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PENSIONS, THE OPTION VALUE OF WORK, AND RETIREMENT

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ABSTRACT

The paper develops a model of retirement based on the option value of continuing to work. Continuing to work maintains the option of retiring on more advantageous terms later. The model is used to estimate the effects on retirement of firm pension plan provisions. Typical defined benefit pension plans in the United States provide very substantial incentives to remain with the firm until some age, often the early retirement age, and then a strong incentive to leave the firm thereafter. (This may be a major reason for the rapidly declining labor force participation rates of older workers in the United States.) The model fits firm retirement data very well; it captures very closely the sharp discontinuous jumps in retirement rates at specific ages. The model is used to simulate the effect on retirement of potential changes in pension plan provisions. Increasing the age of early retirement from 55 to 60, for example, would reduce firm departure rates between ages 50 and 59 by almost forty percent.

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The typical firm pension plan presents very large incentives to retire from the firm at an early age, often as young as 55. Although the labor supply effects of Social Security provisions have been the subject of a great deal of analysis, much less attention has been directed to the implications of firm pension plans. Yet the retirement inducements in the provisions of firm plans are much greater than the incentives inherent in Social Security benefit formulae, as demonstrated by Kotlikoff and Wise [1985, 1987]. Indeed, the provisions of most firm plans are at odds with the planned increase in the Social Security retirement age. This paper presents a new model of retirement and uses it to estimate the effects of pension plan provisions on the departure rates of older salesmen from a large Fortune 500 firm. An important goal is to develop a model that can be used to predict the effects on retirement of potential changes in pension plan provisions. The analysis is based on longitudinal personnel records from the firm.

The option value of continued work is the central feature of the model. Pension plan provisions typically provide a large bonus if the employee works until a certain age, often the early retirement age, and then a substantial inducement to leave thereafter. Employees who retire later may do so under less advantageous conditions. If the employee retires before the early retirement age, the option of a later bonus is lost. Continuing to work preserves the option of retiring later, hence the terminology: the "option value" of work.

The provisions of firm pension plans that have motivated our work are described in the next section. The option value model is described in section II. Results are presented and the model fit is discussed in section III.

Simulations of illustrative potential changes in pension plan provisions are presented in section IV. A summary and conclusions are in section V.

I. Background

A. Firm Pension Plan Provisions.

Approximately 75 percent of American workers are covered by defined benefit pension plans. These plans promise the employee a benefit at retirement that is typically based on age, years of service, and his final salary (or an average of earnings in the last few years of employment). Within this general framework, the benefit formulas of most plans provide a large incentive to remain with the firm until some age and then a substantial incentive to leave the firm at some later age. The specific provisions of firm plans, however, vary enormously. Thus the incentives for retirement or departure from the firm vary widely among firms. The incentives of plan provisions and their variation among plans are described in detail by Kotlikoff and Wise [1985].[1] Because the incentives vary so greatly among plans, to analyze the effects of plan provisions on retirement, it is necessary to account for the precise provisions of an employee's plan. It is also critical to have information on past and current earnings in the firm. For these reasons, we rely on firm personnel records for this analysis.

The easiest way to understand the incentive effects of pension plans is to consider the relationship between age and total compensation -- including

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[1] See also Bulow [1981], Lazear [1983], Clark and McDermed [1984], Fields and Mitchell [1985], Frant and Leonard [1987].

wage earnings, the accrual of future pension benefits, and the accrual of future Social Security benefits. Table 1 presents forecasts of wage earnings and projected pension and Social Security benefits for a representative employee drawn from our data set. Based on the forecasts, this individual, who is 50 years old, will have slightly declining real wage earnings over the next 15 years, with more rapidly declining earnings in his late 60's. The annual pension and Social Security benefits he would receive, were he to retire at the indicated age, are given in the final three columns.

Figure 1 shows the present value of the future wage earnings and retirement benefits presented in table 1, graphed against age of retirement. The curve labelled earnings is the present value, at 50, of cumulated earnings; thus the slope is the discounted annual wage rate. (Forecasted future earnings are based on the experience of other employees in the firm, and on the past earnings of this individual. The estimation procedure is described below.) The retirement benefits curve shows the present value of expected pension plus Social Security benefits, by age of retirement. The slope of this curve indicates the annual accrual of retirement benefits. The accrual of firm pension benefits is negative for this individual after age 60. The top curve shows total compensation, the sum of wage earnings and the accrual of retirement benefits. After age 62 or 63, total compensation from working an additional year is essentially zero. The sharp kinks in the total compensation curve are due to the discontinuous accrual of pension benefits.

The pension accrual is the result of several important provisions of the firm's pension plan. Possibly the most important provision pertains to early retirement benefits. If a person leaves this firm before age 55, early retirement benefits can be taken beginning at age 55, but the benefit will be the normal retirement benefit -- that would be received at age 65 --

Table 1

Earnings, Pension Benefits, and SS Benefits for a Representative Individual

Age	Earnings Forecast	Annual Pension Benefits Adjusted	Annual Pension Benefits Normal	Social Security
50	22317	--	2764	4533
51	22389	--	2900	4723
52	22327	--	3010	4914
53	22330	--	3221	5102
54	22275	--	3271	5288
55	22172	9522	6251	5465
56	22024	10884	7047	5640
57	21832	12465	8006	5771
58	21593	14173	9035	5798
59	21310	16272	10394	5822
60	20981	18546	11861	5842
61	20610	19739	12647	5863
62	20196	20989	13473	5885
63	19742	22309	14346	6305
64	19250	23652	15219	6538
65	18721	--	15756	6757
66	18158	--	16809	6546
67	17564	--	17847	6349
68	16943	--	18862	6167

Notes: All values are in 1980 dollars, calculated assuming a 5 percent inflation rate. Income forecasts were computed using the estimated income forecasting equation shown in appendix B. The individual was 50 years old on January 1, 1981, the date to which these calculations correspond; he will accumulate 30 years of experience during the year in which he reaches age 60. The adjusted pension benefits are paid until he is 65, and are only available if he retires between ages 55 and 65. The social security benefits are unavailable until he is 62; the reported benefits correspond to the benefits he would receive if he collected them starting at age 62, or during the first year of his retirement, whichever is later. Source: Authors' calculations.

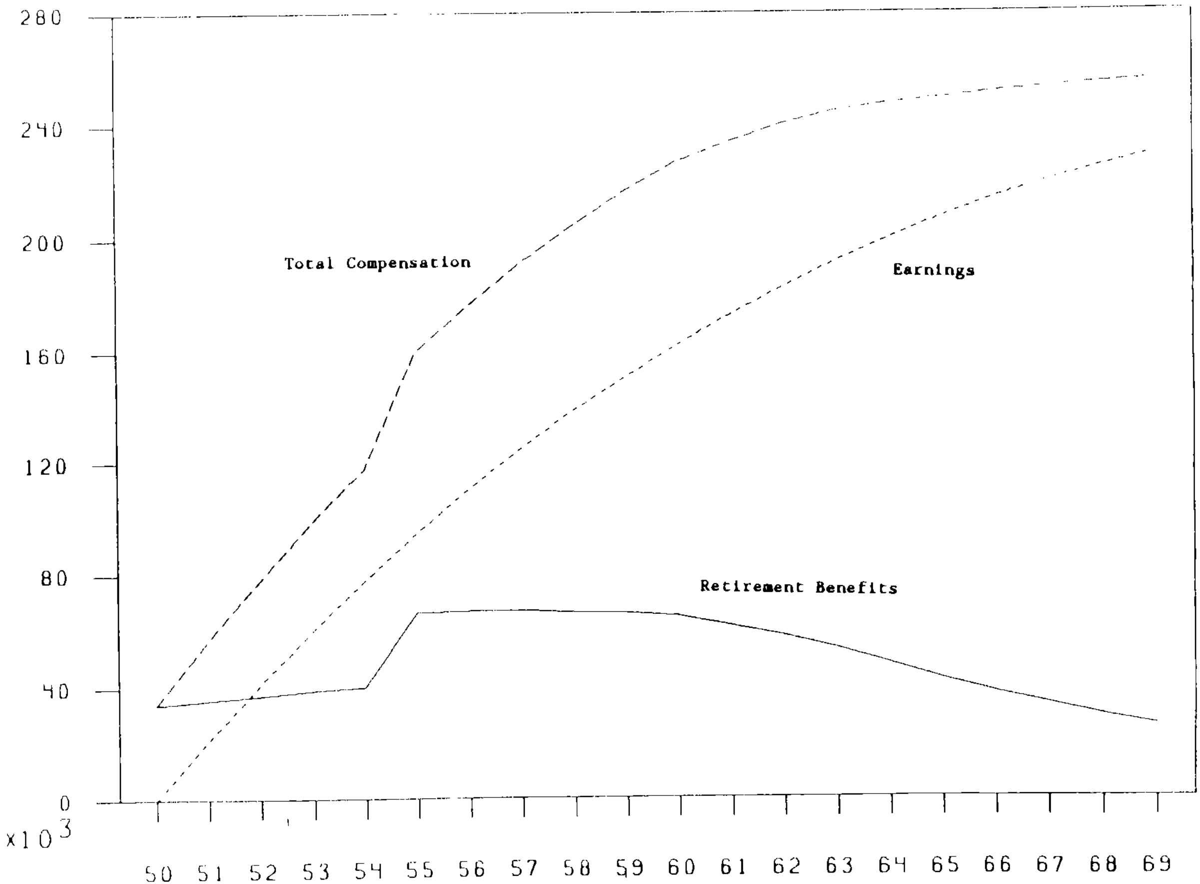

Figure 1. Present discounted values of income while working and pension benefits as a function of date of retirement, based on a real discount rate of 5%. See the notes to Table 1.

actuarially reduced to age 55. If, however, the person stays in the firm until age 55, early retirement benefits can be taken immediately and the early retirement reduction factor is much less than actuarial. In addition, the benefit formula includes a Social Security offset after age 65; pension benefits are reduced depending on the person's Social Security benefits. But the offset is not applied to benefits received between 55 and 65, the normal retirement age. This is a second important feature of the plan. These provisions mean that there is a large incentive to stay in the firm until 55. After 55, the incentive is reduced. For the person represented in the graph, there is a sharp reduction in the accrual of pension benefits at age 60, due to the third important feature of the plan. If the person has 30 years of service at age 60, he is entitled to full normal retirement benefits. That is, by continuing to work he will no longer gain from fewer years of early retirement reduction, as he did before age 60. Other plan provisions are not discussed here, but are described in detail in Kotlikoff and Wise [1987].

