Pauli Nonlocality in Heavy-Ion Rainbow Scattering: A Further Test of the Folding Model

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Nonlocal interactions are an intrinsically quantum phenomenon. In this work we point out that, in the context of heavy ions, such interactions can be studied through the refractive elastic scattering of these systems at intermediate energies. We show that most of the observed energy dependence of the local equivalent bare potential arises from the exchange nonlocality. The nonlocality parameter extracted from the data was found to be very close to the one obtained from folding models. The effective mass of the colliding, heavy-ion, system was found to be close to the nucleon effective mass in nuclear matter. [S0031-9007(97)02958-X]

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Of fundamental importance in nuclear physics are the effects arising from the Fermi nature of the nucleons. When calculating interaction potentials between nuclei, these effects translate into a nonlocality. This Pauli nonlocality has been discussed in the context of the nucleon-nucleus scattering [1,2]. A fully microscopic calculation of the nucleus-nucleus interaction is quite complicated and one relies here on procedures such as the resonating-group method [3]. Other methods rely on relating the nucleus-nucleus nonlocality to that of the nucleon-nucleus one using folding procedure [4]. However, the prediction of Jackson and Johnson [4], namely, the nonlocality range in the nucleus-nucleus systems, is smaller than that in the nucleon-nucleus one by about the inverse of the reduced mass in the former, was never really subjected to tests. In this Letter we supply such a test through a careful analysis of the elastic scattering of the systems $^{12}C + {}^{12}C$ and $^{16}O + {}^{12}C$ at intermediate energies.

Before we set the stage for our analysis of exchange effects in the ion-ion interaction, we first say a few words about this interaction. The effective, one-body interaction that determines the elastic scattering between two nuclei can be written in a schematic way as

$$V(\vec{r},\vec{r}\,') = V_{\rm bare}(\vec{r},\vec{r}\,') + \sum_i V_i(\vec{r}) G_i^{(+)}(\vec{r},\vec{r}\,';E) V_i(\vec{r}\,') \,. \qquad (1)$$

The first term, $V_{\rm bare}(\vec{r},\vec{r}\,')$, is usually called the bare interaction. It represents the ground state expectation value of the interaction operator, which contains as basic input the average effective nucleon-nucleon force (G matrix). The nonlocality here is solely due to the Pauli exclusion principle and in what follows we refer to it as the Pauli nonlocality. The second term contains the contribution arising from virtual transitions to intermediate states i (inelastic channels, transfer channels, etc). The corresponding nonlocality arises almost entirely from the polarizations that ensue in the heavy-ion system owing to the propagation in the intermediate channels. This is exemplified by the channel Green's function $G_i^{(+)}(\vec{r},\vec{r}\,';E)$, which contains an explicit energy dependence. This latter contribution is called the Feshbach term and thus we refer to its nonlocality as the Feshbach nonlocality.

When confronting theory with experiment one usually relies on a one-body optical model with a local potential. This brings into light immediately the issue of extracting from Eq. (1) a local equivalent potential. This potential is thus defined through the equation

$$\int V(\vec{r},\vec{r}\,')\psi_E^{(+)}(\vec{r}\,')\,d\vec{r}\,' \equiv V(\vec{r},E)\psi_E^{(+)}(\vec{r})\,, \qquad (2)$$

where $\psi_E^{(+)}(\vec{r})$ is the exact wave function that describes the elastic scattering of the nucleus-nucleus system.

Clearly, from the structure of Eq. (1) for $V(\vec{r},\vec{r}\,')$, the energy-dependent local equivalent potential is

$$V(\vec{r},E) = V_{\rm PAULI}(\vec{r},E) + V_{\rm FESHBACH}(\vec{r},E)\,, \qquad (3)$$

where $V_{\rm PAULI}$ is the local equivalent of $V_{\rm bare}$ and $V_{\rm FESHBACH}$ is the corresponding one for the second term on the right-hand side of Eq. (1). Here $V_{\rm FESHBACH}$ is manifestly complex, whereas $V_{\rm PAULI}$ is taken to be predominantly real. A small imaginary component in $V_{\rm PAULI}$ may be present due to the complex nature of the underlying effective nucleon-nucleon interaction (G matrix) and as a consequence of Eq. (2) through the wave function. The energy dependence of $V_{\rm PAULI}$ would have two origins: the nucleon-nucleon G matrix and, more importantly, the Pauli nonlocality.

