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# PERCEPTUAL WATERMARKING FOR DISCRETE SHEARLET TRANSFORM

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# ABSTRACT

This paper presents a new perceptual watermarking model for Discrete Shearlet transform (DST). DST provides the optimal representation [10] of the image features based on multi-resolution and multi-directional analysis. This property can be exploited on for watermark embedding to achieve the watermarking imperceptibility by introducing the human visual system using Chou's model. In this model, a spatial JND profile is adapted to fit the sub-band structure. The combination of DST and the Just-Noticeable Distortion (JND) profile improves the levels of robustness against certain attacks while minimizing the distortion; by assigning a visibility threshold of distortion to each DST sub-band coefficient in the case of grey scale image watermarking.

*Index Terms*—Digital image watermarking, Frequency domain watermarking, Discrete Shearlet Transform (DST), Just-Noticeable Distortion (JND).

# **1. INTRODUCTION**

During recent years, popularity in the transmission of digital information through the internet has created a new set of challenges. The huge amount of transmitted information has led to a need in terms of digital multimedia authentication and content integrity verification of the digitized properties [5]. Digital image watermarking is one method that has been developed to address these problems. However, efficient digital watermarking methods should meet some criteria such as robustness, imperceptibility and capacity as the three conflicting attributes of data hiding systems. Currently, the most challenging issue is how to solve the trade-off between robustness and imperceptibility, since enhancing robustness implies necessarily increasing the watermark strength and therefore produces a loss of transparency [4].

Finding such an optimized solution still reminds a challenge among the watermarking community. The Least Significant Bit (LSB) [9] is one of the embedding techniques developed in the spatial domain. This technique is based on modifying the least significant bit of an image. However such simple techniques have relatively low bit capacity and poor robustness.

Watermarking algorithms based on transform domain such as the DFT (Discrete Fourier Transform) [8], DCT (Discrete Cosine transform) and DWT (Discrete Wavelet Transform) [6] have been proposed to overcome the drawback of spatial domain watermarking.

The Discrete Cosine Transform is a technique for converting and dividing a signal (or image) in terms of the sum of sinusoids with different frequencies and amplitudes. However, the DCT-based watermarking techniques have shortcomings in terms of higher compression levels and attack strengths [5]. Similarly, DWT transform provides a time-frequency representation of the signal. Wavelet functions have the ability to capture data at different scales or resolutions, which makes this transform widely used in image compression, denoising and texture analysis. However, it has shortcomings in terms of having limited directionality in its filtering structure. This fact reduces its data embedding capacity for watermarking when preserving the imperceptibility condition [10, 11].

In this regard, initial research on DST has shown its properties for capturing directional features more precisely than previous methods. This makes it a good candidate for watermarking applications [5].

Many image watermarking algorithms that utilize visual models to increase the robustness and transparency can be found in [1, 12]-[15].

This paper aims to explore further the usage of DST for watermarking and to achieve new standards of imperceptibility by combining visual models and the discrete Shearlet transforms for watermarking.

## 2. THE DISCRETE SHEARLET TRANSFORM

The DST is a new discrete multi-scale directional representation with two potentially interesting capabilities for watermarking: using the power of multi-scale methods and capturing the geometry of multidimensional data [5]. The disadvantage of this transform is the increased redundancy [10]. The Shearlet transform is implemented by applying the Laplacian pyramid scheme and directional filtering [16].

