Performance Analysis of Cloud Radio Access Networks with Distributed Multiple Antenna Remote Radio Heads

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F. A. Khan, H. He *Student Member, IEEE*, J. Xue *Member, IEEE* and T. Ratnarajah *Senior Member, IEEE*

***Abstract*—In this paper, the performance of cloud radio access networks (CRANs) where spatially distributed remote radio heads (RRHs) aid the macro base station (MBS) in transmission is analysed. In order to reflect a realistic scenario, the MBS and the RRHs are assumed to be equipped with multiple antennas and distributed according to a Poisson point process. Both, the MBS and the RRHs, are assumed to employ maximal ratio transmission (MRT) or transmit antenna selection (TAS). Considering downlink transmission, the outage performance of three schemes is studied; first is the selection transmission (ST) scheme, in which the MBS or the RRH with the best channel is selected for transmission. In the second scheme, all the RRHs participate (ARP) and transmit the signal to the user, whereas in the third scheme, a minimal number of RRHs, to attain a desired data-rate, participate in transmission (MRP). Exact closed-form expression for the outage probability is derived for the ST scheme. For the ARP and MRP schemes, analytical approximations of the outage probability are derived which are tight at high signal-to-noise ratios. In addition, for the MRP scheme, the minimal number of RRHs required to meet a target data rate is also calculated which can be useful in characterizing the system complexity. Furthermore, the derived expressions are validated through numerical simulation. It is shown that the average diversity gains of these schemes are independent of the intensity/number of RRHs and only depend on the number of antennas on the MBS. Furthermore, the ARP scheme outperforms the ST scheme when the MBS/RRHs transmit with maximum power. However, in case of a sum power constraint and equal power allocation, the ST scheme outperforms the ARP scheme.**

***Index Terms*—Cloud radio access networks, maximum ratio transmission, MISO, Poisson point process, stochastic geometry, transmit antenna selection.**

I. INTRODUCTION

Cell densification is one of the key technologies proposed to improve the capacity and area spectral efficiency of existing networks [1]. A major drawback of increasing the cell/base station (BS) density is that the overall interference in the network also increases resulting in a limited capacity gain [2], [3]. In addition, deploying more BSs is neither cost efficient nor power efficient [4].

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Cloud radio access networks (CRANs) have been proposed as a low-cost and power-efficient solution to meet the increasing capacity demand. In the existing networks, the baseband units (BBUs), which consume high power, and the radio units are situated together. The idea in CRANs is to move the BBUs to a central location/data centre and connect it to the radio units, also called remote radio heads (RRHs), via optical fibres [4]. Moving the BBUs to a central location results in improved power efficiency. In addition, the cost of network expansion is lowered because only low cost RRHs/BSs need to be deployed for improving the coverage as well as the capacity of the network. Furthermore, it has been shown that through coordinated multipoint processing (CoMP), the overall interference can be limited. CoMP is very efficient when all the RRHs are connected with each other and possess the data information of each other [1], [5]. CoMP can easily be adopted in CRANs, to reduce the interference and improve the network capacity.

A. Existing Relevant Work:

When each of the RRH has a single antenna, the CRAN model becomes similar to the distributed antenna system (DAS). There have been several studies to analyse the performance of the DAS see [6]–[9] and the references therein. In [7] it was shown that the average spectrum efficiency per sector and the cell edge spectrum efficiency in the traditional system with co-located BS antennas (TS-CBA) is better than that of a DAS without frequency reuse. However, when the frequency reuse is considered the DAS outperforms TS-CBA. In [6], it was shown that DAS reduces inter-cell interference in a multicell environment and significantly improves capacity particularly in case of the users near the cell boundaries. In [8], the cell average ergodic capacity for a DAS in a composite fading channel model was analysed. An antenna selection strategy to maximize the energy efficiency under a pre-defined target rate constraint was proposed in [9]. In [10], it was shown that, for a CRAN with distributed RRHs with multiple antennas, the optimal distributed beamforming scheme had a form of maximum ratio transmission (MRT) at each RRH and the outage probability and ergodic capacity under Rayleigh fading channels was also analyzed. A joint strategy to select the antenna, the regularization factor, and the transmit power to maximize the average weighted sum-rate was proposed in [11].

In these previous works, the RRHs were assigned fixed regular locations. However, in many practical situations, this is difficult to do so and the RRHs are located randomly [12]–[17]. When the RRHs are assumed to be randomly placed, it can give a reasonable lower bound on the performance of an actual system. In [12], the authors proposed a low-complexity power allocation scheme among the distributed transmit antennas. In [13], the antennas were distributed according to a binomial point process, the users were distributed according to a Poisson point process (PPP) and a composite fading channel model was assumed. The authors derived analytical expressions for the outage performance of only selection transmission, where the antenna with the best channel was selected to serve the user, under different scenarios. It was shown in [14], that the DAS yields a higher capacity gain compared to the TS-CBA, provided the channel state information (CSI) is available at the transmitter and the receiver. The ergodic capacity of a multi-cell distributed RRH system, where the RRH locations were modelled as a spatial PPP was studied in [15], and it was shown that this system provides better cell-edge performance and can even provide higher capacity in a user-centric configuration. In [16], the RRHs were distributed according to a PPP and a Rayleigh fading channel with a standard path loss model was assumed. Different from the work in [12], the outage performance of selection transmission scheme, in which the macro BS (MBS) or the RRH with the best channel is selected for transmission, was compared to the scheme where all the RRHs employ distributed beamforming and aid in transmission [16]. In addition, the minimal number of RRHs required to meet a predefined quality of service (QoS) was also studied. In [17], it was shown that the uplink sum capacity increases as a result of reduction in the inter-cell interference of a DAS.

B. Our Contribution:

In [12]–[17], the RRHs were assumed to be equipped with a single antenna. However, in the proposed CRAN model, the RRHs will be equipped with multiple antennas. Therefore, different from the models in [12]–[17], in this work, a more general and realistic scenario is considered, and we analyze the performance of a network where several multiple antenna RRHs are distributed randomly (according to a PPP) over a circular region and serve the user along with a multiple antenna macro base station (MBS). To the authors best knowledge, for this network setup, the performance of a CRAN with multiple antenna RRHs has not been analysed previously. Having multiple antennas at the MBS and RRHs leads to a new and more involved analysis compared to the one presented in [12]–[17] because the distribution of the signal-to-noise ratio (SNR) from the RRHs to the users is no longer an exponential distribution[1]. In addition, in [16] the path loss coefficient was fixed to 2. However, in this work, the performance is analysed for arbitrary value of the path loss coefficient which also results in a more involved analysis.

The performance of this network is studied under the scenario when the MBS and the RRHs have varying complexity. Specifically, two levels of complexity are considered. The MBS and RRHs with higher complexity consist of multiple radio frequency (RF) chains and employ maximum-ratio-transmission (MRT) whereas the MBS and RRHs with lower complexity consist of a single RF chain and employ transmit antenna selection (TAS). Furthermore, three different transmission schemes are considered; 1) the MBS or the RRH with the best channel participates in transmission, also called selection transmission (ST), 2) all the RRHs participate (ARP) and aid the MBS in transmission and 3) minimal number of RRHs to attain a desired data-rate participate in transmission (MRP). Employing more RRHs results in a higher cost in terms of higher power expenditure and requires more control and data processing for synchronizing the transmissions. Therefore, MRT can help improve the power efficiency and reduce the overhead compared to ARP scheme as was also discussed in [9], [16]. From among the three schemes considered, ST has the lowest cost and ARP has the highest cost.

The performance of these schemes is analysed in terms of outage probability. Exact closed-form expression for the outage probability is derived for the ST scheme, whereas, a tight approximation of the outage probability at high signal-to-noise ratio is obtained for the ARP scheme. In the worst case scenario, the MRP scheme becomes the same as the ARP scheme and therefore it has the same outage probability as the ARP scheme. In addition, in order to quantify the complexity of the MRP scheme, we also analyze and obtain expressions for the minimum number of RRHs required to achieve a certain QoS. These expressions are obtained for two cases, one in which the RRHs have fixed transmit power and the other in which the RRHs can adapt power and compensate the pathloss. Furthermore, the derived expressions are verified through numerical simulations. Our simulation results show the effective trade-off between number of RRHs and number of antennas at each RRH. For example, considering the ARP scheme with MRT, increasing the number of antennas at each RRHs is more effective than employing more RRHs, as increasing the number of RRHs results in a lower transmit power at the MBS/RRHs and a lower coding gain. In addition, it is also shown that at high SNR regime, ST, ARP and MRP achieve a diversity order m_T, where m_T is the number of antennas at the MBS, and this diversity order does not depend on the number/intensity of the RRHs.

The rest of the paper is organized as follows. The system model is explained in section II. The statistics of the channel used in the performance analysis of each scheme are derived in Section III. The analytical expressions for the outage performance for the ST and ARP schemes as well as the complexity analysis of the MRP scheme is presented in Section IV. The numerical results are presented in Section V. Finally, the main results are summarized in the concluding Section VI.

[1]The distribution of the large-scale fading gain (LSFG) and the small-scale fading gain (SSFG) in the case of multiple antenna systems is well known. The *method* to obtain the distribution of the received SNR using the distributions of the LSFG and the SSFG is also straightforward and comes from basic probability theory and has been reported in many existing works eg see [12], [13], [16], [18] and references therein. However, to the authors best knowledge, even with the known fading distributions and the analysis method, the distribution of the received SNR from the multiple antenna RRHs to the users has not been reported previously. Moreover, it is more challenging to obtain the performance expressions for the transmission schemes considered in this work using the derived distribution of the received SNR.

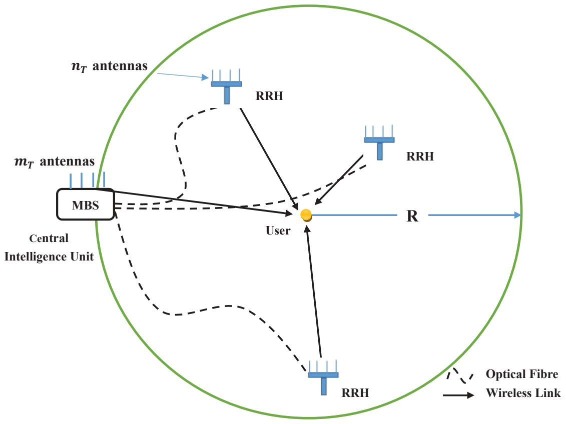

Fig. 1. System model

II. System Model

Consider a network shown in Fig 1, where a user is being served by a central intelligence unit (can also be termed as a macro-cell base station (MBS)), at a distance R from the user, and a group of N RRHs distributed randomly over a circular region $\mathcal{D}$, of radius R, around the user location[2]. The MBS is equipped with m_T antennas whereas each RRH has n_T antennas. It is assumed that the location of the RRHs obey a homogeneous Poisson point process (PPP) with intensity λ_{RRH}, therefore, N, is Poisson distributed, i.e., $\Pr\{N \text{ RRH in Disc } \mathcal{D}\}=\frac{\alpha^N}{\Gamma(N+1)}e^{-\alpha}$ where $\alpha = \pi R^2 \lambda_{RRH}$.

The channel vector between the n-th RRH and the user U can be written as $\mathbf{g}_n = [g_{n,1} ... g_{n,n_T}]^T$, where $g_{n,t}$ is the channel gain between the t-th antenna of n-th RRH and the user, $(\cdot)^T$ denotes the transpose operator. Assuming Rayleigh fading channel $g_{n,t} \sim \mathcal{CN}(0,\mu)$ where $\mathcal{CN}(x,y)$ denotes complex Gaussian random variable (RV) with mean x and variance y and μ denotes the mean power of the channel. Similarly, the channel between the user and the MBS is denoted as $\mathbf{g}_0$, where $\mathbf{g}_0 = [g_{0,1} ... g_{0,m_T}]^T$. The distance of the n-th RRH from the centre is denoted by d_n. As the locations of the RRHs are random, d_n is a RV with distribution

$$f_{d_n}(x) = \frac{2x}{R^2} \quad ; 0 \leq x \leq R. \tag{1}$$

Transmission Schemes:

For the system under consideration, three transmission schemes are studied, namely; 1) selection transmission (ST), 2) all the RRHs participate (ARP) and 3) minimal number of RRHs participate (MRP). In ST, the MBS or the RRH with the best channel is selected for transmission whereas in ARP, all the RRHs transmit to the user[3]. The ST scheme has lower overhead compared to the ARP scheme as it does not require coordination among the RRHs. However, this lower overhead is possible at the cost of some performance loss as will be discussed later.

