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Perfect equilibria in budget-constrained sequential auctions: an experimental study

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This article presents an experimental study of bidding behavior in sequential auctions in which there are budget constraints and perfect information. Our experiments test both the properties of such auctions and the predictive power of a refinement of the Nash equilibrium concept. We find that budget constraints affect the behavior of bidders and that the trembling-hand perfect equilibrium is generally a good predictor of prices.

1. Introduction

■ We investigate the behavior of laboratory subjects in budget-constrained, sequential auctions with complete information. In addition to testing the properties of such auctions, these experiments present what we believe is the first test of the predictive power of a refinement of the Nash equilibrium concept. This article thus contributes both to the field of experimental game theory and to the debate concerning the appropriateness of a refinement of the set of Nash equilibria as the solution concept for games.[1]

The article is organized as follows. In the next section we motivate our experiments by outlining the problem at hand. We present a summary of the theoretical results (derived in Pitchik and Schotter (1986)) in Section 3, and explain the intuition behind these results with the aid of an example. The hypotheses we test are stated in Section 4. We present the experimental design in Section 5, the experimental methods in Section 6, and our results in Section 7. Finally, in Section 8 we offer some concluding comments and some conjectures about future research.

2. Motivation

■ The existing theoretical auction literature concentrates on the allocation of one good to one of many bidders.[2] Parts of this theory are tested in some recent experimental articles

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[1] For a theoretical discussion of some of these refinements, see Myerson (1978), Kreps and Wilson (1982), Binmore (1985), and Kohlberg and Mertens (1986).

[2] Revenue-maximizing (one good) auctions are analyzed in Maskin and Riley (1984), Myerson (1981), Harris and Raviv (1981), and Riley and Samuelson (1981), and welfare-maximizing auctions are analyzed in

(Cox, Smith, and Walker, 1986; Kagel, Levin, Battalio, and Meyer, 1983). In real-world auctions, however, heterogeneous goods are often allocated sequentially to a set of budget-constrained bidders and, as a result, the prices and allocation of the goods in these auctions may be interdependent.

Interdependence can result in a number of ways. If the goods sold sequentially are complementary, then a bidder's value for a good sold later may depend on the allocation of earlier goods. If firms with limited plant capacities bid on projects let by the government, the results of letting any given contract will depend on the available capacity of firms in the industry. Finally, if bidders are budget-constrained (owing to the existence of imperfect capital markets),[3] these constraints may limit their ability to bid for later goods if earlier allocations and prices deplete their resources.

It is our hypothesis that sophisticated bidders recognize this interdependence and exploit it when possible. In particular, we are interested in the strategic behavior that occurs if bidders are budget-constrained. Intuition suggests that bidders may exploit the budget constraints of others by bidding up the price of goods offered early in the auction. This relatively high price for an early good depletes the winner's budget. A later good can then be won at a relatively low price.

The literature dealing with the allocation of multiple goods to multiple bidders is sparse. Ortega-Reichert (1968) analyzes sequential auctions with incomplete information in which the bidders are not budget-constrained. Bids on earlier goods are signals of bidders' valuations of later goods. Palfrey (1980) considers multiobject, budget-constrained auctions in which a set of heterogeneous goods are sold simultaneously to identical bidders. He shows that symmetric, pure-strategy Nash equilibria exist only if the number of goods and buyers are less than or equal to 2. Engelbrecht-Wiggans and Weber (1979) find the equilibrium allocation of a fixed budget across a set of independent auctions. Finally, Weber (1983) analyzes sequential and simultaneous multigood auctions in which the bidders are not budget-constrained. In general, buyers are restricted to purchase only one unit of the good. He considers private-value and common-value auctions. In the former he obtains revenue-equivalence theorems in the case of simultaneous sale of the homogeneous goods. If the homogeneous goods are sold sequentially, he shows that the revenue is independent of whether the price is revealed as each good is sold.

Our experiments test the effects of budget constraints in sequential auctions.[4] The theory of budget-constrained, sequential auctions (under first- and second-price rules) is developed in Pitchik and Schotter (1986). These auctions possess unique trembling-hand perfect equilibria (Selten, 1975) amid multiple subgame-perfect equilibria. (We refer to trembling-hand perfect equilibria simply as "perfect equilibria" through the rest of this article.) Thus, our experiments are a good setting in which to test the predictive power of the perfect Nash equilibrium, one of the most commonly used refinements of the Nash equilibrium.

3. The model

■ Consider an auction with two bidders, 1 and 2. There are two goods, α and β, to be sold sequentially. Each bidder i has monetary valuations $V^i(\alpha)$, $V^i(\beta)$, respectively.[5] Each

Chatterjee and Samuelson (1983), Myerson and Satterthwaite (1983), and Leininger, Linhart, and Radner (1986). Revenue-equivalence theorems are proved in Milgrom and Weber (1982) and Wilson (1984).

[3] Alternatively, Engelbrecht-Wiggans (1987) uses a principal-agent context in an attempt to rationalize the use of limited exposure or budget constraints.

[4] Little theoretical or experimental work has been done on sequential auctions. For a survey of the existing theoretical literature see Weber (1983), Milgrom (1987), and McAfee and McMillan (1987). For a survey of much of the experimental literature, see Smith (1982a).

[5] Different monetary valuations can result if bidders have private uses for the goods.

bidder i is constrained not to spend more than I^i (i's income) in the auction.[6] All units (i.e., bids, budgets, and valuations) are multiples of an indivisible unit, .01. Ties are broken by the flip of a fair coin. The above is common knowledge between the bidders. The sealed-bid auction works as follows. In the first stage one of the goods, α or β, is brought up for sale. We refer to the first good sold as good 1, and to the second good sold as good 2. The participants bid for good 1 (in multiples of .01), but cannot submit a bid greater than their budgets. The bidder who submits the higher bid obtains the first good. The price paid for the good is the higher bid if first-price rules are in effect, and is the lower bid if second-price rules prevail. After good 1 is sold, the winner's budget is reduced by the price paid for good 1. Good 2 is then allocated in an identical way in the second stage of the auction.

□ **Equilibria.** The second-stage game is a one-good, sealed-bid auction with complete information. As we want to find subgame-perfect equilibria of the complete two-stage game, we need to analyze the equilibria of the second stage. We maintain the standard assumption that the only reasonable equilibrium outcome in this stage is that in which the bidder with the higher reservation price obtains the good at the lower reservation price under second-price rules (plus .01 under first-price rules). Using backward induction, we can replace the second stage of our two-stage game with the equilibrium payoffs in this standard equilibrium outcome. Then we have a finite strategy normal form game G that depends on the valuations and resources of each bidder. It remains for us to solve G for the equilibrium bids on good 1.

Pitchik and Schotter (1986) investigate the theory of such auctions under complete information, and also study the effect of incomplete information. To state their results for the parameters used in this experiment, we need to introduce the notion of *critical values* for the first good. These critical values play a role in the first-stage bidding that is similar, but not identical, to that of reservation prices when only a single object is offered for sale. Let us denote these critical values by c^i, $i = 1, 2$. (Later we shall give the values of c^i, $i = 1, 2$, in terms of the parameters of our model.) For the parameters used in this experiment the critical values have the following three properties.

Property 1. At any price below c^i, bidder i *strictly* prefers to obtain good 1; at any price above c^i, bidder i *strictly* prefers to lose good 1 to bidder $j \neq i$; at a price of c^i bidder i is *indifferent* between obtaining good 1 and losing good 1 to bidder $j \neq i$.

Property 2. Under second price rules, bidder i strictly prefers to lose good 1 to bidder $j \neq i$ at higher prices rather than at lower prices so long as the higher price is at least as great as c^i.

Property 3. The payoffs to bidder i at the pair (b, b) of bids for good 1 are the same for any $b \geq c^i - .01$.

We state and prove our results under the assumption that the critical values satisfy Properties 1–3. We later show that the properties are satisfied by the critical values for the parameters used in our experiment.

Proposition 1. Consider the game G in which the critical values satisfy Properties 1 and 2. Suppose that $c^j > c^i$ and let $W = [c^i, c^j]$. Then, under second-price rules (b^i, b^j) is a Nash equilibrium pair of bids for good 1 if and only if $(b^i, b^j) = (b, b + .01)$ from some $b \in W$. Under first-price rules, (b^i, b^j) is a Nash equilibrium pair of bids for good 1 if and only if $(b^i, b^j) = (b - .01, b)$ for some $b \in W$. Under either rule player j obtains good 1, and the set of equilibrium prices is W.

[6] Imperfect capital markets explain the existence of such budget constraints.

Proof. We first show that no price outside the interval W can be an equilibrium price for good 1 in G. Since c^k, $k = 1, 2$ is bidder k's reservation price for good 1, we see that neither player wants the good at a price above c^j, and both want the good at any price below c^i. Hence, any equilibrium price for good 1 must belong to the interval W.

Now we show that any price belonging to W is an equilibrium price supported by a unique equilibrium pair of bids. If bidder i bids $b^i \in W$ $(\cup(c^i - .01)$ under first-price rules), then (by Property 1) bidder j's best response is to bid above b^i (immediately above, if first-price rules are in effect). If bidder j bids $b^j \in W$ $(\cup(c^j + .01)$ under second-price rules), then (by Property 1) bidder i's best response is to bid below b^j (immediately below if second-price rules are in effect (by Property 2)). *Q.E.D.*

Thus, the interval W of equilibrium prices for good 1 is identical under both price rules. If we refine our solution concept to that of (trembling-hand) perfect equilibrium, however, this is no longer so. We state our result below.

Proposition 2. Consider the game G in which the critical values satisfy Properties 1–3. Under both price rules there is a unique perfect-equilibrium price, equal to the highest price in W under second-price rules, and equal to the lowest price in W under first-price rules.

Proof. Assume that $c^j > c^i$. In two-person games the set of perfect equilibria is identical to the set of equilibria in which no (weakly) dominated strategies are used (van Damme, 1984). We show below that under second-price rules a bid by player k of $c^k + .01$ is undominated and dominates all bids less than or equal to c^k; under first-price rules a bid by player k of $c^k - .01$ is undominated and dominates all bids greater than or equal to c^k, $k = 1, 2$. But all equilibrium prices belong to W and are supported by bids that are separated by .01. Thus, there is a unique perfect equilibrium in G; under second-price rules it is $(b^j, b^i) = (c^j + .01, c^j)$ (with price equal to c^j); under first-price rules it is $(b^j, b^i) = (c^i, c^i - .01)$ (with price equal to c^i).

First, consider second-price rules. If bidder k's opponent bids less than c^k, then bidder k strictly prefers to obtain the good (by Property 1). If k's opponent bids above $c^k + .01$, then bidder k strictly prefers to bid $c^k + .01$ than to bid less (by Property 2). If the other bidder bids either c^k or $c^k + .01$ then bidder k is indifferent between bidding c^k and $c^k + .01$ (by Properties 1 and 3, respectively). Thus, $c^k + .01$ dominates all lower bids.

