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Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) has been adopted as a standard for many modern wireless applications requiring high data rate due to bandwidth efficiency, resistance to frequency selective fading and simple digital realization using IFFT/FFT operations. However, physical implementation of the OFDM system suffers from several difficulties. One of the major limitations of OFDM is that it suffers from high peak-to-average power ratio (PAPR), which results in inter-carrier interference (ICI), high out-of-band radiation, and degradation of bit error rate performance. In this paper, different OFDM PAPR reduction techniques are reviewed and analyzed based on their computational complexity, bandwidth requirement and error performance.

Keywords

OFDM, PAPR, Companding, PTS, SLM, TR and TI

1. INTRODUCTION

Conventional single carrier modulation techniques have a major limitation that these schemes can achieve only small data rates mainly owing to the multipath nature of the wireless channel. These days many wireless multimedia applications require high data-rates. However, with increase in data-rate of a communication system, the period of symbol is reduced, which leads to severe inter-symbol interference (ISI), thereby necessitating a complex equalization procedure at the receiver. Orthogonal Frequency Division Multiplexing (OFDM) is a special kind of modulation technique, in which whole of the frequency selective fading multipath channel is divided into numerous narrow sub channels which are orthogonal and offer flat fading. In OFDM system, a stream containing high-speed data is transmitted using a number of parallel lower data rate subcarriers. As the individual data rate is much less the total data rate, ISI is avoided owing to the long symbol duration [2].

Due to its advantages, OFDM modulation has been employed in many wireless applications such as Wireless Personal Area, Local Area and Metropolitan Area Networks, Digital Audio and Video Broadcasting [3]. It is also being used as a modulation technique for IEEE 802.20, IEEE 802.16 and 3GPP-LTE [3]. A simple one tap equalizer is required at the receiver side as the effect of ISI is eliminated by introducing cyclic prefix (CP) [2].

Despite its advantages, some problems still need to be resolved in the design of OFDM systems. One of the prominent problems is high Peak-to-Average Power Ratio (PAPR) of the transmitted OFDM signals. The transmit signals in an OFDM system can have high peak values in the time domain as many subcarriers are added due to IFFT operation at the transmitter. Therefore, OFDM systems have a high PAPR as compared with single-carrier communication systems. As a result of this problem, the Signal-to-Noise Ratio of Analog-to-Digital Converter and Digital-to-Analog Converter is reduced, which further degrades the efficiency of the high power amplifier at the transmitter side. As more efficient Power Amplifier is essential in a mobile terminal due to the limited battery power, the PAPR problem is more troublesome in the uplink design. Hence it is essential to reduce PAPR in OFDM based systems.

The methods used to reduce PAPR in OFDM can be classified based on various criteria. First, these schemes can be classified as Distortion based techniques and Non-distortion techniques [3]. Distortion based techniques introduce re-growth of spectral components in the signal. The Peak Clipping and Filtering is the simplest distortion based technique, but it can cause in-band as well as out of band interference as it may destroy the orthogonality among subcarriers (SC) in the OFDM signal [4]. Modifications to this technique are proposed in [5]. Another important distortion based scheme to reduce PAPR is companding. Various types of companding techniques e.g. μ-law companding [6], exponential companding [7], and trapezoidal companding [8], [9] are proposed in literature.

Non-distortion PAPR reduction schemes include Coding technique [10], Partial Transmit Sequence (PTS) [11] and Selective Mapping (SLM) [12]. These techniques do not distort the OFDM signal as a result of which no spectral re-growth takes place.

Secondly, the PAPR reduction techniques can also be classified according to the computational operations performed in the frequency domain. Based on this criterion, these schemes can be termed as multiplicative and additive schemes [13]. Multiplicative schemes include SLM and PTS, and Additive schemes include Tone reservation (TR) [14], Tone Injection (TI) [15], and clipping.