Using all RRHs in the ARP scheme provides the optimal reception reliability but at the price of increasing system complexity. However, in some instances, optimal performance is not always needed and only a certain data-rate requirement is to be satisfied. In such cases, it is possible to achieve the pre-defined data-rate using only a subset of the available RRHs [9]. Using a minimal number of RRHs is beneficial as it yields a practical scheme with reduced complexity and ensuring desired system performance. Thus, in MRP scheme, the minimal number of RRHs required to meet a pre-defined data rate are used for transmission. In addition, in case of all these schemes, it is assumed that the multiple antenna MBS and RRHs employ MRT or TAS for transmission of the signal.

In this sequel we analyse the performance of these transmission schemes in terms of outage probability. In the next section, we derive the required statistics of the channel for analysing the outage probability.

III. Finding Statistics of the Channel

A. Statistics of SNR for MRT

When MRT is employed at the MBS and the RRHs, the received signal-to-noise ratio (SNR) from the n-th RRH to the user can be given as [16]

$$\gamma_n = \frac{P}{N_0}\left(\frac{1}{1+d_n^v}\right)\|\mathbf{g}_n\|^2 \tag{2}$$

where P is the transmit power at the MBS and each RRH, N_0 denotes the noise power at the user, term $\delta_n = (1+d_n^v)$ denotes the pathloss, v is the path loss coefficient, and $\|\cdot\|$ denotes the 2-norm[4]. When N RRHs transmit using MRT, the overall SNR at the user is given as

$$\gamma = \sum_{n=1}^{N}\gamma_n = \frac{P}{N_0}\sum_{n=1}^{N}\left(\frac{1}{1+d_n^v}\right)\|\mathbf{g}_n\|^2. \tag{3}$$

Similarly, the SNR of the MBS can be given as [16]

$$\gamma_0 = \frac{P}{N_0}\left(\frac{1}{1+R^v}\right)\|\mathbf{g}_0\|^2. \tag{4}$$

When the MBS and the N RRHs transmit using MRT, the overall SNR at the user is given as

$$\gamma_{MBS} = \sum_{n=0}^{N}\gamma_n = \frac{P}{N_0}\sum_{n=0}^{N}\left(\frac{1}{1+d_n^v}\right)\|\mathbf{g}_n\|^2 = \gamma_0 + \gamma \tag{5}$$

where $d_0 = R$. In order to analyse the performance of this scheme, the statistics of γ_{MBS} and γ, such as the cumulative distribution function (CDF) and the probability distribution function (PDF), are required. For deriving the statistics of γ_{MBS} and γ, the statistics of γ_n are required. Therefore, the CDF γ_n is given by following Proposition.

[2]In our analysis, we condition on the location of the user. The MBS at a distance R and the RRHs within a distance R from the user, serve the user. In this model, if the user is displaced and comes closer to the MBS, it implies that R will reduce. As a result the area of the circular region will also reduce. The converse is also valid. The distribution of the distance of the user from the RRHs depends on R and thus, it will change when R changes.

[3]In both these schemes, the MBS also participates in transmission. The performance expressions derived in this paper are derived for this scenario. The performance expressions for the scenario in which the MBS does not transmit, can easily be obtained by substituting $m_T = 0$ in the derived expressions.

[4]In deriving (2), it is assumed that the transmitted signal vector is $\mathbf{x} = P\frac{\mathbf{g}_n^H}{|\mathbf{g}_n|}s$, where s is the transmitted symbol having mean zero and unit variance. Therefore, the average transmit power is $\mathbb{E}[\|\mathbf{x}\|^2] = P$, where $\mathbb{E}[\cdot]$ is the expectation operator. Moreover, in this paper, it is assumed that the MBS and the RRHs employ equal power allocation and, therefore, transmit with the same power P.

$$F_{\gamma_n}(\Phi)=1-\sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\binom{i}{j}\frac{2}{\Gamma(i+1)\,vR^2}e^{-\frac{N_0\Phi}{P\mu}}\left(\frac{N_0\Phi}{P\mu}\right)^{i-\left(j+\frac{2}{v}\right)}\zeta\left(j+\frac{2}{v},\frac{N_0\Phi}{P\mu}R^v\right) \tag{6}$$

Proposition 1. *When all the RRHs transmit using MRT, the CDF of the SNR of the n-th RRH received at the user, denoted by γ_n, is given in* (6).

Proof: See Appendix A. ∎

The CDF in (6) is obtained in closed-form and is given in terms of incomplete Gamma function which can be easily evaluated using existing mathematical packages. In addition, the obtained CDF is valid for arbitrary value of the pathloss exponent, v, unlike [16] in which the pathloss exponent was assumed fixed i.e. $v=2$. Therefore, this expression in (6) is more general and the CDF for the scenario considered in [16] can be obtained by substituting $n_T=1$ and $v=2$. By substituting, $\Phi=2^{\mathcal{R}}-1$ in (6), where $\mathcal{R}$ denotes the data-rate, the outage probability for the n-th RRH can be obtained. Furthermore, the PDF of the SNR can be easily obtained by taking the derivative of (6) w.r.t. Φ.[5]

It is not trivial to obtain the statistics of γ using the CDF derived in (6). Therefore, we derive an approximation of the CDF of γ_n, using which one can obtain the statistics of γ. The approximate CDF of γ_n is given in following Proposition.

Proposition 2. *When all the RRHs transmit using MRT, the CDF of the SNR of the n-th RRH received at the user, denoted by γ_n, can be approximated as*

$$F_{\gamma_n}(\Phi)\approx\sum_{p=0}^{K}\Xi_p\left(\frac{N_0\Phi}{P\mu}\right)^{n_T+p} \tag{7}$$

where the infinite series is truncated to $K+1$ terms, K is any positive integer and $\Xi_p=\sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\sum_{u=l+k=n_T+p-i}\frac{2\binom{i}{j}(-1)^{u+1}(R^v)^{k+i}}{v\left(k+j+\frac{2}{v}\right)\Gamma(i+1)\Gamma(l+1)\Gamma(k+1)}$.

Proof: See Appendix B. ∎

It can be noted that the CDF in (7) is a polynomial function and has been limited to $K+1$ terms. As $K\to\infty$, the approximation becomes closer to the exact CDF given in (6). Furthermore, this approximation in (7) is tight at high SNRs[6]. Our simulation results show that even for, $K\leq 10$, the simulation results match the analytical results at high SNRs. Moreover, the approximate PDF of the SNR can be easily obtained by taking the derivative of (7) w.r.t. Φ. In addition, using (7), the approximate CDF of γ can be derived and is given in following Proposition.

Proposition 3. *When N RRHs transmit using MRT, the CDF of the overall SNR received at the user, denoted by γ, can be approximated as*

$$F_\gamma(\Phi)\approx\sum_{I_N}\frac{\varkappa_{I_N}}{\Gamma(Nn_T+\xi_{I_N}+1)}\left(\frac{N_0\Phi}{P\mu}\right)^{Nn_T+\xi_{I_N}} \tag{8}$$

where $\sum_{I_N}$ is shorthand notation of $\sum_{i_1=0}^{K}\sum_{i_2=0}^{K}\cdots\sum_{i_N=0}^{K}$ and $\xi_{I_N}=\sum_{l=1}^{N}i_l$ and $\varkappa_{I_N}=\prod_{l=1}^{N}\Xi_{i_l}\Gamma(n_T+i_l+1)$.

Proof: See Appendix C. ∎

The approximation given in (8) closely approximates the exact CDF at high SNRs. The CDF is in form of a polynomial function and can be easily implemented in existing mathematical packages. For a CRAN system with N participating RRHs, the CDF in (8) can be used to obtain the approximate outage probability. By substituting, $\Phi=2^{\mathcal{R}}-1$ in (8), the approximate outage probability at the user can be obtained when N RRHs transmit using MRT. The approximate PDF of γ can be easily obtained by taking the derivative of (8) w.r.t. Φ.

Diversity Order: (8) is in form of a polynomial function and at high SNRs, it can be approximated by its lowest order term, which is obtained by taking $\xi_{I_N}=0$ and (8) can be approximated as

$$F_\gamma(\Phi)\approx F_\gamma^\infty(\Phi)=\frac{(\Xi_0\Gamma(n_T+1))^N}{\Gamma(Nn_T+1)}\left(\frac{N_0\Phi}{P\mu}\right)^{Nn_T}. \tag{9}$$

Using (9), it can be easily shown that the diversity gain[7] achieved when N RRHs transmit using MRT is Nn_T. When N is a RV, the diversity gain is different as will be shown in Section IV.

B. Statistics of SNR for TAS

In case of TAS, the MBS and the RRHs transmit using the antenna providing the highest SNR. The SNR from the n-th RRH can be given as

$$\Upsilon_n=\frac{P}{N_0}\max_t\left\{\left(\frac{1}{1+d_n^v}\right)|g_{n,t}|^2\right\} \tag{10}$$

where $1\leq t\leq n_T$. When N RRHs transmit using the best antenna, the overall SNR at the user is given as

$$\Upsilon=\frac{P}{N_0}\sum_{n=1}^{N}\left\{\max_t\left\{\left(\frac{1}{1+d_n^v}\right)|g_{n,t}|^2\right\}\right\}. \tag{11}$$

Similarly, the SNR of the MBS can be given as [16]

$$\Upsilon_0=\frac{P}{N_0}\max_t\left\{\left(\frac{1}{1+R^v}\right)|g_{0,t}|^2\right\}. \tag{12}$$

When the MBS and N RRHs transmit after selecting the best antenna, the overall SNR at the user is given as

$$\Upsilon_{MBS}=\frac{P}{N_0}\sum_{n=0}^{N}\left\{\max_t\left\{\left(\frac{1}{1+d_n^v}\right)|g_{n,t}|^2\right\}\right\}=\Upsilon_0+\Upsilon \tag{13}$$

[5]Note that using the CDF and PDF expressions derived in this work, the expressions for the moment-generating-function (MGF) of the SNR as well as symbol error rate (SER) performance can be obtained for the system under consideration. However, it is omitted due to space limitation.

[6]High SNR implies that $z=\frac{N_0\Phi}{P\mu}$ is very small i.e. $z\approx 0$. The CDF in (7), $F_{\gamma_n}(\Phi)\approx\sum_{p=0}^{K}\Xi_p(z)^{n_T+p}$, is thus, a summation of powers of z. When z is very small, eg. in the high SNR regime, it implies $z^b\ll z^a\leq z$ where a and b are any positive integers and $b>a$. Therefore, in this case, the summation result is only influenced by lower powers and the terms with higher powers have minimal contribution and can be neglected. Therefore, the CDF in (7) approximates the CDF at high SNR accurately. However, at low SNRs, i.e. large z, this approximation might not be accurate.

[7]Diversity gain can be obtained as $d=\lim_{\frac{N_0}{P}\to 0}\frac{\log\left(F\left(\frac{N_0}{P}\right)\right)}{\log\left(\frac{N_0}{P}\right)}$ where $F(\cdot)$ denotes the CDF.

where $d_0 = R$. Similar to the case of MRT, in order to analyse the performance of this scheme, the statistics of Υ_{MBS} and Υ, such as the CDF and PDF, are required. For deriving the statistics of Υ_{MBS} and Υ, the statistics of Υ_n are required. Therefore, the CDF of Υ_n is obtained and is given in following Proposition.

Proposition 4. *When all the RRHs transmit using TAS, the CDF of the SNR of the n-th RRH received at the user, denoted by Υ_n, is given in* (14).

Proof: See Appendix D. ■

Similar to the expression in case of MRT, the CDF in (14) is obtained in closed-form and is given in terms of incomplete Gamma function which can be easily evaluated using existing mathematical packages. In addition, the obtained CDF is valid for arbitrary value of the pathloss exponent, v. By substituting, $\Phi = 2^{\mathcal{R}} - 1$ in (14) the outage probability for the n-th RRH can be obtained. Furthermore, the PDF of the SNR can be easily obtained by taking the derivative of (14).

Again, in this case, it is not trivial to obtain the statistics of Υ using the CDF derived in (14). Therefore, we derive an approximation of the CDF of Υ_n, using which one can obtain the statistics of Υ. The approximate CDF of Υ_n is given in following Proposition.

