dagger \sigma^a D^\mu H W_{\nu\rho}^a B^{\nu\rho} \quad (\text{C.12})$$

contribute to processes with two transverse and two longitudinal modes, although eq. (C.10) and eq. (C.12) are forbidden in the model of section 4.2.1 because they break the global symmetry \mathcal{G} , but are allowed if the Higgs originates from the $\text{ISO}(4)/\text{SO}(4)$ coset. The structure $(H^\dagger \sigma^a D_\mu D_\nu H - D_\mu D_\nu H^\dagger \sigma^a H) W_{\mu\rho}^a B^{\nu\rho}$ is instead forbidden in both models as it breaks both \mathcal{G} and ISO .

$D^3 H^2 \psi^2$. If the fermions are pseudo-Goldstini,

$${}_8\mathcal{O}_{TH} = \mathcal{T}^{\mu\nu} D_\mu H^\dagger D_\nu H \quad (\text{C.13})$$

mediates the leading interaction between fermions and two longitudinal gauge bosons, including effects in $Z_L Z_L$. In non-supersymmetric models, the structure ${}_8\mathcal{O}_{JH}' = J^{a\nu} D^\mu H^\dagger \cdot \sigma^a \overleftrightarrow{D}_\nu D_\mu H$ also arises, but is of limited interest as it clearly only contributes to $W_L^+ W_L^-$ production and is always subdominant w.r.t. dimension-6 effects from $\mathcal{O}_{L,R}^\psi$ and $\mathcal{O}_L^{(3)\psi}$.

$DH^2 \psi^2 X_{\mu\nu}$. Finally, for completeness, we mention contributions to $\bar{q}q \rightarrow V_T Z_L'$ when fermions and Higgs are composite and gauge bosons are dipole-strong. The operator $J_H^\mu J_\psi^\nu B_{\mu\nu}$ (with $J_H^\mu = H^\dagger \overleftrightarrow{D}^\mu H$) is forbidden by custodial symmetry. Instead $J_H^\mu J_\psi^{a\nu} W_{\mu\nu}^a$, $J_H^{a\mu} J_\psi^\nu W_{\mu\nu}^a$ and $J_H^{a\mu} J_\psi^{b\nu} W_{\mu\nu}^c \epsilon^{abc}$ are suppressed in the $\text{SO}(5)/\text{SO}(4)$ model as they break the global symmetry and in ISO models as they break the shift symmetry.

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