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Peculiarities of Acoustooptic Transformation of Bessel Light Beams in Gyrotropic Crystals (*)

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Abstract The peculiarities have been studied of acoustooptic (AO) diffraction of quasi- nondiffracting vector Bessel light beams (BLB) on the ultrasound waves in optical gyrotropic cubic crystals. The system of coupled equations describing the process of acoustooptic interaction is solved, diffraction efficiency has been calculated. The mathematical description of AO interaction, which differs from the similar description for the plane optical waves means of two types of synchronism, is conducted. It is shown that besides the usual longitudinal synchronism realized at the equality of phase velocities transmitted and diffracted waves, for Bessel beams it is also necessary to perform the so-called transverse synchronism. It is related with the fact, that Bessel beams with differing cone angles have different spatial structure and, consequently, various values of overlap integral with the input beam. The possibility has been investigated of transformation of the order of phase dislocation of Bessel beams wave front due to AO diffraction. It is proposed to use the process of acoustooptic diffraction in gyrotropic cubic crystals as a method for dynamic manipulation of polarization state of output Bessel beam, particularly for transformation of left- to right (and vice versa) polarization states.

Keywords Gyrotropic Crystals, Transversal Synchronism

1. Introduction

An important area of Bessel light beams (BLB) research is the elaboration of the methods for transformation of phase dislocation order of their wave front. The acousto-optic (AO) interaction is a promising one for these purposes, because it allows one to control dynamically the transformation process, unlike, for example, the well-known methods of transformation of spatial structures and order of BLBs developed in papers [1-3].

It should be noted that whereas the acoustooptic diffraction of light field in the plane-wave approximation or Gaussian light beams has received a rather good study [4-6], there are only few papers on the AO transformation of BLB [7, 8]. As with plane waves, Bessel beams can be represented as rigorous solutions of Maxwell equations [7, 8]. This is of importance for studying the vector AO interactions because they give us the exact knowledge of the polarization states of such beams [9, 10]. Among the various polarization states of BLB, the well-known ones are the radial (ρ-) and azimuthal (φ-) polarizations. The beams with such polarization states are more effective in some applications as compared to the beams with the linear or circular polarization. For example, a number of papers are devoted to applications of such beams in laser technology (see, for example, [11-13]). Due to their non-diffractive nature and a narrow dark central region, the high-order Bessel beams can be used for atom guiding over extended distances [14-16], as well as for focusing cold atoms [13]. In papers [14-15] the orbital angular momentum of BLB is calculated with the demonstration of the transfer of the orbital angular momentum to a low-index particle trapped in optical tweezers with the help of a high-order Bessel beam. Some properties of interfering high-order Bessel beams and BLBs with z-dependent cone angle were examined in [17-20]. The use of such beams for controlling the rotation of microscopic particles in optical tweezers and rotators is demonstrated. The self-healing properties of interfering Bessel beams allow the simultaneous manipulation and rotation of particles in spatially separated sample cells [21]. Thus, the development of methods for generating and transforming Bessel vortices is of both scientific and practical interest.

The using of acousto-optic interaction for separation of transverse electric (TE-) and transverse magnetic (TH-) polarized components of Bessel beams in non gyrotropic crystals have been proposed in [7-8]. In paper [8] the theory of AO diffraction of Bessel light beams on plane acoustic wave in optically anisotropic nongyrotropic crystals is developed.

In this paper the process of the transformation of Bessel beam order is investigated at AO interaction in optically gyrotropic crystals. It should be noted that at AO interaction in uniaxial or biaxial crystals an optical gyrotropy essential only for directions of light propagation in the vicinity of optical axe. But in isotropic medium and in cubic crystals optical gyrotropy should be taken into account at any direction of propagation of light. In spite of relative low value of gyrotropy parameter, its taking into account is important for correct description of the AO interaction.