Proposition 5. *When all the RRHs transmit using TAS, the CDF of the SNR of the n-th RRH received at the user, denoted by Υ_n, can be approximated as*

$$F_{\Upsilon_n}(\Phi) \approx \sum_{i=0}^{K} \chi_i \left(\frac{N_0 \Phi}{P\mu}\right)^{n_T + i} \tag{15}$$

where $\chi_i = \sum_{t=1}^{n_T} \sum_{u=k+l=n_T+i} \binom{n_T}{t} \frac{2(-1)^{u+t} R^{vk} t^u}{v\Gamma(l+1)\Gamma(k+1)\left(k+\frac{2}{v}\right)}$ *and the infinite series is truncated to $K+1$ terms.*

Proof: Proof follows similar steps to the proof of Proposition 2 and thus has been omitted due to space limitation. ■

Similar to MRT scheme, the CDF in (15) is a polynomial function and has been limited to $K+1$ terms. As $K \to \infty$, the approximation converges to the exact CDF. Furthermore, this approximation in (15) is tight at high SNRs. Again, the approximate PDF of the SNR can be easily obtained by taking the derivative of (15) w.r.t. Φ. Furthermore, using (15), the approximate CDF of Υ can be derived and is given in following Proposition.

Proposition 6. *When N RRHs transmit using TAS, the CDF of the overall SNR received at the user, denoted by Υ, can be approximated as*

$$F_{\Upsilon}(\Phi) \approx \sum_{I_N} \frac{\kappa_{I_N}}{\Gamma(Nn_T + \xi_{I_N} + 1)} \left(\frac{N_0 \Phi}{P\mu}\right)^{Nn_T + \xi_{I_N}} \tag{16}$$

where $\sum_{I_N}$ is shorthand notation of $\sum_{i_1=0}^{K} \sum_{i_2=0}^{K} \cdots \sum_{i_N=0}^{K}$ and $\xi_{I_N} = \sum_{l=1}^{N} i_l$ and $\kappa_{I_N} = \prod_{l=1}^{N} \chi_{i_l} \Gamma(n_T + i_l + 1)$.

Proof: Proof follows similar steps to the proof of Proposition 3 and thus has been omitted due to space limitation. ■

Again, in this case, the CDF given in (16) is a polynomial function which is tight at high SNRs. The approximate PDF of Υ can be easily obtained by taking the derivative of (16) w.r.t. Φ. By substituting, $\Phi = 2^{\mathcal{R}} - 1$ in (16), the approximate outage probability at the user can be obtained when all the RRHs transmit using TAS.

Diversity Order: Again, in this case, (16) is in form of a polynomial function and at high SNRs it can be approximated by its lowest order term, which is obtained by taking $\xi_{I_N} = 0$ and (16) can be approximated as

$$F_{\Upsilon}(\Phi) \approx F_{\Upsilon}^{\infty}(\Phi) = \frac{(\chi_0 \Gamma(n_T + 1))^N}{\Gamma(Nn_T + 1)} \left(\frac{N_0 \Phi}{P\mu}\right)^{Nn_T}. \tag{17}$$

Using (17), it can be easily shown that the diversity gain achieved when N RRHs transmit using TAS is Nn_T.

IV. Performance Analysis

A. ST Scheme

In this scheme, the MBS or the RRH with the best channel is selected for transmission. Therefore, the outage event occurs when the channels of both the MBS and the best RRH are in outage.

When the MBS and N RRHs transmit using MRT, the outage probability can be given as

$$\mathcal{P}_{MRT,ST}(\Phi|N) = (F_{\gamma_0}(\Phi))(F_{\gamma_n}(\Phi))^N \tag{18}$$

where $F_{\gamma_0}(y) = \frac{1}{\Gamma(m_T)} \zeta\left(m_T, \frac{y}{\beta\mu}\right)$ and $\beta = \frac{P}{N_0(1+R^v)}$. As N is a RV, the overall outage probability can be given as

$$\mathcal{P}_{MRT,ST}(\Phi) = \sum_{N=0}^{\infty} (F_{\gamma_0}(\Phi))(F_{\gamma_n}(\Phi))^N \frac{\alpha^N}{\Gamma(N+1)} e^{-\alpha}. \tag{19}$$

Similarly, when the MBS and N RRHs transmit using TAS, the outage probability can be given as

$$\mathcal{P}_{TAS,ST}(\Phi|N) = (F_{\Upsilon_0}(\Phi))(F_{\Upsilon_n}(\Phi))^N \tag{20}$$

where $F_{\Upsilon_0}(\Phi) = 1 + \sum_{t=1}^{m_T} (-1)^t \binom{m_T}{t} e^{-\frac{\Phi}{\beta\mu} t}$. As N is a RV, the overall outage probability can be given as

$$\mathcal{P}_{TAS,ST}(\Phi) = \sum_{N=0}^{\infty} (F_{\Upsilon_0}(\Phi))(F_{\Upsilon_n}(\Phi))^N \frac{\alpha^N}{\Gamma(N+1)} e^{-\alpha}. \tag{21}$$

(19) and (21) give the average probability of outage of the system with Poisson distributed RRHs.

Diversity Order:

1) Number of RRHs is N: In the case of MRT at the MBS/RRHs, at high SNRs, (7) can be approximated as

$$F_{\gamma_n}^{\infty}(\Phi) \approx \Xi_0 \left(\frac{N_0 \Phi}{P\mu}\right)^{n_T} \tag{22}$$

and $F_{\gamma_0}(\Phi) = \frac{1}{\Gamma(m_T)} \zeta\left(m_T, \frac{\Phi N_0(1+R^v)}{\mu P}\right) \approx F_{\gamma_0}^{\infty}(\Phi) = \frac{\Phi^{m_T}(1+R^v)^{m_T}}{\Gamma(m_T) m_T \mu^{m_T}} \left(\frac{N_0}{P}\right)^{m_T}$. When the MBS and N RRHs transmit using MRT, at high SNRs the outage probability can be approximated as

$$\begin{aligned} \mathcal{P}_{MRT,ST}^{\infty}(\Phi|N) &\approx \left(F_{\gamma_0}^{\infty}(\Phi)\right)\left(F_{\gamma_n}^{\infty}(\Phi)\right)^N \\ &= \frac{\Phi^{m_T + Nn_T}(1+R^v)^{m_T} \Xi_0^N}{\mu^{m_T + Nn_T}\Gamma(m_T) m_T} \left(\frac{N_0}{P}\right)^{m_T + Nn_T}. \end{aligned} \tag{23}$$

Using (23), it can be easily shown that, when N RRHs are present and the MBS/RRHs employ MRT, the diversity gain achieved by ST scheme is $(Nn_T + m_T)$.

$$F_{\Upsilon_n}(\Phi) = 1 + \frac{2}{v}\sum_{t=1}^{n_T}(-1)^t\binom{n_T}{t}e^{-\frac{N_0 t}{P\mu}\Phi}\left(R^v\frac{N_0 t}{P\mu}\Phi\right)^{-\frac{2}{v}}\zeta\left(\frac{2}{v},\frac{N_0 t}{P\mu}\Phi R^v\right) \tag{14}$$

Similarly, when the MBS/RRHs employ TAS, at high SNRs the outage probability of the ST scheme can be approximated as

$$\begin{aligned}\mathcal{P}^{\infty}_{TAS,ST}(\Phi|N) &\approx \left(F^{\infty}_{\Upsilon_0}(\Phi)\right)\left(F^{\infty}_{\Upsilon_n}(\Phi)\right)^N \\ &= \frac{\Phi^{m_T+Nn_T}(1+R^v)^{m_T}\chi_0^N}{\mu^{m_T+Nn_T}}\left(\frac{N_0}{P}\right)^{m_T+Nn_T}.\end{aligned} \tag{24}$$

Using (24), it can be easily shown that, when N RRHs are present and the MBS/RRHs employ TAS, the diversity gain achieved by ST scheme is $(Nn_T + m_T)$.

2) Number of RRHs is random: At high SNRs, using (23), (19) can be approximated as

$$\begin{aligned}\mathcal{P}^{\infty}_{MRT,ST}(\Phi) &= \mathcal{G}_\gamma(0)\left(\frac{N_0}{P}\right)^{m_T}e^{-\alpha} \\ &+ \mathcal{G}_\gamma(1)\frac{\alpha e^{-\alpha}}{\Gamma(2)}\left(\frac{N_0}{P}\right)^{m_T+n_T} + \ldots \approx \mathcal{G}_\gamma(0)\left(\frac{N_0}{P}\right)^{m_T}e^{-\alpha}\end{aligned} \tag{25}$$

where $\mathcal{G}_\gamma(N) = \frac{\Phi^{m_T+Nn_T}(1+R^v)^{m_T}\Xi_0^N}{\mu^{m_T+Nn_T}\Gamma(m_T)m_T}$. Using (25), it can be easily shown that, when the number of RRHs is random and the MBS/RRHs employ MRT, the diversity gain achieved by ST scheme is m_T[8].

Similar derivations can be done for ST scheme, where the MBS/RRHs employ TAS, which will yields the expression of outage probability at high SNR, as $\mathcal{P}_{TAS,ST} \approx \mathcal{G}_\Upsilon(0)\left(\frac{N_0}{P}\right)^{m_T}e^{-\alpha}$ where, $\mathcal{G}_\Upsilon(N) = \frac{\Phi^{m_T+Nn_T}(1+R^v)^{m_T}\chi_0^N}{\mu^{m_T+Nn_T}}$. Again using this expression, it can be easily shown that, when the number of RRHs is random and the MBS/RRHs employ TAS, the diversity gain achieved by ST scheme is m_T.

This shows that the diversity order of the ST scheme is m_T which is the number of antennas on the MBS. This indicates that the diversity order can be increased by increasing m_T and vice versa. Furthermore, it can be deduced that the parameters of the RRHs do not affect the diversity order. For example, by varying the intensity λ_{RRH} or the number of antennas, n_T, the diversity order cannot be varied. However, (25) and the corresponding expression in the case of TAS involves the term $e^{-\alpha}$ where α depends on λ_{RRH}. This means that the intensity of the RRHs, λ_{RRH}, does impact the outage probability. For example, a larger λ_{RRH} implies a larger α and thus, a lower outage probability.

B. ARP Scheme

In this scheme, all the RRHs are selected for transmission. Therefore, the outage event will occur if the overall SNR from the MBS and the RRHs is in outage.

When the MBS and N RRHs transmit using MRT, the outage probability can be given as

$$\mathcal{P}_{MRT,ARP}(\Phi|N) = Pr\{\gamma + \gamma_0 < \Phi\} = Pr\{\gamma < \Phi - \gamma_0\}. \tag{26}$$

$\mathcal{P}_{MRT,ARP}(\Phi)$ can be expressed as

$$\mathcal{P}_{MRT,ARP}(\Phi|N) = \int_0^{\Phi} F_\gamma(\Phi - \gamma_0) f_{\gamma_0}(\gamma_0)\, d\gamma_0. \tag{27}$$

Substituting $F_\gamma(\cdot)$ and $f_{\gamma_0}(\cdot)$, applying the binomial theorem and solving the resulting integral, yields the outage probability expression given in (28), where $\mu_\beta = \frac{P\mu}{N_0(1+R^v)}$ and $\mu_\alpha = \frac{P}{N_0}\mu$. The overall outage probability can be given as

$$\mathcal{P}_{MRT,ARP}(\Phi) = \sum_{N=0}^{\infty}\mathcal{P}_{MRT,ARP}(\Phi|N)\frac{\alpha^N}{\Gamma(N+1)}e^{-\alpha}. \tag{29}$$

Similarly, when the MBS and N RRHs transmit using TAS, the outage probability can be given as

$$\mathcal{P}_{TAS,ARP}(\Phi|N) = \int_0^{\Phi} F_\Upsilon(\Phi - x) f_{\Upsilon_0}(x)\, dx. \tag{30}$$

Again, substituting $F_\Upsilon(\cdot)$ and $f_{\Upsilon_0}(\cdot)$, and solving the resulting integral using [19, eq. (3.381.1)] gives the outage probability expression in (31). The overall outage probability is given as

$$\mathcal{P}_{TAS,ARP}(\Phi) = \sum_{N=0}^{\infty}\mathcal{P}_{TAS,ARP}(\Phi|N)\frac{\alpha^N}{\Gamma(N+1)}e^{-\alpha}. \tag{32}$$

Diversity Order: For this scheme, the probability of outage can be upper bounded by the outage probability of the ST scheme. This implies that the diversity order achieved by ARP scheme is also $(Nn_T + m_T)$ when N RRHs serve the user along with the MBS, and is m_T when random number of RRHs serve the user.