Thirdly, these techniques can be classified as deterministic or probabilistic schemes [13]. In deterministic techniques such as Peak Clipping, the PAPR of the signal is kept below a prefixed threshold level. In this process, the OFDM signal gets distorted. In probabilistic techniques, several OFDM signals are generated and the one having smallest PAPR is transmitted. This method includes PTS, SLM, TI and TR techniques.

Lastly, the PAPR reduction techniques can be classified depending on whether these transform the original OFDM signal before or after the multi carrier modulation at the transmitter [9]. Techniques like coding, SLM and PTS fall under the first category and different types of companding techniques belong to the second type as they change the formation of the signal after multi carrier modulation.

In the remaining paper, the distribution of PAPR is analyzed based on the characteristics of OFDM signals. Then various techniques to suppress PAPR in OFDM systems are described and reviewed. Various criterions for the choice of appropriate PAPR reduction technique are also discussed in this paper.

2. OFDM MODEL AND PAPR

2.1 OFDM System Model

OFDM is a block transmission scheme. At the transmitter side, a block of *N* data symbols represented as $X = \{X_k, k = 0,1,....,N-1\}$ is first converted from serial to parallel stream. Each symbol modulates separately one of the subcarriers taken from $\{f_k, k = 0,1,.....,N-1\}$.

To ensure, the subcarrier frequencies, $f_k = k\Delta f$, are separated equally with subcarrier spacing $\Delta f = 1/NT_s$. Here T_s is the original symbol period and NT_s denotes the OFDM block period.

The OFDM signal is generated by the summation of the orthogonal and modulated subcarriers. Hence the complex baseband OFDM signal in continuous time domain can be represented as [3],

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{2\pi f_k t}, \qquad 0 \le t \le NT_s \qquad (1)$$

The discrete time baseband OFDM signal which is sampled at the Nyquist rate $t = nT_s$ can be expressed as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{N} n}, \quad n = 0,\ 1,\ ....,\ N-1. \qquad (2)$$

The discrete time OFDM signal is needs to be sampled at more than Nyquist rate for better approximation of the continuous time signal. Let it be oversampled by a factor of L i.e. the sampling frequency $f_s = L / T_s$. The discrete time OFDM signal can then be expressed as,

$$x_L[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{nk}{LN}},\ n = 0,\ 1,\ ....,\ LN-1. \qquad (3)$$

It is to be noted that the time-domain samples of the OFDM signal which is *L*-times oversampled, are actually *LN*-point IFFT of the data block with $(L-1)N$ zero-padding. An oversampling factor of $L = 4$ is enough to successfully represent the peak value of the continuous time OFDM signal [16].

2.2 Peak-to-Average Power Ratio

The PAPR of an OFDM signal $x(t)$ in continuous time domain is defined as the ratio of maximum instantaneous power to its average power

$$PAPR[x(t)] = \frac{\max\limits_{0 \le t \le NT_s} \left[|x(t)|^2 \right]}{E\left\{ |x(t)|^2 \right\}} \qquad (4)$$

where $E\{\cdot\}$ denotes the expectation operator.

The PAPR of the OFDM signal after *L* times oversampling is given by

$$PAPR\left(x_L[n]\right) = \frac{\max\limits_{0 \le n \le NL-1} \left[|x_L[n]|^2 \right]}{E\left\{ |x_L[n]|^2 \right\}} \qquad (5)$$

2.3 Complementary Cumulative Distribution Function (CCDF) of PAPR

The CCDF can be used to estimate the bounds of the PAPR and is generally used as a performance indicator for most of the PAPR reduction schemes.

As per probability theory, the CCDF represents the probability that a random variable *X* will be having a value greater than or equal to real valued number *x* [14]. Central limit theorem suggests that the real and imaginary parts of the discrete OFDM signal follow Gaussian distributions for sufficiently large number of subcarriers, assuming that each of imaginary and real parts have zero valued mean and a variance of 0.5. This implies that the amplitude of an OFDM signal follow Rayleigh distribution.