Notwithstanding the fact that the major part of the nonlocality in the potential is related to channel couplings (Feshbach nonlocality), we take the view that the effect of these couplings is embedded in the energy-dependent imaginary potential. Further energy dependence may be expected in the real part of the potential that comes from the dispersion relation. However, the part of the ion-ion interaction which contains this Feshbach nonlocality-related energy dependence is concentrated in the surface. Therefore, in the inner region the ion-ion potential is expected to have its energy dependence arising predominantly from the Pauli nonlocality alluded to above. The

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probe of the inner region is made possible through the elastic scattering at intermediate energies [5].

In a recent review article [5] the phenomenon of rainbow scattering seen in several heavy-ion elastic scattering data was discussed. It was emphasized that by measuring the angular distribution of systems such as $^{12}C + ^{12}C$, $^{16}O + ^{16}O$, etc., in the energy region where the far-side amplitude dominates, one is able to extract unambiguously the depth of the real part of the ion-ion potential. This has been shown in detail in [6–9]. Further, by tracing the nucleus-nucleus interaction to its underlying, density-dependent, effective nucleon-nucleon interaction, one is eventually able to extract the compressibility of nuclear matter,

$$K = 9\rho_0^2 \frac{\partial^2}{\partial \rho^2}\left(\frac{E}{A}\right)_{\rho=\rho_0}, \tag{4}$$

where ρ is the density and E/A is the average binding energy per nucleon of the cold system. The extracted value of K from the $^{16}O + ^{16}O$ system [9] at several center of mass energies was found to be roughly 220 MeV, indicative of a soft equation of state. What other physics may one extract from these angular distributions? The question which we raise here is certainly asked by many other heavy-ion physicists. We shall give sufficient evidence in the affirmative to the above question. In particular, we show below that the same data that were analyzed by Khoa *et al.* [7–9] for the purpose of the extraction of K can be used to extract the nonlocality parameter b which measures the nonlocal spread in configuration space where the ion-ion force is operative. The extracted value of b is very close to the one predicted 22 years ago by Jackson and Johnson [4], who showed within the single folding model that $b \approx b_0 m/\mu$, where b_0 is the nucleon-nucleus nonlocality parameter, m is the nucleon mass, and μ is the reduced mass of the nucleus-nucleus system. Theoretically, it was estimated [10,11] that $b_0 \sim 1$ fm. By an extensive fit of nucleon-nucleus elastic scattering data, Perey and Buck [1] have found $b_0 = 0.85$ fm.

As stressed in the introduction, we assume that the bare nucleus-nucleus real potential is nonlocal and we adopt the following form for it:

$$V_{\mathrm{bare}}(\vec{r}, \vec{r}\,') = V_{\mathrm{NL}}\left(\frac{|\vec{r} + \vec{r}\,'|}{2}\right)\exp\left[-\frac{(\vec{r} - \vec{r}\,')^2}{b^2}\right], \tag{5}$$

where the nonlocal potential $V_{\mathrm{NL}}(|\vec{r} + \vec{r}\,'|/2)$ is of a density-density folding inspired form. Using the Perey prescription [1], based on Eq. (2), for finding a local equivalent potential, we obtain

$$V_{\mathrm{LE}}(r,E) = V_{\mathrm{NL}}(r)\exp\left\{-\frac{\mu\ b^2}{2\ \hbar^2}[E_{\mathrm{c.m.}} - V_{\mathrm{LE}}(r,E) - V_{\mathrm{C}}(r)]\right\}, \tag{6}$$