It should be noted that the specific character of mathematical description of the indicated form of the AO interaction is in the necessity of fulfillment of two types of the phase-matching. Besides the usual longitudinal phase-matching realizable at the equality of the phase velocities of transmitted and diffracted waves, BLBs necessitate at the same time the so-called transversal phase-matching. The latter is related to the fact that BLBs with different cone angles have also the different spatial structure and, consequently, various values of the overlap integral with the diffracted beam. As a result, the AO interaction can effectively be realized only at the maximum of the overlap integral. The calculations of the appropriate integrals should be carried out and conditions should be determined when the overlap integrals are maximal.

The paper is structured as follows. In Section 2 the geometry of AO interaction of vector Bessel light beams in a gyrotropic medium is considered. In Section 3 the tensor of the dielectric permittivity at the presence of the AO interaction in the cylindrical coordinates is presented. In Section 4 the equations for the slowly varying amplitudes of interacting Bessel beams at AO interaction of BLBs are considered. The analysis of overlap integrals and peculiarities of AO interaction of Bessel beams in gyrotropic cubic crystals of different symmetry will be considered in Section 5. A conclusion is given in Section 6.

2. Bessel Beams in a Gyrotropic Medium

Let us consider the geometry of AO interaction when TH-polarized Bessel beam incidents from isotropic medium with the refractive index n_1 on the boundary with a gyrotropic medium (optically gyrotropic crystal of a cubic class of symmetry) along a crystallographic axis z (4-fold,2-fold, or 3-fold axis) of cubic crystals (Fig. 1). It follows from the boundary conditions that in the crystal two Bessel beams with various phase velocities and polarization states will be propagate. Further they will be denoted with symbols right (+) and left (–) [22]. Cones of the wave vectors of these beams are circular ones. Wave vectors $\vec{k}_+$ and $\vec{k}_-$ belonging to the pointed cones and lying within the plane (x, z) are shown in Fig.1.

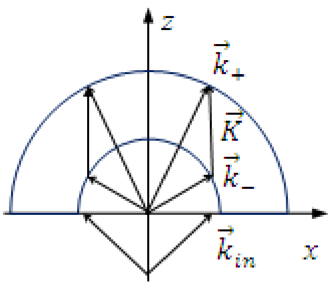


Figure 1. Orientation of wave vectors at acousto-optic interaction in gyrotropic cubic crystal. $\vec{k}_{in}$, $\vec{k}_{\pm}$, are wave vectors of incident (in) and two refracted (+) and (–) Bessel beams propagating in gyrotropic medium, $\vec{K}$ is the wave vector of plane ultrasonic wave.

Thus, at incidence of TH-polarized Bessel beam on the boundary with gyrotropic medium, two BLBs of right (+) and left (–) with various phase velocities propagate [22].

The vectors of the electric field for these beams can be written as

$$\vec{E}_m^{\pm} = A_m^{\pm}\vec{e}_{\pm}(\rho)\exp[i(k_{\pm z}z + m\varphi)] \qquad (1)$$

where the following denotations for vector mode functions $\vec{e}_{\pm}(\rho)$ are introduced [7]

$$\vec{e}_+ = \begin{pmatrix} \left(\frac{im}{qr}J_m(x)(1+\cos(\gamma_+)) - i\cos(\gamma_+)J_{m+1}(x)\right)\vec{e}_\rho + \\ \left(J_{m+1}(x) - \frac{m}{qr}J_m(x)(1+\cos(\gamma_+))\right)\vec{e}_\varphi + \sin(\gamma_+)J_m(x)\vec{e}_z \end{pmatrix}$$
$$\vec{e}_- = \begin{pmatrix} \left(\frac{-im}{qr}J_m(x)(1-\cos(\gamma_-)) - i\cos(\gamma_-)J_{m+1}(x)\right)\vec{e}_\rho + \\ \left(-J_{m+1}(x) + \frac{m}{qr}J_m(x)(1-\cos(\gamma_-))\right)\vec{e}_\varphi + \sin(\gamma_-)J_m(x)\vec{e}_z \end{pmatrix}, \qquad (2)$$