The CCDF of the PAPR denotes the probability that the PAPR of the transmitted OFDM symbols exceeds a preset threshold value.

For an OFDM system with *N* subcarriers, in which sampling is done at Nyquist rate, the CCDF of PAPR represented as [14]

$$\Pr ob\left\{PAPR > \lambda\right\} = 1 - \left(1 - e^{-\lambda}\right)^N \qquad (6)$$

where, λ is the fixed threshold level. The above relation is valid if the *N* time domain samples are mutually independent as well as uncorrelated. The above expression does not hold when oversampling is applied and when the number of subcarriers is not large as the assumption of Gaussian distribution does not hold in that case.

For an OFDM system with sufficiently large number of subcarriers, the preceding expression does not hold good. For such a system, an approximate expression of CCDF can be given as [16]

$$\Pr ob\left\{PAPR > \lambda\right\} = 1 - \left(1 - e^{-\lambda}\right)^{2.8N} \qquad (7)$$

Figure 1 represents the distribution of the PAPR of the OFDM signal with 256 number of subcarriers and different values of oversampling factor *L*. As evident from the figure, the PAPR does not increase much beyond *L* = 4.

3. PAPR REDUCTION TECHNIQUES

Several techniques have been proposed till date in the literature for the reduction of PAPR in OFDM systems. PAPR reduction technique should be appropriately selected as per the system requirements, as no single technique may be suitable for all systems.

3.1 Amplitude Clipping and Filtering

The idea of clipping technique is to clip the peak of the OFDM signal below a prefixed threshold level [5]. The output of the amplitude clipper can be expressed as

$$C(x) = \begin{cases} x, & |x| \le \mu \\ \mu, & |x| > \mu \end{cases} \qquad (8)$$

where μ is the prefixed threshold level selected for clipping and is a positive real number.

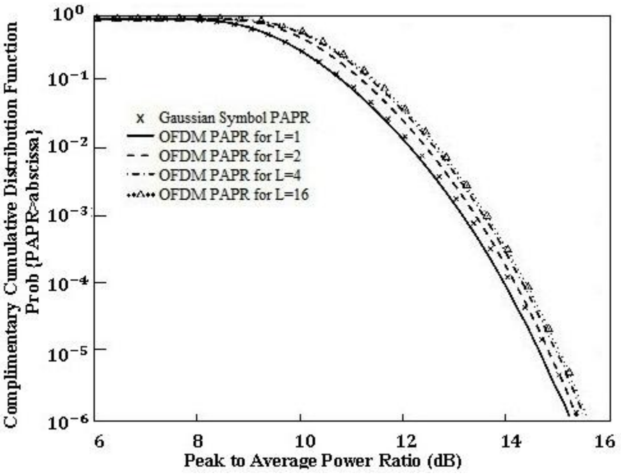

Fig. 1 Distribution of PAPR of *L* times oversampled OFDM signal

The task of the OFDM receiver is to estimate clipping done at the transmitter to undo it at the receiving end accordingly. As it is very difficult for the receiver, the signal gets distorted, since both in-band and out-of band distortion is introduced. Due to in-band distortion, orthogonality among the subcarriers is lost, which degrades the BER performance. In addition loss of spectral efficiency also takes place due to out-of band distortion.

An easy way to reduce out-of band distortion is to filter out the signal after clipping operation, but it cannot reduce in-band distortion. Moreover, the effect of clipping is undone to some extent as filtering can cause peak re-growth. Hence in practice, a repeated dual operation of clipping followed by filtering is employed [5], [17]. But it also comes at the cost of increased computational complexity.

The clipping and filtering methods also includes peak windowing technique in which the OFDM signal is multiplied with a correcting function. Gaussian, Kaiser and cosine filters can be used as correcting functions as these filters have nearly rectangular frequency spectrum. As a result, out-of band frequency spectrum is greatly suppressed [18].