where we use the notation V_{LE} to designate V_{PAULI}. Here $V_{\mathrm{C}}(r)$ is the Coulomb interaction given as usual by

$$V_{\mathrm{C}}(r) = \begin{cases} (3R_{\mathrm{C}}^2 - r^2)Z_1Z_2e^2/2R_{\mathrm{C}}^3 & r < R_{\mathrm{C}} \\ Z_1Z_2e^2/r & r \geq R_{\mathrm{C}} \end{cases}, \tag{7}$$

where R_{C} is the Coulomb radius. Thus a plot of $\ln V_{\mathrm{LE}}(r,E)$ against $E_{\mathrm{c.m.}} - V_{\mathrm{LE}}(r,E)$ should yield a straight line whose slope is just $-\mu b^2/2\hbar^2$.

As said above, ample proof has been accumulated over the last several years, which indicates that the real part of the ion-ion interaction, at very short distances, can be unambiguously extracted from the refractive scattering of heavy ions at intermediate energies. Elastic scattering angular distributions for the system $^{12}C + ^{12}C$ at intermediate energies were analyzed in Ref. [12]. It was shown that the region of radial sensitivity, where the optical potential is probed, is $r \simeq 4$ fm. Those angular distributions are dominated by the far side component in the region of momentum transfer $q \geq 400$ MeV/c. Theoretically, using the semiclassical approach, it is possible to demonstrate [13] that for this value of momentum transfer the real part of the potential is probed at interacting distances around 4 fm.

Having spelled out the possible limitations of our model, we turn now to the extraction of the nonlocality range parameter b, using Eq. (6) and the potential values in the probed radial region, which were unambiguously determined from the elastic data analyses of Refs. [7,12,14–16]. In Fig. 1(a) is shown $\ln V_{\mathrm{LE}}(r)$ versus $E_{\mathrm{c.m.}} - V_{\mathrm{LE}}(r)$ for the system $^{12}C + ^{12}C$ at $r = 4$ fm (circles). The extracted value of b is 0.14 fm to be compared to 0.15 fm expected from single folding result of Ref. [4], $b \approx b_0 m/\mu$, where b_0 is about 0.9 fm. Also shown in the inset in Fig. 1(a) is the energy dependence of $V_{\mathrm{LE}}(r = 4\ \mathrm{fm})$. Similar analysis was made on the system $^{16}O + ^{12}C$. This is shown in Fig. 1(b) (circles). Here b was found to be 0.11 fm, whereas $b_0 m/\mu$ is 0.13 fm. We point out that within about 10% accuracy, as shown in Fig. 1, the energy dependence of V_{LE} extracted from data analyses is described by Eq. (6). There is also an excellent agreement for the extracted b values and those predicted by the folding model. These findings are consistent with our hypothesis that associates the main energy dependence of the local equivalent potential to exchange nonlocal effects.

We should mention that the region of radial sensitivity, where the potential is unambiguously extracted from refrative elastic data, is rather system dependent. For example, nucleon, deuteron, and alpha-nucleus systems present radial sensitivity near $r \simeq 0$ fm [17] while, as discussed above, for systems like $^{12}C + ^{12}C$ this region is around 4 fm. Thus, with the aim to extend our analyses to other systems, we have considered the potential values at $r = 0$ fm. The potential extrapolation from the