C. MRP Scheme

In this scheme, only a subset of the available RRHs is employed to meet a specified data-rate requirement. Using this scheme, a minimal number of RRHs are used, which is beneficial as it yields a practical scheme with reduced complexity and ensuring desired system performance. The outage probability of this scheme will be same as that of the MRT scheme, because the outage event will only occur when the overall SNR from the MBS and all the RRHs is in outage. Therefore, in this section, we derive expression for the average number of RRHs that are required to meet a pre-defined data rate. This expression is beneficial as it gives information to the network operators about the minimal average number of RRHs that are needed to be activated to achieve a certain data-rate requirement.

We consider the same network of Fig. 1. However, for mathematical tractability, the MBS is not considered in this case and $N \geq 2$ [9]. Assuming that the SNR for each RRH is denoted as γ_i where $i \in [1, \ldots, N]$. All SNRs are ordered as $\gamma_{(N)} \geq \gamma_{(N-1)} \geq \ldots \geq \gamma_{(1)}$. The subset of RRHs that is minimally sufficient to meet a pre-defined data rate should correspond to the $\mathcal{S}$ largest SNRs. Without loss of generality,

[8]If the MBS does not participate in transmission, it can be shown, that the average diversity gain achieved is n_T.

[9]Note that, when $N = 1$, the single RRH must always transmit and it determines the outage performance.

$$\mathcal{P}_{MRT,ARP}\left(\Phi|N\right)=\sum_{I_N}\sum_{l=0}^{Nn_T+\xi_{I_N}}\binom{Nn_T+\xi_{I_N}}{l}\left(\frac{1}{\mu_\alpha}\right)^{Nn_T+\xi_{I_N}}\left(\frac{1}{\mu_\beta}\right)^{-l}\frac{\varkappa_{I_N}\left(-1\right)^l\Phi^{Nn_T+\xi_{I_N}-l}\zeta\left(m_T+l,\frac{\Phi}{\mu_\beta}\right)}{\Gamma\left(m_T\right)\Gamma\left(Nn_T+\xi_{I_N}+1\right)} \tag{28}$$

$$\mathcal{P}_{TAS,ARP}\left(\Phi|N\right)=\sum_{I_N}\sum_{t=1}^{m_T}\binom{m_T}{t}\left(\frac{1}{\mu_\alpha}\right)^{Nn_T+\xi_{I_N}}\frac{\kappa_{I_N}\left(-1\right)^t e^{-\frac{t}{\mu_\eta}\Phi}}{\Gamma\left(Nn_T+\xi_{I_N}+1\right)}\left(-\frac{t}{\mu_\eta}\right)^{-Nn_T-\xi_{I_N}}\zeta\left(Nn_T+\xi_{I_N}+1,-\Phi\frac{t}{\mu_\eta}\right) \tag{31}$$

denote $\theta_{N,n}=\sum_{i=1}^{n}\gamma_{(N-i+1)}$, the averaged minimal number of RRHs to meet a pre-defined data rate is given as [16]

$$\bar{n}_N=\sum_{n=1}^{N}n\cdot\Pr(\mathcal{S}=n|N) \tag{33}$$

where

$$\Pr(\mathcal{S}=n|N)=\begin{cases}1-F_{\theta_{N,1}}\left(\epsilon\right) & ;\ n=1\\ F_{\theta_{N,n-1}}\left(\epsilon\right)-F_{\theta_{N,n}}\left(\epsilon\right) & ;\ 2\le n\le N-1\\ F_{\theta_{N,N-1}}\left(\epsilon\right) & ;\ n=N.\end{cases} \tag{34}$$

From (33) and (34) it is clear that in order to obtain the average minimal number of RRHs required, the CDF of $\theta_{N,n}$ needs to be derived for the MRT and TAS based systems. In the following we derive the CDF of $\theta_{N,n}$ for both the MRT and TAS based systems. The CDF of $\theta_{N,n}$ is also beneficial in obtaining the outage performance when n RRHs with the best channels are selected for transmission.

1) MRT With Fixed Transmit Power:

In case of the MRT scheme, we denote the SNR for each RRH as $\bar{\gamma}_n=U_n$, the ordered SNRs are expressed as $\bar{\gamma}_{(N)}\ge\bar{\gamma}_{(N-1)}\ge\ldots\ge\bar{\gamma}_{(1)}$ and $\bar{\theta}_{N,n}=\sum_{i=1}^{n}\bar{\gamma}_{(N-i+1)}$.

Proposition 7. *The approximate CDF of the sum of n largest SNRs in the case of MRT, denoted by $\bar{\theta}_{N,n}$, is given in* (35), *where $\sum_J$ is shorthand notation of $\sum_{j_1=0}^{K}\cdots\sum_{j_{N-n-1}=0}^{K}$, $\varkappa_j=\prod_{l=1}^{N-n-1}\Xi_{j_l}$, $\varkappa_k=(n_T+k)\,\Xi_k$ and $\xi_j=\sum_{l=1}^{N-n-1}j_l$, $\sum_M$ is shorthand notation of $\sum_{m_1=0}^{n_T+i_1-1}\cdots\sum_{m_n=0}^{n_T+i_n-1}$, $\xi_m=\sum_{l=1}^{n}m_l$, $\varkappa_{m,i}=\frac{\prod_{l=1}^{n}\Gamma(n_T+i_l)}{\prod_{l=1}^{n}\Gamma(m_l+1)}$, $\sum_I$ is shorthand notation of $\sum_{i_1=0}^{K}\cdots\sum_{t_n=1}^{n_T}$ and $\varkappa_i=\prod_{l=1}^{n}(n_T+i_l)\,\Xi_{i_l}$ and $\xi_i=\sum_{l=1}^{n}i_l$.*

Proof: See Appendix E. ∎

Proposition 7 gives a tight approximation of the CDF of the sum of n largest SNRs, in the high SNR regime. By substituting (35) into (34) and substituting $\epsilon=\frac{N_0\Phi}{P}$ the PDF of $\mathcal{S}$ can be obtained. Substituting the resulting PDF of $\mathcal{S}$ into (33) gives the expression for averaged minimal number of RRHs to meet a pre-defined data rate in case of MRT.

2) TAS With Fixed Transmit Power:

In case of the TAS scheme, we denote the SNR for each RRH as $\bar{\Upsilon}_i=H_n$, the ordered SNRs are expressed as $\bar{\Upsilon}_{(N)}\ge\bar{\Upsilon}_{(N-1)}\ge\ldots\ge\bar{\Upsilon}_{(1)}$ and $\bar{\Theta}_{N,n}=\sum_{i=1}^{n}\bar{\Upsilon}_{(N-i+1)}$.

Proposition 8. *The approximate CDF of the sum of n largest SNRs in the case of TAS, denoted by $\bar{\Theta}_{N,n}$, is given in* (36), *where $\sum_J$ is shorthand notation of $\sum_{j_1=0}^{K}\cdots\sum_{j_{N-n-1}=0}^{K}$, $\kappa_j=\prod_{l=1}^{N-n-1}\chi_{j_l}$, $\kappa_k=(n_T+k)\,\chi_k$, $\xi_j=\sum_{l=1}^{N-n-1}j_l$, $\sum_J$ is shorthand notation of $\sum_{j_1=0}^{K}\cdots\sum_{j_{N-n-1}=0}^{K}$, $\kappa_j=\prod_{l=1}^{N-n-1}\chi_{j_l}$, $\kappa_k=(n_T+k)\,\chi_k$ and $\xi_j=\sum_{l=1}^{N-n-1}j_l$.*

Proof: Proof follows similar steps to the proof of Proposition 7 and thus has been omitted due to space limitation. ∎

Similar to the case of MRT, (36) gives a tight approximation of the CDF of the sum of n largest SNRs in the high SNR regime and by substituting (36) into (34) and substituting $\epsilon=\frac{N_0\Phi}{P}$, the PDF of $\mathcal{S}$ in case of TAS is obtained. Substituting the resulting PDF of $\mathcal{S}$ into (33) gives the expression for averaged minimal number of RRHs to meet a pre-defined data rate for TAS based system.

So far we have considered the scenario in which each RRH transmits with power P. Now we consider the special case in which the RRHs are able to adapt their transmit power. There exist multiple schemes for power adaptation, however, in this work, for a tractable analysis, we consider a scheme in which the power is adjusted such that the pathloss is compensated. Therefore, in this case, the transmit power of the n-th RRH will be $P_n=(1+d_n^v)$. Note that, the transmit power is dependent on the distance of the RRH from the user and, therefore, it is different for different RRHs. Moreover, in this case, the SNR at the user will only depend on the channel fading gain. For this power allocation scheme, the outage performance of ST and ARP schemes has been extensively studied in literature. However, to the best of the authors knowledge, the CDF of the SNR for the MRP scheme has not been reported before.

3) MRT With Adaptive Transmit Power:

If each RRH possesses the ability to adapt its power, then the pathloss can be compensated by varying the transmit power inversely to the pathloss. In this scenario, the received SNR only depends on the channel fading gain. In case of the MRT scheme with adaptive power, we denote the SNR for the n-th RRH as $\hat{\gamma}_n=\frac{\|\mathbf{g}_n\|^2}{N_0}$, the ordered SNRs are expressed as $\hat{\gamma}_{(N)}\ge\hat{\gamma}_{(N-1)}\ge\ldots\ge\hat{\gamma}_{(1)}$ and $\hat{\theta}_{N,n}=\sum_{i=1}^{n}\hat{\gamma}_{(N-i+1)}$.

Proposition 9. *The exact CDF of the sum of n largest SNRs in the case of MRT with adaptive transmit power, denoted by $\hat{\theta}_{N,n}$, is given in* (37), *where $m=1+nn_T-\xi_m+(\xi_m+n_T+\xi_j)$, $p=1+nn_T-\xi_m+(\xi_m+n_T+\xi_j)$, $\bar{b}_N=$*

$$\left\{0,\underbrace{1,1,...,1}_{nn_T-\xi_m},\underbrace{1+\frac{(j+1)}{n},1+\frac{(j+1)}{n},...,1+\frac{(j+1)}{n}}_{(\xi_m+n_T+\xi_j)}\right\},$$

$a_N=\{\}$, $b_D=\{\}$, $\bar{a}_D=\bar{b}_N+1=$

$$F_{\bar{\theta}_{N,n}}(y) \approx \sum_{I}\sum_{M}\sum_{J}\sum_{k=0}^{K} \frac{N!\varkappa_i\varkappa_{m,i}\varkappa_k\varkappa_j}{(N-n-1)!(n)!n^{((N-n)n_T+\xi_m+\xi_j+k)}} \frac{\Gamma\left((N-n)\,n_T+\xi_m+\xi_j+k\right)}{\Gamma\left(Nn_T+\xi_j+\xi_i+k+1\right)} \left(\frac{y}{\mu}\right)^{Nn_T+\xi_i+\xi_j+k} \tag{35}$$

$$F_{\bar{\Theta}_{N,n}}(y) = \sum_{I}\sum_{M}\sum_{J}\sum_{k=0}^{K} \frac{N!\kappa_i\kappa_{m,i}\kappa_k\kappa_j}{(N-n-1)!(n)!n^{((N-n)n_T+\xi_m+\xi_j+k)}} \frac{\Gamma\left((N-n)\,n_T+\xi_m+\xi_j+k\right)}{\Gamma\left(Nn_T+\xi_j+\xi_i+k+1\right)} \left(\frac{y}{\mu}\right)^{Nn_T+\xi_i+\xi_j+k} \tag{36}$$

$$F_{\hat{\theta}_{N,n}}(y) = \sum_{M}\sum_{j=0}^{N-n-1}\sum_{I}\kappa_I \binom{N-n-1}{j} \frac{(-1)^j\, N!n^{-(\xi_m+n_T+\xi_j)}\Gamma\left(\xi_m+n_T+\xi_j\right)\kappa_m}{\Gamma\left(n_T\right)(N-n-1)!(n)!} G_{p,q}^{m,n}\left(e^{-y},\mu\,\middle|\, \begin{matrix} a_N, & \bar{a}_D \\ \bar{b}_N, & b_D \end{matrix}\right) \tag{37}$$

$$\left\{1,\underbrace{2,2,...,2}_{nn_T-\xi_m},\underbrace{2+\frac{(j+1)}{n},2+\frac{(j+1)}{n},...,2+\frac{(j+1)}{n}}_{(\xi_m+n_T+\xi_j)}\right\},$$

$\sum_I = \sum_{i_1=0}^{n_T-1}\cdots\sum_{i_j=0}^{n_T-1}$, $\kappa_I = \prod_{o=1}^{j}\frac{1}{\Gamma(i_o+1)}$, $\xi_j = \sum_{o=1}^{j} i_o$, $\sum_M = \sum_{m_1=0}^{n_T-1}\cdots\sum_{m_n=0}^{n_T-1}$, $\xi_m = \sum_{i=1}^{n} m_i$ *and* $\kappa_m = \frac{1}{\prod_{i=1}^{n}\Gamma(m_i+1)}$.