3.2 Companding

Companding is a good technique as it can effectively reduce the PAPR with less complexity irrespective of the number of subcarriers in the OFDM signal. The principle of companding is to decrease the dynamic range of signal by compressing high peaks and/or by increasing the level of small signals. Compressed original signal can be reconstructed at the receiver side by performing the reverse operation.

The companding technique makes the distribution of the signal quasi-uniform as it compresses the signal, ensuring that the signal peak does not exceed system's limitations. It has been reported in literature that the companding techniques have better performance than clipping [6].

In OFDM signal, large peaks occur very rarely just similar to speech signals; hence the same companding technique can be used to improve the performance of OFDM at transmitter side.

However the reduction in PAPR is obtained at the cost of degraded BER. Two factors are responsible for this problem: Firstly, companding shifts the data symbols at the transmit side from their original constellation locations. Secondly, the decompanding process expands the channel noise at the receiver resulting in increase in BER.

A large number of companding techniques have been reported and studied in literature. All these techniques distort the shape of the OFDM signal.

In μ-law companding, the peak value of the OFDM signal remains unchanged, but the average power of the OFDM signal after companding is increased and hence the PAPR is reduced [6]. Exponential companding [7] transforms Rayleigh distributed amplitude of the signal into a uniformly distributed OFDM signal by employing an exponential function.

Another companding technique called Trapezoidal companding is a good method to suppress the PAPR of signal with comparatively low error rate [8]. It employs a piecewise function which is defined in three different magnitude intervals of the OFDM signal.

In [9], a general design criterion was proposed to make an optimum tradeoff between the PAPR reduction and error performance. It is proved that appreciable PAPR reduction is feasible by a proper choice of the companding transforms and their parameters.

3.3 Coding

Block coding is one of the well-known techniques for decreasing PAPR in which input data is encoded into a codeword having low PAPR. In [10], a method is demonstrated to decrease the value of PAPR of OFDM signal which has four subcarriers. The three-bit input data is mapped into four-bit codeword, the fourth bit being the parity bit in the frequency domain. Later the use of Cyclic Code was demonstrated in [19] with good results. An efficient Simple Block Code was used in [20], but it is not useful for large frame size. To overcome this limitation, two methods namely Complement Block Coding [21] and Modified Complement Block Coding techniques [22] were proposed. These techniques also offer flexibility in choosing the frame size, coding rate and low implementation complexity. In these schemes, the probability of OFDM signals with large peaks is reduced by using the extra bits which are added to the original bits containing information.

Table 1 shows the comparisons of the PAPR reduction achieved by using various coding schemes [16]. Here N, n and R represent the number of subcarriers, frame size and coding rate for the concerned coding scheme. It is clear from the table that a PAPR reduction of nearly 3-dB is achievable for coding rate $R > (\mathrm{N}-2)/N$ by using Simple Block Coding technique (CBC) with large frame size. In addition the results obtained for various values of coding rate $R \le (\mathrm{N}-1)/N$ are almost same for CBC. Further, additional 3-dB more PAPR reduction can be achieved by using Modified Complement Block Coding (MCBC) as compared to other coding methods for any frame size, if the coding rate is 3/4. This feature of flexibility in the selection of coding rate and less complexity makes the CBC and MCBC methods suitable for OFDM based systems loaded with large frame size and high coding rates.

A PAPR of more than 3 dB can be attained by employing Golay complementary sequences as code words [23]. Although, these sequences have better error correction capabilities and low PAPR, large data loss occurs. In general, the reduction in PAPR in all coding schemes can be achieved only at the cost of data rate.

