# Notes and Comment 

# Perceptual constancy during ocular pursuit: A quantitative estimation procedure 

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Perceptual constancy of visual motion is usually described as the degree of correspondence between physical and perceived characteristics of motion in the external world. To study it, one has to assess the relationship between physical motion, its retinal image, and its perception. We describe a quantitative estimation procedure for a measure K denoting the degree of perceptual constancy of background target motions noncollinear to the eye movements during ocular pursuit. The calculation of K is based on three vectors describing the target motion (1) as it is physically, (2) as it is mapped to the retina, and (3) as it is perceived, but only the direction of the perceptual motion vector has to be determined experimentally. K allows for quantitative comparison between experiments with a variety of parameters in visual motion displays.

Human perceptual systems are capable of stabilizing the surrounding world despite the continuous displacement of the retinal image due to eye and body movements. Perceptual stability or constancy is usually determined by the degree of correspondence between perceived and physical stationarity or motion of the objects in the external world. A variety of factors related to visual stimulation and self-motion influence perceptual constancy. To study the compensatory processes responsible for relating selfmotion information to retinal information, it is necessary to assess the relationship between physical event, retinal image, and perception. If physical events differ from their retinal images, the degree of constancy is higher, the closer the percept is to the physical event.

A typical example of incomplete perceptual constancy is the Filehne illusion (Filehne, 1922). It is observed during visual tracking of a moving object, and consists of the apparent motion of another stationary background target in the direction opposite to that of the pursuit movement. In this case, the background target is physically stationary while its retinal image is moving with a velocity equal to the velocity of the tracking eye. If no background

[^0]motion is perceived, the constancy is complete, whereas if the background target is perceived to move with the same velocity as that of the eyes, there is a complete loss of constancy.

To determine the degree of constancy (or of loss of constancy) during pursuit eye movements, it is necessary to measure the perceived velocity of background motionthat is, the strength of the Filehne illusion. Several authors have measured the perceived velocity of motion of background targets during eye tracking by using a "compensation' method (de Graaf \& Wertheim, 1988; Ehrenstein, Mateeff, \& Hohnsbein, 1985, 1987; Mack \& Herman, 1978; Wertheim, 1987; Wertheim \& Bles, 1984). This consists of presenting a background target during ocular pursuit that moves in the same direction as the pursuit movement. The velocity of the background target is adjusted so that it compensates for the velocity of the illusory background motion, and the target is perceived as stationary. It is assumed that if the background target is physically stationary during ocular pursuit, it will be perceived to move with the same "compensation'" velocity but in the opposite direction. Thus, the magnitude of the 'compensation" velocity ( $V_{c}$ ) at which a moving background target is perceived as stationary during smooth pursuit eye movements is assumed to be equal to the magnitude of the "velocity of the Filehne illusion'" $\left(V_{\mathrm{f}}\right)$, or $V_{\mathrm{c}}=V_{\mathrm{f}}$.
The validity of the compensation method relies on the idea of collinear vector summation. The perceived velocity of motion of the background target $V_{\mathrm{f}}$ is compensated by the real velocity of motion $V_{t}$ of the target. Therefore, if the velocity $V_{t}$ is equal to the velocity of ocular pursuit, $V_{e}$, constancy is zero because the subject's percept of immobility corresponds to the immobility of the target on the retina-that is, the subject processes nothing but his retinal image. If, on the other hand, $V_{c}=0$, constancy is complete because perception corresponds to immobility of the background target in the external physical space.

Mack and Herman (1978) proposed measuring the 'constancy loss'' by the ratio $V_{\mathrm{c}} / V_{\mathrm{e}}$ (which they assumed to be the same as $V_{\mathrm{f}} / V_{\mathrm{e}}$ ). Ehrenstein et al. (1987) introduced a similar measure, $K$, expressing the "degree of constancy' in percent.

$$
\begin{equation*}
K=\left(1-V_{\mathrm{c}} / V_{\mathrm{e}}\right) \cdot 100 \% \tag{1}
\end{equation*}
$$

where $K=100 \%$ if constancy is complete.
Measuring the constancy is not quite as simple when the background target moves noncollinearly with the eye movement. In this case, the trajectory of the retinal image motion, determined by the vectorial difference of the motion of the background target and the eye movement,
might be much more complicated than it is in the case of the Filehne illusion. To establish the degree of constancy (or the constancy loss), it is necessary to compare the motion pattern perceived by the subject with the physical and retinal trajectories of the background target motion. The possibilities of quantitative measurements with curvilinear motion trajectories are rather restricted. Situations in which the trajectories of the background target on the screen and on the retina are straight lines are more convenient for estimating the degree of constancy. Such displays were used by, for example, Festinger, Sedgwick, and Holtzman (1976), Hansen (1979), Mateeff (1980), Swanston and Wade (1988), and Wallach, Becklen, and Nitzberg (1985).
Since different methods were used to estimate constancy in these studies, the data cannot be easily compared quantitatively. Here, we propose a general measurement procedure and a way in which to calculate the degree of constancy that includes the case when a background target motion of constant velocity is not collinear with ocular pursuit.
Let us assume that the eyes smoothly follow an object, O , moving with a (constant) velocity $\vec{V}_{\mathrm{c}}$ along a linear path. During the same time, a background target moves with a (constant) velocity $\vec{V}_{\text {, at }}$ an angle $\alpha \neq 0$ with respect to $\vec{V}_{\mathrm{e}}$. The vector $\vec{V}_{\mathrm{r}}=\vec{V}_{\mathrm{t}}-\vec{V}_{\mathrm{e}}$ represents the velocity of motion of the retinal image of the background target (Figure 1). The angle between $\vec{V}_{\mathrm{r}}$ and $\vec{V}_{\mathrm{e}}$ is $\beta \neq 0$. Simple considerations of the geometrical interrelations between $\vec{V}_{\mathrm{t}}, \overrightarrow{\mathrm{r}}_{\mathrm{r}}$, and $\vec{V}_{\mathrm{e}}$ lead to the following expressions:

$$
\begin{gather*}
V_{\mathrm{r}}=V_{\mathrm{I}} \cdot \sin \alpha / \sin \beta,  \tag{2}\\
V_{\mathrm{r}}=V_{\mathrm{e}} \cdot \sin \alpha / \sin (\beta-\alpha), \tag{3}
\end{gather*}
$$

and

$$
\begin{equation*}
V_{\mathrm{t}}=V_{\mathrm{e}} \cdot \sin \beta / \sin (\beta-\alpha), \tag{4}
\end{equation*}
$$

where $V_{\mathrm{t}}, V_{\mathrm{r}}$, and $V_{\mathrm{e}}$ are the magnitudes of the vectors $\vec{V}_{\mathrm{t}}, \vec{V}_{\mathrm{r}}$, and $\vec{V}_{\mathrm{e}}$.
From Expression 2 it follows that the velocity $V_{\mathrm{r}}$ of the retinal image motion can be kept constant while the orien-


Figure 1. When the eye is following an object, O , moving with $\stackrel{a}{V}^{\text {a velocity }} \vec{V}_{e}$ while a background target, $T$, moves with a velocity $\vec{V}_{i}$, the velocity of the retinal motion of the background is $\vec{V}_{r}=$ $\vec{V}_{t}-\vec{V}_{e} \cdot \alpha=$ angle between $\vec{V}_{e}$ and $\vec{V}_{i} ; \beta=$ angle between $\vec{V}_{e}$ and $\vec{V}_{r}$.


Figure 2. If the velocity of the apparent background motion during ocular pursuit of object $O$ is $\vec{V}_{t}$, the velocity of the perceived subjective motion $\vec{V}_{\mathbf{a}}$ of a background target $T$, moving with velocity $\vec{V}_{V}$, will be $\vec{V}_{i}=\vec{V}_{t}+\vec{V}_{f}, \gamma$ determines the direction of perceived motion with respect to $\overrightarrow{V_{c}}$.
tation $\beta$ of the motion trajectory is varied by appropriately varying either $V_{\mathrm{t}}$ or $\alpha$. This equation was used by Mateeff (1980). In his experiment, he controlled the velocity of the retinal image and demonstrated that changing $V_{\mathrm{r}}$ from 6 to $13 \mathrm{deg} / \mathrm{sec}$ did not affect the constancy.
Equation 3 shows that if the parameters $V_{\mathrm{r}}$ and $\beta$ of the retinal image motion are to be kept constant while the eye velocity, $V_{c}$, is manipulated, the angle $\alpha$ between $\vec{V}_{c}$ and $\vec{V}_{\mathrm{t}}$ has to be appropriately varied as well.
Equation 4 can help if manipulation of $V_{\mathrm{e}}$ is needed while the parameters $V_{\mathrm{t}}$ and $\alpha$ of the motion of the background target are to be kept constant.
Solving Equation 4 for $\beta$, we obtain:

$$
\begin{equation*}
\beta=\arctan \left[V_{\mathrm{t}} \cdot \sin \alpha /\left(V_{\mathrm{t}} \cdot \cos \alpha-V_{\mathrm{e}}\right)\right] . \tag{5}
\end{equation*}
$$

From Equations 5 and 2, the parameters $V_{\mathrm{r}}$ and $\beta$ of the retinal image motion can be calculated when the eye velocity and the parameters of the target motion are given.
We suppose that Swanston and Wade (1988) had intended a similar formula for describing the deviation from the vertical of the retinal path of their background targets during horizontal eye movement, but that formula, 1 in their paper (p. 560 ), has been misprinted. ${ }^{1}$

So far, the analysis deals with the control of physical variables such as $\vec{V}_{\mathrm{t}}, \vec{V}_{\mathrm{r}}$, and $\overrightarrow{\mathrm{V}}_{\mathrm{e}}$ in Figure 1. The perceived motion of the background target can be represented by the velocity vector $\vec{V}_{\mathrm{s}}$. If $\vec{V}_{\mathrm{s}} \equiv \vec{V}_{\mathrm{t}}$, the target motion on the screen is veridically perceived and the constancy is complete. If $\vec{V}_{\mathrm{s}} \equiv \vec{V}_{\mathrm{r}}$, the perception is entirely determined by the retinal image and there is a total loss of constancy.