For an image *I*, the Shearlet transform is a mapping

$$I \rightarrow S \mathcal{H}_{\psi} I(a, s, x)$$

depending on the scale a > 0, the orientation s and the location x. The Shearlet transform can be expressed as

$$SH_{\psi}I(a,s,x) = \int I(y)\psi_{as}(x-y)dy = I * \psi_{as}(x)$$
(1)

The affine systems with composite dilations are the collections of the form:

$$\psi_{j,k,l}(x) = |\det A|^{\frac{l}{2}} \psi(S^k A^j x - l); j, k \in \mathbb{Z}, l \in \mathbb{Z}^2$$
(2)

where  $\psi \in L^2(\mathbb{R}^2)$ , A and S are invertible 2×2 matrices which represent dilation and geometrical transform as follows:

$$A = \begin{pmatrix} a & 0 \\ 0 & \sqrt{a} \end{pmatrix}, S = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$
(3)

Hence the Discrete Shearlet transform (DST) is defined as below:

$$\mathcal{SH}\{\psi_{i,k,l} = 2^{\frac{3}{2}j}\psi(s_k A_{2^j} - l): j, k \in \mathbb{Z}, l \in \mathbb{Z}^2\}$$
(4)

The Shearlet coefficients are given by

$$X = \iint 2^{\frac{-3}{2}j} g_j(u,v) (\overline{W(2^jv-l)} \exp\left(2\pi i \left(\frac{n_1+ln_2}{4^j}\xi_1 + \frac{n_2}{2^j}\xi_2\right)\right) d\xi_1 \ d\xi_2$$
(5)

where W is a window function localized on a pair of trapezoids,  $l = -2^{j}$  (or  $2^{j} - I$ ) is the junction of the horizontal cone and u and v are the pseudo-polar coordinates.  $g_{j}(n_{1},n_{2})$  are the values of the DFT on a pseudo-polar grid.  $n_{1}$  and  $n_{2}$  are finite sequence of values for a given image  $N_{rows} * C_{olumns.}$  More details are given in [10, 17].

#### **3. THE VISUAL MODEL**

To fulfil the imperceptibility requirement of watermarking system, the characteristics of the human visual system (HVS) can be exploited. With this idea in mind, a just-noticeable-distortion (JND) model or its equivalent minimally noticeable distortion (MND) profile, were proposed by Chou and Li [1] to quantify the "perceptual redundancy"[2]. In this model, each individual coefficient is assigned a value that quantifies the maximum distortion that can be applied to that coefficient before creating an unacceptable level of visual distortion.

The full band JND profile is described by the following expressions [1]:

$$JND_{fb}(x,y) = max\left\{f_1\left(b_g(x,y), m_g(x,y)\right), f_2\left(b_g(x,y)\right)\right\}$$
(6)

where the values bg(x,y) and mg(x,y) are the average background luminance and luminance contrast around the pixel at (x,y). The spatial masking effect and the visibility threshold based on back ground luminance are given by the functions  $f_1(x,y)$  and  $f_2(x,y)$  respectively [2].

In order to apply this model to the multisclae multidirectional decomposition structure of DST some modifications need to be applied. To reflect the directionality of the DST, a set of filters are designed to obtain the value of the mg(x,y) and therefore,  $f_1(x,y)$ , across the DST scales and direction so the resulting values can be directly assigned to the DST coefficients. A set of

M operators  $G_k$  are calculating by rotating the original filter G.

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	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Ĺ
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Ĺ
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Ĺ
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Ĺ
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Ĺ
	1	3	5	7	9	11	13	15	13	11	9	7	5	3	1	Ĺ
G =	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Ĺ
u –	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
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u –	-1 0 0 0	-3 0 0 0	-5 0 0 0	-7 0 0 0	-9 0 0 0	$-11 \\ 0 \\ 0 \\ 0 \\ 0$	-13 0 0 0	-15 0 0 0	$-13 \\ 0 \\ 0 \\ 0 \\ 0$	-11 0 0 0	-9 0 0 0	-7 - 0 0 0	-5 0 0 0	-3 0 0 0		

Each  $G_k$  corresponds to a DST sub-band and it is obtained by rotating and then cropping the original filter G for different orientations  $\forall \theta \in s$ .

$$G_{k}(x',y') = G(x,y), \text{ where } [x',y'] = R^{*}[x,y] \text{ and } R = \begin{bmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{bmatrix}$$
(7)

Similarly,  $m_g$  and  $f_l$  are then calculated using equations (8) and (9). The parameters  $\alpha$  and  $\beta$  are background dependent functions which specify the slope of the line and the intersection with the visibility threshold axis [3].