B. Prior Emphasis on Social Security Provisions.

The incentive effects inherent in the firm pension plan provisions are much more important than those resulting from Social Security provisions. Indeed, this is typically the case. Yet most prior research on retirement behavior has been directed to the effects of Social Security provisions. Recent examples are Blinder, Gordon and Wise [1980], Burkhauser [1980], Hurd and Boskin [1981], Gustman and Steinmeier [1986], Burtless and Moffitt [1984], Burtless [1986], Hausman and Wise [1985]. With few exceptions, (Hurd and Boskin [1981] and, to some extent, Hausman and Wise [1985]), these studies suggest only a modest effect of Social Security provisions on retirement behavior. In contrast, there has been very little work relating retirement behavior of covered workers to the retirement incentives provided by their pension plans.[2] The apparent reason for this lack of attention has been the absence of appropriate data.

Figure 1 suggests three key requirements for analysis of the retirement effects of pension provisions. First, the data must include the precise provisions of the individual's pension plan, together with information on prior earnings records. Second, the estimation method must account for sharp jumps or drops in pension accrual in future years. For example, in considering whether a person will leave the firm at age 50, it is critical to account for the large "bonus" that he will get if he remains until age 55; consideration only of total compensation at age 50 is not sufficient. Third, the estimation method needs to account for the fact that individual circumstances, such as the level of wage earnings, change over time. Such changes in turn affect future pension and Social Security accrual. The combination of firm data and the estimation method proposed here is intended to meet these requirements.

C. Prior Estimation Methods.

To date, in addition to least squares regression, two basic approaches have been used to analyze retirement behavior. The first is the method of estimation developed to analyze the choices of individuals who face discontinuous or kinked budget constraints. The second approach is the continuous time failure rate or hazard model. Since retirement is typically a discrete outcome, but also has a time dimension (age) which not only

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[2] Exceptions are Fields and Mitchell [1982], Kotlikoff and Wise [1988], Burkhauser [1979], Hogarth [1988].

characterizes retirement but may also affect the desire for it, it is natural to describe retirement within the context of a continuous time qualitative choice model. These two approaches are described briefly in turn.

The adaptation of non-linear budget constraint analysis to retirement may be called the "lifetime budget constraint" approach. The central feature of this method is a lifetime budget constraint analogous to the standard labor-leisure budget constraint, but with annual hours of work replaced by years of labor force participation, and annual earnings replaced by cumulative life-time compensation. The optimal age of retirement is determined by a utility function defined over years of work (post-retirement years of leisure) and cumulative compensation. A careful application of this approach to retirement is by Burtless [1986], who analyzed the effects of changes in Social Security benefits on the retirement.[3]

While appealing in many respects, this procedure has an important drawback. It implicitly assumes that individuals know with certainty the opportunities -- like wage rates -- that will be available to them in the distant future. Although it is plausible to assume that an individual knows his wage rate for the purposes of estimating annual labor supply, the simple extension of this idea to construct a lifetime budget constraint is not as plausible. How much does a 50-year-old person know about his wage at 67? Concomitant with this assumption, the method makes no allowance for updating of information about future opportunities as the individual ages.

[3] An analogous model was used by Venti and Wise [1984] to describe the rent paid by low income families faced with discontinuous budget constraints. Earlier papers that develop these techniques are Hausman and Wise [1980] and Burtless and Hausman [1978].

The hazard model approach as implemented to date is essentially a reduced-form technique designed to capture the effects on retirement of movements in variables such as Social Security wealth. Implementations of the hazard model have not been as "forward looking" as the non-linear budget constraint specifications. It is natural under this specification, however, to update information as individuals age. For example, if an individual has not retired at age t it is convenient to describe the probability of retirement by age t+1 in terms of variables such as annual wage earnings and private pension accruals up to age t and in terms of these values in the period t to t+1; but it is not natural to consider values of these variables in future years. Thus in Hausman and Wise [1985], for example, changes from the current period to the next, in earnings, pension wealth, and the increment to pension wealth, are allowed to affect the decision to retire in the next period, but these values several years hence are not. On the other hand, it is easy within this framework to allow a flexible specification. In particular, different forms of monetary compensation can be entered separately with no increase in computational complexity. And possibly more important for retirement, unexpected shocks, like sudden changes in earnings enter the analysis very naturally.

The hazard model is commonly thought to have no apparent utility maximization interpretation.[4] It is shown in Appendix A, however, that, in fact, it does have such an interpretation, and that it is a special case of the model developed in this paper.

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[4]The Brownian motion model described in Hausman and Wise [1985] can be assigned such an interpretation, but it is difficult to estimate with variables that change over time.

II. The Option Value Model

The model proposed here incorporates the advantages of both of the approaches described above. It allows updating of information, as does the traditional hazard model, but also considers potential compensation many years in the future, as does the nonlinear budget constraint approach. Antecedents of our work begin with Lazear and Moore [1988], who argue that the option value of postponing retirement is the appropriate variable to enter in a regression equation explaining retirement.[5] Our model is close in spirit to the stochastic dynamic programming model of Rust [1988a].[6] A dynamic

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[5]Indeed it was their work and analysis of military retirement rates by Phillips and Wise [1987] that motivated us to pursue this approach.

[6]Rust's [1988a] model poses substantially greater numerical complexity than ours and has not yet been estimated for retirement. In principle, he observes not only the individual's retirement age, but subsequent consumption decisions as well. Thus his model allows the individual to optimize over age of retirement and future consumption jointly. The choices are assumed to be equivalent to the solution to a dynamic programming problem. As in our case, the individual's expectations are conditioned on current known variables like income. The idea is to recover the parameters of a utility function specified in terms of these choice variables. In practice, though, he uses income to describe consumption (Rust [1988b]), with a value function similar to ours, specified in terms of income. To simplify the solution to the dynamic programming problem in his model, he assumes that random unobserved individual components are independent over time, whereas we allow such terms in our model (representing differences among individuals in health status, desire for leisure, and the like) to be correlated. In short, Rust has described a solution to a more complicated choice than ours, but with uncorrelated errors, whereas ours is a solution to a less complex problem, but with correlated errors.

programming model of employment behavior has also been proposed by Berkovec and Stern [1988].[7] Neither Rust nor Berkovec and Stern, however, have information on private pension plan provisions, the focus of our analysis. Our strategy is to simplify the general stochastic programming problem to facilitate its otherwise burdensome econometric implementation. These simplifications reduce the computational requirements substantially while retaining the key forward-looking features of the dynamic programming approach.

The key ideas of the model can be summarized briefly. It is intended to capture an important empirical regularity, the irreversibility of the retirement decision. Although it is not uncommon to work -- at least part-time -- after "retirement," it is rare to return to the job from which one has retired. The model focuses on the opportunity cost of retiring or, equivalently, on the value of retaining the option to retire at a later date. It has two key aspects. The first is that a person will continue to work at any age if the option value of continuing work is greater than the value of

[7]Berkovec and Stern's analysis is also in progress. They consider transitions among three employment states over time. To simplify the solution to their optimization problem, they, like Rust, assume that disturbance terms are uncorrelated over time. Their analysis is in terms of individual attributes like education, race, health status, and age. Government benefits like Social Security are not explicitly modeled, whereas these benefits, as well as firm pension benefits play the central role in our analysis. We estimate a discount, or weighting factor, whereas they obtain estimates of other parameters conditional on an assumed discount rate. Age itself is used explicitly to estimate retirement. As will be emphasized below, age is not a direct determinant of retirement in our model. This has important implications if the model is to be used to predict the effect of changes in firm pension plan or Social Security provisions on retirement.

immediate retirement. In effect, the person compares the best of expected future possibilities -- the option value of continuing to work -- with the value of retiring now. The second is that the individual reevaluates this retirement decision as more information about future earnings -- and thus future retirement benefits -- becomes available with age. For example, a decline in the wage between ages 56 and 57 will cause the individual to reassess future wage earnings, and thus future pension benefits and Social Security accrual as well. Thus retirement may seem more advantageous upon reaching 57 than it was expected to be at age 56. Retirement occurs when the value of continuing work falls below the value of retiring.