Proof: See Appendix F. ■

Unlike the case of MRT with fixed transmit power, (37) is an exact and accurate expression of the CDF of the sum of n largest SNRs from the RRHs. By substituting (37) into (34), one can obtain the PDF of $\mathcal{S}$ and $\epsilon = N_0\Phi$ in case of MRT. Substituting the resulting PDF of $\mathcal{S}$ into (33) gives the expression for averaged minimal number of RRHs to meet a pre-defined data rate for MRT based system with adaptive power.

4) TAS With Adaptive Transmit Power:
Similarly, in case of the TAS scheme with adaptive power, we denote the SNR for the n-th RRH as $\hat{\Upsilon}_i = G_n = \max_t |g_{i,t}|^2$, the ordered SNRs are expressed as $\hat{\Upsilon}_{(N)} \geq \hat{\Upsilon}_{(N-1)} \geq \ldots \geq \hat{\Upsilon}_{(1)}$ and $\hat{\Theta}_{N,n} = \sum_{i=1}^{n}\hat{\Upsilon}_{(N-i+1)}$.

Proposition 10. *The exact CDF of the sum of* n *largest SNRs in the case of TAS with adaptive transmit power, denoted by* $\hat{\Theta}_{N,n}$, *is given in* (38), *where* $m = n+2$, $p = n+2$, $\bar{b}_N = \left\{0,\frac{m}{n}+\frac{\xi_l}{n}+\frac{\xi_t}{n},\underbrace{t_1,t_2,...,t_n}_{n}\right\}$, $a_N = \{\}$, $b_D = \{\}$, $\bar{a}_D = \left\{1,\frac{m}{n}+\frac{\xi_l}{n}+\frac{\xi_t}{n}+1,\underbrace{t_1+1,t_2+1,...,t_n+1}_{n}\right\} = \bar{b}_N+1$, $\sum_T = \sum_{t_1=1}^{n_T}\cdots\sum_{t_n=1}^{n_T}$, $\kappa_t = \prod_{i=1}^{n}(-1)^{t_i+1}\binom{n_T}{t_i}t_i$ *and* $\xi_t = \sum_{i=1}^{n} t_i$, $\sum_L = \sum_{l=0}^{n_T}\cdots\sum_{l_{N-n-1}=0}^{n_T}$, $\kappa_l = \prod_{i=1}^{N-n-1}(-1)^{l_i}\binom{n_T}{l_i}$, $\xi_l = \sum_{i=1}^{N-n-1} l_i$ *and* $\kappa_m = (-1)^{m+1}\binom{n_T}{m}m$.

Proof: Proof follows similar steps to the proof of Proposition 9 and thus has been omitted due to space limitation. ■

Similarly, for the case of TAS with adaptive transmit power, (38) is an exact and accurate expression of the CDF of the sum of n largest SNRs from the RRHs and by substituting (38) into (34) one can obtain the PDF of $\mathcal{S}$ and $\epsilon = N_0\Phi$ in case of TAS. Substituting the resulting PDF of $\mathcal{S}$ into (33) gives the expression for averaged minimal number of RRHs to meet a pre-defined data rate for TAS based system with adaptive power.

V. Numerical Results

In this section, numerical simulation results are shown to corroborate the derived analytical results. In the simulations, we assume a macro-cell with radius $R = 1000$m, average channel power $\mu = 1$ and $N_0 = 10^{-6}$. The parameters are fixed unless stated. The intensity can be expressed as $\lambda_{RRH} = \frac{\Lambda}{\pi R^2}$, where Λ is any integer, and it implies that the average number of RRHs in a region of πR^2 is Λ. The performance of two practical power allocation schemes is examined; 1) there is a maximum power constraint on each MBS and RRH in the network and 2) there is a total power constraint on the MBS and RRHs in the network. In order to analyse the best performance offered by the first scheme, the MBS and RRHs are assumed to transmit with same maximum power P. In case of the second scheme, for demonstration purposes, the total power, P_T, is equally distributed among the MBS and the RRHs[10]. Furthermore, in order to get insights on whether collocated antennas are better or distributed antennas, N and n_T are chosen such that the total number of antennas on the RRHs is same.

In all the figures (except Fig. 8), the blue dashed-dot lines indicate the results obtained via Monte-Carlo simulations and the remaining lines/curves are plotted using the expressions derived in this paper and depict the analytical results. Specifically, the black dashed lines denote the analytical results for the ARP scheme, the black solid lines indicate the analytical results for the ST scheme and the maroon dotted lines denote the asymptotic results for each scheme. In Fig. 8, the simulation results are indicated by green squares and the lines/curves are plotted using the expressions derived in this paper.

First we consider the scenario in which there is an individual power constraint on each MBS and RRH in the network. In this case, the MBS and RRHs are assumed to transmit with same maximum power P. Fig. 2 and Fig. 3 show the probability of outage of both the ST and ARP schemes with varying the transmit power, P, when the MBS/RRHs employ MRT and TAS, respectively. The simulation results are shown when N

[10]Note that other power allocation schemes can also be considered. However, in this work, for corroboration of our results, we presented the performance of these sub-optimal power allocation policies. The derivation of the optimal power allocation policy will be considered in a future work.

$$F_{\hat{\Theta}_{N,n}}(y) = \frac{N!}{(N-n-1)!(n)!}\sum_{T}\sum_{L}\sum_{m=1}^{n_T}\frac{\kappa_t}{n}\kappa_l\kappa_m G^{m,n}_{p,q}\left(e^{-y},\mu \,\middle|\, \begin{matrix} a_N, & \bar{a}_D \\ \bar{b}_N, & b_D \end{matrix}\right) \tag{38}$$

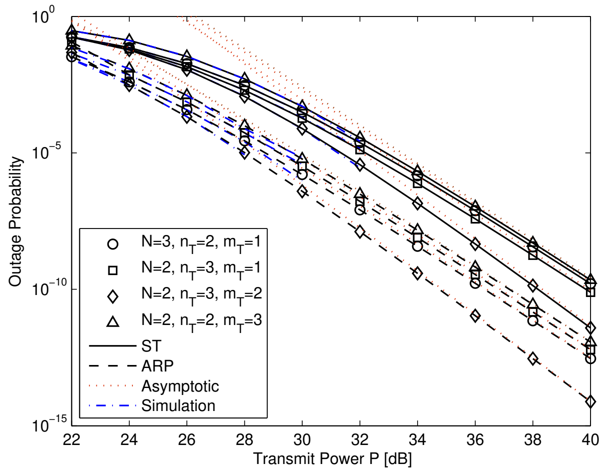

Fig. 2. Probability of outage for ST and ARP schemes with varying transmit power for a fixed number of RRHs with MRT where $v = 3$ and $\mathcal{R} = 1$.

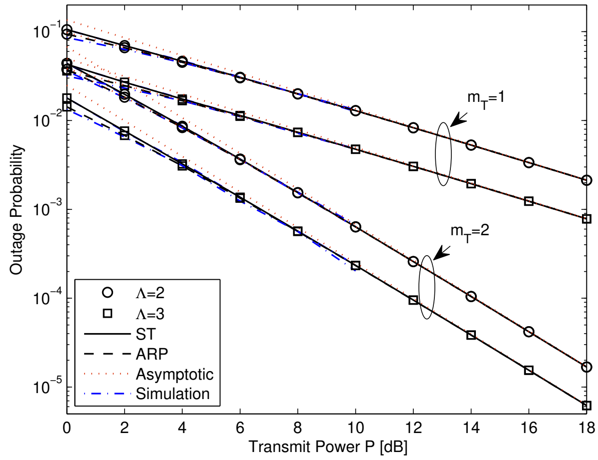

Fig. 4. Probability of outage for ST and ARP schemes with varying transmit power for N RRHs with MRT where N is a Poisson RV, $v = 2$ and $\mathcal{R} = 1$.

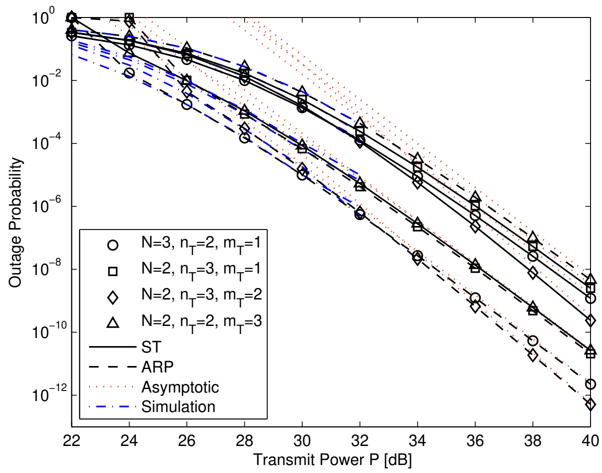

Fig. 3. Probability of outage for ST and ARP schemes with varying transmit power for a fixed number of RRHs with TAS where $v = 3$ and $\mathcal{R} = 1$.

RRHs are serving the user in the region[11]. The performances of ST and ARP schemes are compared for different antenna allocations. It can be seen that the outage probability of both schemes decrease with increasing transmit power and the ARP scheme has lower outage probability compared to the ST scheme. It is worth noting that the ARP scheme gives better performance but at a cost of higher system complexity. In addition, the outage probability decreases as the number of antennas increases, i.e. more antennas or more RRHs provide more diversity and array gain and thus, result in a lower outage probability. For the ARP scheme, with a fixed number of total transmitting antennas (i.e. $Nn_T + m_T = 7$), the system with $(N, n_T, m_T) = (2, 3, 1)$ gives lower outage probability compared to the system with $(N, n_T, m_T) = (2, 2, 3)$, implying that it is better to distribute antennas on the RRHs rather than collocating them on the MBS. Distributing antennas on the RRHs diversifies the path loss and therefore gives better performance. The system with $(N, n_T, m_T) = (3, 2, 1)$ gives lower outage probability compared to the system with $(N, n_T, m_T) = (2, 3, 1)$, since increasing N implies an increase in the overall transmission power as well as it diversifies the path loss resulting in improved performance. However, for the ST scheme with MRT, comparing the system having $(N, n_T, m_T) = (3, 2, 1)$ with the system having $(N, n_T, m_T) = (2, 3, 1)$, the system with larger n_T gives lower outage probability which suggests that for ST with MRT, fewer RRHs with more antennas gives better performance due to a higher coding gain. Whereas, for the ST scheme with TAS, diversifying the pathloss, i.e. employing more RRHs with fewer antennas gives larger coding gain and thus, improved performance. It can be observed from Fig. 2 and Fig. 3 that the analytical results for ST match the simulation results exactly. Whereas the analytical results for the ARP scheme and the asymptotic results match well with the simulation results at high SNRs[12].

The probability of outage with varying transmit power for the system with random N and $\mathcal{R} = 1$ BPCU is shown in Fig. 4 for MBS/RRHs with MRT. Again in this case, the MBS and RRHs are assumed to transmit with maximum transmit power P. It can be observed that, at low SNRs the ARP scheme

[11] Note that, in the network, the number of RRHs, N, is random, However, here for the purpose of analysis, we show the performance of the network when N RRHs are transmitting to the user. Later, we will show the overall average performance of the network when the number of RRHs is random.

[12] This match is good at high SNRs, because as was discussed in footnote 3, the approximation used in the derivations is accurate in the high SNR regime. At low SNRs, this approximation is not accurate and therefore, there is mismatch between analytical and simulation results.

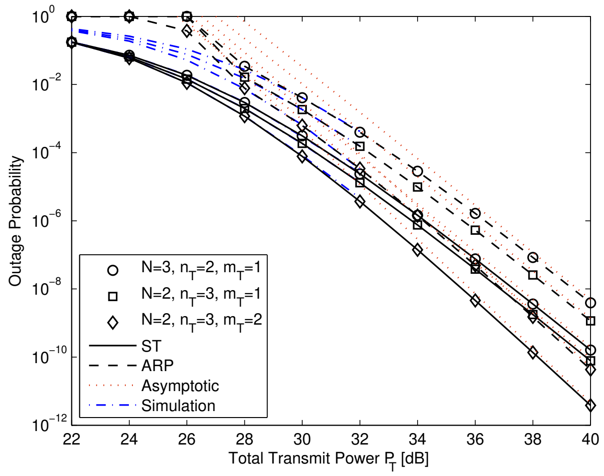

Fig. 5. Probability of outage for ST and ARP schemes with varying total transmit power for a fixed number of RRHs with MRT where $\upsilon = 3$ and $\mathcal{R} = 1$.