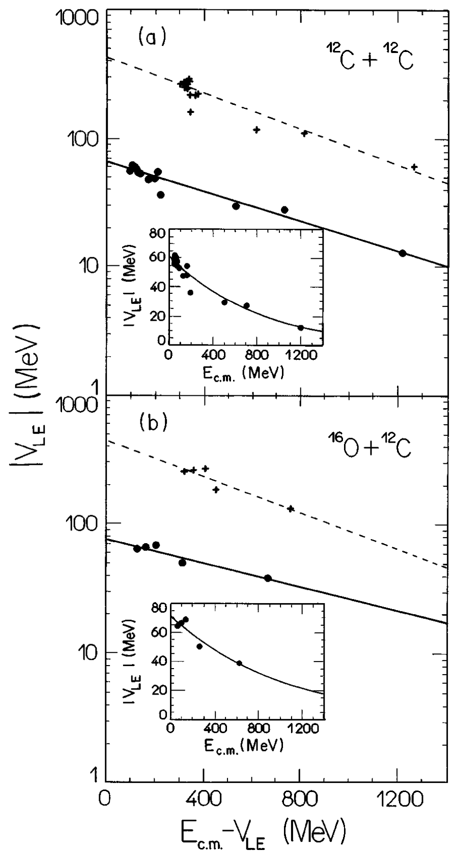

FIG. 1. The energy dependence of the local equivalent potential, V_{LE} at $r = 4$ fm (circles) and $r = 0$ fm (crosses), in the energy range $10 \leq E_{lab}/A \leq 200$ MeV/nucleon for (a) $^{12}C + ^{12}C$ and (b) $^{16}O + ^{12}C$. The solid and dashed lines represent the Perey prescription at $r = 4$ fm and $r = 0$ fm, respectively (see text for details).

sensitivity region to $r = 0$ is shape dependent. Therefore, in our analyses we have considered only realistic shapes, such as those provided by the DDM3Y folding calculations. Using this extrapolation for the ^{12}C, $^{16}O + ^{12}C$ systems, the energy dependence of $V_{LE}(r = 0)$ (crosses in Fig. 1) is well described by Eq. (6) and the corresponding b values are also very close to those obtained through the potential values at $r = 4$ fm.

We have used expression (6) and the data analysis from Refs. [1,7–9,12,14,16–18] to extract $V_{NL}(0)$ as a function of μ. For the systems which the data energy range was not sufficiently extensive, we have used the b values from Ref. [4]. As is shown in Fig. 2, $V_{NL}(0)$ increases linearly with μ.

A further test of the consistency of our analysis method is supplied by a look at the effective reduced mass μ^* of the combined system. Back in 1956, Frahn established the following simple relation [19]:

$$\frac{\mu^*(r)}{\mu} = \frac{1}{1 + \frac{\mu b^2}{2\hbar^2} \mid V_{NL}(r) \mid}. \tag{8}$$

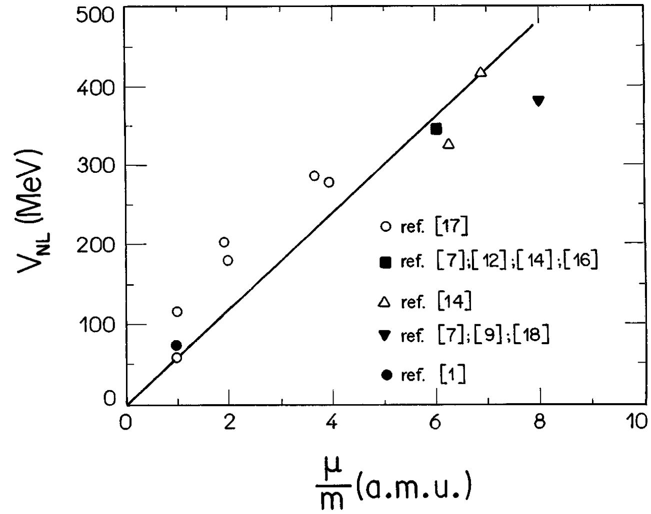

FIG. 2. The resulting nonlocal potential $V_{NL}(r = 0)$, using the Perey prescription [1] and the elastic data analyses as a function of the reduced mass of the system. The data point [•] is the result of the Perey and Buck analyses of several nucleon-nucleus systems. The solid line in the figure serves only as a guide to the eye.