Table 1. Performance Comparison of some coding Techniques

N	n	R	PAPR Reduction (dB)			
			CBC	SBC	MCBC	CC
4	1	3/4	3.56	3.56	-	3.56
8	1	7/8	2.59	2.52	-	3.66 (R=3/4)
	2	3/4	2.67	3.72	2.81	
16	1	15/16	2.74	1.16	-	3.74 (R=3/4)
	2	7/8	2.74	2.52	-	
	3	13/16	2.74	-	-	
	4	3/4	2.74	2.98	3.46	
32	1	31/32	1.16	0.55	-	-
	2	15/16	1.16	1.16	-	
	3	29/32	2.75	-	-	
	4	7/8	2.50	2.51	-	
	5	27/32	2.75	-	-	
	8	3/4	2.75	3.00	3.45	

3.4 Selective Mapping and Partial Transmit Sequence

In SLM, the transmit side produces a set of OFDM signals, where all signals represent the same information as the original signal. Then the one having the lowest PAPR is transmitted [12]. However this method is more computationally complex as more than one IFFT blocks are required at the transmitter. In addition, the useful data rate is suffered as additional overhead bits will be required to get back the original data at the receiver. In [24], modified SLM is suggested to reduce the computational complexity and to decrease the amount of side information required to be transmitted.

In PTS, the original input data block is divided into multiple disjoint sub-blocks. All subcarriers belonging to each sub-block are scaled by a phase factor. All these phase factors are statistically independent. Subsequently, the signal with the lowest PAPR is selected for transmission. There are different methods for dividing the data blocks into multiple sub-blocks. These include pseudorandom partition, interleaved partition and adjacent partition. Pseudo-random partitioning is the best among these methods. An improvement over existing PTS in MIMO-OFDM systems was proposed recently in [25], in which the authors used constant modulation algorithm (CMA) to solve the phase optimization problem. The resulting method has less computational complexity without any degradation in error performance compared with existing methods.

Similar to SLM, the PTS also suffers from computational complexity and low data rate. However PTS is less complex than SLM. Moreover it performs better than SLM so far as PAPR reduction is concerned. However PTS requires more bits for sending the side information than SLM [26].

3.5 Tone Reservation and Tone Injection

Both these schemes reserve a subset of subcarriers (also called tones) for the generation of a PAPR reducing signal [14], [15]. These reserved tones are not meant for data representation. The common block diagram of TR and TI is shown in figure 2.

In TR, the goal is to determine signal *c* which must be added to the original signal *x* to decrease PAPR.

In TI, the constellation size is extended in such a manner that every signal point in the original constellation can be represented by many equivalent points in the expanded constellation. This results in extra degrees of freedom and hence can be utilized for reducing PAPR. This method is named so because the process of replacing a signal point in the original constellation by a new point in the expanded constellation is same as applying a subcarrier of appropriate frequency and phase to the original OFDM signal.

Unlike TI, it is essential to send side information along with the transmit signal in case of TR.

4. CRITERION FOR PAPR REDUCTION TECHNIQUES

PAPR reduction can be achieved only at the expense of bandwidth, data rate, BER performance, complexity and/or power efficiency. An efficient trade off of these conflicting factors must be made before selecting an appropriate PAPR reduction technique as follows:

4.1 Decrease in PAPR

Obviously this factor is the most important point in selecting an appropriate PAPR reduction technique. But care must be taken to avoid other unwanted associated drawbacks such as distortion introduced by the techniques.

4.2 Increase in Power of Transmit Signal

Certain techniques require an increase in the average power of the transmit signal after using PAPR reduction method. TR and TI need more signal power at the transmitter side. This necessitates a HPA with large linear operating range resulting in degraded BER performance.

4.3 Effective Loss in Data Rate

Some PAPR reduction techniques decrease the effective data rate of the OFDM signal. These include block coding, SLM and PTS. In block coding, parity check bits are required to be added. In SLM and PTS, transmit side information is needed to be sent to enable the receiver to bring back the signal to its original constellation size. When channel coding is also applied, further loss of data rate takes place.

4.4 BER Performance Degradation

This important factor is closely linked to the increase in power of the transmit signal. In some techniques, if the transmit signal power is fixed; there might be rise in BER at the receiver. In some techniques such as SLM and PTS, if the side information is not received properly, the entire data block may be lost. This will also degrade the BER performance at the receiver.