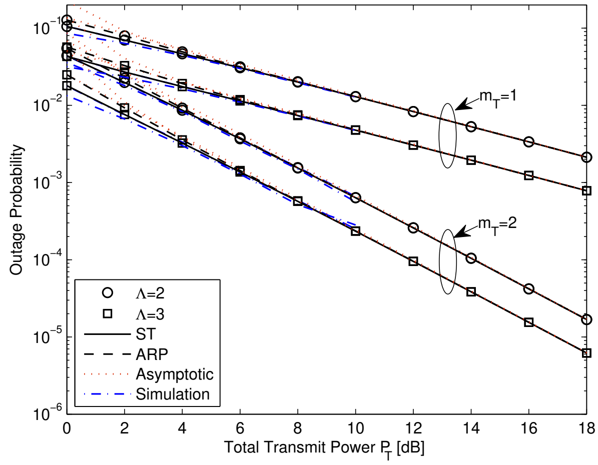

Fig. 7. Probability of outage for ST and ARP schemes with varying total transmit power for N RRHs with MRT where N is Poisson RV, $\upsilon = 2$ and $\mathcal{R} = 1$.

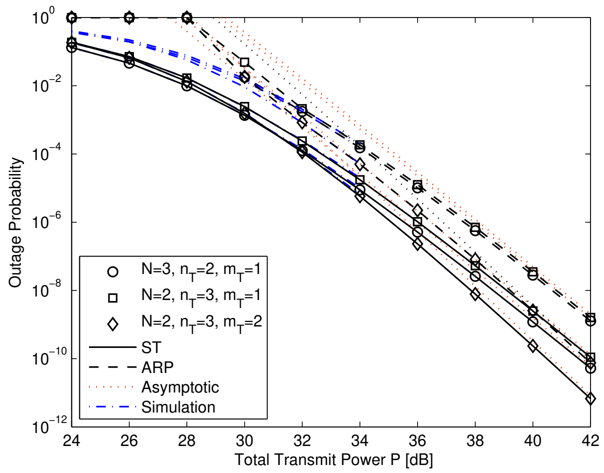

Fig. 6. Probability of outage for ST and ARP schemes with varying total transmit power for a fixed number of RRHs with TAS where $\upsilon = 3$ and $\mathcal{R} = 1$.

performs better compared to the ST scheme. However, at high SNRs both schemes give similar performance, implying that in actual networks it might be better to adopt the ST scheme due to its lower complexity. In addition, Fig. 4 also shows that when n_T is fixed, a larger density λ_{RRH} can provide a better outage performance and increasing the number of antennas at the MBS, m_T, gives significant performance gain and also a higher diversity gain as was discussed previously. It can be observed from Fig. 4 that the analytical results for ST match the simulation results exactly. Whereas the analytical results for the ARP scheme and the asymptotic results match well with the simulation results at high SNRs.

Next, we consider the scenario in which there is a total power constraint on the network and the total power, P_T, is equally distributed among the MBS and the RRHs. In case of the ST scheme, as only the MBS/RRH with the best channel is selected for transmission, all transmission power will be allocated to it. Whereas, for the ARP scheme, when N RRHs aid the MBS in transmission, the power allocated to each MBS/RRH is $P = \frac{P_T}{N+1}$. Fig. 5 and Fig. 6 show the probability of outage of both the ST and ARP schemes with varying the total transmit power, P_T, when the MBS/RRHs employ MRT and TAS, respectively. Again, the performances of ST and ARP schemes are compared for different antenna allocations. The outage probability decreases with increasing the total transmit power or the total number of antennas. However, due to a total power constraint and equal power allocation policy, the ST scheme performs better compared to the ARP scheme. This was not the case in Fig. 2 and Fig. 3, where ARP outperformed ST scheme because each additional RRH increased the overall system power. By employing other power allocation schemes, the performance of ARP can be improved. However, deriving the optimal power allocation policy for ARP scheme will be considered in a future work. Furthermore, for the ARP scheme with MRT, comparing the system with $(N, n_T, m_T) = (3, 2, 1)$ with the system with $(N, n_T, m_T) = (2, 3, 1)$, the system with higher N gives higher outage probability, as increasing N results in a lower transmit power at the MBS/RRHs and a lower coding gain. However, for the ARP scheme with TAS, increasing N gives higher coding gain that is sufficient to overcome the lower transmit power and thus, give better outage performance. Moreover, for the ST scheme, the outage performance is the same as that in Fig. 2 and Fig. 3. It can be observed from Fig. 5 and Fig. 6 that the analytical results for ST match the simulation results exactly. Whereas the analytical results for the ARP scheme and the asymptotic results match well with the simulation results at high SNRs.

The probability of outage with varying total transmit power, P_T, for the system with random N and $\mathcal{R} = 1$ BPCU is shown in Fig. 7 for MBS/RRHs with MRT. Similar to Fig. 4, it can be observed in Fig. 7 that at high SNRs both schemes give similar performance. Furthermore, the performance can be improved by increasing the density of the RRHs or increasing the

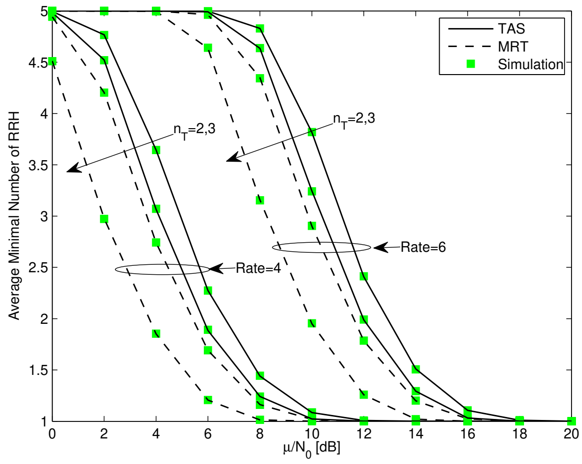

Fig. 8. The averaged minimal number $\bar{n}_N$ to meet a pre-defined data-rate with adaptive transmit power at the RRHs where $N = 5$.

number of antennas at the MBS/RRHs. Similarly, the diversity gain of the system can be increased only by increasing the number of antennas at the MBS. It can be observed from Fig. 7 that the analytical results for ST match the simulation results exactly. Whereas the analytical results for the ARP scheme and the asymptotic results match well with the simulation results at high SNRs.

Next we consider the scenario, where the RRHs transmit with adaptive transmit power and compensates the path loss. In Fig. 8, the averaged minimal numbers of RRHs, with adaptive power policy, to meet different pre-defined data rates are plotted against mean channel power μ, where $N = 5$. It can be observed from Fig. 8, for a fixed data rate, the minimal number of RRHs required decreases with increase in SNR and vice versa. When the SNR is fixed, more RRHs are needed to meet a higher data rate. Furthermore, using MRT fewer RRHs need to be employed to achieve a certain target rate compared to TAS. This happens because MRT offers higher array gain compared to TAS. It can be observed that the analytical results match the simulation results quite well.

VI. Conclusion

In this work, the downlink performance of CRAN with randomly distributed multiple antenna RRHs was investigated. The MBS and the RRHs were assumed to employ MRT or TAS for transmission. For this system, the performance of three downlink protocols, namely, ST, ARP and MRP were analysed and the analytical expressions for the outage probability were obtained. Furthermore, for the MRP scheme, the minimal number of RRHs required to meet a pre-defined data rate was also studied. The derived analytical expressions were validated through numerical simulations. Our results showed, that in the case of power constraint per MBS/RRHs, the ARP scheme outperformed the ST scheme, whereas in case of the sum power constraint, the ST scheme outperformed the ARP scheme. In addition, at high SNRs, the diversity could only be improved by increasing the number of antennas employed on the MBS. On increasing the density of the RRHs, the outage probability was reduced, but the diversity order was not impacted.

VII. Appendix

A. Statistics of SNR from n-th RRH

The CDF of γ_n can be obtained as

$$F_{\gamma_n}(\Phi) = \Pr\left\{\frac{P}{N_0}\left(\frac{1}{1+d_n^v}\right)\|\mathbf{g}_n\|^2 < \Phi\right\} = \Pr\left\{\frac{P}{N_0}U_n < \Phi\right\} \quad (39)$$

where $U_n = \left(\frac{1}{1+d_n^v}\right)\|\mathbf{g}_n\|^2$. In order to obtain the CDF of U_n, first we need to find the statistics of $\|\mathbf{g}_n\|^2 = \sum_{t=1}^{n_T}|g_{n,t}|^2$. $g_{n,t}$ is $\mathcal{CN}(0,\mu)$, therefore, $|g_{n,t}|^2$ is an exponential RV with mean μ and $\|\mathbf{g}_n\|^2 = \sum_{t=1}^{n_T}|g_{n,t}|^2$, is thus, Erlang distributed. The PDF of $\|\mathbf{g}_n\|^2$ is given as $f_{\|g_n\|^2}(y) = \frac{1}{\mu^{n_T}\Gamma(n_T)}y^{n_T-1}e^{-\frac{y}{\mu}}$ and the CDF of $\|\mathbf{g}_n\|^2$ is $F_{\|g_n\|^2}(y) = \frac{1}{\Gamma(n_T)}\zeta\left(n_T,\frac{y}{\mu}\right) = \left(1-\sum_{i=0}^{n_T-1}\frac{1}{\Gamma(i+1)}e^{-\frac{y}{\mu}}\left(\frac{y}{\mu}\right)^i\right)$ where $\zeta(\cdot,\cdot)$ denotes the lower incomplete Gamma function [19, Eq. (8.350.1)] and $\Gamma(\cdot)$ is the Gamma function [19, Eq. (8.310.1)]. The CDF of $U_n = \left(\frac{1}{1+d_n^v}\right)\|\mathbf{g}_n\|^2$ can be derived using the statistics of $\|\mathbf{g}_n\|^2$ as

$$F_{U_n}(y) = \int_0^R F_{\|\mathbf{g}_n\|^2}\left(y\left(1+x^v\right)\right)f_{d_n}(x)\,dx. \quad (40)$$

Substituting the CDF of $\|g_n\|^2$ and the PDF in (1) into (40) and doing some simplification yields

$$F_{U_n}(y) = 1 - \sum_{i=0}^{n_T-1}\frac{e^{-\frac{y}{\mu}}}{\Gamma(i+1)}\left(\frac{y}{\mu}\right)^i\frac{2}{R^2}\int_0^R xe^{-\frac{y}{\mu}x^v}\left(1+x^v\right)^i dx. \quad (41)$$

Using the binomial theorem, doing some simplification yields and making change of variable $z = x^v$ yields

$$F_{U_n}(y) = 1 - \sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\binom{i}{j}\frac{e^{-\frac{y}{\mu}}}{\Gamma(i+1)}\left(\frac{y}{\mu}\right)^i\frac{2}{vR^2}\int_0^{R^v} z^{j-\left(1-\frac{2}{v}\right)}e^{-\frac{y}{\mu}z}dz \quad (42)$$

which can be expressed in terms of lower incomplete gamma function as

$$F_{U_n}(y) = 1 - \sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\binom{i}{j}\frac{2e^{-\frac{y}{\mu}}}{\Gamma(i+1)vR^2}\left(\frac{y}{\mu}\right)^{i-\left(j+\frac{2}{v}\right)}\zeta\left(j+\frac{2}{v},\frac{y}{\mu}R^v\right). \quad (43)$$

Finally the CDF of γ_n can be obtained by substituting $y = \frac{N_0}{P}\Phi$ in (43) to yield (6).