Our basic assumption is that during ocular pursuit every background target, irrespective of whether it is moving or not, is subjected to an apparent motion component in a direction opposite to that of the pursuit movement. In the case of a stationary background target, this apparent motion component is the Filehne illusion. Hence, the perceived motion of the background target is the vector sum of an apparent and a real motion component, both determined relative to the stationary observer's head. This as-
sumption is based on the same logic that justifies the measurement of constancy when the target motion is collinear with the pursuit motion. In this case, the apparent backward motion $V_{\mathrm{f}}$ is compensated by the real motion $V_{\mathrm{t}}$ in the direction of the eye movement, until subjective stationarity is achieved when $V_{\mathrm{t}}=V_{\mathrm{c}}$. In other words, the perception in the collinear case is also regarded as determined by the vector sum of a real and an apparent motion. We simply apply the same logic in the twodimensional case of noncollinear motion of the background target and pursuit motion.

In Figure 2, the velocity of the apparent background motion is labeled $\vec{V}_{\mathrm{f}}$. The sum of $\vec{V}_{\mathrm{f}}$ and $\vec{V}_{\mathrm{t}}$ determines the vector of the perceived subjective motion $\vec{V}_{\mathrm{s}}$ :

$$
\begin{equation*}
\vec{V}_{\mathrm{s}}=\vec{V}_{\mathrm{t}}+\vec{V}_{\mathrm{f}} . \tag{6}
\end{equation*}
$$

From Equation 1, we have

$$
\begin{equation*}
V_{\mathrm{f}}=V_{e} \cdot(1-K) \tag{7}
\end{equation*}
$$

and in Figure 2, it is easy to show that

$$
\begin{equation*}
V_{\mathrm{s}} \cdot \cos \gamma=V_{\mathrm{t}} \cdot \cos \alpha-V_{\mathrm{f}} \tag{8}
\end{equation*}
$$

and that

$$
\begin{equation*}
V_{\mathrm{s}}=V_{\mathrm{t}} \cdot \sin \alpha / \sin \gamma . \tag{9}
\end{equation*}
$$

Then, from Equations 7, 8, and 9, we get

$$
\begin{align*}
V_{\mathrm{t}} \cdot \cos \gamma \cdot \sin \alpha & \alpha \sin \gamma \\
& =V_{\mathrm{t}} \cdot \cos \alpha-V_{\mathrm{e}} \cdot(1-K) \tag{10}
\end{align*}
$$

Finally, substituting $V_{\mathrm{t}}$ from Equation 4 in Equation 10, solving for $K$, and simplifying, we get

$$
\begin{equation*}
K=1-\frac{\sin (\gamma-\alpha)}{\sin (\beta-\alpha)} \cdot \frac{\sin \beta}{\sin \gamma} \tag{11}
\end{equation*}
$$

This is the measure of the degree of constancy that we propose. All angles in Equation 11 are measured counterclockwise, as shown in Figures 1 and 2. The calculation of $K$ is based on the comparison between the orientation of the three motion vectors describing (1) the target motion relative to the screen $\left(\vec{V}_{t}\right)$, (2) the target motion relative to the moving line of sight-that is, the target's retinal motion ( $\vec{V}_{\mathrm{I}}$ ), and (3) the subjective motion of the target $\left(\vec{V}_{\mathrm{s}}\right)$. When the subjective motion coincides with the target motion relative to the line of sight, $\gamma=\beta$ and $K=0$. When the physical target motion on the screen is perceived veridically, $\gamma=\alpha$, and therefore $K=1-$ that is, we have $100 \%$ degree of constancy.

According to Equation 11, $K$ is entirely determined by the angles between velocity vectors irrespective of their magnitude. The same measure $K$ can also be used in cases of sinusoidal eye tracking combined with in-phase target motion. This measure allows for a quantitative compari-
son between data from experiments with a great variety of parameters in linear motion displays to be made.
The experimental procedure requires the estimation of only one parameter-the orientation angle $\gamma$ of perceived motion of the background target, as in the studies of Hansen (1979), Mateeff, Ehrenstein, and Hohnsbein (1987), and Wallach et al. (1985). Another possibility is to apply psychophysical matching of the subjective direction of motion to the vertical, as has been done by Mateeff (1980). In such a case, $\gamma=90^{\circ}$, and the entries of $\alpha$ and $\beta$ in Equation 11 are those at which a vertical motion of the background target is experienced.

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## NOTES

1. Their formula should read:

$$
\tan p=(M+d \cdot \sin (t)) / d \cdot \cos (t)
$$

2. Note that $V_{\mathrm{f}}$ and $V_{\mathrm{c}}$ denote the magnitudes of the vectors $\vec{V}_{\mathrm{f}}$ and $\vec{V}_{\mathrm{e}}$. Therefore Equation 7 holds, although the velocities are in different directions-that is, $\vec{V}_{\mathrm{f}}=-\overrightarrow{V_{c}}$.
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