It remains to show that a bid by player k of $c^k + .01$ is not dominated by a higher bid, say b^k. If bidder k's opponent bids above $c^k + .01$ but less than or equal to b^k, then bidder k strictly prefers to bid $c^k + .01$ than to bid b^k (by Property 1).

Now consider first price rules. If bidder k's opponent bids anything greater than c^k, then bidder k strictly prefers to lose good 1. Any bid that loses the good results in the same payoff to the loser of good 1. If k's opponent bids anything below $c^k - .01$, then bidder k strictly prefers to bid $c^k - .01$ to anything above $c^k - .01$. If the other bidder bids either c^k or $c^k - .01$, then bidder k is indifferent between bidding c^k and $c^k - .01$ (by Properties 1 and 3, respectively).

It remains to show that a bid by player k of $c^k - .01$ is not dominated by a lower bid, say b^k. If bidder k's opponent bids below $c^k - .01$ but greater than or equal to b^k, then bidder k strictly prefers to bid $c^k - .01$ than to bid b^k. *Q.E.D.*

It remains to show that the critical values derived from the parameters used in our experiment satisfy Properties 1–3. We make the argument for the following example in which the parameters match those of experiment 2. The argument is analogous for the parameters in the other experiments. Consider Table 1.

We recall that the second-stage equilibrium is one in which the bidder with the higher reservation price obtains the good at the lower reservation price under second-price rules (plus .01 under first-price rules). For example, if bidder 2 has a budget of 150 in the second stage, then bidder 2's reservation price is 150, while bidder 1's reservation price remains at

TABLE 1 **Example**

Bidder	Good 1 Valuation	Good 2 Valuation	Budget
1	240	200	400
2	300	200	400

200. Thus, bidder 1 obtains good 2 at a price of 150 under second-price rules (150.01 under first-price rules). We now argue that under either price rule the critical value is 250 for bidder 2 and is 220 for bidder 1. (In the auctions we study in our experiments, the critical values are always given by $c^i = [V^i(1) - V^i(2) + I^j]/2$ for $i = 1, 2$.) We now discuss how these critical values satisfy the properties discussed earlier.

We need to show that, under either auction rule, bidder 2 strictly prefers to win good 1 if the price is less than 250, strictly prefers to lose good 1 if the price is greater than 250, and is indifferent between winning and losing if the price is 250. If the price of good 1 is less than 250, then bidder 2 can obtain at least 50 by acquiring it. If instead, bidder 2 loses good 1 at such a price, then bidder 2 can obtain no more than 50 in any equilibrium in the second stage (since bidder 1 has at least 150 left). At a price of 250, bidder 2 obtains 50 whether good 1 is won or lost. Thus, 250 is a critical value for bidder 2 which satisfies Property 1.

If bidder 2 loses good 1, then bidder 2's marginal gain is one unit for every unit increase in the price for every price above 160. (Note that 160 is the price that leaves bidder 1 with exactly 200 to bid on good 2.) Thus, bidder 2's critical value satisfies Property 2 since $250 > 160$.

It remains to show that the payoff to bidder 2 is the same at all pairs (b, b), where $b > 250$ is each player's bid on good 1. For every price above 160, the marginal gain of losing the good and the marginal loss of winning the good is one unit for every unit increase in the price. But the payoff to bidder 2 at (b, b) is the sum of the gain from winning the good at b with probability .5 and the gain from losing the good at b with probability .5. Thus, the payoff to bidder 2 is identical for all pairs of bids (b, b), where $b \geq 250$, since $250 > 160$. The argument is analogous for the critical value 220 of bidder 1, so we omit it.

□ **Predictions.** The theory outlined above results in price and bid predictions in budget-constrained, sequential auctions. These are stated below. Recall that, for the parameters used in our experiments, the critical values satisfy $c^i = (V^i(1) - V^i(2) + I^j)/2$ for $i = 1, 2$. The individual, say j, with the higher critical value is the winner of good 1 in equilibrium. The higher critical value, c^j, is the perfect equilibrium price of good 1 under second-price rules; the lower critical value, c^i, is the perfect-equilibrium price under first-price rules. Thus, under second-price rules, the perfect-equilibrium price is increasing in $V^j(1)$ and I^j, and is decreasing in $V^j(2)$. Under first-price rules the perfect equilibrium price is increasing in $V^i(1)$ and I^i and is decreasing in $V^i(2)$.

The set of equilibrium prices is the interval W. The interval W is bounded on the left by the lower critical value and on the right by the higher critical value. Thus, the theory of Nash equilibrium predicts that the price of good 1 must lie in W. Further, this theory predicts the bids the players will make. A weak form of the equilibrium bid prediction requires that all bids b^i for bidder i belong to the set, say B^i, of strategies used by bidder $i = 1, 2$ in some equilibrium. A strong form of the equilibrium bid prediction requires that all pairs of bids (b^1, b^2) be such that $b^i \in B^i$ for $i = 1, 2$ and $b^j - b^i = .01$ (where $c^j > c^i$).

Lastly, the theory of perfect equilibrium predicts that, under second-price rules, changing the order in which the goods are sold affects the prices. To illustrate this assume that α is sold first. In this case $c_1^i = (V^i(\alpha) - V^i(\beta) + I^j)/2$ for $i = 1, 2$. (Note that the subscript denotes the order (α, β) in which the goods are sold.) In the example discussed above, assume that α is the good valued at 240 by bidder 1 and at 300 by bidder 2. The perfect equilibrium price of α is the maximum of the critical values. In this example $c_1^2 = 250$ and $c_1^1 = 220$, so that the perfect equilibrium price of α is 250, and the winner of α is bidder 2 if the goods are sold in the order (α, β).

Now suppose that β is sold first. In that case $c_2^i = (V^i(\beta) - V^i(\alpha) + I^j)/2$. (Here the subscript denotes the order (β, α) in which the goods are sold.) The perfect equilibrium price of β is the maximum of the critical values. Thus, $c_2^1 = 180$ and $c_2^2 = 150$ if β is sold first, so that the perfect-equilibrium price of β is 180 and the winner of β is bidder 1. We obtain the second-stage price of α as follows. Bidder 1 has only 220 remaining to bid on α, while bidder 2 has 400. Hence, bidder 2 obtains α at a price of 220 if the goods are sold in the order (β, α). Thus, under second-price rules, the perfect equilibrium price depends on the order in which the goods are sold. The later a good is sold, the lower is its price.

4. Hypotheses

■ We first consider (in three variations) whether the results of our experiments are consistent with Nash equilibrium. In the first variation, the strong form stated in Hypothesis 1a below, we test whether all (first-good) bids of paired members are separated by only .01 (as predicted by the theory). In addition, we test whether the bids of bidder i belong to the interval B^i of strategies used in equilibrium. In the second variation of the equilibrium hypothesis, the weak form stated in Hypothesis 1b below, we test only whether all bids of bidder i are in B^i. The last variation of the equilibrium hypothesis, stated in Hypothesis 1c below, focuses on prices. In it we test whether all prices are in W, the equilibrium interval.

□ Equilibrium hypotheses.

Hypothesis 1a. All first-stage bids of bidder i belong to B^i, for $i = 1, 2$. Further, the difference between bids satisfies $b^j - b^i = .01$, where j is the bidder with the higher critical value.

Hypothesis 1b. All first-stage bids of bidder i belong to B^i, for $i = 1, 2$.

Hypothesis 1c. All prices for good 1 lie in the interval W.

Since perfection predicts that the bids and prices will be at the endpoints of B^i and W, respectively, we have only two variations on the predictions that perfect equilibrium imposes. In the first variation, stated in Hypothesis 2a, we test whether bidder i's bids are at the endpoint of B^i, the interval of equilibrium strategies, and are thus separated by .01. In the second, stated in Hypothesis 2b, we test whether the prices formed are at the endpoint of W, the interval of equilibrium prices.

□ Perfect-equilibrium hypotheses.

Hypothesis 2a. All first-stage bids of bidder i are at the endpoint of B^i relevant for the price rule under consideration. Under first-price rules the relevant endpoint is the lower one, and under second-price rules it is the upper one.

Hypothesis 2b. All first-good prices are at the endpoint of W relevant for the price rule under consideration. Under first-price rules the relevant endpoint is the lower one, and under second-price rules it is the upper one.

As shown in Section 3, the interval of equilibrium prices is invariant to the auction rule used. The restriction to perfect equilibria results in different prices under different price rules, as summarized by the next hypothesis.

□ **Price-rule hypothesis.**

Hypothesis 3. The price of good 1 is higher under second-price rules than under first-price rules.

An interesting aspect of budget-constrained, sequential auctions is that the sequence in which the goods are sold can affect the prices and thus the revenues of the owners of these goods. The results predict that, under second-price rules, the earlier a good is sold, the higher is its price.

□ **Sequence hypothesis.**

Hypothesis 4. Under second-price rules, the price of good α is higher if the goods are sold in the sequence (α, β) than in the sequence (β, α).

The effects of increases in values and incomes are in Hypotheses 5 and 6.

□ **Valuation hypothesis.**

Hypothesis 5. Suppose that $c^i > c^j$ so that bidder i is the equilibrium good 1 winner. The price of good 1 increases if $V^i(1)$ increases, but is independent of $V^j(1)$.

□ **Income hypothesis.**

Hypothesis 6. Suppose that $c^i > c^j$, so that bidder i is the equilibrium good 1 winner. The price of good 1 increases if I^j increases, but is independent of I^i.

As discussed earlier, in the second stage we restrict to the standard equilibrium outcome in which the second good is allocated to the player with the higher reservation price at the lower reservation price (+.01 under first-price rules). The equilibrium bids that result in this outcome are listed in Hypothesis 7.

□ **Hypothesis on second-stage bids.**

Hypothesis 7. Suppose that bidder i is the bidder with the lower reservation price in the second stage. Call the lower reservation price m. Under first- and second-price rules, bidder i bids m, the minimum of $V^i(2)$ and $I^i - p$, for good 2 in the second stage, where p is the price paid by bidder i for good 1 ($p = 0$ if bidder i is not allocated good 1.) Under first-price rules, bidder $j \neq i$ bids .01 above this minimum. Under second-price rules, bidder $j \neq i$ bids at least this minimum.

In any equilibrium of our experiments, the bidder with the higher critical value is the predicted winner of good 1. We list the hypothesis concerning the allocation of good 1 below.

□ **Allocation hypothesis.**

Hypothesis 8. If $c^i > c^j$, then bidder i is the winner of good 1.