$$m_{g}(x, y) = max_{k=1,2,..,M}\{|grad_{k}(x, y)|\}$$
(8)

$$f_1\left(b_g(x,y),m_g(x,y)\right) = m_g(x,y)\,\alpha(b_g(x,y)) + \beta\left(b_g(x,y)\right) \quad (9)$$

Finally, the adapted JND sub-band structure that replicate the DST structure is calculated as follows:

$$JND_{q}^{2}(x, y) = \left[\sum_{l=0}^{4^{t}-1} \sum_{j=0}^{4^{t}-1} JND_{fb}^{2}(i+x.4^{t}, j+y.4^{t})\right] \cdot \omega_{q}$$
(10)  
for  $0 \le x \le N/4^{t}, 0 \le y \le N/4^{t}$   
and 
$$\begin{cases} t = 4 - \left[\frac{p-1}{4}\right] & (if \ 0$$

where  $JND_q(x,y)$  represents the JND value at position (x,y) of the  $q^{\text{th}}$  sub-band and the weighting factor  $\omega_q$  for the  $q^{\text{th}}$  sub-band is defined by the following expression:

$$\omega_q = \left(S_q \cdot \sum \forall j \, S_j^{-1}\right)^{-1} \tag{11}$$

where  $S_j$  denotes the average sensitivity of the HVS to spatial frequencies in the  $j^{th}$  sub-band, more detail is given in [1]. The reduction factor  $4^t$  is introduced since the DST sub-samples by four at each level of resolution.

An example of the modified JND profile decomposition corresponding to the frequency content of DST sub-bands is shown in Fig.1.This decomposition allows assigning a maximum distortion level to each DST coefficient, which indicates where and with what strength the watermarking can be embedded at individual basis.

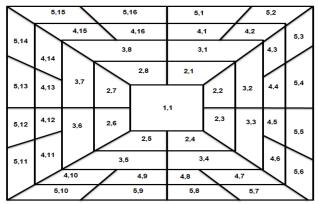


Fig.1. JND profile structure for DST sub-bands using five decomposition levels 16 orientations and 49 sub-bands. First number represents the decomposition level while second number depicts the orientation within the level.

#### 4. IMAGE WATERMARKING USING JND PROFILES OF DISCRETE SHERALET TRANSFORM

Using the perceptual model proposed in Section 3, the following embedding rule is applied for the watermarking system, as depicted in Fig.2.

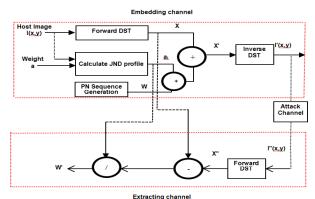


Fig.2. Proposed Watermarking System. Upper block describes the watermarking process while lower block depicts the extraction process.

First, the host Image is decomposed using discrete Shearlet transform. Then JND values for each individual coefficient in the decomposition are estimated using Chou's visual models. Once this has been calculated, the watermark sequence W is embedded in the largest and most significant C values of JND, using the following additive-multiplicative rule:

$$X_i' = X_i \cdot (1 + a_i \cdot W_i) \tag{12}$$

Parameter  $a_i$  is the JND profile weight based on the perceptual model proposed in this paper and is calculated as follows:

$$a_i = a * JND_{q \in X_i} (x \in X_i, y \in X_i)$$
(13)

Finally, the watermarked image I'(x, y) is obtained by performing the inverse DST of the watermarked coefficients x'.

In order to extract the watermark, the original image is needed and the extraction is performed by reversing the insertion process. The Watermark extraction process can be described by following equation:

$$W_i' = (X_i'' - X_i)(1/a_i)$$
(14)

where W' is extracted watermark, X'' are coefficients related to the DST decomposition of the received watermarked image and X are the original coefficients related to the DST decomposition of the original Image.

