Performance Analysis of Closed–Loop Transmit Diversity in the Presence of Feedback Errors

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ABSTRACT

Transmit diversity techniques have received a lot of attention recently, and open–loop and closed–loop downlink transmit diversity modes for two transmit antennas have been included into 3GPP WCDMA Release 4. System capacity can be increased from that of open–loop modes if the transmitter is equipped with additional side information of the downlink channel. In a frequency division duplex system this means that the receiver has to provide the information through a feedback mechanism.
In this paper, we study the performance loss caused by feedback errors using the bit error probability (BEP) as a performance measure. It is shown that the asymptotic diversity of closed-loop schemes is reduced to one in the presence of feedback errors, *i.e.*, the performance is similar to a single transmit antenna system with some SNR gain. The proof is given in a very general manner and it is valid for a large variety of feedback schemes. Two example schemes, selection and co-phasing algorithms, are discussed in more detail to verify the results.

1 Introduction

The capacity of wireless cellular systems is limited by several different physical constraints like co-channel and adjacent channel interference, propagation loss, and ¤at or multipath fading. Deploying multiple antennas at base stations presents one possible approach to the problem, while multiple receive antennas at mobile terminals do no present as attractive solution due to the increase in signal processing complexity and power consumption.

Several different open–loop downlink transmit diversity techniques based on space–time coding have been developed in recent years, and the simplest space-time block code [1] has been adopted into 3GPP WCDMA Release 4 as an open-loop transmit diversity method for two transmit antennas. Unfortunately, full rate full diversity orthogonal space–time codes for complex modulation alphabets do not exist when the number of transmit antennas is larger than two, and when extending open–loop schemes to more than two antennas it is necessary to £nd a suitable balance between diversity order, rate, and orthogonality. Retaining the latter reduces the code rate thereby introducing some coding gain to the system due to the space–time code. However, the space–time codes are rather weak, and the increase in spatial diversity is typically not able to compensate the reduction in code diversity because the rate of the error correcting code has to increase to match the reduced rate of the channel. Another approach is to retain the rate of the space–time code and give up orthogonality which leads to more complicated decoding and loss in asymptotic diversity [2].

Closed–loop transmit diversity techniques can provide full diversity and increase the received SNR without space–time coding. When uplink and downlink operate in different frequency bands the side information related to the downlink channel requires additional signaling, and the design of signaling formats optimizing some performance measure, *e.g.* BEP in the mobile terminal while simultaneously minimizing the amount of uplink signaling makes the problem challenging. Closed–loop techniques typically outperform the open–loop ones particularly within low–mobility environments when the delay of the feedback signaling does not exceed channel coherence time. Different quantization strategies for the feedback message using SNR gain as a performance measure have been studied in [3, 4, 5].

The 3GPP FDD WCDMA speci£cation currently includes two different closed–loop transmit diversity modes for two transmit antennas with slightly different tradeoffs in effective constellation resolution and signaling robustness. In Mode 1 the length of the feedback word is one bit, and the base station interpolates between two consecutive feedback words making the transmit weight to follow a time-varying QPSK constellation. In Mode 2 the feedback word consists of four bits where three bits are assigned to phase and one bit to gain. Thus, Mode 1 maintains equal power transmission from both antennas while with Mode 2 the antennas transmit with different powers so that the better channel is assigned 6dB more transmit power than the weaker one. A detailed description of frame and slot structures can be found in [6]. The performance of WCDMA closed–loop schemes has been addressed in [7, 8] without taking into account quantization of the feedback messages and errors in the feedback link.

In this paper we study bit error probabilities of some general closed–loop transmit diversity schemes and show that in the presence of feedback errors the asymptotic diversity of the feedback schemes is equal to one which is similar to a single antenna transmission. This may seem to be a severe drawback of feedback schemes, but with two transmit antennas and relatively low SNR operation points typical to WCDMA systems feedback errors do not dominate the performance. However, increasing the number of transmit antennas necessarily implicates longer feedback words, and the effect of feedback errors becomes more pronounced.

The paper is structured as follows: Section 2 introduces the system model and feedback algorithms which are further analyzed in Section 3. Concluding remarks are presented in Section 4.