5. Experimental design

■ The parameters of the five experiments we ran to test the theory described above are listed in Table 2.

Experiment 1 is the baseline experiment. Experiment 2 is different from 1 only in the valuation of bidder 1 for good 1; bidder 1 values the first good more highly in experiment 1. This change expands the interval of Nash equilibrium prices from a unique price of 250 to the interval [220, 250]. But the perfect equilibrium price remains unchanged.

TABLE 2 Experimental Design

Experiment	Valuations	Incomes	Equilibrium Prices of Good 1*	Number of Subjects
Baseline (1)	$v_1^1 = 300$, $v_2^1 = 200$ $v_1^2 = 300$, $v_2^2 = 200$	$I^1 = 400$ $I^2 = 400$	$W = 250$	18
Valuation Change (2)	$v_1^1 = 240$, $v_2^1 = 200$ $v_1^2 = 300$, $v_2^2 = 200$	$I^1 = 400$ $I^2 = 400$	$W = (220, 250^a)$	10
Income Change (3)	$v_1^1 = 300$, $v_2^1 = 200$ $v_1^2 = 300$, $v_2^2 = 200$	$I^1 = 450$ $I^2 = 400$	$W = (250, 275^a)$	20
Sequence Change (4)	$v_1^1 = 200$, $v_2^1 = 240$ $v_1^2 = 200$, $v_2^2 = 300$	$I^1 = 400$ $I^2 = 400$	$W = (150, 180^a)$	16
First-Price Rule (5)	$v_1^1 = 240$, $v_2^1 = 200$ $v_1^2 = 300$, $v_2^2 = 200$	$I^1 = 400$ $I^2 = 400$	$W = (220^a, 250)$	16

* β is good 1 in experiment 4, α is good 1 in experiments 1, 2, 3, and 5.
[a] Perfect equilibrium price of good 1.

These experiments can be used to test whether perfection is a good predictor among Nash equilibria.

Experiment 3 differs from 1 only in that bidder 1 has more income in experiment 3. This experiment tests the effect of income changes. It also tests the predictive power of the perfect equilibrium.

FIGURE 1

GOOD 1: BIDS AND MEAN BIDS BY ROUND AND BY TYPE OF BIDDER BASELINE EXPERIMENT (EXPERIMENT 1)

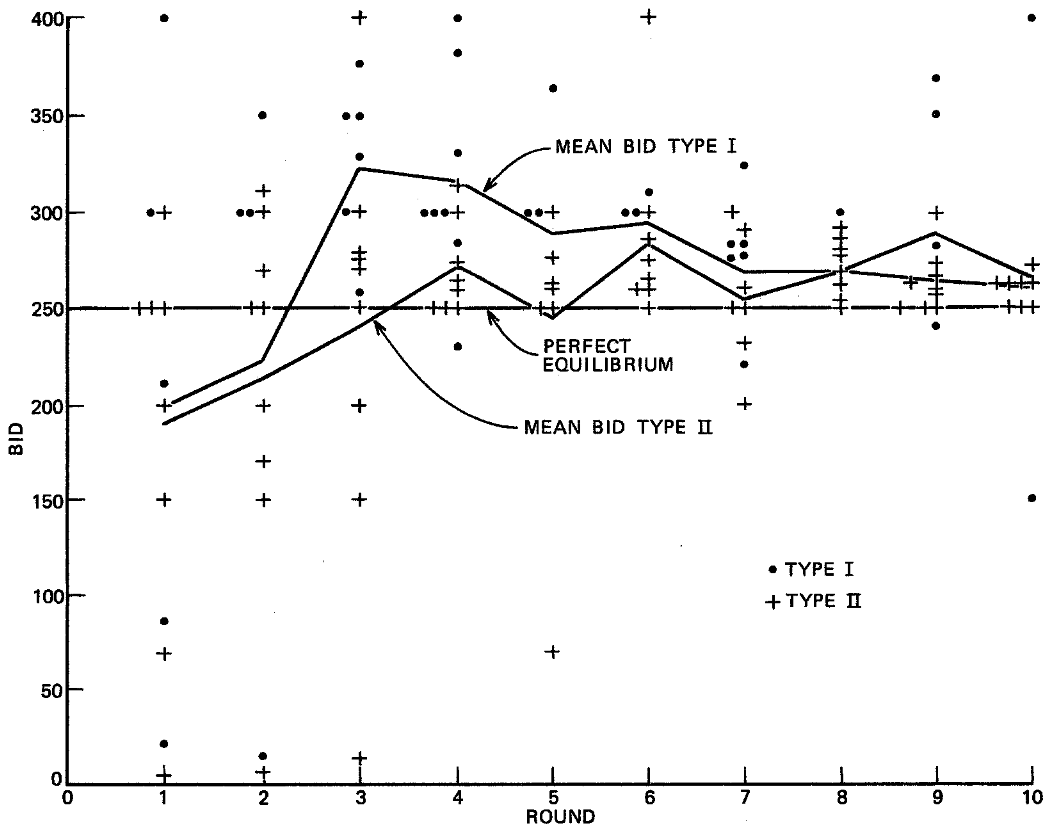
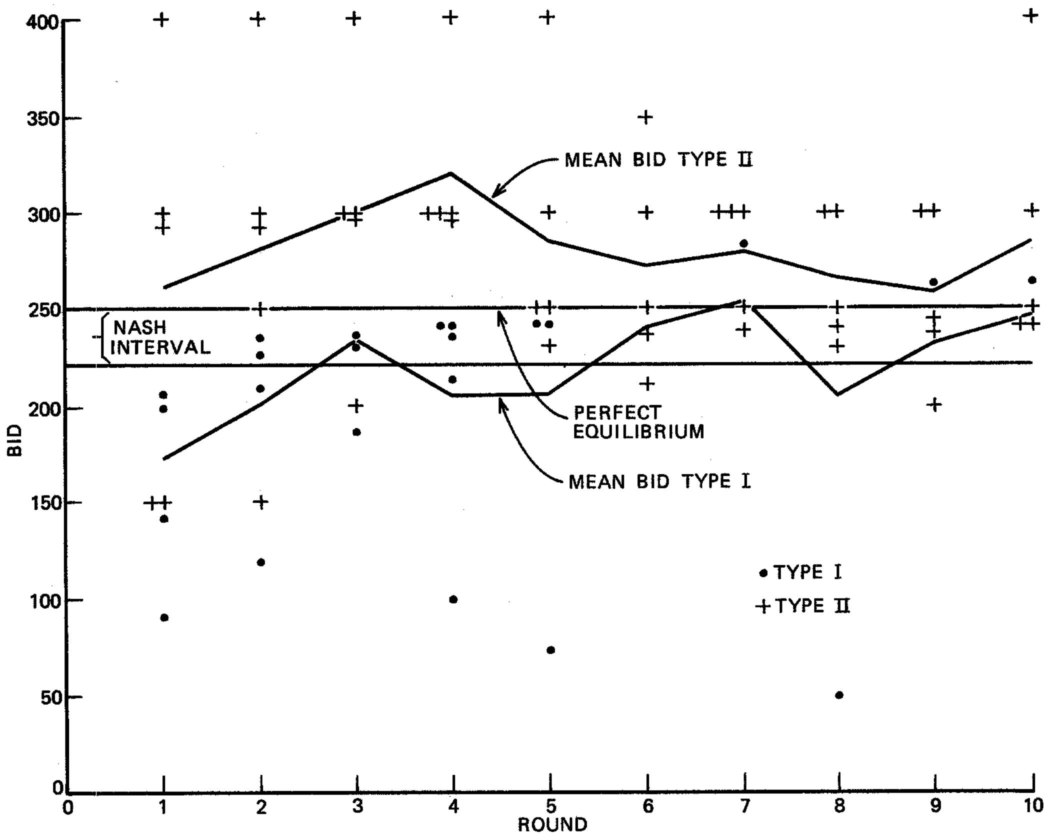


FIGURE 2

GOOD A: BIDS AND MEAN BIDS BY ROUND AND BY TYPE OF BIDDER
VALUE CHANGE EXPERIMENT (EXPERIMENT 2)



Experiment 4 differs from 2 only in that, in 4, β is sold first. These experiments are used to test the effect of sequence on prices.

Finally, experiment 5 differs from 2 only in the price rule used. Experiment 5 uses the first-price rule. We use these experiments to test whether revenue and the price of the first good sold are invariant to the auction rule used.

6. Experimental methods

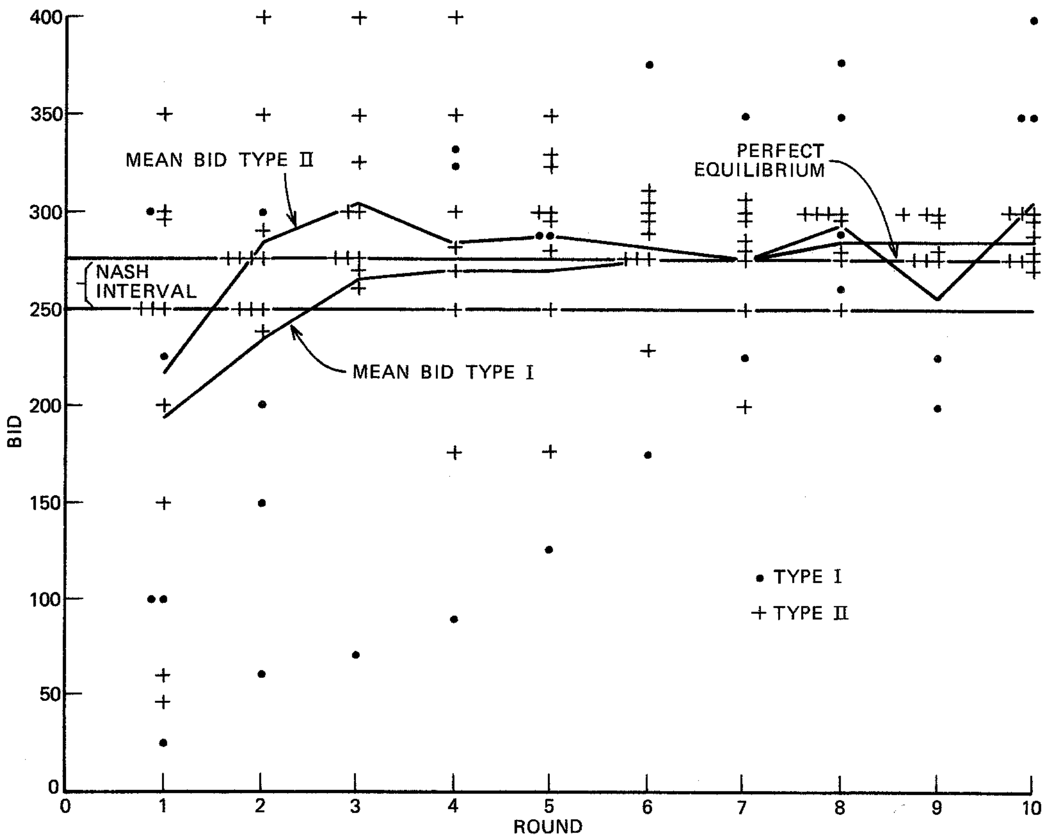
■ The experiments used paid volunteers recruited from undergraduate classes at New York University. All experiments were performed within a three-day period. This minimized the potential for word to spread to incoming subjects. Students earned an average of $10.00 for an hour and thirty minutes. Motivation was observed to be high.