Because the model is somewhat complex in its details, it is useful to know whether a simpler model could capture the important features of this one; in particular, whether a model that is easier to implement could predict retirement outcomes as well as the more complex model. It is shown in appendix A that a simplified version of the model developed here has an almost direct hazard model counterpart. Indeed, as is shown in the appendix, the proportional hazard model can be interpreted in terms of utility maximization, contrary to a common misperception. Hazard models are very simple to estimate. Unfortunately, the hazard model is obtained only after imposing strong restrictions on several important features of the option value model.

In addition to the general rationale for the option value model, the precise specification as set forth in this paper is guided by two considerations: first, by the features of the firm data that are used in estimation, and second, by the primary goal of the model, to predict the result of changes in firm pension plan provisions. In particular, some individual attributes that might be expected to affect retirement behavior -- such as assets other than pension and Social Security wealth -- are unknown to us.[8] Our retirement decision function is therefore based on wage earnings and retirement benefit income. In the absence of additional information, we propose an error structure that is intended to capture the effects of persistent unobserved individual attributes.

A. The model.

Consider an individual at the beginning of year t, who has not yet retired. Looking ahead, he will receive wage income Y_s in year s as long as he continues to work; if he is retired in year s, he will receive real retirement benefits B_s. (We adopt the convention that if s is the first calendar year during which the person has no wage earnings, he is assumed to have left the firm during the previous year, at the age that he was on January 1 of year s.) Let r denote the first full year of the individual's retirement (that is, the first year in which the individual has no wage earnings). As described above, these benefits will depend on the person's age and years of service at retirement, and on his earnings history; thus we typically write

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[8]While assets other than retirement annuity wealth (the present value of firm pension and Social Security benefits) should in principle affect retirement, prior analysis shows that their effect is small relative to Social Security wealth, as demonstrated in Hausman and Wise [1985], for example. In addition, prior work has shown that a large majority of the elderly have very little wealth other than housing and firm pension and Social Security annuities (e.g. Diamond and Hausman [1984], Hurd and Shoven [1983], Hurd and Wise [1988]) and that housing wealth is typically not consumed as the elderly age (Merrill [1984], Venti and Wise [1988], Feinstein and McFadden [1988]). Indeed, non-housing bequeathable wealth is reduced very little as the elderly age (e.g. Venti and Wise [1988], Bernheim [1987]). Thus there is substantial evidence that the typical retired person is living largely from Social Security and pension benefits.

$B_s(r)$.

To develop a decision function relative to retirement, suppose that the individual indirectly derives utility $U_w(Y_s)$ from the real income earned while working, and utility $U_r(B_s(r))$ from the pension benefits received while retired. Suppose that in deciding whether to retire the individual weights future income (or utility) by the discount factor β, and that with probability one he will die by year S. If he retires at age r, the weighted, or discounted, value received over the remainder of his life is:

$$(2.1) \qquad V_t(r) = \textstyle\sum_{s=t}^{r-1}\beta^{s-t}U_w(Y_s) + \sum_{t=r}^{S}\beta^{s-t}U_r(B_s(r)) \ .$$

Thus the value function $V_t(r)$ depends on future earnings and retirement benefits, which in turn depend on the age r at which he retires.

The individual must choose either to work during year t, so that $r > t$, or to retire, so that $r = t$. Assume that he makes the decision by comparing the expected value he would receive were he to retire now, at $r = t$, with the greatest of the expected values from possible retirement dates $r > t$ in the future. Let $E_t(\cdot)$ denote the individual's expectation about future circumstances, based on information available to him at the beginning of year t. (With this convention, real income y_t earned during year t is not known at the beginning of year t.)

The expected gain, at time t, from postponing retirement to age r is then given by

$$(2.2) \qquad G_t(r) = E_tV_t(r) - E_tV_t(t).$$

In the firm that provided our data, retirement is mandatory at age 70. Thus we assume that the individual considers potential retirement ages between t+1 and the year of his seventieth birthday, t_{70}. Let r* be the retirement age with the highest expected value, that is

$$(2.3) \qquad r^* \text{ solves } \max_{r \in \{t+1, t+2, \ldots, t_{70}\}} E_t V_t(r) \; .$$

The individual retires if there is no expected gain from continued work, that is, if $G_t(r^*) = E_t V_t(r^*) - E_t V_t(t) < 0$. Otherwise he postpones retirement. In short, he

$$(2.4) \qquad \text{Retires at } r > t \text{ if:} \quad G_t(r^*) = E_t V_t(r^*) - E_t V_t(t) > 0 \; .$$

We assume that the utility derived indirectly from annual income has a constant relative risk aversion form, with additive individual disturbance terms distributed independently of income and age. Specifically,

$$(2.5a) \qquad U_w(Y_s) = Y_s^{\gamma} + \omega_s$$

$$(2.5b) \qquad U_r(B_s) = (kB_s(r))^{\gamma} + \xi_s$$

where ω_s and ξ_s are individual-specific random effects. They are intended to capture several unobserved determinants of retirement. For example, ω_s and ξ_s could reflect individual preferences for work versus leisure. Or, they could reflect health status. They could reflect differences among individuals in unobserved wealth and other variables that may affect retirement decisions. Given the nature of our data, they are also likely to reflect the fact that for some persons the alternative to continued work in the firm is not retirement, but another job, an issue that we return to below. We presume that, for a given individual, there should be considerable persistence in these random effects over time. For example, a disability that affects the burden of working, and thus corresponds to a negative ω_s, is likely to yield a negative ω_{s+1} as well. Such persistence is captured by assuming that the random individual effects follow a Markovian or first order autoregressive process:

$$(2.6a) \qquad \omega_s = \rho\omega_{s-1} + \epsilon_{\omega s}\ ,\ E_{s-1}(\epsilon_{\omega s}) = 0,$$

$$(2.6b) \qquad \xi_s = \rho\xi_{s-1} + \epsilon_{\xi s}\ ,\ E_{s-1}(\epsilon_{\xi s}) = 0,$$

for $s=t+1,\ldots,S$. We give particular attention in the empirical work to the case with $\rho = 1$, with the individual effects evolving according to a random walk. We adopt the convention that at time s the individual knows ω_s and ξ_s, but not their values at s + 1 and subsequent ages; future forecasts of ω and ξ are based on (2.6).

With the parameterization (2.5), $G_t(r)$ in (2.3) becomes,

$$\begin{aligned}
(2.7) \qquad G_t(r) &= E_t\textstyle\sum_{s=t}^{r-1}\beta^{s-t}[(Y_s^\gamma)+\omega_s] \\
&\quad + E_t\textstyle\sum_{s=r}^{S}\beta^{s-t}[(kB_s(r))^\gamma+\xi_s] \\
&\quad - E_t\textstyle\sum_{s=t}^{S}\beta^{s-t}[(kB_s(t))^\gamma+\xi_s] \\
&= E_t\textstyle\sum_{s=t}^{r-1}\beta^{s-t}(Y_s^\gamma) + E_t\sum_{s=r}^{S}\beta^{s-t}(kB_s(r))^\gamma \\
&\quad - E_t\textstyle\sum_{s=t}^{S}\beta^{s-t}(kB_s(t))^\gamma \\
&\quad + E_t\textstyle\sum_{s=t}^{r-1}\beta^{s-t}\omega_s + E_t\sum_{s=r}^{S}\beta^{s-t}\xi_s \\
&\quad - E_t\textstyle\sum_{s=t}^{S}\beta^{s-t}\xi_s \\
&= g_t(r) + \phi_t(r)
\end{aligned}$$

where $g_t(r)$ and $\phi_t(r)$ distinguish the terms in $G_t(r)$ containing the random effects, ω and ξ, from the other terms.

If whether the person is alive in future years is statistically independent of his earnings stream and the individual effects ω_s and ξ_s, then $g_t(r)$ and $\phi_t(r)$ become

$$(2.8) \qquad g_t(r) = \sum_{s=t}^{r-1}\beta^{s-t}\pi(s|t)E_t(Y_s^{\gamma}) + \sum_{s=r}^{S}\beta^{s-t}\pi(s|t)[E_t(kB_s(r))^{\gamma}] - \sum_{s=t}^{S}\beta^{s-t}\pi(s|t)[E_t(kB_s(t))^{\gamma}]$$

and

$$(2.9) \qquad \phi_t(r) = \sum_{s=t}^{r-1}\beta^{s-t}\pi(s|t)E_t(\omega_s-\xi_s),$$

where $\pi(s|t)$ denotes the probability that the person will be alive in year s, given that he is alive in year t. Given the random Markov assumption (2.6), $\phi_t(r)$ can be written as

$$(2.10) \qquad \phi_t(r) = \sum_{s=t}^{r-1}\beta^{s-t}\pi(s|t)\rho^{s-t}(\omega_t-\xi_t) = K_t(r)\nu_t ,$$

where $K_t(r) = \sum_{s=t}^{r-1}(\beta\rho)^{s-t}\pi(s|t)$ and $\nu_t = \omega_t - \xi_t$. The simplification results from the fact that at time t the expected value of $\nu_s = \omega_s - \xi_s$ is $\rho^{s-t}\nu_t$, for all future years s. Thus the individual random component $\phi_t(r)$ depends only on the random effect at time t (together with β and ρ). The term $K_t(r)$ cumulates the deflators that yield the present value in year t of the future expected values of the random components of utility. The further r is in the future, the larger is $K_t(r)$. That is, the more distant the potential retirement age, the greater the uncertainty about it, yielding a heteroskedastic disturbance term. This heteroskedastic property is apparently an important determinant the model's ability to predict departure rates accurately for both younger and older employees, as shown below.