# 5. PERFORMANCE EVALUATION

To verify the effectiveness of the proposed algorithm, series of experiments were conducted. In these experiments, thirty 512 ×512 sized grayscale images were used as host images. Watermarking in the DST domain was performed by embedding the watermark in the all level DST sub-bands of the host image. The Shearlet Matlab toolbox was used for the embedding and extracting procedure [10]. The sizes of the shearing filters are 16x16 and 32x32 for all 8 and 16 directions. A set of operators which are based on DST sub-band structure were fixed to M=16, q=49, a=[1, 5], s=[1, 16] and  $\theta=[0 \pm 11.25 \pm 22.5 \pm 33.75 \pm 45 \pm 67.5 \pm 78.75 \pm 90]$ ; for all the experiments. These parameters were used to provide a better level of resolution.

In the following sections imperceptibility and robustness, two key measurements, are examined for watermarking performance. Root-mean squared error (RMSE), Peak signal to noise (PSNR) and Structural similarity (SSIM) are the used metrics for measuring the similarity between two images. In particular SSIM measures the quality of the image using an initial distortion-free image as reference. SSIM is designed to improve traditional methods such as PSNR and RMSE, which have proved to be inconsistent with human eye perception. The resulting SSIM index is a decimal value between -1 and 1, where 1 is only reachable in the case of two identical sets of data. The SSIM metric is calculated on various windows of an image. SSIM is calculated using the following equation

$$SSIM(x, y) = \frac{(2\mu_x \,\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}$$
(15)

where  $\mu_x$  and  $\mu_y$  are the average of x and y,  $\sigma_x^2$  and  $\sigma_y^2$  are variance of x and y.  $\sigma_{xy}$  is the covariance matrix.  $c_1$  and  $c_2$  are two variables to stabilize the division with weak denominator. More details can be found in [7].

#### 5.1. Imperceptibility

In order to validate the impact of the JND addition for watermarking in terms of imperceptibility, the JND model was added straight to the DST coefficients as in [2]. The images were then recomposed and PSNR, RMSE and SSIM between the original and watermarked images were measured. Results are compared against the spread spectrum scheme using Discrete Wavelet Transform (DWT) [2] and Dual Tree Complex Wavelet Transform (DTCWT) [3].The distortions are given in table 1 and 2. By comparing the results, it is concluded that the proposed algorithm based on DST has a better imperceptibility as reflected in a smaller RMSE and higher similarity SSIM between original and watermarked images.

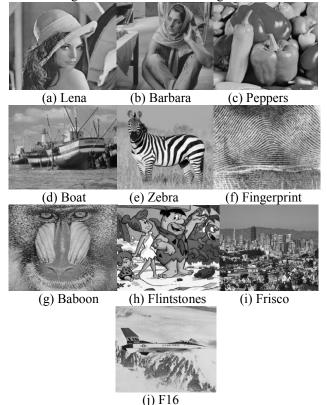


Fig.3. Set of ten images used for embedding watermark

In this section the effect of parameters of proposed method are introduced. In this regard, trade-off between imperceptibility (measured with PSNR), watermarking strength (modified with input weight an in eq 12 and Fig 2) and capacity (modified by parameter C defining the watermark length) are investigated. Different watermark lengths C=[1000, 10000, 100000] with different watermark strengths a=[0.1 5] were tested for all 30 images. The average PSNRs are shown in Fig.4.

Table1. Comparison between Average RMSE distortions based on all JND Coefficients

RMSE	DWT	DTCWT	DST
Baboon	2.438	11.499	1.721
Barbara	2.151	6.629	1.266
Boat	2.188	4.756	1.488
F16	2.629	4.358	2.965
Fingerprint	2.919	5.568	3.242
Flintstone	3.285	8.316	3.993
Frisco	2.880	9.890	2.698
Lena	2.082	3.765	1.317
Peppers	2.094	3.664	1.363
Zebra	2.816	5.392	3.065
Bunny	2.575	1.880	2.027
Cameraman	2.246	5.392	1.710
Clock	2.676	4.132	2.089
Elaine	2.039	5.796	1.594
Flower	1.901	4.885	1.470
Girl	1.796	1.895	1.414
House	2.160	1.930	1.697
Jelly Beans	2.333	1.830	1.829
Lake	2.404	5.451	1.797
Living room	2.212	7.446	1.679