Subjects reported to a seminar room. Chairs were arranged in a circle with alternate subjects facing outside to minimize the potential for signalling among subjects. Students were randomly assigned to be either a bidder of type 1 or 2. Written instructions (see Appendix A) were distributed and reviewed. Any questions were then answered.

Each of the five experiments consisted of ten rounds. In each of these rounds two goods were sold sequentially. We used the Smith (1976) method to induce the subjects to have the right valuations for the goods. Thus, subjects were told that any goods purchased during the auction were redeemable afterwards for a specific amount of francs. These francs were

FIGURE 3

GOOD 1: BIDS AND MEAN BIDS BY ROUND AND BY TYPE OF BIDDER
VALUE CHANGE EXPERIMENT (EXPERIMENT 3)



later converted into dollars at a known conversion rate. All the parameters of the experiment were common knowledge among the subjects.

In each round each subject wrote a bid for the first good on a piece of paper. The bids of bidders of type 1 and 2 were then collected separately (and quickly). They were brought to an experimental administrator who randomly drew pairs of bids, one from each type. This determined each bidder's opponent for both goods in this particular round of the experiment. It also determined who was allocated the first good among each pair. Price determination depended on the rule being used.

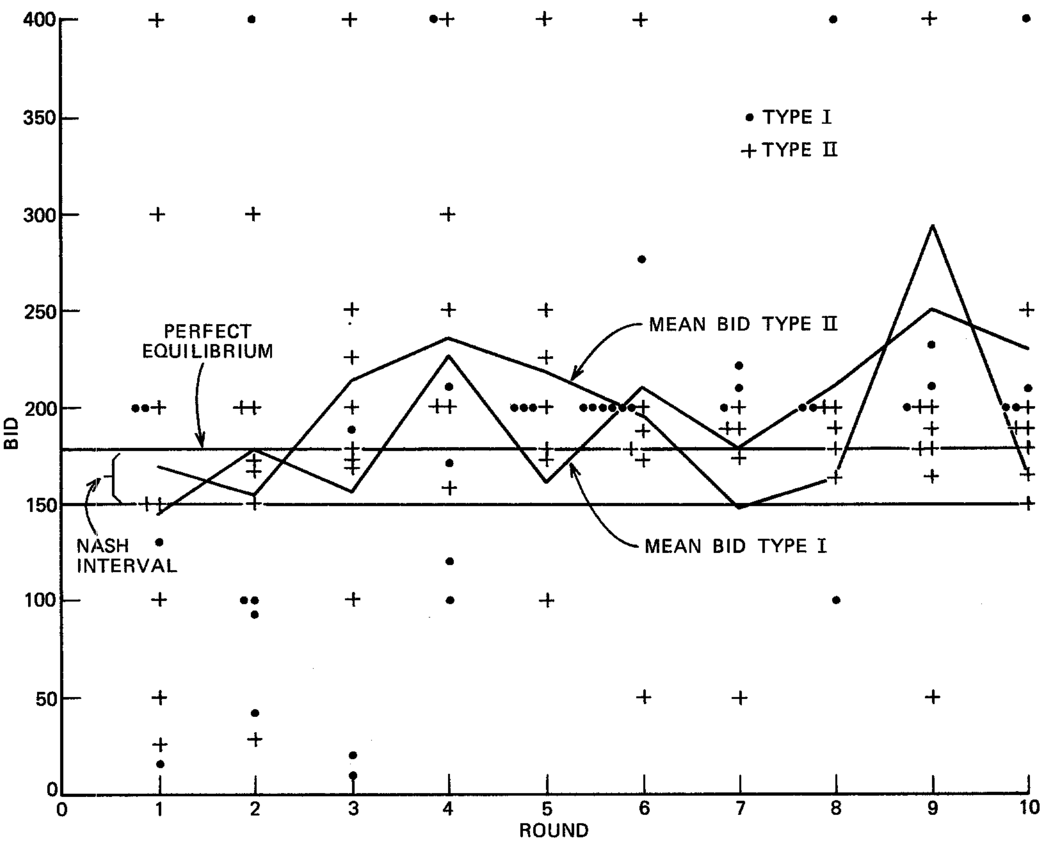
If second-price rules were used, the bidder with the higher bid was allocated the good at the lower bid. This bidder was then informed of this price by receiving a note to this effect. Those bidders who did not receive notes knew their opponent was allocated the good at a price equal to their bid. Thus, they could calculate the budget remaining to their opponent to bid for the other good. If first-price rules were in effect, bidders were also informed of the price paid for the good. Subjects recorded whether they were allocated the good and, if so, at what price.

Although the price information revealed was identical under both rules, the bid information was not. Under second-price rules, the bid of the winning bidder is not revealed to the loser. Under first-price rules, the bid of the loser is not revealed to the winner.

Bidding for the second good then began. Bidders knew the remaining budget of their matched opponents. The second good was then allocated to the higher bidder (of each pair).

FIGURE 4

GOOD 1: BIDS AND MEAN BIDS BY ROUND AND BY TYPE OF BIDDER
SEQUENCE CHANGE EXPERIMENT (EXPERIMENT 4)



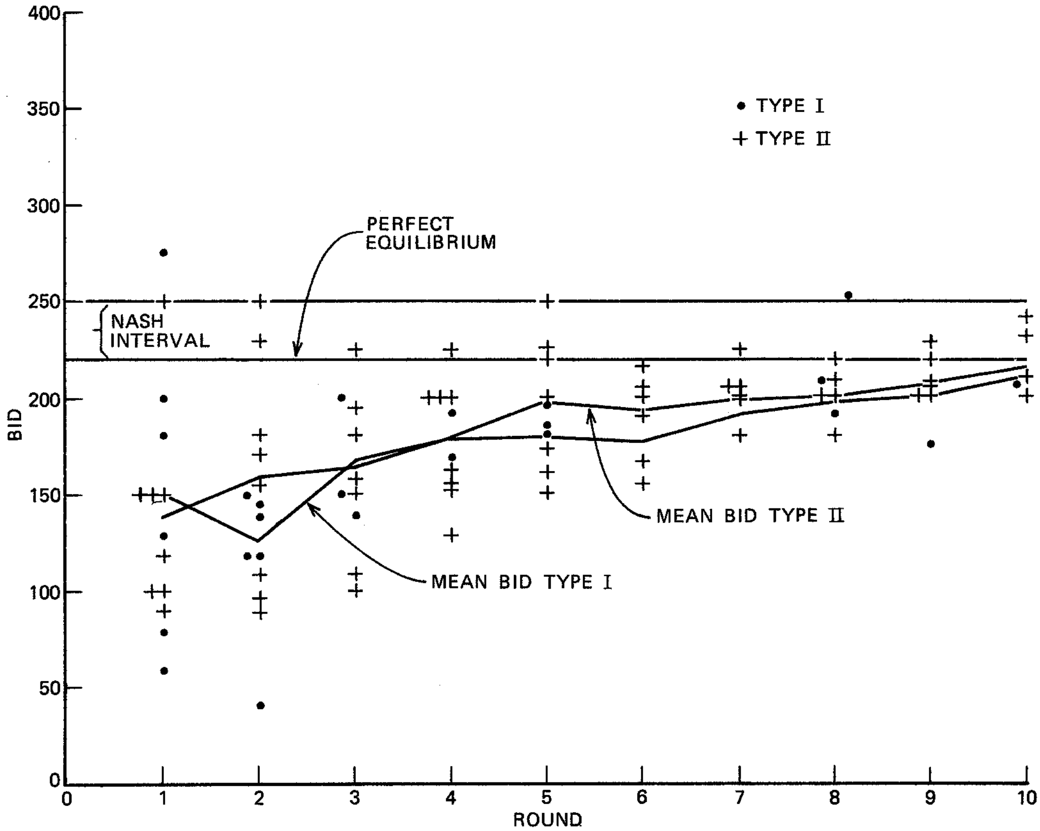
After these results were distributed, subjects recorded their payoffs. Each bidder's payoff on each good was simply the difference between the bidder's redemption value and the price paid if the bidder was allocated the good; zero, if not. To prevent actual losses from occurring, subjects were given a lump-sum dollar amount at the beginning of the experiment from which any negative payments could be subtracted. With this lump-sum adjustment, only one subject out of eighty made an actual loss.

Each round was run identically. Budgets were as in Table 1 at the beginning of each round, and the random pairing of bidders changed from round to round. At the end of ten rounds payoffs were determined as follows. First, the subjects added their ten-round franc payoffs and converted them into dollars by using a proportional conversion factor (given to them at the beginning of the experiment). The conversion was usually 1 franc = \$.01. Then, the lump-sum payment of \$3.00 was given to each bidder. This paid them for participating in the experiment and covered losses incurred during any of the rounds. Finally, to focus subjects' attention on their own payoffs without regard to the payoffs of their competing bidders, we paid a \$2.00 prize to the bidder of each type, 1 or 2, with the greatest cumulative franc payoff.