Combining (2.7)-(2.10), $G_t(r)$ may be written simply as

$$G_t(r) = g_t(r) + K_t(r)\nu_t \ . \quad (2.11)$$

B. The probability of retiring.

1. Retirement probabilities for a single year. The year in which an individual retires is a random variable; we call it R. The probability that an individual in the sample in year t-1 retires in year t, that is $Pr[R = t]$, is the relevant probability when using cross-sectional data for a single year t. The probability that the individual chooses not to retire is $Pr[R > t]$. From (2.4), an individual will retire in year t if $G_t(r) < 0$ for all $r \in \{t+1,\ldots,T\}$. Thus:

$$\begin{aligned} Pr[R=t] &= Pr[G_t(r) < 0 \ \forall \ r \in \{t+1,\ldots,T\}] \\ &= Pr[g_t(r)+K_t(r)\nu_t < 0 \ \forall \ r \in \{t+1,\ldots,T\}] \\ &= Pr[g_t(r)/K_t(r) < -\nu_t \ \forall \ r \in \{t+1,\ldots,T\}]. \end{aligned} \quad (2.12)$$

Alternatively, the final expression in (2.12) can be written:

$$Pr[R=t] = Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) < -\nu_t] \quad (2.13)$$

where $r_t^\dagger$ is the value of r that solves

$$(2.14) \qquad \max_{r\in\{t+1,\ldots,T\}} g_t(r)/K_t(r).$$

Because the individual either retires at t or he does not, $\Pr[R>t] = 1 - \Pr[R=t]$.

2. Retirement probabilities for multiple years. The data set analyzed below contains data on individual retirement decisions for several consecutive years. To analyze these data requires computing the probability that the individual retires in year τ. In general, suppose that the retirement status is observed for years $t,\ldots,T$. An individual in the sample in year $t-1$ retires in year $\tau\in\{t,\ldots,T\}$ if there is no earlier age when he considers it optimal to retire, and if it is optimal to retire in year τ based on equation (2.4). If it had not been optimal to retire in year t, there would have been, at time t, at least one future r with $G_t(r) > 0$. This would occur if and only if it were true for the $r^\dagger$ that maximized $g_t(r)/K_t(r)$, evaluated at year t. That is, it requires that $g_t(r_t^\dagger)/K_t(r_t^\dagger) > -\nu_t$. The same would have to be true for every year t through year $\tau-1$. In year τ, however, retirement is optimal, so that $g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger) < -\nu_\tau$. Thus

$$(2.15) \qquad \Pr[R=\tau] = \Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) > -\nu_t, \ldots, g_{\tau-1}(r_{\tau-1}^\dagger)/K_{\tau-1}(r_{\tau-1}^\dagger) > -\nu_{\tau-1}, g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger) < -\nu_\tau] .$$

Equation (2.15) can be used to compute the probability that $R = \tau$ for $\tau=t,\ldots,T$. The remaining possible event is that the individual does not retire during the period of the data. The probability of this event is

$$(2.16)\qquad \Pr[R>T] = \Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) > -\nu_t, \ldots,$$
$$g_{T-1}(r_{T-1}^\dagger)/K_{T-1}(r_{T-1}^\dagger) > -\nu_{T-1},$$
$$g_T(r_T^\dagger)/K_T(r_T^\dagger) > -\nu_T] .$$

The retirement model thus reduces to a multinomial discrete choice problem, with dependent error terms ν_s. Thus far, the only assumption about the individual effects is that they are Markov. Empirical implementation, however, requires additional distributional assumptions. We assume that ν_s follows a Gaussian Markov process, with

$$(2.17)\qquad \nu_s = \rho\nu_{s-1} + \epsilon_s, \quad \epsilon_s \text{ i.i.d. } N(0,\sigma_\epsilon^2),$$

where the initial value, ν_t, is i.i.d. $N(0,\sigma_\nu^2)$ and is independent of ϵ_s, $s=t+1,\ldots,S$. The covariance between ν_τ and $\nu_{\tau+1}$ is $\rho\text{var}(\nu_\tau)$, and the variance of ν_τ for $\tau \geq t$ is $\rho^{2(\tau-t)} + (\sum_{j=0}^{\tau-t-1}\rho^{2j})\sigma_\epsilon^2$. In the random walk case, with $\rho = 1$, the covariance between ν_τ and $\nu_{\tau+1}$ is $\text{var}(\nu_\tau)$, and the variance of ν_τ for $\tau \geq t$ is $\sigma_\nu^2 + (\tau-t)\sigma_\epsilon^2$. Thus there are two equivalent ways to see that uncertainty about the future is reduced as the planning horizon is shortened, presumably as the person approaches typical retirement ages. First, their are fewer future random components of utility to cumulate in the $K_t(r)$ term (see equation 2.10). Second, the uncertainty about the value of future random effects is reduced -- the Markov assumption yields decreasing $\text{var}(\nu_\tau)$ as the planning horizon is shortened. In particular, in a given calender year, the uncertainty about the retirement decisions of younger persons is greater than the uncertainty about older employees. This property plays a key role in providing the flexibility that allows the model to fit the departure behavior of younger as well as older employees.

In summary: conditional on $\{g_s(r_s^\dagger)/K_s(r_s^\dagger)\}$, $s=t,\ldots,\tau$, the probability that year τ is the first year of retirement is given by (2.15), while (2.16) gives the probability that the person does not retire during the years $t,\ldots,T$. These probabilities are evaluated by computing the appropriate integrals over a multivariate normal density, where the error term follows the Markov process (2.17). The unknown parameters of the model are γ, k, β, and the variance parameters σ_ν^2, σ_ϵ^2, and ρ.

C. Evaluation of $g_t(r_t^\dagger)/K_t(r_t^\dagger)$.

To determine $g_t(r_t^\dagger)/K_t(r_t^\dagger)$ requires evaluation of the expectations $E_t(Y_s^\gamma)$ and $E_t(kB_s(r_t^\dagger))^\gamma$ for $s \geq t$. In the empirical work, the conditional expectation of the first of these terms is approximated by the conditional expectation of its second order Taylor series expansion around the mean of a stream of earnings forecasts computed for each individual.[9] The pension and Social Security benefits depend on the entire earnings stream of the individual through his last year of work. The expectation $E_t(kB_s(r))^\gamma$ was approximated by $(k\hat{B}_s(r))^\gamma$, where $\hat{B}_s(r)$ is the pension benefit calculated using the mean earnings forecasts for the individual through year r-1, based on observed earnings through year t-1.[10]

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[9] $E_t(Y_s)^\gamma \doteq (1 + (1/2)\gamma(\gamma-1)E_t[(Y_s-E_tY_s)/E_tY_s]^2)(E_tY_s)^\gamma$. This term is evaluated assuming that $E[(Y_s-E_tY_s)/E_tY_s]^2 = (s-t)\text{var}(e_t)$, where $\text{var}(e_t) = \text{SEE}^2$ from the $\Delta\ln Y_t$ regression, explained in appendix B.

[10] In principle, the expectation could be evaluated using Monte Carlo methods to determine the variance of $B_s(r)$. Then a Taylor series expansion, like the procedure used to evaluate $E_t(Y_s^\gamma)$, could be used to evaluate $E_t(kB_s(r_t))^\gamma$. The Monte Carlo procedure would entail computation of the benefits that an individual would receive for a given income stream, where the future part of the income stream is drawn from an estimated distribution of

The income forecasts for each individual were generated by a second order autoregression. The autoregression was estimated using the individual earnings histories of all salesmen employed at least three years, with earnings converted to 1980 dollars using the Consumer Price Index. The parameters of the forecasting model depend on age, A_t, years of service, S_t, and an interaction term, with

$$(2.18) \qquad \Delta \ln Y_t = \delta_0(A_t,S_t) + \delta_1(A_t,S_t)\Delta \ln Y_{t-1} + \delta_2(A_t,S_t)\Delta \ln Y_{t-2} + e_t \ .$$

The estimated equation exhibits regression toward the mean; $\delta_1 + \delta_2 < 0$ for typical values of A_t and S_t in the sample. The estimated parameters of equation (2.18) are shown in appendix B.

IV. Results

The option value model has been estimated based on a sample of 1500 salesmen 50 years of age or older on January 1, 1981, selected at random from the firm data. All persons in the sample are men performing similar jobs. To facilitate earnings forecasts, the sample was restricted to persons who had at least three years of service before 1980, the first year of our retirement

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future income streams. However, the pension and Social Security calculations are quite cumbersome. Beyond its substantial computational advantages, a justification for the approximation that we use is that the benefits calculations involve the entire earnings history of the individual. At least for values of r in the near future, the unknown elements in these calculations are small.

analysis.