2 System Model

Consider a system with M transmit antennas in the base station and a single receive antenna in the mobile station. For the analysis we adopt a single path Rayleigh fading channel model. Since $\mathbf{h}_m$ is now a complex scalar rather than vector, we denote h_m instead of $\mathbf{h}_m$ and $\mathbf{h} = (h_1, h_2, \ldots, h_M)^T$ instead of $\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_M)^T$. Mobile station receives the signal from the dedicated channel as

$$r = (\mathbf{w} \cdot \mathbf{h})s + n,$$

where s is the transmitted symbol, n is zero-mean Gaussian noise, $\mathbf{w} = (w_1, w_2, ..., w_M)$ consists of transmit weights selected based on the feedback from mobile station, and components of the channel vector $\mathbf{h} = (h_1, h_2, ..., h_M)^T$ are samples of a zero-mean Gaussian process with the common variance $\sigma^2 = \frac{1}{2}$. We assume that channel coef£cients $h_m(t)$ are samples from independent processes.

Let us de£ne $\mathbf{W} = \{\mathbf{w} \in \mathbb{C} : \|\mathbf{w}\| = 1\}$ and denote by $\mathbf{W}_K$ a quantization set that is a subset of $\mathbf{W}$ and consists of K points. Usually K is selected such that $\kappa = \log_2(K) \in \mathbb{N}$. Now we can introduce an optimal feedback algorithm given the quantization $\mathbf{W}_K$.

Optimal Algorithm. Assume that κ bits of side information are available in the transmitter. The problem of £nding the optimal transmit weight vector $\hat{\mathbf{w}}$ becomes

$$\text{Find } \hat{\mathbf{w}} \in \mathbf{W}_K : \qquad |\hat{\mathbf{w}} \cdot \mathbf{h}| = \max_{\mathbf{w} \in \mathbf{W}_K} |\mathbf{w} \cdot \mathbf{h}|. \tag{1}$$

The given algorithm is optimal in the sense that it selects the optimal quantization point from SNR maximization point of view. We remark that in the forthcoming discussion we will use 'feedback word' and 'transmit weight' interchangeably. In practice they are, of course, not the same but there is a one-to-one mapping between the quantization set and the set of feedback words.

When the number of transmit antennas is large, the complexity of this algorithm can be very high, and the selection of the optimal quantization points $\mathbf{W}_K$ is not straightforward. Thus, there is a need for simple suboptimal solutions, and in the sequel we give two examples of such schemes, the selection algorithm and the co–phasing algorithm. First we de£ne

Selection Algorithm. In this case quantization consists of vectors $\mathbf{w} = (0, \ldots, 0, 1, 0, \ldots, 0)$, where the nonzero component indicate the best channel in terms of received power. Hence

$$|\mathbf{w} \cdot \mathbf{h}| = \max\{|h_m| : 1 \le m \le M\}.$$

Since the quantization set has M points, $\lceil \log_2(M) \rceil$ feedback bits are needed.

Selection algorithm is very simple and it requires a relatively low feedback capacity. However, the corresponding SNR gain is proportional to $\log M$ for large M, and we will later see that the selection algorithm is also sensitive to feedback errors. As another example of a suboptimal algorithm we give

Co-Phasing Algorithm. Now quantization set $\mathbf{W}_K$ has a product form,

$$\mathbf{W}_K = \prod_{m=2}^{M} \{e^{-j2\pi(n-1)/2^N} / \sqrt{M} : \ n = 1, 2, \ldots, 2^N\}.$$

Feedback bits are selected using the condition

$$|\hat{w}_1 h_1 + \hat{w}_m h_m| = \max_{w_m} |w_1 h_1 + w_m h_m|, \tag{2}$$

where $2 \le m \le M$ and $\hat{w}_1 = w_1 = 1/\sqrt{M}$. That is, we adjust each phase independently against the phase of the £rst channel. It should be noticed that the complexity of the example algorithm increases linearly with additional antennas, *i.e.*, complexity is proportional to $(M-1)2^N$. Furthermore, it can be shown that the SNR gain increases linearly as well [4].

We note that when $M = N = 2$ the co–phasing algorithm resembles FDD WCDMA transmit diversity Mode 1. The only difference between Mode 1 and the example algorithm is that in Mode 1 the feedback word results from the interpolation between two consecutive one–bit feedback words. However, this difference is irrelevant here, because the delay in feedback signaling is not taken into account.