4.5 Computational Complexity

Generally, more complex techniques perform better as far as reduction in PAPR is concerned.

4.6 Bandwidth Expansion

The bandwidth of any communication system is a valuable and scarce resource. Those PAPR reduction techniques should be preferred which required less bandwidth.

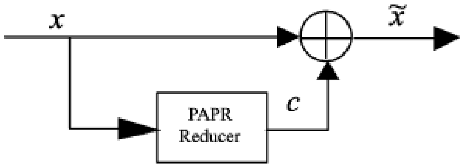

Fig. 2. Block diagram of TR/TI

A good PAPR reduction scheme should offer minimum of PAPR for the smallest possible level of BER. In Table 2, some PAPR reduction techniques are compared.

Table 2. Performance Comparison of some PAPR Reduction Techniques

	Power rise	**Compl-exity**	**Extra band-width required**	**BER degradation**
Clipping	No	Low	No	Yes
Coding	No	Low	Yes	No
Compand-ing	No	Low	No	No
PTS/SLM	No	High	Yes	No
TR/TI	Yes	High	Yes	No

5. CONCLUSION

OFDM has been proved to be a suitable technique for wireless communication due to its spectral efficiency and resistance to channel impairments. However it suffers from high PAPR. Despite two decades of continuous and intensive research, this problem stands in the way of successful implementation of practical OFDM based communication systems. Many PAPR reduction techniques have been reported and proposed so far. In this paper, these techniques are described and compared. Most of these techniques can achieve low PAPR at the cost of bandwidth efficiency, more complexity and poor error performance. Hence no single technique can be considered as best technique. PAPR reduction technique should be appropriately selected as per the system requirements after making optimum tradeoff between different requirements. In addition compatibility with existing standards should also be taken into account.

6. REFERENCES

[1] G. Wunder, Robert F. H Fisher, H. Boche and S. Litsyn, "The PAPR problem in OFDM transmission," IEEE Signal Processing Magazine, pp. 130–144, Nov. 2013.

[2] Y. Wu and W. Y. Zou, "Orthogonal frequency division multiplexing: A multi-carrier modulation scheme," IEEE Trans. Consumer Electronics, vol. 41, no. 3, pp. 392–399, Aug. 1995.

[3] Y. Soo Cho, J. Kim, W. Y. Yang and C. G. Kang "MIMO-OFDM Wireless Communication with MATLAB," 1st Edition., Wiley-IEEE Press, Aug. 2010.

[4] R.O'Neill and L. B. Lopes, "Envelope variations and spectral splatter in clipped multicarrier signals," in Proc. IEEE PIMRC'95, Toronto, Canada, Sept. 1995, pp. 71–75.

[6] J. Amstrong, "Peak to average power reduction for OFDM by repeated clipping and fequency domain filtering," IEE Electron. Lett., vol. 38, no. 5, pp. 246–247, Feb. 2002.

[7] X. B.Wang, T. T. Tjhung, and C. S. Ng, "Reduction of peak-to-average power ratio of OFDM system using A companding technique," IEEE Trans. Broadcasting, vol. 45, no. 3, pp. 303–307, Sept. 1999.

[8] T. Jiang, Y. Yang, and Y. Song, "Exponential companding transform for PAPR reduction in OFDM systems," IEEE Trans. Broadcasting, vol. 51, no. 2, pp. 244–248, Jun. 2005.

[9] J. Hou, J. H. Ge and J. Li, "Trapezoidal companding scheme for peak to average power ratio reduction of OFDM systems," Electron. Letters, vol. 45, no. 25, pp. 1349-1351, Dec. 2006.

[10] S. S. Jeng, and J. M. Chen, "Efficient PAPR reduction in OFDM systems based on a companding technique with trapezium distribution," IEEE Trans. Broadcasting, vol. 57, no. 2, pp. 291–298, June 2011.