$x = q\rho$, $\vec{e}_{\rho,\varphi,z}$ are the unite vectors of the cylindrical system, $k_{\pm z} = k_z \pm \beta$, $\beta = \alpha\, k_0 / \cos(\gamma_0)$ is the specific polarization rotation, γ_0 is the cone angle of the refracted BLB without gyrotropy, α is the parameter of medium gyrotropy, m is the integer, ρ, φ, z are the cylindrical coordinates,

$$\cos(\gamma_{\pm}) = \cos(\gamma_0)[1 \pm (\alpha/\sqrt{\varepsilon})tg^2(\gamma_0)].$$

Here the amplitudes of beam equally depend on the transverse coordinate, namely, as it follows from Eqs. (1), for m-order BLB this dependence is described by Bessel function of m and m+1 orders, respectively. Moreover, the electric fields contain the longitudinal component proportional to the m-order Bessel function.

Note that the vector electric fields (1), (2) are the rigorous solutions of Maxwell or Helmholtz equations.

3. The tensor of Dielectric Permittivity at the AO Interaction

At the presence of AO transformation the interacting fields are described by the Helmholtz equation

$$\left(\Delta - c^{-2}\partial_t^2(\varepsilon + \Delta\varepsilon)\right)\vec{E} = 0, \qquad (3)$$

where $\Delta\varepsilon_{ij} = -\varepsilon_{ik}\varepsilon_{jl}p_{kemn}u_{mn}$ is the change of the dielectric permittivity tensor induced by the acoustic wave, ε_{ik} is the dielectric permittivity of the crystal in the absence of ultrasound, p_{kemn} are the components of the photoelastic tensor, u_{mn} are the components of the elastic deformations tensor.

In the studied case of the field propagation along the crystallographic axes of cubic crystal the symmetry of the problem is an axially symmetric one. That is why the solution of the problem of AO transformation is suitable for fulfillment in the cylindrical system of coordinates.

The undisturbed tensor of the dielectric permittivity for cubic crystals in the Cartesian coordinates has the well-known diagonal form with components $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon$. In the cylindrical coordinates with the z-axis parallel to the beam optical axis, this tensor is also diagonal and has the same components.

The tensor $\Delta\varepsilon_{ij}$ depends on the polarization state of an acoustic wave. We consider the case when the acoustic wave is transversely polarized along the y-axis and propagates along the z-axis, i.e. $\vec{u} = \vec{e}_2 u_0 \exp(iKz - i\Omega t)$, where u_0 is the amplitude,

K is the wavenumber, Ω is the angular frequency. In this case the diagonal components of the dielectric tensor do not change, but nondiagonal components arise. For the gyrotropic crystals belonging to the symmetry classes of 23, 432 the nondiagonal components are:

$$\Delta\varepsilon_{\rho z} = \Delta\varepsilon_{z\rho} = -\frac{1}{2} i\varepsilon^2 p_{44} u_0 K \sin(\phi), \qquad (4)$$

$$\Delta\varepsilon_{\phi z} = \Delta\varepsilon_{z\phi} = -\frac{1}{2} i\varepsilon^2 p_{44} u_0 K \cos(\phi) \qquad (5)$$

From the Eqs. (4), (5) it follows that photo-elasticity caused by a plane acoustic wave changes essentially the effective tensor of dielectric permittivity ε . Particularly, when the acoustic wave is transversely polarized along the y-axis, tensor $\Delta\varepsilon$ has azimuthally depending non-diagonal components.

4. Equations for Slowly Varying Amplitudes (SVA)

It is assumed that the AO interaction of Bessel beams, similar to that of plane wave, leads, first of all, to a z–modulation of the scalar amplitudes of $A^{\pm}$ in Eq. (1). At the same time the mode functions $\vec{e}_m^{(\pm)}(\rho,\phi)$ are considered to be unchanged. Such a regime of the AO transformation means the absence of transformation of spatial structure of BLBs in the process of energy exchange and can be explained physically. Firstly, due to the linearity of the AO process, its efficiency does not depend on the local intensity of the beams, but without local-inhomogeneous disturbances BLBs preserve their transverse profile due to the known nondiffracting property. Secondly, all plane-wave components of BLBs are transformed under identical conditions of the longitudinal and transverse phase-matching, due to the cylindrical symmetry of the problem resulting from the propagation of beams along the crystallographic axis.