3 Analysis

Let us now study the receiver BEP when selection and co-phasing are employed. For that purpose we recall the well known BEP of the selection algorithm when BPSK modulation is used (see, for example [10]),

$$P_{sc}(0) = \frac{1}{2}\left(1 - \sum_{m=1}^{M} \binom{M}{m} (-1)^{m-1} \sqrt{\frac{\gamma}{m+\gamma}}\right), \tag{3}$$

where $\gamma = E_b/N_0$ is the SNR per bit, and argument 0 emphasizes that the given BEP is valid when no feedback errors occur. It is known that the selection algorithm provides full diversity, *i.e.* the slope of the asymptotic BEP curve in a logarithmic scale is M when M transmit antennas are employed.

Intuitively the co-phasing should also give full diversity bene£t. This has also been shown in [9], where the following result has been proved.

Proposition 1 *Assume that the expected signal-to-noise ratio $\gamma_M = E_b/(MN_0)$ per bit and per antenna is large ($\gamma_M >> 1$) and co–phasing transmit diversity with $(M-1)N$ feedback bits is employed. Then the bit error probability for BPSK signal is given by*

$$P_{cp}(0) = \binom{2M-1}{M}\left(\frac{C_{M,N}}{4\gamma_M}\right)^M, \qquad (4)$$

where the constant $C_{M,N}$ attains the form

$$C_{M,N} = \Big\{ \int_{\mathbb{R}^{M-1}} q_\theta(\theta) \Big(\int_{\mathbb{R}_+^{M-1}} \frac{(M-1)!2^{M-1}\pi(\mathbf{r})}{R(\mathbf{r},\theta)^{2M}} d\mathbf{r} \Big) d\theta \Big\}^{\frac{1}{M}},$$

and $\mathbf{r} = (1, r_1, \ldots, r_{M-1})$, $d\mathbf{r} = dr_1 \ldots dr_{M-1}$.

Function $R(\alpha,\theta)$ refers to the the amplitude of the adjusted sum channel and it is de£ned by

$$R(\alpha,\theta) = \Big| \sum_{m=1}^{M} \alpha_m e^{j\theta_m} \Big|,$$

where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_M)$, $\alpha_m = |h_m|$ and component amplitudes follow the Rayleigh distribution. The component phases of $\theta = (\theta_1, \theta_2, \ldots, \theta_M)$ are uniformly distributed on $(-\frac{\pi}{2^N}, \frac{\pi}{2^N})$ if $m > 1$ and $\theta_1 = 0$. The joint distribution of θ is denoted by $q_\theta(\cdot)$. We remark that in the case of the co–phasing algorithm the SNR is given per bit and per antenna while in the case of the selection algorithm SNR is given per bit. This is due to the fact that the selection algorithm transmits all power through a single antenna, while the co-phasing algorithms divides the transmit power between all M antennas. It is noticed that the given asymptotic formula is very similar to the known asymptotic BEP of receiver maximal ratio combining [11]. The closed–form expressions for $C_{M,N}$ are available for $M = 2$ [9].

Lemma 1 *Let the assumptions of Proposition 1 be valid. Then there holds*

$$C_{2,N} = \Big\{ \frac{1 - \frac{2^{N-1}}{\pi} \sin \frac{\pi}{2^{N-1}}}{2 \sin^2 \frac{\pi}{2^N}} \Big\}^{\frac{1}{2}}.$$

For ideal feedback ($N \to \infty$) we have $C_{ideal} = 1/M$ while the BEP gain for ideal co–phasing is of the form

$$C_{M,\infty} = \Big\{ \frac{2^{M-1}(M-1)!}{(2M-1)!} \Big\}^{\frac{1}{M}}.$$

Using the results of Proposition 1 and Lemma 1 the asymptotic BEP performance of co–phasing algorithm is obtained. Moreover, from Proposition 1 it is clear that co–phasing attains full M–fold diversity. In the following we show that this is not true any more in the case of feedback bit errors. We assume that the feedback bit error probability is constant and bit errors are uniformly distributed in time. This model can be considered to be approximately valid in FDD WCDMA since the fast uplink power control is applied to the control channel carrying the feedback information. Of course, this assumption does not hold any more with high mobile speeds when the delay of the feedback loop exceeds the coherence time of the channel. However, the assumption is well justi£ed within low mobility environments.