It is one of the tenets of experimental economics (Smith, 1982b; Wilde, 1980) that an experimenter must control the subjects' incentives. More precisely, subjects should be made to concentrate on the size of their absolute payoffs and not on their relative payoffs. Otherwise, the subjects' motives are not under the control of the experimenter. This effect can be

FIGURE 5

GOOD 1: BIDS AND MEAN BIDS BY ROUND AND BY TYPE OF BIDDER
FIRST PRICE EXPERIMENT (EXPERIMENT 5)



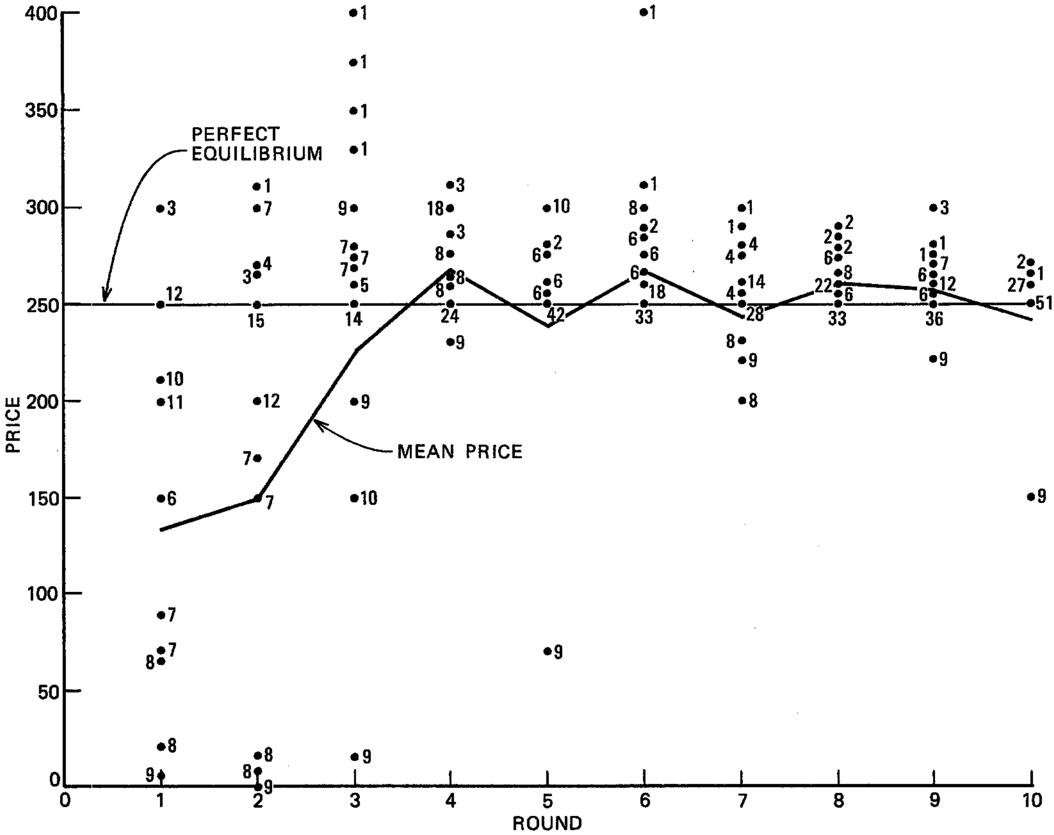
controlled by only allowing subjects incomplete information about the payoffs of others. Information was complete in our experiment, however. Two attempts were made to overcome this problem. First, we randomized the pairs of bidders over rounds and preserved the anonymity of opponents in each pair.[7] Second, we awarded the $2.00 prize as discussed above.

7. The results

■ We are interested in whether the data from round ten of each of the experiments confirm our hypotheses. But observed price data are an artifact of the random matchings used in each round. Different matchings would have resulted in different observed prices. The arbitrary nature of the observed price data compels us to consider the "potential" prices that are generated by all feasible pair matches. These data have the merit that they are independent of the actual matches made, though unfortunately, they exaggerate the impact of outliers since their influence is magnified by matching them repeatedly. But the fraction of outliers remains the same under either method. We use these potential price data for descriptive and inferential purposes. In fact, the means of both actual and potential prices are practically identical in all of our experiments. The bid data do not depend on the actual matchings that occurred in each round.

[7] This also made our design more suitable to test a static theory. Although individuals repeatedly played the game over a series of ten rounds, the randomization of bidding pairs and the anonymity of bidding opponents reduced the problems of testing a static theory in a repeated setting.

FIGURE 6
POTENTIAL AND MEAN PRICES BASELINE EXPERIMENT (EXPERIMENT 1)



□ **Equilibrium results.** In general, the data were quite supportive of Hypotheses 1b and 1c, the weak forms of the equilibrium hypotheses, but were less supportive of the strong form, Hypothesis 1a. The data in Figures 1–10 present the bids and the potential prices in each round. The numbers indicate the number of potential prices formed at the levels indicated. We use these data for our statistical tests.

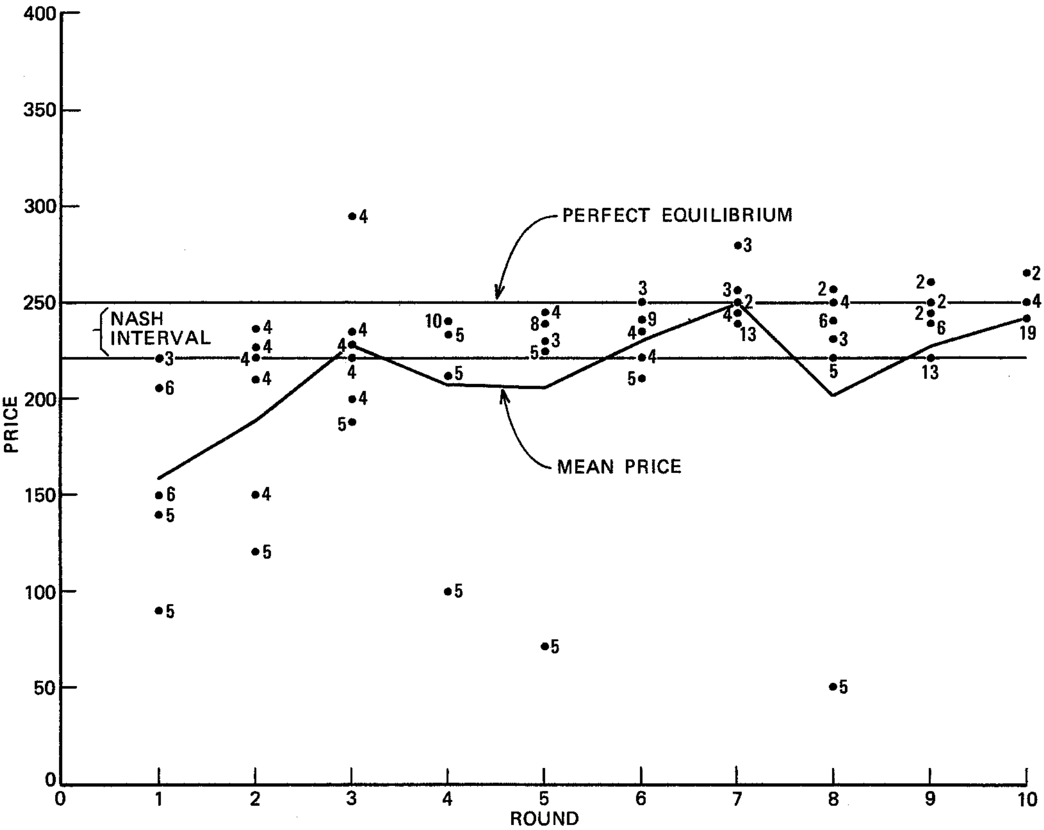
Figures 1–10 indicate a clear tendency for bids, potential prices, and, more particularly, mean potential prices, to be in the equilibrium intervals by round 10. Consider the baseline experiment (1) in which the unique equilibrium bid and price for the first good is 250 for each player. Of the 18 round-ten bids, sixteen are between 240 and 270 and nine are between 250 and 260. Of the 72 potential prices, all are between 250 and 270, 23 are exactly 250, and 63 are between 250 and 265. The mean potential price in round 10 was 242.78. If an outlier bid of 150 is removed, however, the mean is 254.38.

The income experiment (3) also performed well. In this experiment the interval of equilibrium prices was [250, 275], the perfect equilibrium price was 275, and the perfect equilibrium bids of bidders of type 1 and 2 were 275 and 275.01, respectively. In round ten 51 of the 100 potential prices were equal to 275, while 65 were between 270 and 280. Seven of the twenty bids were either 274 or 275, while 16 were between 270 and 300.

Under first-price rules (experiment 5), Figures 5 and 10 confirm that bids and prices of the first good tended to converge, from below, to 220. The equilibrium interval of prices was [220, 250], the perfect-equilibrium price was 220 and, the perfect-equilibrium bids of bidders of type 1 and 2 were 220.01 and 220, respectively. The mean potential price converged almost monotonically to 220.57. Bids approached the 220 level with means of 211 for bidders of type 1, and 215 for bidders of type 2.

FIGURE 7

POTENTIAL AND MEAN PRICES VALUE CHANGE EXPERIMENT (EXPERIMENT 2)



In the value experiment (2), the equilibrium interval is [250, 275]. Here the movement is in the direction of 240, the valuation of bidders of type 1 for good 1.

In summary, there is substantial support for Hypotheses 1b and 1c, the weak forms of the equilibrium hypothesis. The strong form of the equilibrium hypothesis (1a) is a hypothesis about the differences in bids of bidders of each type. It asks that *all* observed bids be separated by .01 and that they belong to the set of equilibrium strategies. The data do not support such a severe prediction. Less than 6% of the differences between bids of potential bidding pairs in all five experiments were .01 or 0 (11.6% were less than 1.0). The mean difference ranged from a low of 14.01 in the first-price experiment (5) to a high of 46.73 in the value experiment (2).[8] Despite this, the data do support a weaker test of Hypothesis 1a. In particular, a Wilcoxon-Mann-Whitney U-test does not allow us to reject the null hypothesis that the bids of each type of bidder came from populations with the same mean in experiments 1, 3, and 5.[9] But this hypothesis can be rejected in experiments 2 and 4. The results in

[8] The high occurs in experiment 2 if the outliers in experiment 4 are removed.