To derive SVA-equations for the AO interaction of Bessel beams, the solutions represented in the form of Eqs. (1) and (2) with z-dependent amplitudes $A^{\pm}$ are substituted into Helmholtz equation. For illustration of the approach we considered the case of the scattering of an incident (–) mode on a linearly-polarized along the y-axis acoustic wave in crystals of symmetry class of 23 and 432.

In this case the incident m-order BLB will scatter into two Bessel beams having $m \pm 1$ orders. The dielectric tensors $\Delta\varepsilon_{m,m\pm 1}$ describing two scattering channels are of the form

$$\Delta\varepsilon_{m,m\pm 1}(\varphi) = \begin{bmatrix} 0 & 0 & \mp\Delta\varepsilon_{\rho z}^0 \\ 0 & 0 & -i\Delta\varepsilon_{\phi z}^0 \\ \mp\Delta\varepsilon_{\rho z}^0 & -i\Delta\varepsilon_{\phi z}^0 & 0 \end{bmatrix} \exp(\pm i\phi), \qquad (6)$$

where $\Delta\varepsilon_{\rho z}^0 = \Delta\varepsilon_{\phi z}^0 = \varepsilon^2 p_{44} u_0 K / 4$.

As it is seen from Eq. (6), the tensors of AO scattering into channels of $m \pm 1$ have both imaginary and real components.

The above results are enough to derive the following SVA-equations:

$$2ik_{+z} \frac{dA_{m\pm 1}^{+}}{dz} = -k_0^2 g_{m,m\pm 1}^{+,-} A_m^{-} \exp(i\Delta k_z z). \qquad (7)$$

Here the values $g_{m,m\pm 1}^{+,-}$ are the efficient parameters of AO interaction. The upper and lower indices designate the type of interaction and change of the order of Bessel functions ($- \to +$ and $m \to m\pm 1$ in the studied case). The parameters $g_{m,n}^{-,+}$ have the convolution form (overlap integral)

$$g_{m,n}^{-,+} = \frac{\iint \vec{e}_n^{+}(\rho,\phi)^{*} \Delta\varepsilon_{m,n}(\phi) \vec{e}_m^{-}(\rho,\phi) \rho d\rho d\phi}{\iint \left|\vec{e}_n^{+}(\rho,\phi)\right|^2 \rho d\rho d\phi}. \qquad (8)$$

The equation describing the reverse energy transfer can be written as

$$\frac{dA_m^{-}}{dz} = \left(\chi_{m+1,m}^{-,+} A_{m+1}^{+} + \chi_{m-1,m}^{-,+} A_{m-1}^{+}\right) \exp(-i\Delta k_z z) \qquad (9)$$

In Eq. (9) the following denotations $\chi_{m,n}^{-,+} = ik_0^2 g_{m,n}^{-,+} / 2k_z$ for effective AO parameters are introduced. Further at calculations a small difference of the components k_{+z} and k_{-z} in SVA-equations will be omitted. For scattering in channel $m \to m + 1$ the effective AO parameter is given by the expression

$$\chi_{m,m+1}^{-,+} = \frac{i\alpha_1 (1+\cos(\gamma_+)) \int \left[J_{m+2}(x) - 2(m+1)/x\, J_{m+1}(x)\right] \sin(\gamma_-) J_m(x_{in}) \rho d\rho}{\int \left[F_1^2(x) + F_2^2(x) + \sin^2(\gamma_+) J_{m+1}^2(x)\right] \rho d\rho}, \qquad (10)$$