Moon surface	1.823	3.231	1.425
Pirate	2.096	6.755	1.593
Scientist	1.893	4.174	1.460
Splash	1.970	3.521	1.540
Straw	2.936	12.433	2.131
Tree	2.360	3.230	1.813
Truck	1.736	5.579	1.338
Walk bridge	2.305	7.931	1.687
Woman-blonde	2.150	7.432	1.645
Woman-dark hair	2.054	4.692	1.614
Average	2.305(0.385)	5.448(2.671)	1.889(0.655)

Table2. SSIM between original and watermarked images based on all JND Coefficients.

JND Coefficients.			
SSIM	DWT	DTCWT	DST
Baboon	0.997	0.966	0.999
Barbara	0.995	0.977	0.999
Boat	0.995	0.984	0.999
F16	0.989	0.981	0.999
Fingerprint	0.998	0.995	0.999
Flintstone	0.993	0.987	0.999
Frisco	0.995	0.975	0.999
Lena	0.994	0.984	0.999
Peppers	0.994	0.984	0.999
Zebra	0.991	0.990	0.999
Bunny	0.975	0.982	0.993
Cameraman	0.989	0.979	0.997
Clock	0.986	0.969	0.996
Elaine	0.994	0.977	0.998
Flower	0.990	0.967	0.997
Girl	0.993	0.992	0.997
House	0.988	0.990	0.997
Jelly Beans	0.984	0.988	0.995
Lake	0.992	0.986	0.998
Living room	0.995	0.973	0.999
Moon surface	0.995	0.980	0.999
Pirate	0.995	0.978	0.999
Scientist	0.994	0.984	0.998
Splash	0.991	0.988	0.998
Straw	0.998	0.976	0.999
Tree	0.991	0.990	0.998
Truck	0.997	0.977	0.999
Walk bridge	0.996	0.980	0.999
Woman-blonde	0.991	0.975	0.998
Woman-dark hair	0.988	0.979	0.998
Average	0.992( 0.005)	0.981( 0.007)	0.998(0.001)

5.2 Balance between strength, imperceptibility and capacity

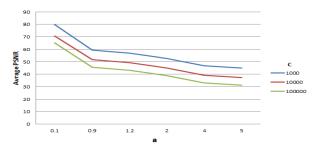


Fig.4. PSNR value response to different randomly generated watermarks length C and watermarking strength a.

As expected, the imperceptibility decreases when the watermarking length or the watermarking strength increases. On the other hand, by increasing C a bigger capacity available in proposed method. Similarly, by increasing the strength a, proposed method should be able to cope better with attacks and errors in the transmission channel.

#### 5.3. Effects of Attacks on watermarking algorithm

Robustness is a measure of the watermarking method's resistance against different types of digital signal processing attacks. In this section different tests have been carried out to prove the performance of the proposed method. The watermark to be embedded is a simple pseudo-random sequence  $(\pm 1)$  that is generated to get the spread-spectrum modulated watermark. Results are again compared against DWT and DTCWT.

In order to have a fair comparison, given that every method has a different imperceptibility/robustness balance, all the methods were tuned to provide a nearly 43db PSNR value before the attack. The effect of five attacks Additive white Gaussian noise (AWGN), Compression, Blurring, Cropping and Rotation are tested on the watermarked image Baboon and the visual results are shown in Figure 5.

Figure 6, 7, 8, 9 and 10 illustrates the Bit error rate (BER) obtained when the different attacks are performed. Every attack is analyzed at different levels, from the weakest strength to the maximum in the horizontal axis.