[11] A. E. Jones, T. A. Wilkinson, and S. K. Barton, "Block Coding Scheme for Reduction of Peak to Mean Envelope Power Ratio of Multicarrier Transmission Scheme," Elect. Lett., vol. 30, no. 22, pp. 2098–2099, Dec. 1994.

[12] A. Alavi, C. Tellambura, and I. Fair, "PAPR reduction of OFDM signals using partial transmit sequence: An optimal approach using sphere decoding," IEEE Trans. Communications Letters, vol. 9, no. 11, pp. 982–984, Nov. 2005.

[13] S. H. Wang and C. L. Pir, "A low complexity PAPR reduction scheme for SFBC MIMO-OFDM systems," IEEE Signal Processing Letters, vol. 16, no. 11, pp. 941–944, Nov. 2009.

[14] D. W. Lim, S. J. Heo and J. S. No, "An overview of PAPR reduction schemes for OFDM signals," Journal of Communications and Networks, vol. 11, no. 3, pp. 229–239, June 2009.

[15] D.-W. Lim, H.-S. Noh, J.-S. No, and D.-J. Shin, "Near optimal PRT set selection algorithm for tone reservation in OFDM system," IEEE Trans Broadcast., vol. 54, no. 3, pp. 454–460, Sept. 2008.

[16] B. S. Krongold and D. L. Jones, "PAR reduction in OFDM via active constellation extension," IEEE Trans. Broadcasting, vol. 49, no. 3, pp. 258–268, Sept. 2003.

[17] T. Jiang and Y. Wu, "An overview: Peak to average power ratio reduction techniques for OFDM signals," IEEE Trans. Broadcasting, vol. 54, no. 2, pp. 257–268, June 2008.

[18] H. Chen and A. M. Haimovich, "Iterative estimation and cancellation of clipping noise for OFDM signals," IEEE Commun. Lett., vol. 7, pp. 305–307, July 2003.

[19] M. Pauli and H. P. Kuchenbecker, "Minimization of the intermodulation distortion of a nonlinearly amplified OFDM signal," Wireless Pers. Commun., vol. 4, no. 1, pp. 93–101, Jan. 1997.

[20] D. Wulich, "Reduction of peak to mean ratio of multicarrier modulation using cyclic coding," IEE Electronics Letters, vol. 32, no. 29, pp. 432–433, Feb. 1996.

[21] S. Fragicomo, C. Matrakidis, and J. J. OReilly, "Multicarrier transmission peak-to-average power reduction using simple block code," IEEE Electronics Letters, vol. 34, no. 14, pp. 953–954, May 1998.

[22] T. Jiang and G. X. Zhu, "Complement block coding for reduction in peak-to-average power ratio ofOFDMsignals," IEEE Communications Magazine, vol. 43, no. 9, pp. S17–S22, Sept. 2005.

[23] T. Jiang and G. X. Zhu, "OFDM peak-to-average power ratio reduction by complement block coding scheme and its modified version," in The 60th IEEE Vehicular Technology Conference 2004 Fall, Los Angeles, USA, pp. 448–451, Sept. 2004.

[24] H. Lee and S. W. Golomb, "A new construction of 64-QAM Golay complementary sequences," IEEE Trans. Inf. Theory, vol. 52, no. 4, pp. 1663–1670, Apr. 2006.

[25] H. Breiling, S. H. Muller-Weinfurtner, and J. B. Huber, "SLM Peak-Power Reduction without Explicit Side Information," IEEE Communications Letters, vol. 5, no. 6, pp. 239–241, Jun. 2001.

[26] S. Khademi, and A. J. Veen, "Constant modulus algorithm for PAPR reduction in MIMO OFDM/A," IEEE Signal Processing Letters, vol. 20, no. 5, pp. 531-534, May 2013.

[27] S. H. Muller and J. B. Huber, "A comparison of peak power reduction schemes for OFDM," in Proceeding of IEEE Global Telecommunications Conference, Nov. 1997, vol. 1, pp. 1–5.