B. Approximation of Statistics of SNR from n-th RRH

The CDF of U_n is given as $F_{U_n}(y) = 1 - \sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\binom{i}{j}\frac{2}{\Gamma(i+1)vR^2}e^{-\frac{y}{\mu}}\left(\frac{y}{\mu}\right)^{i-\left(j+\frac{2}{v}\right)}\zeta\left(j+\frac{2}{v},\frac{y}{\mu}R^v\right)$. Replacing the lower incomplete Gamma function and the exponential function with its series representation in [19, eq. (8.354.1) and eq. (1.211.1)] yields

$$\begin{aligned} F_{U_n}(y) = & 1 - \sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\binom{i}{j}\frac{2}{\Gamma(i+1)v}\left(\sum_{l=0}^{\infty}\frac{(-1)^l}{\Gamma(l+1)}\left(\frac{y}{\mu}\right)^l\right)\times \\ & \left(\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)\left(k+j+\frac{2}{v}\right)}\left(\frac{y}{\mu}R^v\right)^{k+i}\right). \end{aligned} \quad (44)$$

After rearranging the terms and doing some simplification, (44) can be expressed as

$$F_{U_n}(y) = 1 + \sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\frac{2\binom{i}{j}(-1)^{l+k+1}\left(R^v\right)^{k+i}\left(\frac{y}{\mu}\right)^{k+i+l}}{v\left(k+j+\frac{2}{v}\right)\Gamma(i+1)\Gamma(l+1)\Gamma(k+1)}. \quad (45)$$

In the above series representation, the terms with power of y less than n_T are zero, therefore $F_{U_n}(y)$ can be expressed as

$$F_{U_n}(y)=\sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\sum_{u=l+k\geq n_T-i}\frac{2v^{-1}\binom{i}{j}(-1)^{u+1}(R^v)^{k+i}\left(\frac{y}{\mu}\right)^{u+i}}{\left(k+j+\frac{2}{v}\right)\Gamma(i+1)\Gamma(l+1)\Gamma(k+1)} \tag{46}$$

which can be compactly expressed as

$$F_{U_n}(y)=\Xi_0\left(\frac{y}{\mu}\right)^{n_T}+\Xi_1\left(\frac{y}{\mu}\right)^{n_T+1}+\ldots+\Xi_K\left(\frac{y}{\mu}\right)^{n_T+K}\ldots \tag{47}$$

where $\Xi_p=\sum_{i=0}^{n_T-1}\sum_{j=0}^{i}\sum_{u=l+k=n_T+p-i}\frac{2v^{-1}\binom{i}{j}(-1)^{u+1}(R^v)^{k+i}}{\left(k+j+\frac{2}{v}\right)\Gamma(i+1)\Gamma(l+1)\Gamma(k+1)}$. Limiting to $K+1$ terms, and approximation of $F_{U_n}(y)$ is obtained as

$$F_{U_n}(y)\approx\sum_{p=0}^{K}\Xi_p\left(\frac{y}{\mu}\right)^{n_T+p}. \tag{48}$$

Again the approximate CDF of γ_n can be obtained by substituting $y=\frac{N_0}{P}\Phi$ in (48) to yield (7).

C. Statistics of overall SNR when all RRHs transmit

We need to find statistics of $\gamma=\sum_{n=1}^{N}\gamma_n=\frac{P}{N_0}\sum_{n=1}^{N}U_n$. Unfortunately, it is not trivial to obtain the statistics of γ. However, the approximate statistics of γ can be derived using the approximation of the CDF of γ_n. First step is to obtain the statistics of $T=\sum_{n=1}^{N}U_n$. Using the MGF approach to obtain the distribution of T. The idea is to obtain the CDF of T can be obtained by taking the inverse Laplace transform of the MGF of T. The MGF of U_n can be obtained as

$$\mathcal{M}_{U_n}(s)=s\int_0^{\infty}e^{-sx}F_{U_n}(x)\,dx. \tag{49}$$

Substituting the CDF from (48) into (49) and solving the resulting integration yields the MGF as

$$\mathcal{M}_{U_n}(s)=\sum_{i=0}^{K}\Xi_i\left(\frac{1}{\mu}\right)^{n_T+i}\frac{\Gamma(n_T+i+1)}{s^{n_T+i}}. \tag{50}$$

As the channel is assumed to be i.i.d., the MGF of $T=\sum_{n=1}^{N}U_n$ can be obtained as

$$\mathcal{M}_T(s)=\left(\sum_{i=0}^{K}\Xi_i\left(\frac{1}{\mu}\right)^{n_T+i}\frac{\Gamma(n_T+i+1)}{s^{n_T+i}}\right)^N. \tag{51}$$

Representing the product of sum in terms of sum of products yields

$$\mathcal{M}_T(s)=\sum_{i_1=0}^{K}\sum_{i_2=0}^{K}\ldots\sum_{i_N=0}^{K}\prod_{l=1}^{N}\Xi_{i_l}\left(\frac{1}{\mu}\right)^{n_T+i_l}\frac{\Gamma(n_T+i_l+1)}{s^{n_T+i_l}} \tag{52}$$

which can be compactly expressed as

$$\mathcal{M}_T(s)=\sum_{I_N}\varkappa_{I_N}\left(\frac{1}{\mu}\right)^{Nn_T+\xi_{I_N}}s^{-Nn_T-\xi_{I_N}} \tag{53}$$

where $\sum_{I_N}$ is shorthand notation of $\sum_{i_1=0}^{K}\sum_{i_2=0}^{K}\cdots\sum_{i_N=0}^{K}$ and $\xi_{I_N}=\sum_{l=1}^{N}i_l$ and $\varkappa_{I_N}=\prod_{l=1}^{N}\Xi_{i_l}\Gamma(n_T+i_l+1)$. Finally, the CDF of T can be obtained by taking inverse Laplace Transform of $\frac{\mathcal{M}_T(s)}{s}$ to yield

$$F_T(y)=\sum_{I_N}\frac{\varkappa_{I_N}}{\Gamma(Nn_T+\xi_{I_N}+1)}\left(\frac{1}{\mu}\right)^{Nn_T+\xi_{I_N}}y^{Nn_T+\xi_{I_N}}. \tag{54}$$

The CDF of γ can be obtained by substituting $y=\frac{N_0}{P}\Phi$ in (54) to yield (8).

D. Statistics of SNR from n-th RRH

The CDF of Υ_n can be obtained as

$$F_{\Upsilon_n}(\Phi)=\Pr\left\{\frac{P}{N_0}\left(\frac{1}{1+d_n^v}\right)\max_t\left\{|g_{n,t}|^2\right\}<\Phi\right\}=\Pr\left\{\frac{P}{N_0}H_n<\Phi\right\} \tag{55}$$

where $H_n=\left(\frac{1}{1+d_n^v}\right)\max_t\left\{|g_{n,t}|^2\right\}$. In order to obtain the CDF of H_n, first we need to find the statistics of $G_n=\max_t\left\{|g_{n,t}|^2\right\}$. The CDF of $G_n=\max_t|g_{n,t}|^2$ can be obtained as

$$F_{G_n}(y)=\left(F_{|g_{n,t}|^2}\left(\frac{y}{\mu}\right)\right)^{n_T}=1+\sum_{t=1}^{n_T}(-1)^t\binom{n_T}{t}e^{-\frac{y}{\mu}t} \tag{56}$$

and the PDF of G_n can be obtained as

$$f_{G_n}(y)=-\sum_{t=1}^{n_T}(-1)^t\binom{n_T}{t}\frac{t}{\mu}e^{-\frac{y}{\mu}t}. \tag{57}$$

Given d_n, the CDF of $H_n=\left(\frac{1}{1+d_n^v}\right)G_n$ is

$$F_{H_n|d_n}(y|d_n)=1+\sum_{t=1}^{n_T}(-1)^t\binom{n_T}{t}e^{-\frac{y}{\mu}(1+d_n^v)t}. \tag{58}$$

The CDF of H_n can be obtained by averaging the CDF over the PDF of d_n as

$$F_{H_n}(y)=1+\frac{2}{R^2}\sum_{t=1}^{n_T}(-1)^t\binom{n_T}{t}\int_0^R xe^{-\frac{y}{\mu}(1+x^v)t}dx. \tag{59}$$

Substituting $z=x^v$ and then $x=\frac{y}{\mu}tz$ yields

$$F_{H_n}(y)=1+\frac{2}{R^2v}\sum_{t=1}^{n_T}(-1)^t\binom{n_T}{t}e^{-\frac{y}{\mu}t}\int_0^{\frac{y}{\mu}tR^v}\frac{\left(\frac{x}{\frac{y}{\mu}t}\right)^{\frac{2}{v}-1}}{\frac{y}{\mu}t}e^{-x}dx. \tag{60}$$

The above integral can be expressed in terms of lower incomplete Gamma function to yield

$$F_{H_n}(y)=1+\frac{2}{v}\sum_{t=1}^{n_T}(-1)^t\binom{n_T}{t}e^{-\frac{y}{\mu}t}\left(R^v\frac{y}{\mu}t\right)^{-\frac{2}{v}}\zeta\left(\frac{2}{v},\frac{y}{\mu}tR^v\right). \tag{61}$$

Finally the CDF of Υ_n can be obtained by substituting $y=\frac{N_0}{P}\Phi$ in (61) to yield (14).

E. CDF of $\bar{\theta}_{N,n}$

Conditioned on the $(n+1)$-th largest order statistics, the n largest order statistics are i.i.d [20]. Therefore, conditioned on the $(n+1)$-th largest order statistic, the Laplace transform of the n-th largest order statistic is

$$\mathcal{M}_{\bar{\gamma}(n)}(s)=\frac{1}{1-F_{\bar{\gamma}_n}(y)}\int_y^{\infty}e^{-sz}f_{\bar{\gamma}_n}(z)\,dz. \tag{62}$$

Substituting the approximate PDF of $\bar{\gamma}_n$ yields

$$\mathcal{M}_{\bar{\gamma}(n)}(s)=\sum_{i=0}^{K}\left(\frac{1}{\mu}\right)^{n_T+i}\frac{(n_T+i)}{1-F_{\bar{\gamma}_n}(y)}\Xi_i s^{-(n_T+i)}\Gamma(n_T+i,sy). \tag{63}$$

The Laplace transform of the PDF of the sum of n largest order statistics is

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s)=\int_0^{\infty}\frac{f_{\bar{\gamma}(N-n)}(y)}{(1-F_{\bar{\gamma}_n}(y))^n}\times\left(\sum_{i=0}^{K}\left(\frac{1}{\mu}\right)^{n_T+i}(n_T+i)\,\Xi_i s^{-(n_T+i)}\Gamma(n_T+i,sy)\right)^n dy. \tag{64}$$

Expressing the product of sum as the sum of products yields

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s) = \sum_I \varkappa_i \left(\frac{1}{\mu}\right)^{n_T n+\xi_i} s^{-(n_T n+\xi_i)} \times \int_0^\infty \prod_{l=1}^n \Gamma(n_T + i_l, sy) \frac{f_{\bar{\gamma}(N-n)}(y)}{(1-F_{\bar{\gamma}_n}(y))^n} dy \tag{65}$$

where $\sum_I$ is shorthand notation of $\sum_{i_1=0}^K \cdots \sum_{t_n=1}^{n_T}$ and $\varkappa_i = \prod_{l=1}^n (n_T + i_l)\, \Xi_{i_l}$ and $\xi_i = \sum_{l=1}^n i_l$. Using summation representation of $\Gamma(n+1, x) = \Gamma(n+1) e^{-x} \sum_{m=0}^n \frac{x^m}{\Gamma(m+1)}$ yields

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s) = \sum_I \varkappa_i \left(\frac{1}{\mu}\right)^{n_T n+\xi_i} s^{-(n_T n+\xi_i)} \left(\prod_{l=1}^n \Gamma(n_T + i_l)\right) \times \int_0^\infty e^{-nsy} \left(\prod_{l=1}^n \sum_{m=0}^{n_T+i_l-1} \frac{(sy)^m}{\Gamma(m+1)}\right) \frac{f_{\bar{\gamma}(N-n)}(y)}{(1-F_{\bar{\gamma}_n}(y))^n} dy. \tag{66}$$

Again expressing the product of sum as the sum of products yields

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s) = \sum_I \sum_M \varkappa_i \varkappa_{m,i} \left(\frac{1}{\mu}\right)^{n_T n+\xi_i} s^{-(n_T n+\xi_i-\xi_m)} \times \int_0^\infty e^{-nsy} y^{\xi_m} \frac{f_{\bar{\gamma}(N-n)}(y)}{(1-F_{\bar{\gamma}_n}(y))^n} dy \tag{67}$$

where $\sum_M$ is shorthand notation of $\sum_{m_1=0}^{n_T+i_1-1} \cdots \sum_{m_n=0}^{n_T+i_n-1}$, $\xi_m = \sum_{l=1}^n m_l$ and $\varkappa_{m,i} = \frac{\prod_{l=1}^n \Gamma(n_T+i_l)}{\prod_{l=1}^n \Gamma(m_l+1)}$. Substituting PDF $f_{\bar{\gamma}(N-n)}(y)$ yields

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s) = \frac{N!}{(N-n-1)!(n)!} \sum_I \sum_M \varkappa_i \varkappa_{m,i} \left(\frac{1}{\mu}\right)^{n_T n+\xi_i} \times s^{-(n_T n+\xi_i-\xi_m)} \int_0^\infty e^{-nsy} y^{\xi_m} F_{\bar{\gamma}_n}(y)^{N-n-1} f_{\bar{\gamma}_n}(y) dy. \tag{68}$$

Substituting the CDF $F_{\bar{\gamma}_n}(y)$ and PDF $f_{\bar{\gamma}_n}(y)$ and simplifying yields

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s) = \sum_I \sum_M \frac{N! \varkappa_i \varkappa_{m,i}}{(N-n-1)!(n)!} \left(\frac{1}{\mu}\right)^{n_T n+\xi_i} s^{-(n_T n+\xi_i-\xi_m)} \times \int_0^\infty e^{-nsy} y^{\xi_m} \left(\sum_{j=0}^K \Xi_j \left(\frac{y}{\mu}\right)^{n_T+j}\right)^{N-n-1} \sum_{k=0}^K (n_T+k) \frac{\Xi_k y^{n_T+k-1}}{\mu^{n_T+k}} dy. \tag{69}$$

Expressing the product of sum as the sum of products and simplifying yields (71).