[9] The Z-statistics for this test are:

Experiments	1	2	3	4	5
Z-statistic	.58	1.46*	.34	2.26[a] (1.85)[b]	.05

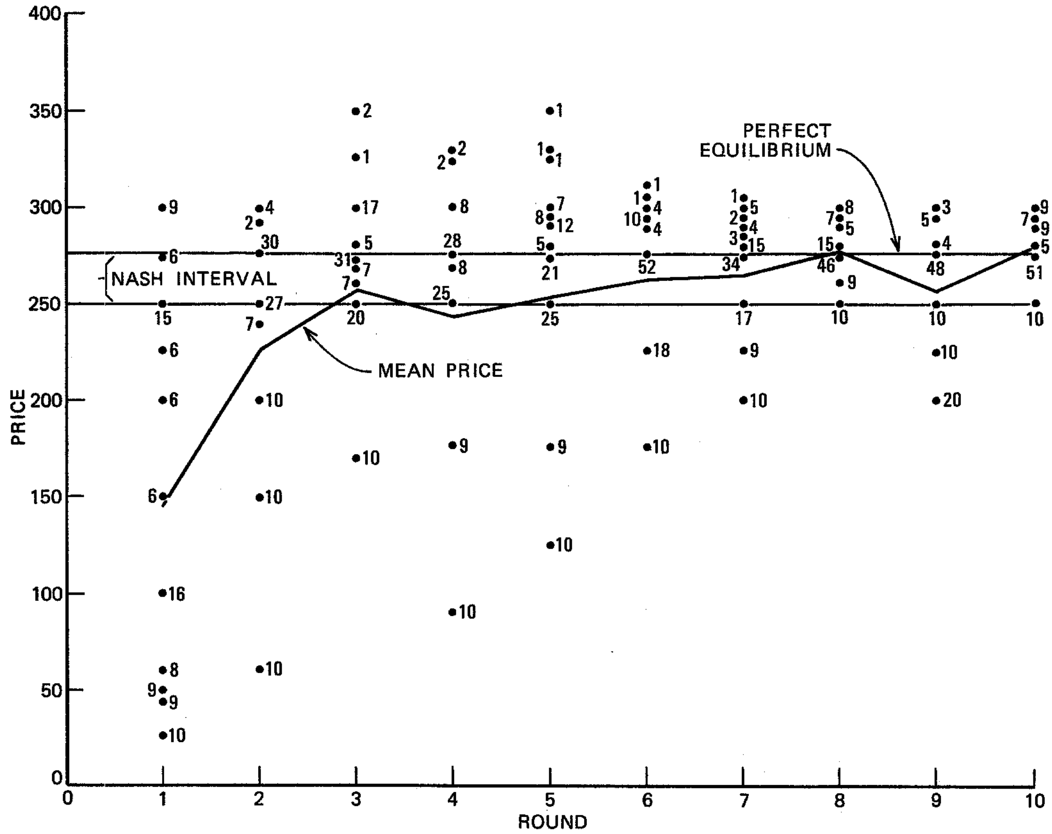
* Significantly different from zero at the 10% level of significance with a one-tailed test.

[a] Significantly different from zero at the 5% level of significance with a one-tailed test.

[b] Derived after the removal of the two outlying bids of 0 and 399.

FIGURE 8

POTENTIAL AND MEAN PRICES INCOME CHANGE EXPERIMENT (EXPERIMENT 3)



experiment 4 are deceiving since two subjects exhibited bizarre bidding strategies of alternating between 0 and 400.[10] If we remove the outliers, the difference is not significantly different from zero at the 5% level of significance. We further discuss the results from experiment 2 below.

□ **Perfect-equilibrium results.** Since the strong form of the perfect-equilibrium hypothesis (2a) is stronger than that of Hypothesis 1a, it met with even weaker support from the data. However, the results of the price form (Hypothesis 2b) were impressive.

In the first-price experiment (5), the perfect-equilibrium price of good 1 is 220. The mean potential price in round ten was 220.57. In the income experiment (3) the perfect-equilibrium price of good 1 was 275. The tenth-round mean potential price was 277.19. In addition, 65 of the 100 tenth-round potential prices were between 270 and 280.

In the baseline experiment (1), 23 out of 81 potential tenth-round prices were exactly at their unique equilibrium level of 250, while 67 were within 10 francs of it.

In the sequence experiment (4) the perfect-equilibrium price was 180. The tenth-round mean potential price was only 160.03. But this is the experiment with two outliers. After removing the outliers, the mean potential price becomes 182.9.

Finally, consider the value experiment (2) in which the perfect-equilibrium price is 250. In this experiment all but one of the tenth-round bids are concentrated around 240,

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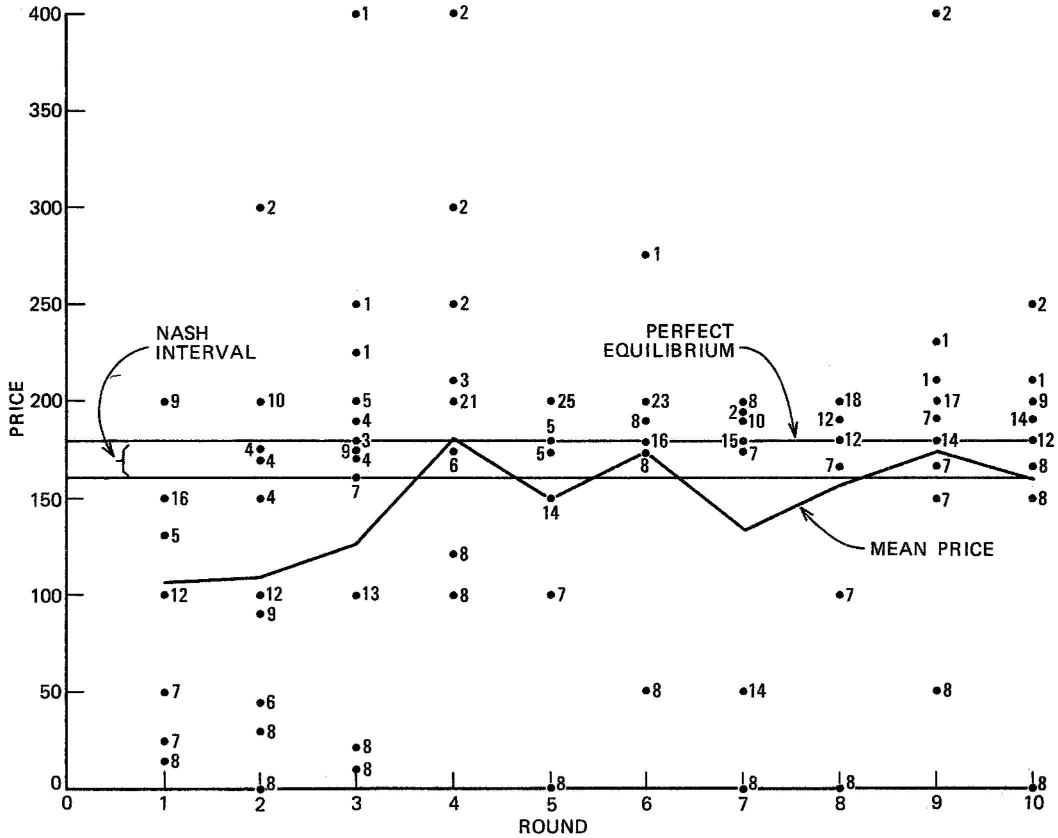
[10] One of these subjects was the only one in all the experiments to obtain a negative overall payoff.

FIGURE 9

POTENTIAL AND MEAN PRICES: SEQUENCE CHANGE EXPERIMENT (EXPERIMENT 4)

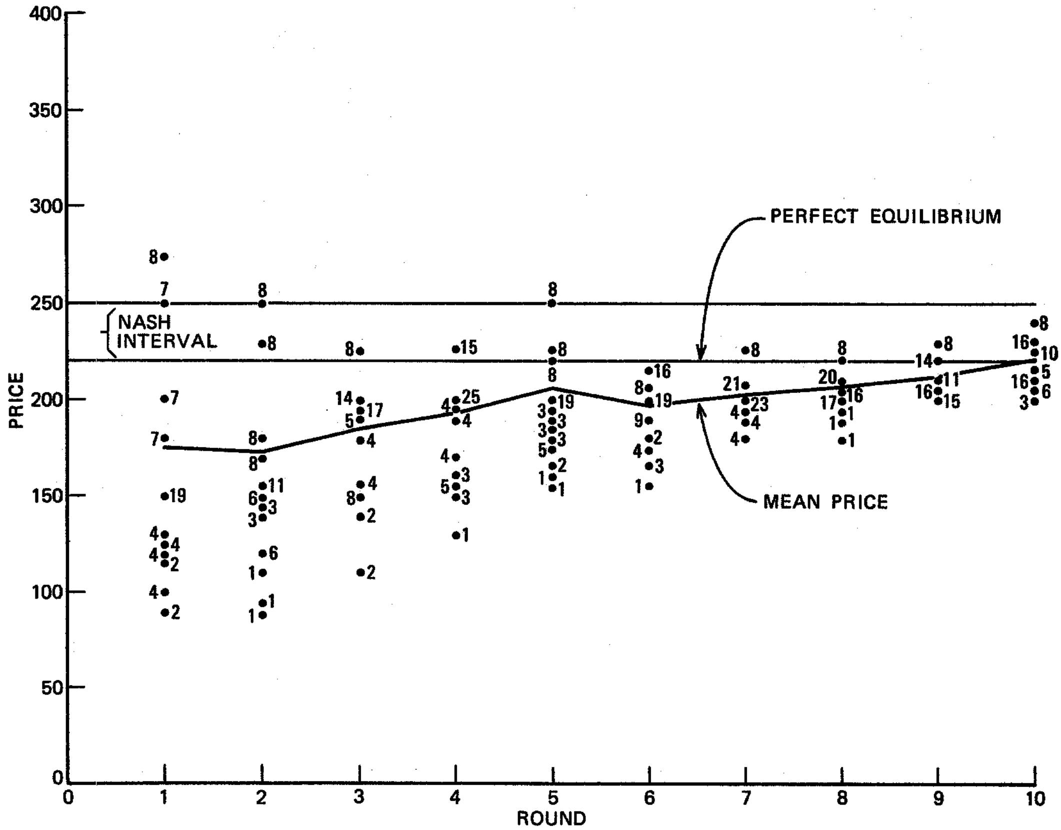


the first-good valuation of bidders of type 1. The mean potential price was 243.04. Of the total 25 potential prices, 15 were between 235 and 240, 4 between 240 and 245, and only 2 above 260. Thus, although the predicted price of good 1 was above the losing bidder's valuation of good 1, there was a clear hesitancy to bid above one's valuation of a good. The prices and bids were above the lower critical value of 220, but did not rise to meet the predicted price of 250. Bidders, therefore, seemed more than willing to bid above their critical value and risk the possibility of an opportunity loss. They seemed unwilling, however, to bid above their induced valuations and risk the possibility of an out-of-pocket loss. (Similar differences between the behavioral effects of opportunity and out-of-pocket losses were discussed by Thaler (1981).) More importantly perhaps, the differences in payoffs are slight for bidders who bid above their induced valuations, so long as their opponents bid equilibrium strategies. On the other hand, the risk associated with bidding above one's induced valuation is great if one's opponent bids out of equilibrium. Thus, the risk outweighs the potential benefit.

□ **Price-rule results.** The data generated in the tenth round by the value (2) and first-price (5) experiments appear in Tables A1 and A5 in Appendix B. We can use these data to test Hypothesis 3 since the two experiments differ only in the price rule. The null hypothesis of the equality of the tenth-round bids in these two auctions is rejected using a Wilcoxon-Mann-Whitney U-test in favor of the alternative (Hypothesis 3) that the population of bids

FIGURE 10

POTENTIAL AND MEAN PRICES FIRST PRICE EXPERIMENT (EXPERIMENT 5)



in a first-price auction is lower.[11] In particular, we note that this systematic difference in bids is predicted by perfect-equilibrium behavior. Thus, support for Hypothesis 3 is support for the perfect-equilibrium outcome as a predictor of behavior.