Gaussian noise is added to the watermarked image with different standard deviations,  $d=[0.01\ 0.8]$ . From these experimental results in Fig.6, it is found that DST provides comparable robustness with the state of the art against AWGN attack, consistently better than DTCWT and similar or better than DWT, especially for severe attacks.

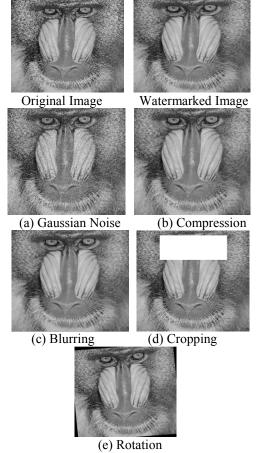


Fig.5. Watermarked image with different types of attacks

The watermarked image is compressed to provide an output quality between 100% and 5% of the original image. No smoothing is applied. According to Fig.7, it can be concluded that DST performs poorly against JPEG compression in comparison with DWT and DTCWT.

Gaussian low pass filter is applied on the watermarked image to analyse the effect of blurring. The standard deviation is varied from 0.1 up to 0.8. From these experimental results in Fig.8, it is found that DST also performs poorly against blurring attack in comparison with DTCWT and DWT.

The watermarked image is cropped by cutting off 5%, 15%, 50% and 75% of some random part of the image. To extract the watermark, the missing part of the image should be replaced with those parts from the original non watermarked image. From these experimental results in Fig.9, it is found that DST provides very good robustness against cropping attack in comparison with DWT and DTCWT.

Finally, the watermarked image is slightly rotated and cropped by applying several angles between 1 to 5 degrees in a counter clockwise direction. According to Fig.10, it can be concluded that DST provides very good robustness against rotation attack in comparison with DWT and DTCWT.

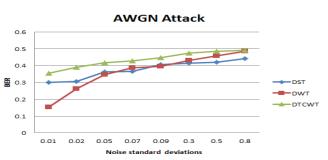


Fig.6. BERs for AWGN attack applied for same 1000 randomly generated watermarks embedded in 512\*512 Baboon Image

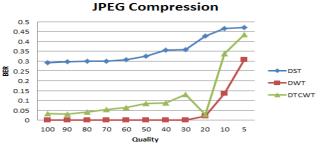


Fig.7. BERs for JPEG compression attack applied for same 1000 randomly generated watermarks embedded in 512\*512 Baboon Image

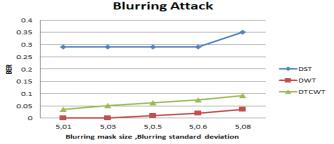


Fig.8. BERs for Blurring attack applied for same 1000 randomly generated watermarks embedded in 512\*512 Baboon Image

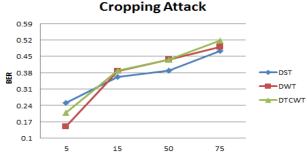


Fig.9. BERs for Cropping attack applied for same 1000 randomly generated watermarks embedded in 512\*512 Baboon Image

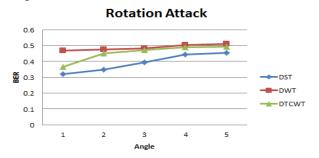


Fig.10. BERs for Rotation attack applied for same 1000 randomly generated watermarks embedded in 512\*512 Baboon Image

#### 6. DISCUSSION AND CONCLUSIONS

In this paper, a perceptual watermarking model combining discrete Shearlet transform and JND profiles is proposed. In the experiments performed using standard metrics and test images, the JND-DST watermarking strategy has proven very good in term of imperceptibility and flexibility to change the balance between capacity, invisibility and watermarking strength. This methodology was also tested against attacks and compare with state of art methodologies, providing good robustness against AWGN, rotation and cropping attacks, but performing poorly against JPG compression and Blurring attacks.

This weakness is probably due to the redundancy problem of Shearlet transform [10]. As feature work the plan is to tackle this problem explicitly by adding more complex coding schemas able to reduce this intrinsic problem [2].

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