Initial estimates were obtained based only on retirement decisions in one year, 1980 -- whether 1981 was the first full year of retirement, by our accounting convention. Expected pension benefits are based on the provisions of the firm plan. Social security benefits were computed according to the provisions in the Social Security Bulletin (1982), based on individual wage histories at the firm.[11] Estimates based on the 1980 retirement decisions are reported first, followed by estimates base on three consecutive years.

A. One year.

1. Parameter Estimates

Maximum likelihood parameter estimates are shown in table 2. Estimated parameters of several variants of the option value model are shown in the second panel of the table. Estimates in this table were obtained under the assumption that the random individual effects follow a random walk; ρ is set to 1. (Unrestricted estimates are reported in the next section.) The first panel reports estimates based on the assumption that all employees have the same constant probability of retiring (model 1), or that all persons of the same age have the same retirement probability (model 2).

Estimates of the parameters of the option value model are shown in the last row of the table. The estimate of γ is 1.00, suggesting that, in deciding whether to retire, individual valuation of income is linear in future earnings. Earnings without work, retirement benefits, are valued at 1.66

[11]To obtain pension and Social Security forecasts for persons who joined the firm before 1969--the first year of our data--backward predictions based on the estimated earnings equation were used to estimate earnings before 1969.

Table 2

Parameter Estimates Based on Retirement Decisions in One Year, 1980[a]

A. Models Without Earnings and Retirement Benefit Terms:

		$\mathcal{L}$
1.	Constant Only	-579.58
2.	Age Dummy Variables	-477.92

B. Option Value Models, with $\rho \equiv 1$:

	γ	k	β	$\sigma_\nu (\times 10^5)$	$\mathcal{L}$
3.	1[b]	1[b]	0.10[b]	.140 (.006)	-413.70
4.	1[b]	1[b]	.782 (.212)	.123 (.020)	-413.191
5.	1.00 (.07)	1.66 (.02)	0.847 (.032)	.119 (.001)	-397.72

[a]The sample size is 1500. A person was counted as having retired if he had no earnings in 1981; that is, he was retired by January 1, 1981. $\mathcal{L}$ is the log likelihood value.

[b]Parameter value imposed.

times wage earnings while employed, based on the estimated value of k. That is, a person would exchange a dollar with work for 60 cents not accompanied by work. The weight given to current versus future income in the retirement decision is indicated by β, estimated to be .847. All of the parameters are measured quite precisely, with the possible exception of β (with a standard error of .032), and each of the estimates seems quite plausible to us. Although a strong interpretation of the parameters of the model might treat β as a general measure of individuals' pure rate of time preference, independent from the decision to which it applies, it is probably more realistic to think of it as a weight specific to the retirement decision. Under either interpretation, a priori judgments about its value surely vary widely. It cannot be observed and can typically be estimated only indirectly. It is, however, estimated directly in the option value model. The estimated value is undoubtedly sensitive to the model specification; but under either interpretation it is nonetheless surprising to us that it is measured as precisely as it is.

Estimated parameters of simplified versions of the model are shown as versions 3 and 4. The only estimated parameter in model 3 is the variance σ_{ν}^2. Yet judging by the large difference in the likelihood values the option value model fits the data much better than the specification based on a full set of age dummy variables, as reported in model 2. Thus there is substantial information in the option value measures. With age dummy variables, the estimated average departure rate for each age matches the actual rate. The option value specification does not assure such a match. But because departure rates vary greatly among persons of the same age, the option value model fits much better. The option value model captures the variation, given age; the dummy variable model does not.

2. *The Model Fit*

The model fit is demonstrated by comparing actual and predicted retirement rates, shown in table 3. Both the predicted annual rates and predicted cumulative retirement are very close to the actual values. In particular, the model captures each of the important jumps in the departure rates. The actual retirement rate at 55 is .078; the predicted rate is .075. Of persons who are employed at age 50, the actual proportion that has left by age 54 is .139, as shown in the forth column of the table. The model prediction is .116. The actual proportion that has left jumps to .206 at 55, the predicted proportion to .182. Again at age 60, the actual proportion jumps from .488 to .599, the predicted proportion from .483 to .583. At age 62, the actual proportion jumps from .675 to .824 and the predicted proportion from .680 to .823. Only at ages 65 and 66 do the predicted rates differ noticeably from the actual ones, but there are very few observations at these ages. In addition, only about 5 percent of persons employed at 50 would still be employed at 65, based on the actual departure rates.

To provide an external check of the predictive validity of the model, parameter estimates based on 1980 retirement decisions were used to predict 1981 departure rates, that on average were higher than in 1980. Actual versus predicted cumulative departure rates, based on actual versus predicted departure rates by age, provide a summary of the results. At ages 60 and 62, they are as follows:[12]

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[12] The 1981 comparison is based on 1305 observations.

Table 3

Predicted versus Actual Retirement Rates by Age, Based on the Single-Year Model, 1980[a]

Age	Number of Observations	Annual Retirement Rates		Cumulative Rates	
		Actual	Predicted	Actual	Predicted
50	36	0.000	0.025	0.000	0.025
51	131	0.053	0.037	0.053	0.061
52	132	0.015	0.026	0.068	0.086
53	123	0.041	0.024	0.106	0.108
54	106	0.038	0.009	0.139	0.116
55	129	0.078	0.075	0.206	0.182
56	137	0.117	0.073	0.299	0.241
57	123	0.089	0.108	0.362	0.323
58	107	0.084	0.102	0.415	0.392
59	120	0.125	0.149	0.488	0.483
60	116	0.216	0.194	0.599	0.583
61	84	0.190	0.233	0.675	0.680
62	70	0.457	0.447	0.824	0.823
63	51	0.412	0.503	0.896	0.912
64	22	0.455	0.491	0.943	0.955
65	14	0.857	0.468	0.992	0.976
66	1	0.000	0.355	0.992	0.985

[a]The retirement rates were computed for the 1500 persons used to obtain the estimates reported in table 2. The predicted retirement rates are based on model 5.

	Age 60		Age 62	
	Actual	Predicted	Actual	Predicted
1980	.599	.583	.824	.823
1981	.674	.667	.868	.876

Thus the model not only matches closely the cumulative departure rates in each year, but also captures the noticeable increase in departure rates between 1980 and 1981. The actual increases were apparently due to changes in expected future earnings or to differences in the distribution of seniority by age, both of which enter the option value caculations.[13]

B. Three Consecutive Years

1. Parameter Estimates

Estimates in this section are based on the same sample of employees used to obtain the single-year estimates reported above, but those who don't leave the firm in the first year are followed for two more years. Four outcomes are possible: a person retires in the first, the second, or the third year, or he does not retire during the three-year period. Estimated parameters of three versions of the model are shown in table 4. The first estimates pertain to the model specification as described in section II-B-2, with $\rho = 1$. In this case, the only difference between the multiple- and single-year versions of the model is that there are two error variances in the multiple-year version: σ_ν^2, the variance in the first of the observation years -- 1980 in this case, and σ_ϵ^2, the variance of the "innovation" ϵ in the relationship $\nu_s = \nu_{s-1} +$

[13]Real earnings of firm employees were in fact declining over this period.

Table 4

Parameter Estimates Based on Retirement Decisions in Three Consecutive Years, 1980-1982

	Parameters								
	γ	k	k_0	k_1	β	ρ	σ_ν	σ_ϵ	$\mathcal{L}$
1.	1.206 (0.005)	1.703 (0.043)	--	--	0.796 (0.004)	1[a]	0.117 (0.003)	0.092 (0.005)	1114.86
2.	1.273 (0.039)	1.716 (0.023)	--	--	0.775 (0.014)	0.795 (0.015)	0.174 (0.008)	0.138 (0.008)	1100.49
3.	1.278 (0.006)	--	1.678 (0.030)	0.233 (0.006)	0.788 (0.002)	0.798 (0.020)	0.176 (0.008)	0.140 (0.007)	1099.75

[a]Parameter value imposed.

ϵ_s. The estimates are close to the single-year estimates (model 5 in table 2), although the estimated value of γ is somewhat larger, and the value of β somewhat smaller. The base error variance is essentially the same as the single-equation estimate. The variance of the innovation is only slightly smaller (.1170 versus .0919) than the base variance. It means that the uncertainty about an individual's valuation of the option value of continued work in future years -- which stems from uncertainty about future values of ν -- is much greater than the uncertainty about the option value of continued work today. This contributes to greater uncertainty about current departure decisions when comparison is made with more distant future retirement ages, that is, when departure rates of younger employees are considered. (See the discussion following equation (2.17).) On the other hand, given ν_{s-1}, the uncertainty about ν_s is less than the uncertainty about ν_{s-1}.