where $x_{in} = q_{in}\rho$,

$$F_1(x) = \frac{(m+1)}{x} J_{m+1}(x)(1+\cos(\gamma_+)) - \cos(\gamma_+) J_{m+2}(x),$$
$$F_2(x) = \frac{-(m+1)}{x} J_{m+1}(x)(1+\cos(\gamma_+)) + J_{m+2}(x), \qquad (11)$$

$$\alpha_1 = \Delta\varepsilon^0 k_0^2 / k_z.$$

As it is seen from Eq. (10), the efficiency of AO interaction is determined by the products of the tensor component $\Delta\varepsilon_{mn}^0$ and the overlap integral. The overlap integrals indicate the level of spatial similarity of the incident and scattered fields and express the feature of so-called spatial synchronism at AO interaction. Thus, AO interaction of BLB is characterized by two types of synchronism:

longitudinal and transverse ones.

The solution of Eqs. (7) and (9) for diffracted BLB corresponding to the boundary condition $A_m^-(z=0)=A_0$ is found to be

$$A_{m+1}^+(z)=A_0\exp\left(-\frac{i\Delta k_z z}{2}\right)\left[\cos\left(\frac{pz}{2}\right)+\frac{i\Delta k_z}{p}\sin\left(\frac{pz}{2}\right)\right], \quad (12)$$

where

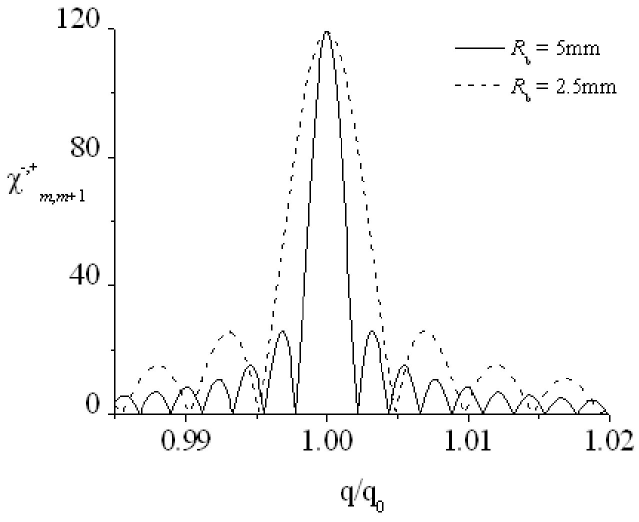
$$p=\left(\Delta k_z^2+4\,|\,\chi_{m,m+1}^{-,+}\,|^2\right)^{1/2} \quad (13)$$

The solution obtained describes the oscillating process of the reversible energy transfer between left (–) and right (+) Bessel beams. The Eq. (12) shows also that efficiency of energy transfer into this channel is determined by the corresponding parameter $\chi_{m,n}^{-,+}$.