Consider £rst an example where $M = 2$ and a single feedback bit is available. Let h_1 and h_2 be uncorrelated zero–mean complex Gaussian variables corresponding to the channel coef£cients of the £rst and the second antenna. First we show an interesting property: Under these assumptions selection and co–phasing algorithm are equivalent. Let us denote $k_\pm = h_1 \pm h_2$. Then it is easily seen that $\mathrm{E}\{k_+ k_-^*\} = \mathrm{E}\{|h_1|^2\} - \mathrm{E}\{|h_2|^2\} = 0$. Thus, k_+ and k_- are uncorrelated zero-mean complex Gaussian random variables, and $\max\{|h_1|, |h_2|\} = \max \frac{1}{\sqrt{2}}\{|k_+|, |k_-|\}$. In [3], it was also remarked that the two quantization strategies result in the same SNR gain, but no justi£cation for the phenomenon was given.

Consider next the effect of feedback bit errors to the BEP performance when $M = 2$. Let P_{sc} be the BEP of two-antenna selection algorithm when the probability of a feedback bit error is p. Then there holds

$$P_{sc} = (1-p)P_{sc}(0) + p \cdot P_{sc}(1),$$

where $P_{sc}(0)$ and $P_{sc}(1)$ refer to error–free and erroneous received feedback bit, respectively. The feedback bits are equally probable, and there holds

$$P_{sa} = \frac{1}{2}P_{sc}(0) + \frac{1}{2}P_{sc}(1),$$

where P_{sa} is the BEP corresponding to the single antenna transmission. The two-antenna system performance is reduced to that of a single transmit antenna if each feedback bit is randomly selected. Combining these two equations gives

$$P_{sc} = (1-2p)P_{sc}(0) + 2p \cdot P_{sa}. \qquad (5)$$

Thus, it is seen that the asymptotic diversity of the co–phasing algorithm is only one when $p > 0$.

Fig. 1 displays the BEP curve of two-antenna selection/co–phasing with $N = 1$. It is seen that the term $2p$ in (5) de£nes the asymptotic difference between the single antenna and the two–antenna bit error probabilities. In practice, the loss of asymptotic diversity is not critical in case of the present WCDMA feedback modes since — as seen from Fig. 1 — the effect of feedback bit errors is not serious at low SNR values where WCDMA usually operates. However, when studying extensions of the closed–loop modes for more than two antennas this phenomenon should be taken into account.

Next we move on to study a more general case: Consider a feedback scheme with quantization $\mathbf{W}_K$ such that all feedback words are equally likely. Moreover we assume that the optimal feedback word $\hat{\mathbf{w}}$ is arbitrary but £xed. The single antenna BEP is given by

$$P_{sa} = \frac{1}{K} \sum_{\mathbf{w} \in \mathbf{W}_K} P_g(\mathbf{w}),$$

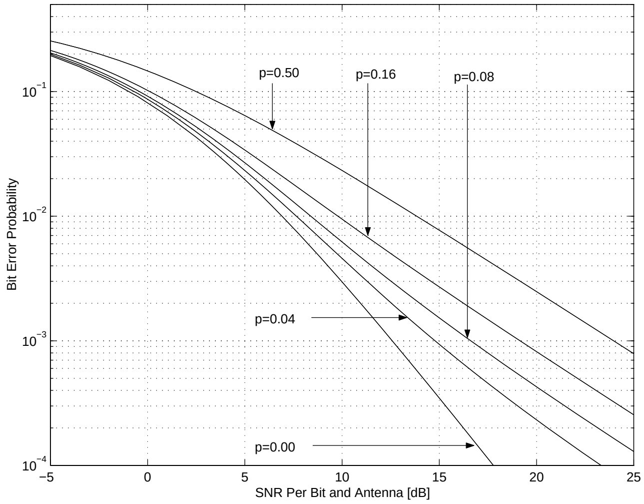

Figure 1: Bit error probability curves as a function of SNR for two-antenna selection/co–phasing with $N = 1$ when $p = 0.00, 0.04, 0.08, 0.16, 0.50$.