□ **Sequence results.** Hypothesis 4 states that the earlier a good is sold, the higher is its price, if second-price rules are used. To test this hypothesis we considered the potential prices from the valuation (2) and sequence (4) experiments, which differ in the order in which the goods are sold. We calculated the *Z*-statistic associated with the Wilcoxon-Mann-Whitney *U*-test to test the null hypothesis that the samples of potential prices came from populations with the same means. The null hypothesis was rejected at the 5% level of significance[12] in favor of the alternative (Hypothesis 4) that prices were higher in the value experiment (2). We note that the mean potential price was 243.04 in the value experiment (2) and was 218.65 (215.598 if the outlier who bid 0 in all rounds is eliminated) in the sequence experiment (4). Since Hypothesis 4 is a product of perfect-equilibrium behavior, support for Hypothesis 4 is further support for the perfect-equilibrium outcome as a predictor of behavior.

[11] The *Z*-statistic for this test has a value of −6.20. This establishes a significant difference between the means using a one-tailed test and a 95% confidence interval.

[12] The *Z*-statistic associated with the one-tailed test is −6.57.

□ **Valuation and income results.** Hypotheses 5 and 6 are comparative-static hypotheses. Hypothesis 5 predicts that the price of good 1 is the same in experiments 1 and 2. Hypothesis 6 states that the price of good 1 increases from experiment 1 to experiment 3. Table 3 presents the tenth-round means of the potential prices formed in these three experiments. It also reports the Z-statistics associated with two Wilcoxon-Mann-Whitney U-tests. One was calculated to test whether the potential prices from experiments 1 and 2 came from populations with the same mean. The other tested whether the potential prices from experiments 1 and 3 came from populations with the same mean.

The Z-statistic in column 3 indicates that we cannot accept the hypothesis that the potential prices formed in experiments 1 and 3 come from populations with the same mean at the 5% level of significance. We reject this null hypothesis in favor of the alternative (Hypothesis 6) that prices were higher in experiment 3. The mean potential price increased from 242.7 in experiment 1 to 277.19 in experiment 3. Hypothesis 6 results from a prediction of perfect-equilibrium behavior. Thus, once more we support perfect equilibrium as a predictor of behavior.

The Z-statistics in column 2 indicate that we must also reject the null hypothesis that the potential prices formed in experiments 1 and 2 come from populations with the same mean. Thus, Hypothesis 5 is rejected. As we indicated earlier in this section, however, this might be attributed to the fear of out-of-pocket losses or the possibility that the risk outweighed any potential benefit.

□ **Result on second-stage bids.** Hypothesis 7 is a prediction regarding the behavior of bidders in the second stage of the auction. It states that if bidder i is the bidder with the lower reservation price, then the unique optimal bid for bidder i is min $(V^i(2), I^i - p)$, where p is the price paid for good 1. If first-price rules prevail, the unique optimal bid of bidder $j \neq i$ is .01 above this minimum.

Table 4 below lists the percentage of bidders who used the optimal strategies discussed above in the tenth round of (the second-price rule) experiments 1, 2, 3, and 4. The evidence strongly supports the use of optimal bids on good 2 under second-price rules.

The evidence is also very strong that the bidders used their optimal strategies under first-price rules. Table 5 provides the details. All but one of the bidders who were allocated good 1 bid their remaining income on good 2. In five of the eight pairs, the bidders who were not allocated good 1 bid just above their opponents' remaining income.

Finally, Hypothesis 8 concerns the allocation of good 1. It states that the bidder whose critical value is higher will win the first good. We report the percentage of times that good 1 was allocated to the predicted winner in Table 6.

We note that the allocation predicted in Hypothesis 8 requires coordination of bids. In our experiments opponents were randomly rotated so that it was difficult to learn about the population of 8 to 10 bidders one was facing in only ten rounds. Thus, perhaps owing to the coordination problem, this hypothesis is not strongly supported.

TABLE 3 **Valuation and Income Results**

	Experiment 1	Experiment 2	Experiment 3
Mean Potential Price	242.7 (254.38)*	243.04	277.19
Z-Statistic		−4.02 (−5.39)*	−10.69 (−10.24)*

* Derived after the outlier of 150 was removed from experiment 1.

TABLE 4 **The Optimality of Bids on Good 2**

Experiment	1	2	3	4
% of Optimal Bids among Bidders Allocated Good 1 in the Tenth Round	100	80	100	87.5
% of Optimal Bids among Bidders Who Lost Good 1 in the Tenth Round	100	87.5	100*	87.5

* Includes one bidder who bid exactly the remaining income of opponent.

TABLE 5 **Bids on Good 2**

	Good 1 Winner		Good 1 Loser
Random Matches	Bid on Good 1	Bid on Good 2	Bid on Good 2
Pair #1	213.10	186.99	187.0
Pair #2	210	190	190.01
Pair #3	210	190	186.6
Pair #4	240.01	158.99	180.0
Pair #5	230.5	170	170.0
Pair #6	205	195	195.1
Pair #7	224	164	176.01
Pair #8	231	169	160.01

TABLE 6 **Allocation of Good 1**

Experiment	1	2	3	4	5
% of Times Bidder with Higher Critical Value is Allocated Good 1	NA*	76%	64%	82%	54%

* In experiment 1 bidders are symmetric so there is no designated winner of good 1.

8. Conclusion

■ This article contributes to the experimental literature on game theory and auctions. The results strongly support the premise that budget constraints affect the behavior of bidders. Bidders do attempt to exploit the constraints of others. The results also support the perfect equilibrium as an attractor for the bids whenever the perfect-equilibrium bids of each player are below their valuations. In all but the valuation experiment (in which the perfect-equilibrium bids might have resulted in out-of-pocket losses, and thus might be viewed as too "risky"), the evidence points to the perfect equilibrium as a good predictor of prices. Since the perfect-equilibrium outcome is unique, any of the current Nash refinements predict identical results. Thus, the experimental results cannot be used to differentiate among them.

Given the surprisingly strong performance of the perfect equilibrium (whenever the perfect-equilibrium bids were below the bidders' valuations), we must ask why it performed so well. As shown in Section 2, the perfect equilibrium in each experiment is a pair of bids that enjoy the following domination property. The perfect-equilibrium bids are not only undominated, but one of the bids in the pair also dominates all other conceivable equilibrium strategies. We conjecture that it is this domination property that accounts for some of the attracting power of the perfect equilibrium. On the other hand, we must also ask why it was that the perfect equilibrium bids were not attracting when the perfect equilibrium required bidders to bid above their valuations. Bidders were not induced to bid above their valuations. Either the risk of out-of-pocket losses, or the fact that any potential benefits were outweighed by the risks, kept bids below their induced valuations. Clearly a different set of game theoretic experiments needs to be done to explore these and related questions.

Appendix A

■ Instructions: baseline experiment.

BUYER TYPE SUBJECT #

□ **Introduction.** You are about to partake in a decisionmaking experiment. Two research institutes have contributed money to support this project, and if you make good decisions, you will earn a good payoff.

□ **Overview.** This experiment involves the sale of two different goods that are sold, one at a time, in sequence. First, good α is sold and then, good β. The subjects in the room have been divided into two types which we call buyers of type 1 and buyers of type 2. (Look at the upper left-hand corner of the instruction sheet to see which you are.) In each round of the experiment you will bid against one and only one bidder of the opposite type. Which one you actually bid against will be determined randomly, and we will not tell you the identity of the subject you bid against. During the experiment all prices are stated in terms of a fictitious currency called francs. These francs are converted into dollars at the end of the experiment in a manner to be described later.

□ **Rules of the sale.** The way the sale works is quite simple. At the beginning of each round, buyers of type 1 are given a budget of *400* francs while buyers of type 2 are given a budget of *400* francs. Then all of the subjects in the room write a bid for good α on a piece of paper. Your bids can be any positive amount of francs written up to 2 decimal points (i.e., an acceptable bid would be something like 6.44 but not 6.447). The only other constraint on your bid is that it may not exceed your available budget. These pieces of paper are then collected separately in two boxes—one box for buyers of type 1 and one for buyers of type 2. The two boxes are then put in the front of the room and pairs of bids are drawn from them, one bid from each box until all pairs of bids are drawn. This drawing determines two things. First, it defines which exact buyer of the opposite type you bid against for good α and good β in this round of the experiment. Next, it determines whether you are allocated good α and, if so, at what price. This is determined as follows: given your bid and the bid of your pair member, good α is allocated to the bidder whose bid is highest at the price of the lower bidder. If both bids are identical, the winner is chosen by a flip of a fair coin and the price equals his bid. For instance, say you bid *80* francs for good α and the bid of the other bidder is *40*. Since your bid is the higher, you are allocated good α and have to pay a price of *40* francs for it. This number of francs is then subtracted from your budget, and you will proceed to bid with this reduced budget for good β against the same bidder. Note, however, that since your pair member did not buy good α, he will have his full budget available to bid for good β. When the bidding for good α is over, if you were allocated good α, the experimental administrator will give you a piece of paper with the price you won the good at written on it. Note that if you do not receive such a piece of paper, it means that you did not receive good α. It also means that you know the price your pair member won at since it is equal to your bid. This information will tell you how many francs he has left to bid against you for good β (i.e. his initial budget minus the price paid on good α).

Before you proceed to bid for good β, fill out part I of your worksheet pertaining to round 1 by entering your starting budget in column 1, your bid in column 2, a yes or no in column 3 (depending on whether you won good α or not), and the price you paid for the good in column 4, if you purchased the good. (If you did not purchase the good, leave this entry blank.) In column 5 place your payoff from good α (described below) and in column 6 place the amount of francs you have left to bid on good β.

You bid for good β in almost the same way you bid for good α. One difference is that, on your bid slip, we ask you to put not only your bid, but also the amount of your budget remaining. This is either your original budget, if you did not buy good α or your original budget minus the price of good α, if you did buy good α. Note that your bid for good β cannot exceed your remaining budget. Any such bid will be rejected by the experimental administrator, and you will be asked to resubmit it. Another difference is that there is no random draw to determine the identity of your pair member—whoever was randomly drawn to be your pair member of good α remains your pair member for good β in round 1 of this experiment. After you submit your bids on good β, you are told whether you are allocated it and at what price. You then fill out part II of your worksheet in round 1 by entering your starting budget (same as in column 6 of part I) in column 1, your bid in column 2, a yes or no in column 3 (depending on whether you won good β), and the price you paid in column 4. In column 5 place your payoff from good β (described below). Finally, in column 6 add up your payoffs on goods α and β to determine your total payoff in round 1.