A measure of the persistence in the individual disturbance is indicated by the correlation between the ν's in the first and second periods of the sample; with the random walk specification; it is given by $\sigma_\nu^2/(\sigma_\nu^2)^{1/2}(\sigma_\nu^2 + \sigma_\epsilon^2)^{1/2}$. This correlation is .748 based on the model 1 estimates in table 4. Less persistence is allowed by estimating ρ. Parameter estimates based on this specification are shown in model 2 in table 4. The estimated value of ρ is .786. Judging by the likelihood values in models 1 and 2, the χ^2 statistic relative to the hypothesis that ρ is 1 is 28.74 with one degree of freedom. Thus the strict random walk assumption is clearly rejected. On the other hand, the estimated variances increase so that the correlation between the first and second disturbance terms does not change much. In this case, it is given by $\rho\sigma_\nu^2/(\sigma_\nu^2)^{1/2}(\rho^2\sigma_\nu^2 + \sigma_\epsilon^2)^{1/2}$. Its value, based on the model 2 estimates in table 4, is .708, compared with a correlation of .786 based on the strict random walk assumption. Consistent with this observation, predicted average departure rates based on the two models are very similar.

Unlike other empirical retirement models, age is not a variable in the option value model; it enters only indirectly through the survival probabilities $\pi(s|t)$, the wage earning forecasts, and the firm pension plan and Social Security rules. A general test of the extent to which retirement behavior is not determined by the monetary variables in the option value model is the gain in the model fit when age itself is added. We implement such a test by parameterizing k as a function of age, allowing the relative value of income without work to income with work to depend on age, independent of the income variables in the model. In addition, this parameterization is a way to recognize that the alternative to work at the firm may be another job, instead of retirement, and thus that the systematic portion of the model may undervalue the "retirement" option for some employees, especially at younger ages.[14] The model 3 estimates in table 4 are based on the specification $k = k_0(Age/55)^{k_1}$. The estimates show virtually no effect of age. For example, at age 65, k is 1.039 times its value at 55, 1.678. The others parameters of the model are essentially unaffected. The likelihood value is increased very little and thus a likelihood ratio test does not reject the hypothesis of no age effect ($\chi^2(1) = 1.48$).

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[14]As emphasized above, however, the Markov specification implies a heteroskedastic disturbance with larger variance the greater the difference between the current age and future contemplated retirement ages. Thus the variance of the individual effects is larger for younger employees. One of the unobserved determinants of departure that the random component captures is valuations of the "retirement" alternative that differ from the average. It is this aspect of the specification that allows the model to fit departure rates at younger ages, as shown in table 3 for the single-year model.

2. The Model Fit

Like the single-year model, the fit of the three-year version can be evaluated by comparing predicted versus actual departure rates. The results are shown in table 5. It is analogous to the comparison presented in table 3, based on the single-year model. The annual retirement rate in table 5, for persons of a given age, is the average of the rates over the three estimation years. The cumulative figures are based on these average annual rates. Three aspects of the results stand out: First, the model fits the data very well. Second, there is little difference between the $\rho = 1$ and the ρ-estimated versions of the model, although the ρ-estimated specification fits somewhat better than the $\rho = 1$ model at older ages and somewhat less well at the younger ages. Thus even though the likelihood values in table 4 indicate that the second version fits the data better, for practical purposes, the strict random walk assumption appears to be as consistent with the data as the more general specification.[15] Third, the model may underpredict the retirement

[15]The χ^2 statistic is a more formal way to compare the model fits. χ^2 statistics have been calculated for each of the three years, based on three methods of estimation: (1) The three-year model with $\rho = 1$, model 1 in table 4. (2) The three-year model with ρ estimated, model 2 in table 4. (3) Independent estimates for each of the three years, with $\rho = 1$. For example, using the sample of persons that is still in the firm after 1980, estimates are obtained for 1981, and similarly for 1982. The statistic is $\sum_a (A_a - E_a)^2/E_a$, where a indexes age, A is the actual number of persons that retired, and E is the expected number, based on the model estimates. The results are as follows:

Year	(1)	(2)	(3)
1980	25.2	20.7	24.4
1981	30.7	29.9	9.2
1982	27.3	19.0	16.1

The comparisons reveal two features of the results: The ρ-estimated fits

Table 5

Predicted versus Actual Retirement Rates by Age, Based on the Three-Year Model, 1980-1982[a]

Age	Number of Observations in 1980	Annual Retirement Rates			Cumulative Rates		
			Predicted			Predicted	
		Actual	$\rho = 1$	ρ Estimated	Actual	$\rho = 1$	ρ Estimated
50	36	0.000	0.007	0.004	0.000	0.007	0.004
51	131	0.036	0.025	0.017	0.036	0.032	0.021
52	132	0.028	0.032	0.022	0.064	0.063	0.043
53	123	0.040	0.031	0.027	0.101	0.092	0.069
54	106	0.043	0.016	0.022	0.140	0.106	0.090
55	129	0.094	0.091	0.075	0.220	0.187	0.158
56	137	0.106	0.084	0.082	0.303	0.256	0.227
57	123	0.077	0.091	0.092	0.357	0.323	0.298
58	107	0.105	0.102	0.108	0.424	0.392	0.374
59	120	0.136	0.124	0.137	0.503	0.468	0.460
60	116	0.201	0.176	0.185	0.603	0.561	0.560
61	84	0.184	0.183	0.198	0.676	0.641	0.647
62	70	0.419	0.416	0.424	0.812	0.791	0.797
63	51	0.435	0.354	0.378	0.893	0.865	0.873
64	22	0.401	0.333	0.359	0.936	0.910	0.919
65	14	0.739	0.320	0.337	0.983	0.939	0.946
66	1	0.000	0.178	0.185	0.983	0.950	0.956

[a]The retirement rates by age are the average of the rates over the three years used in estimation. The cumulative rates are based on these averages.

rates of the few persons that remain in the firm at older ages. In particular, both models underpredict retirement rates at 65. Even with ρ estimated, the model estimates imply substantial persistence in individual valuations of the option value of continued work, consistent with the behavior of the vast majority of the sample. For example, if a person chooses not to retire at 62 when there was a reasonable ex ante probability that he would, the model uses this information to adjust downward the probability that he will retire in the next year. The results seem to suggest that this assumed persistence of tastes may not carry through age 65. There may be an age-65 "customary retirement age" effect. The sample size of older persons is so small, however--only 2.5 percent of the sample is 64 or older--that verification of this possibility will have to await estimation with larger samples of older employees; the current evidence can only be taken as suggestive.[16]

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better than the $\rho = 1$ model. And, although the indpendent estimate for 1981 fits better than the ρ-estimated version of the three-year model_for that year, it is not clear that that would be generally true in repeated replications. The independent estimate is worse for 1980 and only slightly better for 1982. A tentative conclusion is that the ρ-estimated version of the three-year model reproduces the data very accurately.

[16]Another variant of predicted versus actual departure rates shows the probability that a person will retire during the three year period that he is. The conslusions are similar to those discussed above. In general, both specifications fit the data very well. For example, of persons who were age 55 on January 1, 1981, 24.8 percent left the firm between January 1, 1980 and December 31, 1982. The predicted retirement rate based on the specification with $\rho = 1$ is 23.5 percent; it is 22.8 percent based on the specification with ρ estimated.

IV. Illustrative Simulations

To demonstrate the importance of firm pension plan provisions on departure rates, the effects of two alternatives are simulated. The first is a simple variant of the existing plan, increasing the early retirement age from 55 to 60. The second represents a more fundamental change, replacing the existing defined benefit plan with a defined contribution plan. In both cases, the effects are quite dramatic. The simulations are based on the single-year estimates (model 5 in table 2, with $\rho = 1$). Additional simulations, that compare the effects on retirement of changes in pension versus Social Security provisions, are reported in Stock and Wise [1988].

A. *Increasing the Early Retirement Age*

Although retirement rates beginning at age 62 are very high, by that age most of those employed at age 50 have already left the firm. It is evident that this is due in large part to the plan's early retirement provisions. To quantify the importance of early retirement, we have simulated retirement behavior under an alternative provision. Early retirement under the alternative is at 60 instead of 55. Otherwise the alternative is like the existing plan. Persons who are employed at 60 or older face the same options under the alternative as under the existing plan.

The results are reported in table 6. The base retirement rates are the single-year model predictions under the existing plan. Under the existing plan, almost half of those in the firm at age 50 have left before age 60. Only 30 percent would have left if early retirement had been at age 60 instead of 55, according to the simulation results. With the existing plan, 36.7 percent of those employed at 50 leave the firm between 55 and 59. With early

Table 6

Simulation: Early retirement age is 60 instead of 55[a]

Age	Cumulative Retirement Rates			Retirement Rates		
	Base	Simulation	Difference	Base	Simulation	Difference
50	0.025	0.032	0.007	0.025	0.032	0.007
51	0.061	0.078	0.017	0.037	0.047	0.010
52	0.086	0.116	0.030	0.026	0.041	0.015
53	0.108	0.153	0.045	0.024	0.041	0.017
54	0.116	0.185	0.069	0.009	0.038	0.029
55	0.182	0.219	0.037	0.075	0.041	-0 034
56	0.241	0.245	0.004	0.073	0.034	-0.039
57	0.323	0.272	-0.051	0.108	0.036	-0.072
58	0.392	0.289	-0.103	0.102	0.023	-0.079
59	0.483	0.300	-0.183	0.149	0.015	-0.134
60	0.583	0.436	-0.147	0.194	0.194	b
61	0.680	0.568	-0.112	0.233	0.233	b
62	0.823	0.761	-0.062	0.447	0.447	b
63	0.912	0.881	-0.031	0.503	0.503	b
64	0.955	0.939	-0.016	0.491	0.491	b
65	0.976	0.968	-0.008	0.468	0.468	b
66	0.985	0.979	-0.006	0.355	0.355	b

[a]Based on model 5 parameter estimates, reported in table 2. The simulation is described in the text.