where $P_g(\mathbf{w})$ is the BEP of the general algorithm provided that the feedback word $\mathbf{w}$ is used. Hence, irrespective of the optimal feedback word, the performance of the multiantenna system is equal to the performance of a single antenna one when the feedback word is randomly selected. Let us denote by P_g the BEP of the general algorithm in the presence of feedback errors and £nally, let $q_{\mathbf{w}}$ be the probability of a feedback word $\mathbf{w}$ on the condition that $\hat{\mathbf{w}}$ is the optimal (selected) feedback word. Then there holds

$$\begin{aligned} P_g &= \sum_{\mathbf{w}\in\mathbf{W}_K} q_{\mathbf{w}} P_g(\mathbf{w}) \\ &= \sum_{\mathbf{w}\in\mathbf{W}_K} \big(q_{\mathbf{w}} - \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\}\big) P_g(\mathbf{w}) + K\cdot \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\} P_{sa}. \end{aligned}$$

Since the difference in the sum term is always non-negative, we £nd that P_g admits a lower bound

$$P_g \geq K\cdot \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\} P_{sa}.$$

This bound shows that the asymptotic BEP of the general feedback method is always reduced to one in the presence of feedback errors. The assumption that all feedback words occur with equal frequency in the long run is not too restrictive and it can easily be removed. This result is formulated as follows:

Proposition 2 *Assume a feedback scheme for which the quantization* $\mathbf{W}_K$ *is such that all feedback words are equally likely. Then the BEP of the feedback algorithm attains a lower bound*

$$P_g \geq K\cdot \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\} P_{sa} \tag{6}$$

where P_{sa} *is the BEP of a single antenna transmission, and* $q_{\mathbf{w}}$ *is the probability of a feedback word* $\mathbf{w}$ *on the condition that* $\hat{\mathbf{w}}$ *is the optimal (selected) feedback word.*

Consider the bound of Proposition 2 for selection and cophasing algorithms. There holds

$$P_{sc} \geq \frac{M}{M-1}\big(1-(1-p)^{\log_2(M)}\big)P_{sa}, \quad P_{cp} \geq (2p)^{N(M-1)}P_{sa}.$$

These bounds are equal if $M = 2$ and $N = 1$ as expected. If we compare these bounds, for example, when $p = 0.04$, $M = 8$ and $N = 1$ we £nd that the coef£cients in front of P_{sa} are very different: the coef£cient corresponding to the selection algorithm is 0.13 while the coef£cient corresponding to the co-phasing is $2\cdot 10^{-8}$. This difference is only a hint concerning the asymptotic performance of studied algorithms since these coef£cients do not necessarily give the difference in asymptotic curves as was the case in (5). However, for the selection algorithm there holds

Corollary 1 *Assume that the feedback bit error probability is p. Then the BEP of the selection algorithm is given by*

$$P_{sc} = \Big(1-\frac{q}{M-1}\Big)P_{sc}(0) + \frac{q}{M-1}P_{sa}. \tag{7}$$

where P_{sa} *is the BEP of a single antenna transmission and* $q = 1-(1-p)^{\log_2(M)}$ *is the probability of a feedback word error.*

Proof. Consider the proof of Proposition 2. Now $q_{\hat{\mathbf{w}}} = (1-p)^{\log_2(M)}$ while for other weights there holds $q_{\mathbf{w}} = 1/(M-1)(1-(1-p)^{\log_2(M)})$. Hence

$$q_{\mathbf{w}} - \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\} = \begin{cases} q_{\hat{\mathbf{w}}} - \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\}, & \mathbf{w} = \hat{\mathbf{w}}, \\ 0, & \mathbf{w} \neq \hat{\mathbf{w}}. \end{cases}$$

This formula shows the desired result. ■

Figure 2 displays the BEP of the selection algorithm in the presence of different feedback bit error probabilities when $M = 8$. It is seen that already at the feedback bit error level $p = 0.04$ the effect of the errors is signi£cant, and the slope is similar to that of a single transmit antenna ($p = 0.50$).