□ **Payoffs.** As a bidder, you are able to resell any good that you buy during the experiment. In any round, type 1's redemption or resale price of good α is *300* francs. If type 1 purchases good β during the experiment, his resale price is *200* francs. The resale price for a type 2 bidder is *300* francs, if type 2 purchases good α and *200* francs if type 2 purchases good β. Hence if you are a buyer of type 1 and you purchase good α during the experiment for a

TABLE (i)

Bidder Type	Resale Price for Good α	Resale Price for Good β	Budget
Bidder type 1	300	200	400
Bidder type 2	300	200	400

price of *100* francs, your final payoff at the end of the sale is $300 - 100 = 200$ for that good. If your purchase good β at a price of *50*, your final payoff for that good is $200 - 50 = 150$. If you are a buyer of type 2, then your payoff is $300 - 100 = 200$ if you purchase good α for *100* francs and $200 - 50 = 150$ if you purchase good β for *50* francs. If you purchase both good α and good β at these prices, your payoff is the sum of these two payoffs. If you purchase neither good, you earn a payoff of zero. An example is provided below to help clarify how to calculate your franc payoffs in each round.

The relevant information of the auction is summarized in Table (i).

Note that all subjects in the room are given identical instruction sheets so that everybody knows the values of the numbers written on Table (i).

When round 1 is over, all unused francs are removed from your budget. Rounds 2–10 then proceed in exactly the same way as round 1. You begin each round with a budget equal to that with which you started in round 1. You bid for goods α and then β against a randomly chosen bidder of the opposite type. (Hence your pair member changes from round to round.) Your payoff for each round is determined as explained above. Any francs in your budget not used to pay for goods are removed from you at the end of each round. Your final dollar payoff is determined as follows. Your franc payoff is the sum of your franc payoffs from each of the ten rounds of the experiment. To determine the dollar value of this amount of francs, divide the sum by *100*. In other words, you are paid $1 for every *100* francs earned or *1* cent a franc. In addition, at the beginning of the experiment we credit your dollar account with *$3.00*. This payment is to compensate you for actually coming to the experiment and to cover any losses you incur during any round of the experiment. Finally, the bidder of each type whose franc payoff is the highest of his type is also paid a bonus of *$2.00*. Note that you compete only with bidders of your own type for this bonus and not with bidders of the opposite type against whom you bid in each round. Thus, if you earn *500* francs and this is the highest franc payoff of bidders of your type, then your money payoff is $3.00 + (500 ÷ 100) + $2.00 = *$10.00*.

Example. Consider the following market:

	Resale Value for Good α	Resale Value for Good β	Budget
Subject 1	10	4	10
Subject 2	6	9	10

Here subject 1 has a redemption value for good α of 10 francs and a redemption value for good β of 4 francs, while subject 2 has a redemption value for good α of 6 francs and a redemption value for good β of 9 francs. Both subjects have a budget of 10 francs.

Scenario 1. Assume that good α is brought up for sale first and then good β. Say subject 2 bids 7.5 on good α and subject 1 bids 9. Since subject 1's bid is greater than subject 2's, subject 1 wins the first good at a price of 7.5 francs. When good β is brought up for sale, subject 1's remaining budget is $(10 - 7.5) = 2.5$, while subject 2's budget is his original 10. If subject 2 bids 7 on good β and subject 1 bids 2.5, subject 2 wins β at the price 2.5.

Hence, since in this first scenario subject 1 won good α at a price of 7.5 francs and subject 2 won good β at a price of 2.5 francs, subject 1's franc payoff in this round is $10 - 7.5 = 2.5$ francs, while subject 2's payoff is $9 - 2.5 = 6.5$ francs.

Scenario 2. In this scenario again, first good α is brought up for sale and then good β. Now, however, assume that subject 1 bids 7 instead of 9 on good α, while subject 2 maintains his bid on good α at 7.5. Then subject 2 wins good α at a price of 7. When good β is brought up for sale, subject 2 wins good α at a price of 7. When good β is brought up for sale, subject 1 has 10 francs with which to bid, while subject 2 has 3. Now, if subject 1 bids 4 francs for good β and subject 2 bids 3 francs, subject 1 wins good β for a price of 3 francs. In this scenario subject 2 is allocated good α at a price of 7 francs, while subject 1 is allocated good β at a price of 3 francs. The payoff for subject 2 will then be $6 - 7 = -1$, while the payoff for subject 1 is $4 - 3 = +1$.

Appendix B

■ Tables A1–A5 present the tenth-round bids and potential prices for good 1 and their means and variances for the baseline experiment, the value experiment, the income experiment, the sequence experiment, and the first-price experiment, respectively.

TABLE A1 Baseline Experiment

Tenth-Round Bids and Potential Prices for Good 1

Bids: Bidder 1	Bids: Bidder 2	Prices: Potential Prices	Prices: Frequency
400	260.01	150	9
150	250.51	250	23
262.01	259.99	250.23	7
269.99	250	250.5	7
250.5	258	250.51	5
260	260	258	5
265	250	259	5
250.23	259	259.99	5
250	268	260	7
		260.01	4
		262.01	1
		265	1
		268	2

Means and Variances

Round	Mean Bid: Bidder 1	Mean Bid: Bidder 2	Bid Variance: Bidder 1	Bid Variance: Bidder 2	Mean Potential Price	Potential Price Variance
1	199	191	13658	8533	135.123	8460.393
2	220	211	14337	7905	158.9066	11889.69
3	323	237	2290	10434	226.1841	8805.000
4	313	273	2298	516	267.3450	571.5414
5	288	245	1235	4212	239.9135	3880.866
6	294	283	1902	1898	266.3070	513.5990
7	268	254	679	777	245.3108	528.1040
8	268	268	350	182	259.1688	108.5230
9	286	262	1769	218	256.2856	149.8392
10	261	257	3564	32	242.7859	1100.370

TABLE A2 Value Experiment

Tenth Round Bids and Potential Prices for Good 1

Bids: Bidder 1	Bids: Bidder 2	Prices: Potential Prices	Prices: Frequency
238	241	238	5
265	250	239	10
239	400	241	4
239	300	250	2
251	241	251	2
		265	2

Means and Variances

Round	Mean Bid: Bidder 1	Mean Bid: Bidder 2*	Bid Variance: Bidder 1	Bid Variance: Bidder 2	Mean Potential Price	Potential Price Variance
1	171	259	2395	9324	157.12	2790.301
2	201	279	1778	6564	190.2	3253.409
3	233	299	1193	4001	226.44	1163.686
4	214	319	2842	1625	204.6	2842.64

TABLE A2 ***Continued***

Means and Variances

Round	Mean Bid: Bidder 1	Mean Bid: Bidder 2*	Bid Variance: Bidder 1	Bid Variance: Bidder 2	Mean Potential Price	Potential Price Variance
5	204	286	4354	3784	202.6	4442.24
6	238	269	96	2504	231.2	182.8
7	252	278	226	720	248.32	162.2976
8	203	264	6000	904	199.52	5709.209
9	230	257	621	1476	222.24	535.4624
10	246	286	109	3707	243.04	59.7984

* Predicted buyer of good 1.

TABLE A3 **Income Experiment**

Tenth Round Bids and Potential Prices for Good 1

Bids: Bidder 1	Bids: Bidder 2	Prices: Potential Prices	Prices: Frequency
275	300	251	10
275	275	270	9
400	275	274	9
350	275	275	42
275	296	279	5
251	299.99	289	5
350	279	290	4
274	289	294	4
294	270	296	3
290	300	299.99	3
		300	6

Means and Variances

Round	Mean Bid: Bidder 1	Mean Bid: Bidder 2	Bid Variance: Bidder 1	Bid Variance: Bidder 2	Mean Potential Price	Potential Price Variance
1	194	215	1143	9322	147.22	9406.331
2	235	285	5398	2347	225.49	4583.869
3	265	303	4980	1653	254.58	4172.103
4	269	282	4134	3253	245.29	3753.785
5	269	288	3054	2160	252.055	2982.099
6	276	282	2386	517	260.226	1313.545
7	276	274	1098	1076	262.21	806.9059
8	294	285	1254	250	275.9496	174.2482
9	256	285	1203	123	254.4199	1047.594
10	304	286	2777	141	277.1997	158.6063

TABLE A4 **Sequence Experiment**

Tenth Round Bids and Potential Prices for Good 1

Bids: Bidder 1	Bids: Bidder 2	Prices: Potential Prices	Prices: Frequency
200	0	0	8
198	250	150	8

TABLE A4 ***Continued***

Tenth Round Bids and Potential Prices for Good 1

Bids: Bidder 1	Bids: Bidder 2	Prices: Potential Prices	Prices: Frequency
210	190	165	8
250	190	180	5
200	199	180.5	7
180	180.5	190	14
199	150	198	2
299	165	199	7
		200	2
		210	1
		250	2

Means and Variances

Round	Mean Bid: Bidder 1*	Mean Bid: Bidder 2	Bid Variance: Bidder 1	Bid Variance: Bidder 2	Mean Potential Price	Potential Price Variance
1	143	171	3530	13974	104.6093	3872.113
2	179	153	12840	8168	106.7187	5736.889
3	157	212	13243	6725	126.4062	6195.678
4	225	233	11490	5831	178.9062	3609.116
5	162	216	4150	6508	149.4531	4116.279
6	209	196	620	7992	172.8593	2354.589
7	179	148	2547	5261	132.9687	5607.280
8	211	165	6199	4039	154.4843	4249.937
9	248	195	8073	8056	173.2734	3867.655
10	229	165	4455	4664	160.0390	4043.385

* Predicted buyer of good 1.

TABLE A5 **First-Price Experiment**

Tenth Round Bids and Potential Prices for Good 1

Bids: Bidder 1	Bids: Bidder 2	Prices: Potential Prices	Prices: Frequency
200	231	200	1
224	201	201	1
205	200	202	1
210	230.5	205	3
225	240.01	205.6	3
205.6	210	210	16
210	202	213.01	5
213.01	210	224	5
		225	5
		230.5	8
		231	8
		240.01	8

Means and Variances

Round	Mean Bid: Bidder 1	Mean Bid: Bidder 2	Bid Variance: Bidder 1	Bid Variance: Bidder 2	Mean Potential Price	Potential Price Variance
1	150	138	4131	2313	176.6804	3067.909
2	127	159	1252	3144	173.6028	1876.986

TABLE A5 *Continued*

	Means and Variances					
	Mean Bid		Bid Variance		Mean Potential	Potential Price
Round	Bidder 1	Bidder 2	Bidder 1	Bidder 2	Price	Variance
3	166	164	998	1653	186.4	713.7514
4	177	177	828	928	194.3296	590.1590
5	180	197	198	1044	205.3717	595.9554
6	178	193	184	436	198.5217	213.4431
7	191	199	98	189	203.3596	109.1462
8	198	202	110	112	206.4131	56.39941
9	201	208	152	105	211.465	101.3379
10	211	215	69	219	220.5778	147.8470

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