[b]For persons employed at age 60 and older, the simulated alternative is the same as the base case.

retirement at 60, only 11.5 percent would leave at these ages. Almost no one leaves just before age 60. On the other hand, departure rates before 55, are larger under the alternative, with a cumulative rate at 54 of .185, versus .116 under the existing plan. This reflects the longer wait before the early retirement bonus can be claimed. Still, the net reduction in departure rates before age 60 is very substantial.

B. A Defined Contribution versus The Defined Benefit Plan

The incentive effects inherent in the firm's age-compensation profile are largely the result of the provisions of the pension plan. An alternative to a defined benefit plan is a defined contribution plan. Under a typical defined contribution plan an amount equivalent to a certain percentage of an employee's annual wage earnings is put in a pension fund. Once vested, the amount that the employee has in the fund depends only on the contributions on his behalf and on the return on these contributions. Retirement benefits are then based on the employee's accumulated assets in the fund at the time that he retires.

The effect of a change from the existing defined benefit to a defined contribution plan is illustrated under two assumptions. The first assumption is that the defined benefit contribution rate is such that for a person who has 30 years of service at age 60, the fair annuity value of the amount in the defined contribution fund is the same as the present value of the retirement benefits that the person would receive from the defined benefit plan were he to retire at 60. This requires that the contribution to the defined contribution plan be equal to 7.5 percent of earnings. The second assumption is that the contribution is equal to 5 instead of 7.5 percent of earnings. The results are shown in table 7.

Table 7

Simulation: Defined Contribution versus The Defined Benefit Plan[a]

Age	Number of Observations	Annual Retirement Rate Base	Annual Retirement Rate Simulation 7.5	Annual Retirement Rate Simulation 5.0	Cumulative Retirement Base	Cumulative Retirement Simulation 7.5	Cumulative Retirement Simulation 5.0
50	36	0.025	0.081	0.065	0.025	0.081	0.065
51	131	0.037	0.112	0.091	0.061	0.184	0.150
52	132	0.026	0.105	0.084	0.086	0.269	0.221
53	123	0.024	0.105	0.085	0.108	0.346	0.287
54	106	0.009	0.111	0.088	0.116	0.419	0.350
55	129	0.075	0.121	0.097	0.182	0.489	0.413
56	137	0.073	0.116	0.092	0.241	0.548	0.467
57	123	0.108	0.146	0.117	0.323	0.614	0.529
58	107	0.102	0.140	0.112	0.392	0.668	0.582
59	120	0.149	0.180	0.145	0.483	0.728	0.642
60	116	0.194	0.177	0.142	0.583	0.776	0.693
61	84	0.233	0.189	0.151	0.680	0.819	0.739
62	70	0.447	0.382	0.324	0.823	0.888	0.824
63	51	0.503	0.402	0.341	0.912	0.933	0.884
64	22	0.491	0.409	0.344	0.955	0.960	0.924
65	14	0.468	0.538	0.471	0.976	0.982	0.960
66	1	0.355	0.598	0.514	0.985	0.993	0.980

[a]Based on model 5 parameter estimates, reported in table 2. The simulation is described in the text.

Consider first the annual departure rates based on the 7.5 percent contribution level. Again, the comparison is with the predicted departure rates based on the model 5 single-year estimates shown in table 2. There are two important features of the results: First, the discontinuities in the departure rates at 55 and at 60 are eliminated. Departure rates increase smoothly between ages 50 and 61. The jump at 62, due to Social Security provisions, remains, however. The effect of the Social Security provisions at 65 is now noticeable; it was not before. Second, although retirement rates after age 60 are lower under the defined contribution plan, departure rates at earlier ages are much higher. There is now no need to stay in the firm to receive the "retirement bonus" at 55, or to receive full benefits at 60 with 30 years of service. The net result is that more employees have left the firm by age 60 under the defined contribution than under the defined benefit plan. These results are consistent with the view that the defined benefit plan keeps employees in the firm until certain ages and then provides an incentive to leave. The defined contribution plan does not encourage them to stay, but if they do, neither does it encourage them to leave. Like departures under the current firm plan, it should be assumed that a large proportion of persons who leave the firm at the younger ages under the simulated plan do so for another job, whereas at older ages most are leaving the labor force. The results with the 5 percent contribution rate are similar to those with the 7.5 percent level, except that the departure rates are lower.

III. Summary and Conclusions

We have presented a model of retirement based on the option value of continued work. A person continues to work if the option of selecting a better age of retirement in the future is worth more than the value of retiring now. The model is both forward looking at a point in time and allows expectations about future events to be updated as individuals age. It thus incorporated the advantages of non-linear budget constraint formulations of the retirement decision and the advantages of continuous time hazard model formulations. The Markovian specification of the individual random effects, or the random walk special case of it, is an important component of the model. Single- and multiple-year versions of the model yield very similar results.

Predicted departure rates based on the model match actual departure rates very closely. In particular, discontinuous jumps in retirement rates at specific ages are captured by the model predictions. Out of sample predictions lend support to the predictive validity of the model.

Simulations of the effects of alternative pension plans show that plan provisions have very dramatic effects on retirement rates. For example, increasing the early retirement age from 55 to 60 would reduce by almost 40 percent the proportion of those employed at age 50 that has left the firm before age 60. At the same time, it would increase departure rates between 50 and 55, reflecting the longer wait until the early retirement "bonus" can be claimed.

Switching from the defined benefit to a defined contribution plan would have even greater effects on firm departure rates. The defined contribution formulation has no incentive effects. Annual departure rates of persons 60 and over would be reduced substantially. But, the departure rates between 50 and 54 would be increased from around 3 to about 10 percent, close to the departure rate between 55 and 59, after the early retirement age, under the existing plan. It is also close to the departure rate of employees who are under 50 and have just become vested in the existing firm plan. The net effect is to increase significantly the proportion of those employed at age 50 who have left the firm before age 60. These results support the view that defined benefit plans provide a strong and effective incentive for employees to stay in the firm until some age and then a strong and effective incentive to retire at some later age. The defined contribution plan does neither.

Although these results are based on the retirement decisions of employees in only one large firm, it is important to understand that the incentive effects inherent in this firm's pension plan are very typical of defined benefit plans. Nonetheless, we will in future work determine whether the results are supported in similar analysis based on data from other firms.

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Appendix A: A Hazard Model Interpretation

The general option value model is computationally complex. It is therefore of both methodological and substantive interest to demonstrate the relationship between the option value model and the more familiar proportional hazard model, which is straightforward to estimate and is widely used in the empirical analysis of retirement and labor force participation. This appendix demonstrates that the option value model reduces to the conventional proportional hazard model when the stochastic term ν_t in (2.13) is degenerate with a unit exponential distribution, that is, when ν_t corresponds to a single random effect with a unit exponential distribution, and when the "systematic" component of $G_t(r)$, $g_t(r)/K_t(r)$, is non-increasing. These are strong restrictions on the general option value model. The non-increasing requirement is clearly inconsistent with our data, and the degenerate distributional assumption would severely limit the flexibility that the Markovian specification provides in fitting observed retirement behavior. In demonstrating the relationship between the two models, the appendix also draws attention to the natural utility interpretation of the proportional hazard model.

Using the notation in the text, consider the probability that a person retires before age τ, according to the proportional hazard specification, call it $H(\tau)$. It is typically described in continuous time and is given by

$$
\begin{aligned}
\text{(A.1)} \qquad H(\tau) &= \Pr[R \le \tau] \\
&= 1 - \exp[-\textstyle\int_{u=t}^{\tau} \theta(u)du] \\
&= \Pr[e \le \textstyle\int_{u=t}^{\tau} \theta(u)du] \quad ,
\end{aligned}
$$

where θ is required to be non-negative and e is a random variable with a unit exponential distribution. Contemplating the development below, suppose that $\int_{u=t}^{\tau}\theta(u)du$ is the utility gain from retirement at age τ. The hazard rate is $H'(\tau)/[1 - H(\tau)] = [1 - H(\tau)]\theta(\tau)/[1 - H(\tau)] = \theta(\tau)$, which is the derivative of the utility of retirement at time τ.

We adopt the convention that $H(\tau)$ corresponds to the continuous time probability in (A.1), and H_τ corresponds to its discrete time counterpart when τ is an integer. With this notation, for integer τ (A.1) can be written as

$$\text{(A.2)}\qquad H_\tau = 1 - \exp(-\textstyle\sum_{s=t+1}^{\tau}\theta_s^*)$$
$$= \Pr[e \le \textstyle\sum_{s=t+1}^{\tau}\theta_s^*] \ , \text{ where } \theta_s^* = \int_{u=s-1}^{s}\theta(u)du \ .$$

Notice that $\Pr[R>\tau]$ is the probability that at every age before τ the person did not retire. If $-\sum_{s=t+1}^{\tau}\theta_s^* + e$ is the utility gain from retirement, versus continued work, at age τ, then the hazard model has a standard utility interpretation. This interpretation will always be true for some specification of the utility of retirement versus continued work. Suppose that $\sum_{s=t}^{\tau}\theta_s^*$ is monotonically increasing with τ. Then retirement will occur when the value of this expression exceeds e. The random term e in the proportional hazard model is an individual-specific term that remains constant over time; it can be thought of as an individual-specific threshold. The person retires when $\sum_{s=t}^{\tau}\theta_s^*$ crosses the threshold. The utility gain from retirement must be monotonically increasing; it will be if θ_s^* is non-negative for all s. We want to show that under strong restrictions the option value model reduces to this form.