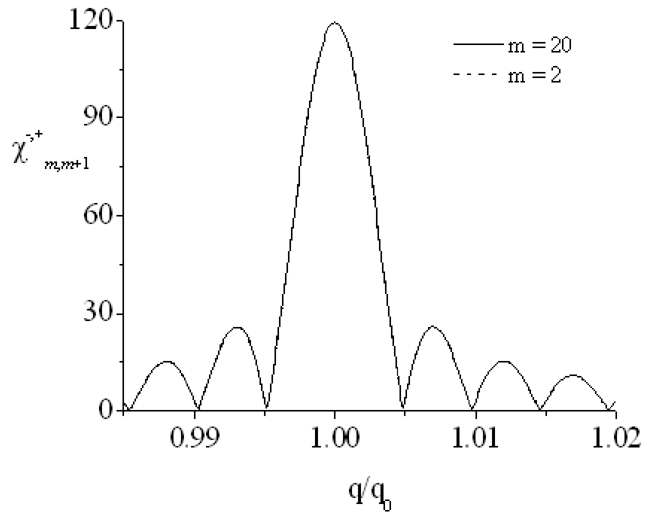
5. Analysis of Overlap Integrals

We have performed the numerical simulation of the AO interaction parameters $\chi_{m,n}^{-,+}=ik_0^2/2k_z\,g_{m,n}^{-,+}$ using Eq. (8). The functions $\chi_{m,n}^{-,+}$ have four arguments: transverse wave number q, cone angle of incident beam γ, radius of the incident beam R_b and phase dislocation order m. The calculation is performed for the gyrotropic cubic bismuth germanate ($Bi_{12}GeO_{20}$) crystal of the 23 class. Such a crystal is characterized by high gyrotropy (parameter of specific rotation $\rho=\alpha k_0=22$ deg/mm at the wavelength $\lambda=0.63$ μm) and good AO efficiency [4,5].

The results are presented in Fig.2. From Fig. 2 it follows that the maximal value of AO parameters is realized at the approximate equality of transverse wave numbers of incident q_0 and scattered q BLB ($q\approx q_0$). Also from Fig 2b it is seen that the condition of spatial synchronism does not depend on the incident BLB m-order. In addition, the width of main maximum decreases with the increase of the Bessel beam radius (Fig. 2a).



a



b

Figure 2. Scattering of left (-) type of BLB into right (+) type BLB on y-polarized acoustic wave in $Bi_{12}GeO_{20}$ crystal. The dependence of the effective AO paramete $\chi_{m,m+1}^{-,+}$ on the ratio of q/q_0 transverse wave numbers. The radius of incident BLB R_b = 2.5mm and 5mm, m =0 (a); R_b = 5mm, m =2 and 20 (b).

The given plots allow one to calculate the length z_0 of energy transfer from incident beam to diffracted ones. As it is seen from Eq. (12), the transfer length is determined by the parameter p (Eq. 13) which for $\Delta k_z=0$ is given by the expression $p=2\,|\,\chi_{m,m+1}^{\mp}\,|$. From the plots in Fig.2 it is possible to find the maximal values of the effective parameters that allow one to calculate value p and transfer length z_0. For example, for the intensity of acoustic wave, at which the change of the dielectric permittivity components is $\Delta\varepsilon_{mn}^o\sim 10^{-4}$ we have $z_0\sim 9$ mm. At the same time for BLB with the radius of 2.5 mm and cone angle of 15 deg BLB nondiffractive length $z_b=R_b/tg(\gamma)$ is equal to approximately 10 mm. Consequently, within the transfer length the BLB is still preserved that means that the described regime of the BLB transformation at AO interaction can be practically realized.

6. Conclusions

The paper develops a theory of AO interaction of Bessel beams with plane acoustic waves in optically gyrotropic crystals. The geometry is considered, wherein incident and diffracted Bessel beam and acoustic wave propagate along the crystallographic axis of a gyrotropic cubic crystal. By going to the cylindrical coordinates it is possible to describe correctly different types of the AO interactions of vector Bessel beams in this geometry.

The mathematical description of the AO interaction is fulfilled, which takes into account the influence on the efficiency of the interaction of two types of phase matching: i) the usual longitudinal synchronism realized at the equality of phase velocities transmitted and diffracted waves, and ii) transverse phase synchronism corresponding to the maximal value of overlap integral. The numerical calculation has been made of overlap integrals for crystals with cubic symmetry.

As a result, the AO interaction is effectively realized only in the vicinity of the maximum of the overlap integral. The calculations of the specified integrals are carried out which allow one to determine the angular width of the diffracted light beams.

The advantage of the proposed method is the possibility of controlling the efficiency of AO diffraction, due to which there arises the possibility of generation of various polarization states of output Bessel beam. When changing the power or frequency of the acoustic wave, it is possible to switch over the right (+) or left (–) Bessel modes, i.e. to obtain arbitrary polarized Bessel light beams at the output face of crystal plate and manipulate the polarization state of the output optical field in time. The acoustooptic process, which has been studied here can be also used as a method for formation of the radially and azimuthally polarized Bessel beams.

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