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s) = \sum_I \sum_M \sum_J \sum_{k=0}^K \frac{N! \varkappa_i \varkappa_{m,i} \varkappa_k \varkappa_j}{(N-n-1)!(n)!} \left(\frac{1}{\mu}\right)^{\xi_i + N n_T + \xi_j + k} s^{-(n_T n+\xi_i-\xi_m)} \int_0^\infty e^{-nsy} y^{(N-n)n_T+\xi_m+\xi_j+k-1} dy \tag{70}$$

where $\sum_J$ is shorthand notation of $\sum_{j_1=0}^K \cdots \sum_{j_{N-n-1}=0}^K$, $\varkappa_j = \prod_{l=1}^{N-n-1} \Xi_{j_l}$, $\varkappa_k = (n_T+k)\, \Xi_k$ and $\xi_j = \sum_{l=1}^{N-n-1} j_l$. Solving the integral and taking the inverse Laplace Transform of (71) yields the CDF in (35).

F. CDF of $\hat{\theta}_{N,n}$

Conditioned on the $(n+1)$-th largest order statistics, the n largest order statistics are i.i.d [20]. Therefore, conditioned on the $(n+1)$-th largest order statistic, the Laplace transform of the n-th largest order statistic is

$$\mathcal{M}_{\hat{\gamma}(n)}(s) = \frac{1}{1-F_{\hat{\gamma}_n}(y)} \int_y^\infty e^{-sz} f_{\hat{\gamma}_n}(z)\, dz. \tag{72}$$

The PDF of $\hat{\gamma}_n = \|\mathbf{g}_n\|^2$ is given as

$$f_{\hat{\gamma}_n}(y) = \frac{1}{\mu^{n_T}\Gamma(n_T)} y^{n_T-1} e^{-\frac{y}{\mu}} \tag{73}$$

and the CDF of $\hat{\gamma}_n$ is

$$F_{\hat{\gamma}_n}(y) = \frac{1}{\Gamma(n_T)} \zeta\left(n_T, \frac{y}{\mu}\right) = \left(1 - \sum_{i=0}^{n_T-1} \frac{1}{\Gamma(i+1)} e^{-\frac{y}{\mu}} \left(\frac{y}{\mu}\right)^i\right). \tag{74}$$

Following a similar procedure as before, the Laplace transform of the order statistic is obtained as

$$\mathcal{M}_{\hat{\gamma}(n)}(s) = \left(\frac{1}{1-F_{\hat{\gamma}_n}(y)}\right) \frac{\left(s+\frac{1}{\mu}\right)^{-n_T}}{\mu^{n_T}\Gamma(n_T)} \Gamma\left(n_T, \left(s+\frac{1}{\mu}\right) y\right). \tag{75}$$

The Laplace transform of sum of largest order statistics is

$$\mathcal{M}_{\hat{\theta}_{N,n}}(s) = \left(\frac{1}{\mu^{n_T}\Gamma(n_T)}\right)^n \times \int_0^\infty \left(s+\frac{1}{\mu}\right)^{-nn_T} \left(\Gamma\left(n_T, \left(s+\frac{1}{\mu}\right) y\right)\right)^n \frac{f_{\hat{\gamma}(N-n)}(y)}{(1-F_{\hat{\gamma}_n}(y))^n} dy. \tag{76}$$

Substituting the PDF of $f_{\hat{\gamma}(N-n)}(y)$ yields

$$\mathcal{M}_{\hat{\theta}_{N,n}}(s) = \left(\frac{1}{\mu^{n_T}\Gamma(n_T)}\right)^n \frac{N!}{(N-n-1)!(n)!} \left(s+\frac{1}{\mu}\right)^{-nn_T} \times \int_0^\infty \left(\Gamma\left(n_T, \left(s+\frac{1}{\mu}\right) y\right)\right)^n F_{\hat{\gamma}_n}(y)^{N-n-1} f_{\hat{\gamma}_n}(y) dy. \tag{77}$$

Substituting the CDF $F_{\hat{\gamma}_n}(y)$ and PDF $f_{\hat{\gamma}_n}(y)$ and simplifying yields

$$\mathcal{M}_{\hat{\theta}_{N,n}}(s) = \frac{\left(\frac{1}{\mu^{n_T}\Gamma(n_T)}\right)^n N!}{(N-n-1)!(n)!} \left(s+\frac{1}{\mu}\right)^{-nn_T} \times \int_0^\infty \Gamma(n_T)^n e^{-\left(s+\frac{1}{\mu}\right)ny} \left(\sum_{m=0}^{n_T-1} \frac{\left(\left(s+\frac{1}{\mu}\right)y\right)^m}{\Gamma(m+1)}\right)^n \times \left(1 - \sum_{i=0}^{n_T-1} \frac{1}{\Gamma(i+1)} e^{-\frac{y}{\mu}} \left(\frac{y}{\mu}\right)^i\right)^{N-n-1} \frac{1}{\mu^{n_T}\Gamma(n_T)} y^{n_T-1} e^{-\frac{y}{\mu}} dy. \tag{78}$$

Expressing product of sum as sum of products yields

$$\mathcal{M}_{\hat{\theta}_{N,n}}(s) = \left(\frac{1}{\mu^{n_T}}\right)^n \frac{1}{\mu^{n_T}\Gamma(n_T)} \frac{N!}{(N-n-1)!(n)!} \left(s+\frac{1}{\mu}\right)^{-nn_T} \times \int_0^\infty e^{-\left(s+\frac{1}{\mu}\right)ny} \left(\sum_M \kappa_m \left(\left(s+\frac{1}{\mu}\right)y\right)^{\xi_m}\right) \times \sum_{j=0}^{N-n-1} \binom{N-n-1}{j} (-1)^j \left(\sum_{i=0}^{n_T-1} \frac{e^{-\frac{y}{\mu}}}{\Gamma(i+1)} \left(\frac{y}{\mu}\right)^i\right)^j y^{n_T-1} e^{-\frac{y}{\mu}} dy \tag{79}$$

where $\sum_M = \sum_{m_1=0}^{n_T-1} \cdots \sum_{m_n=0}^{n_T-1}$, $\xi_m = \sum_{i=1}^n m_i$ and $\kappa_m = \frac{1}{\prod_{i=1}^n \Gamma(m_i+1)}$. Again expressing product of sum as sum of products yields (80), where $\sum_I = \sum_{i_1=0}^{n_T-1} \cdots \sum_{i_j=0}^{n_T-1}$, $\kappa_I = \prod_{o=1}^j \frac{1}{\Gamma(i_o+1)}$ and $\xi_j = \sum_{o=1}^j i_o$. Rearranging terms and solving the resulting integral yields (81). Representing $\mathcal{M}_{\hat{\theta}_{N,n}}(s)$ in terms of Gamma functions yields (82). The CDF can be obtained by taking the inverse Laplace transform of $\frac{\mu}{\mu s}\mathcal{M}_S(s)$ which is given as

$$F_{\hat{\theta}_{N,n}}(y) = \sum_M \sum_{j=0}^{N-n-1} \sum_I \kappa_I \binom{N-n-1}{j} \times \frac{(-1)^j\, N! n^{-(\xi_m+n_T+\xi_j)} \Gamma(\xi_m+n_T+\xi_j)\, \kappa_m}{\Gamma(n_T)\,(N-n-1)!(n)!} \times \frac{\mu}{2\pi\iota} \int_{\gamma-iT}^{\gamma+iT} \frac{\prod_{k=1}^m \Gamma(\mu s + \bar{b}_N(k))}{\prod_{k=1}^p \Gamma(\mu s + \bar{a}_D(k))} e^{sy} ds \tag{83}$$

$$\mathcal{M}_{\bar{\theta}_{N,n}}(s)=\sum_{I}\sum_{M}\sum_{J}\sum_{k=0}^{K}\frac{N!\varkappa_i\varkappa_{m,i}\varkappa_k\varkappa_j\Gamma\left(\left(N-n\right)n_T+\xi_m+\xi_j+k\right)}{(N-n-1)!(n)!n^{\left(\left(N-n\right)n_T+\xi_m+\xi_j+k\right)}}\left(\frac{1}{\mu}\right)^{Nn_T+\xi_i+\xi_j+k}s^{-\left(Nn_T+\xi_j+\xi_i+k\right)} \tag{71}$$

$$\begin{aligned}\mathcal{M}_{\hat{\theta}_{N,n}}(s)=&\left(\frac{1}{\mu^{n_T}}\right)^{n}\frac{1}{\mu^{n_T}\Gamma\left(n_T\right)}\frac{N!}{(N-n-1)!(n)!}\sum_{M}\sum_{j=0}^{N-n-1}\binom{N-n-1}{j}(-1)^j\kappa_m\times\\&\left(s+\frac{1}{\mu}\right)^{-nn_T+\xi_m}\int_0^\infty y^{\xi_m}e^{-\left(s+\frac{1}{\mu}\right)ny}\sum_{I}\kappa_I e^{-\frac{y}{\mu}j}\left(\frac{y}{\mu}\right)^{\xi_j}y^{n_T-1}e^{-\frac{y}{\mu}}dy\end{aligned} \tag{80}$$

$$\mathcal{M}_{\hat{\theta}_{N,n}}(s)=\sum_{M}\sum_{j=0}^{N-n-1}\sum_{I}\binom{N-n-1}{j}\frac{\kappa_I(-1)^j N!\Gamma\left(\xi_m+n_T+\xi_j\right)\kappa_m}{\Gamma\left(n_T\right)(N-n-1)!(n)!n^{\left(\xi_m+n_T+\xi_j\right)}}\left(\mu s+1\right)^{-nn_T+\xi_m}\left(\mu s+\frac{1}{n}\left(n+j+1\right)\right)^{-\left(\xi_m+n_T+\xi_j\right)} \tag{81}$$

$$\begin{aligned}\mathcal{M}_{\hat{\theta}_{N,n}}(s)=&\sum_{M}\sum_{j=0}^{N-n-1}\sum_{I}\kappa_I\binom{N-n-1}{j}\frac{(-1)^j N!n^{-\left(\xi_m+n_T+\xi_j\right)}\Gamma\left(\xi_m+n_T+\xi_j\right)\kappa_m}{\Gamma\left(n_T\right)(N-n-1)!(n)!}\\&\times\frac{\Gamma\left(\mu s+1\right)^{nn_T-\xi_m}\Gamma\left(\mu s+1+\frac{1}{n}\left(j+1\right)\right)^{\left(\xi_m+n_T+\xi_j\right)}}{\Gamma\left(\mu s+1+1\right)^{nn_T-\xi_m}\Gamma\left(\mu s+1+\frac{1}{n}\left(j+1\right)+1\right)^{\left(\xi_m+n_T+\xi_j\right)}}\end{aligned} \tag{82}$$

where $m = 1 + nn_T - \xi_m + \left(\xi_m+n_T+\xi_j\right)$, $p = 1 + nn_T - \xi_m + \left(\xi_m+n_T+\xi_j\right)$, $\bar{b}_N = \left\{0, \underbrace{1,1,...,1}_{nn_T-\xi_m}, \underbrace{1+\frac{(j+1)}{n}, 1+\frac{(j+1)}{n}, ..., 1+\frac{(j+1)}{n}}_{\left(\xi_m+n_T+\xi_j\right)}\right\}$ and $\bar{a}_D = \bar{b}_N + 1 = \left\{1, \underbrace{2,2,...,2}_{nn_T-\xi_m}, \underbrace{2+\frac{(j+1)}{n}, 2+\frac{(j+1)}{n}, ..., 2+\frac{(j+1)}{n}}_{\left(\xi_m+n_T+\xi_j\right)}\right\}$. The CDF in (83) can finally be represented in terms of Meijer-G function as (37).

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