The option value specification of the gain from <u>retirement</u> at age t is $E_tV_t(t) - E_tV_t(r^*)$. As long as $g_t(r_t^\dagger)/K_t(r_t^\dagger) > -\nu_t$ the person does not retire. Recall from (2.16) that in the option value (O) model of retirement, the probability of retiring by τ (let this be O_τ) is

$$\text{(A.3)} \qquad O_\tau = \Pr[R\leq\tau] = 1 - \Pr[R>\tau]$$
$$= 1 - \Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) > -\nu_t, \ldots, g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger) > -\nu_\tau].$$

$\Pr[R>\tau]$ is the probability that at <u>every</u> age from t -- the age that the person is first observed -- until age τ it is better to postpone retirement. That is the value, or utility, of postponing retirement is in each year greater than the value of retirement. Equation (A.3) is the probability of the complement of that event. The proportional hazard specification is obtained as a special case of the option value model by placing restrictions on both the error ν_s and on $g_s(r_s^\dagger)/K_s(r_s^\dagger)$. In particular, assume that: (i) $\nu_\tau = e$ for all τ (so that the only stochastic element in the retirement decision enters through a single time-invariant random effect), where e has a unit exponential distribution, and (ii) $g_s(r_s^\dagger)/K_s(r_s^\dagger)$ (the scaled value of postponing retirement) is non-increasing. Under assumption (i), the expression (A.3) reduces to

$$\text{(A.4)} \qquad O_\tau = 1 - \Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) > -e, \ldots, g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger) > -e]$$
$$= 1 - \Pr[\min(g_t(r_t^\dagger)/K_t(r_t^\dagger), \ldots, g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger)) > -e] \ .$$

That is, the event $R \leq \tau$ occurs only if the minimum gain from continued work was not always greater than zero, if the minimum value of $g_t(r_t^\dagger)/K_t(r_t^\dagger)$ was not always greater than the threshold -e.

If $g_s(r_s^\dagger)/K_s(r_s^\dagger)$ is non-increasing, then its smallest value will be in period τ. With both assumptions (i) and (ii), (A.4) becomes:

(A.5) $$O_\tau = 1 - \Pr[g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger) > -e]$$

$$= 1 - \exp[g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger)]$$

$$= 1 - \exp(-\textstyle\sum_{s=t+1}^{\tau}\bar{\theta}_s)\ \exp(g_t(r_\tau^\dagger)/K_t(r_\tau^\dagger)\ ,$$

$$\text{where } \bar{\theta}_s \equiv -\Delta(g_s(r_s^\dagger)/K_s(r_s^\dagger))$$

$$= g_s(r_s^\dagger)/K_s(r_s^\dagger) - g_{s-1}(r_{s-1}^\dagger)/K_{s-1}(r_{s-1}^\dagger)\ .$$

Note that the assumption that $g_s(r_s^\dagger)/k_s(r_s^\dagger)$ is non-increasing implies that θ_s is non-negative. Given this, the condition for the second expression in (A.5) to be between 0 and 1 is that $g_t(r_t^\dagger)/K_t(r_t^\dagger) \le 0$. This initial term is computed by the option value model. In the hazard model, however, this would correspond to an intercept that can always be chosen so that this condition is satisfied.

Numerical values for the terms in (A.5) would come from evaluating the expressions like those for V in the text, but in the option value model the ΔV terms would clearly not be non-negative. In the proportional hazard model θ_t would typically be expressed as $f(t)\cdot\exp(X\beta)\cdot h(Z_t\alpha)$, where $f(t)$ is a function of age, X is a vector of variables that remain constant over time, and Z_t are variables that change over time. The latter variables could in principle include a variable like our $G_t(r^*)$, but without estimating its parameters. The parameters would be assumed and estimation of the hazard model would yield an estimated coefficient on the computed $G_t(r^*)$ values.

Comparison of (A.2) and (A.5) shows that the option value model reduces to the proportional hazard model if the error ν_t is assumed to be degenerate over time with a unit exponential distribution, if $g_s(r_s^\dagger)/k_s(r_s^\dagger)$ is non-increasing, and if $g_t(r_t^\dagger)/K_t(r_t^\dagger)$ is set to zero. In this case, the unit average hazard rate, θ_t^*, is the negative of the change in the nonstochastic component of the utility, $\bar{\theta}_t$. Equivalently, the hazard model can be thought of as being derived from an underlying optimization problem where the unit averaged hazard rate is equal to the change, during that interval, in the value of being retired.

The assumption that $g_s(r_s^\dagger)/K_s(r_s^\dagger)$ be non-increasing is used to obtain the closed form (A.5). This could be relaxed, however, by using (A.4) to calculate the probabilities, although this would yield a non-standard hazard model. Similarly, the assumption of the unit exponential distribution could be replaced by (say) the assumption of normality, in which case (A.4) implies that $O_\tau = \Phi(-\min[g_t(r_t^\dagger)/K_t(r_t^\dagger),\ldots,g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger)])$, where $\Phi(\cdot)$ denotes the standard normal distribution.

The similarity between (A.2) and (A.5) has two implications. On the one hand, it suggests that an appropriate formulation of the proportional hazard model might be used to describe retirement behavior, avoiding the multiple integrals inherent in (2.15) and (2.16). On the other hand, the derivation makes clear how restrictive the assumptions underlying the proportional hazard specification are. The derivation also provides a link between the utility maximization problem discussed in section II and the conventional proportional hazard model, which is typically presented in an ad-hoc manner with little economic motivation. While this derivation suggests covariates for its estimation (namely those entering $g_t(r)$), it also makes its weaknesses more apparent.

Appendix B: Wage Forecasting Equation

The estimation procedure uses earnings forecasts to compute the expected value of the utility of future income, both when employed and after retirement. Pension benefits depend on the entire earnings history of the individual at the firm up to the date of retirement. Thus pension benefits for future dates of retirement are in general based on both earnings history, known to the individual at the current date, and forecasts of future earnings. For example, in 1981, estimates of the pension benefits that would be received were retirement in 1986 involve known earnings through 1981 and forecasts for the remaining years.

The income forecasting equation, shown in table B-1, was estimated using 98,465 observations, including multiple observations for the same person, taken from a panel of individuals in the same job category in the same firm as the 1500 individuals that were used in the estimation results reported in the text. The earnings data cover 1969-1984. Nominal earnings were converted to 1980 dollars using the consumer price index. S_t and A_t respectively denote years of service at the firm and age; D72, D73, etc. are dummy variables for the indicated years. Income forecasts were computed using the average of the coefficients on the dummy variables for 1978-1980, where the 1980 coefficient is normalized to be zero.

The data set contains earnings from 1969 on. Thus earnings before 1969 (for those who joined the firm before 1969) were back-cast using a specification similar to the forecasting equation in table B-1. The estimates (not reported here) were obtained using the same specification, except that time was reversed in the sense that all lags were replaced by leads.

Table B-1

Estimated income forecasting equation

Dependent variable: $\Delta \ln Y_t$

Regressor	Coefficient	Standard Error	t-statistic
A_t	-0.00107124	0.001002558	-1.069
S_t	0.002456179	0.0009254529	2.654
A_t^2	-0.0001088365	0.00001254898	0.867
S_t^2	0.0001038392	0.00001554057	6.682
$A_t S_t$	-0.000129604	0.00002289325	-5.661
$\Delta \ln Y_{t-1}$	-0.269047	0.0173911	-15.470
$A_t \Delta \ln Y_{t-1}$	0.0001924038	0.0004487285	0.429
$S_t \Delta \ln Y_{t-1}$	0.002951872	0.0005582732	5.288
$\Delta \ln Y_{t-2}$	-0.29905	0.01681537	-17.784
$A_t \Delta \ln Y_{t-2}$	0.003397685	0.0004338587	7.831
$S_t \Delta \ln Y_{t-2}$	0.001986168	0.0005479163	3.625
intercept	-0.0759745	0.01984605	-3.828
D72	0.16442198	0.003250329	50.586
D73	0.13501186	0.003229089	41.811
D74	0.12498651	0.003179938	39.305
D75	0.07275624	0.003142718	23.151
D76	0.13272861	0.003113418	42.631
D77	0.13154143	0.003111171	42.280
D78	0.16778513	0.003076656	54.535
D79	0.07741906	0.003073479	25.189
D81	0.06855253	0.003210239	21.354
D82	0.03074399	0.003277569	9.380
D83	0.05627368	0.003304291	17.030
D84	0.07609989	0.003388655	22.457
SEE = 0.198			