Let us study the effect of feedback word labeling. When different feedback adjustment alternatives are randomly labeled, a single feedback bit error makes the selection of the feedback word random, and in terms of proof of Proposition 2 there holds

$$P_g = \big(q_{\hat{\mathbf{w}}} - \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\}\big)P_g(\hat{\mathbf{w}}) + K\cdot \min_{\mathbf{w}\in\mathbf{W}_K}\{q_{\mathbf{w}}\}P_{sa}.$$

This cannot be avoided in the case of the selection algorithm, but when co–phasing is employed, Gray coding can be used when $N > 1$. Even when $N = 1$, $M > 2$ error probabilities of the feedback words of the co–phase algorithm are not uniformly distributed. Fig. 3 displays the BEP of the co–phasing algorithm when $N = 1$, $M = 8$ and $p = 0.00, 0.04, 0.08, 0.16, 0,50$. Comparing Figs 2 and 3 shows that when $p = 0$ the BEP performance of the co–phasing algorithm is almost the same as the performance of the selection algorithm, and in the case of erroneous feedback the

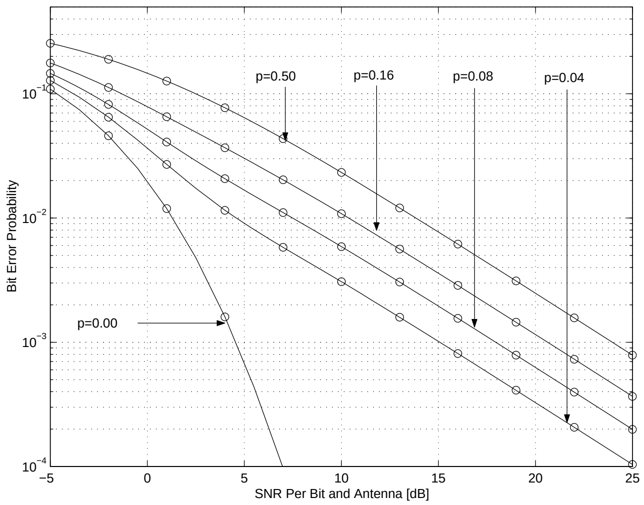

Figure 2: Bit error probability curves as a function of SNR for selection algorithm when $M = 8$ and $p = 0.00, 0.04, 0.08, 0.16, 0.50$.

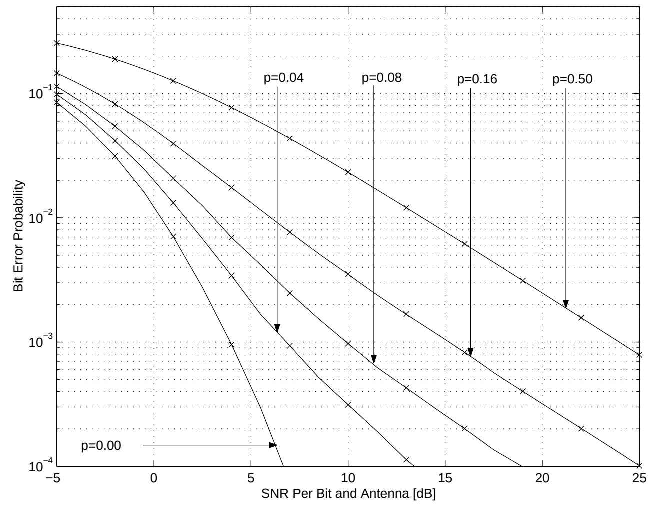

Figure 3: Bit error probability curves as a function of SNR for co–phasing algorithm when $N = 1$, $M = 8$ and $p = 0.00, 0.04, 0.08, 0.16, 0.50$.

co–phasing scheme outperforms the selection algorithm. Finally, we remark that the comparison is not necessarily fair because the number of feedback bits is 3 for selection and 7 for co–phasing. However, our goal has not been to compare the performance of different feedback algorithms but to illustrate the detrimental effects of feedback errors to closed–loop transmit diversity schemes.

4 Conclusions

The effect of feedback errors to the closed–loop transmit diversity techniques suitable to wireless frequency division duplex systems were studied using the bit error probability (BEP) as a performance measure.

It was found out that feedback bit errors reduce the slope of the asymptotic BEP of closed-loop schemes into one. The proof of this result was given in a very general manner and it is valid for a large variety of feedback schemes. Two example schemes, selection and co-phasing algorithms, were studied in more detail showing that the former algorithm is more sensitive to feedback errors than the latter one.

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