We stress at the outset that the concept of effective mass is intimately related to the nonlocal nature of the interaction [19]. We further stress that since the nucleon effective mass determined from the mean free path in the nuclear matter [20,21] is $m^*/m \cong 0.7$, one should find $\mu^*(0)/\mu \approx 0.7$. In Fig. 3 we show the extracted $\mu^*(0)/\mu$, using Eqs. (6) and (8) and the experimentally determined $V_{LE}(0)$. Clearly, our expectations are reasonably met.

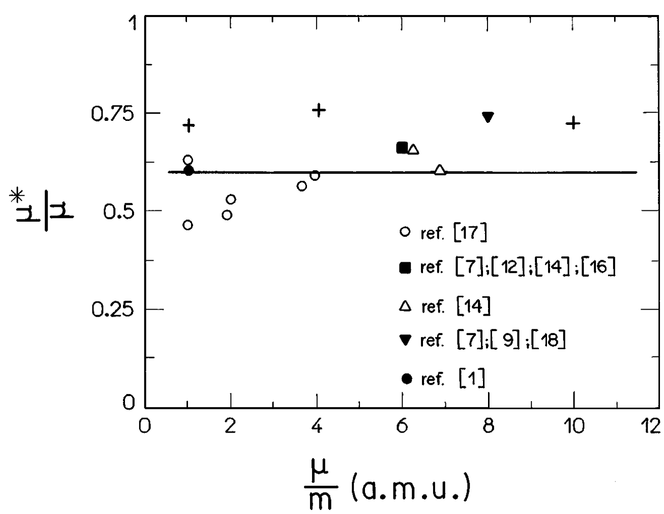

FIG. 3. The dimensionless effective reduced mass μ^*/μ as a function of the reduced mass of the colliding system. The solid line in the figure represents the average value from several systems. The data point [•] is the result of the Perey and Buck analyses of several nucleon-nucleus systems. The [+] points represent the theoretical predictions using the generator coordinate method (see text for details).

It is worth mentioning that the same conclusion can also be attained through theoretical considerations. In fact, in a procedure based on a fully microscopic generator coordinate method, using a Skyrme interaction, aiming at the extraction of the collective potential and effective mass for the giant dipole resonance for light double magic nuclei [22], the proposed behavior was completely verified. The results from these theoretical calculations are represented by crosses in Fig. 3.

It is further worth mentioning that Perey and Buck have analyzed [1] a large set of elastic scattering data for several nucleon-nucleus systems. They have taken into account the effects of exchange nonlocality using a Schrödinger-like integrodifferential equation. It is impressive that the results of those analyses, included in Figs. 2 and 3 as the full circles, are very similar to the results for the heavier systems that we have analyzed.

We turn now to the case of exotic, radioactive, nuclei. Recently, measurements of elastic scattering of ^{11}Li with ^{12}C [23] and ^{28}Si [24] targets, at only one energy, were reported. The phenomenological optical potential in conjunction with coupled channels was employed in the analysis [25–27]. The simple cluster model for the ground state of ^{11}Li implies a structure where the core nucleus ^{9}Li is weakly bound to a dineutron. The folding procedure of Ref. [4] would then lead to a larger b. This in turn will translate into a stronger energy dependence of the bare halo nucleus-ion potential. An extension of the measurement to other energies is urgently called for testing these proposals.

In conclusion, we have considered the exchange nonlocal effects of the nucleus-nucleus interaction arising from the Fermi nature of the nucleons. We have demonstrated in this paper that most of the observed energy dependence of the local equivalent potential, extracted from the refractive elastic scattering data analysis of nucleus-nucleus systems at intermediate energies, arises from nonlocal exchange effects. The obtained values of the parameter b, which measures the range of this Pauli nonlocality, agrees with predictions using the single folding model. The effective mass of the system $\mu^*(0)/\mu$ is found to be very close to the nucleon effective mass of 0.7. The relevance of these findings to the scattering of radioactive, halolike, nuclei is briefly discussed.

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