Performance Analysis of Cooperative Communication Systems with Imperfect Channel Estimation

Yi Wu
Department of ICT, University of Agder
P.O.Box 509, 4898 Grimstad, Norway
yi.wu@uia.no

Matthias Pätzold
Department of ICT, University of Agder
P.O.Box 509, 4898 Grimstad, Norway
matthias.paetzold@uia.no

***Abstract*—This paper investigates the effects of channel estimation errors on the symbol-error-rate (SER) performance of a cooperative communication system operating in an amplify-and-forward (AF) mode. A pilot symbol assisted modulation scheme with linear minimum mean square estimation (LMMSE) is used for the channel estimation. An accurate and easy-to-evaluate SER expression is presented for uncoded cooperative communication systems with quadrature amplitude modulation (QAM) and phase-shift keying (PSK) constellations. Numerical simulations are conducted to verify the correctness of the proposed analytical formulation. It is shown that the performance loss caused by channel estimation errors increases mainly with the normalized maximum Doppler frequency.**

I. Introduction

In wireless communications, fading caused by the movement of the mobile station in a multipath propagation environment often has a severe impact on the system performance. However, the effects of fading can be substantially mitigated by the use of diversity techniques. Among the various forms of diversity techniques, a form of spatial diversity named "cooperative diversity" is particularly attractive, since it provides effective diversity benefits for those devices that cannot be equipped with multiple antennas due to the size, complexity and cost.

According to the way in which the information is transmitted from the source terminal to the relay terminals, and the way it is processed at the relay terminals, the existing cooperative protocols can usually be divided into two types [3]: decode-and-forward (DF) protocols and amplify-and-forward (AF) protocols. In the DF protocol, the relay terminals decode the received signal first before checking whether errors have occurred or not. If the information sequence could be successfully decoded, then the sequence will be re-encoded by either the same or a different code before it will be finally transmitted to the destination terminal. An erroneously decoded information sequence is not relayed to avoid causing extra errors at the destination terminal. On the other hand, in AF protocols, the relay terminals simply re-transmit a scaled version of the signal that they receive from the source terminal to the destination terminal. Depending on the scaling factor, the AF relaying scheme can be further divided into two types which are called fixed gain AF system and variable gain AF system [11].

The performance analysis of AF techniques is one of the active research areas in cooperative communication systems. It has been studied in the past from various aspects. For example, [3]–[5] analyzed the performance of AF systems in terms of the outage probability and diversity gain under different assumptions for the amplifier gain. Various bounds and accurate expressions for the symbol error probability of a cooperative communication system have been derived in [6]–[10]. Thus far, most of the work in performance analysis of AF systems has been carried out under the assumption of perfect channel state information (CSI) at both the relay and destination terminal. In practice, however, the relay and destination terminals never have the perfect knowledge of the CSI. Imperfect CSI occurs either due to an imperfect channel estimation algorithm or due to variations of the channel after it has been accurately estimated. Thus, it is important to investigate the error performance of AF systems with imperfect CSI. In recent works [11, 12], the effects of the channel estimation error on the performance of AF cooperative communication systems have been studied by means of Monte Carlo simulations. While in [13], the error performance of AF system is investigated by using a simple and notional model of the channel estimation error, where the variance of the channel estimation error is assumed to be fixed for all values of the signal-to-noise ratio (SNR). To our best knowledge, accurate analytical symbol-error-rate (SER) expressions for AF cooperative systems, taking into account the varying variance of the channel estimation error for different values of the SNR, have not been derived yet.

In this paper, we focus on variable-gain AF cooperative communication systems with a pilot symbol assisted modulation (PSAM) scheme. We study their end-to-end SER performance by assuming that the linear minimum mean square estimation (LMMSE) is used for the channel estimation. We first investigate the detector at the destination terminal with imperfect channel estimation. We then derive both the probability density function (PDF) and the moment generating function (MGF) of the instantaneous SNR at the destination terminal.

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Finally, these statistical quantities are applied to derive the accurate SER expression for AF cooperative communication systems with quadrature amplitude modulation (QAM) and phase-shift keying (PSK) constellations.

The rest of the paper is organized as follows. In Section II, we describe the system model and introduce some preliminaries on the AF cooperative system with a PSAM scheme. In Section III, we derive the accurate SER expression for the AF cooperative communication scheme with LMMSE. Various simulation results and their discussions are presented in Section IV. Finally, Section V contains the conclusions.

The following notation is used throughout the paper: $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ denote the complex conjugate, vector (or matrix) transpose, conjugate transpose, and matrix inverse, respectively. The symbol $\mathrm{E}[\cdot]$ denotes the expectation operator, $|z|$ represents the absolute value of a complex number z, and the complex Gaussian distribution with mean m and covariance P is denoted by $\mathcal{CN}(m, P)$. Finally, $x \sim \mathcal{CN}(m, P)$ denotes a complex random variable x with distribution $\mathcal{CN}(m, P)$.

II. System Model

We consider an AF-based cooperative communication system which consists of a source, relay, and destination terminal. We assume that each terminal is equipped with a single transmit and receive antenna, and operates in a half-duplex mode, i.e., it cannot transmit and receive simultaneously. Furthermore, we adopt the so-called Protocol II proposed by Nabar et al. [3] as the user cooperative protocol. This means that two time slots are used to transmit one data symbol. In the first time slot, the source terminal communicates with the relay and destination terminal. In the second time slot, only the relay terminal communicates with the destination terminal. This protocol realizes a maximum degree of broadcasting and exhibits no receive collisions [3]. To simplify the following analysis, we consider a symbol-by-symbol transmission, so that the time slot index 1 and 2 can be dropped. Throughout this paper, we assume that the system operates in a Rayleigh flat fading environment with perfect synchronization, no CSI is available at the transmitters, and imperfect channel estimation is assumed at the receiver. As in [11], we use a PSAM scheme for the channel estimation. Pilot symbols are periodically inserted in data symbols with an insertion period of L symbols. Since the design of an optimal channel estimator is very complex, we resort to a suboptimal LMMSE. We further assume that the data information symbols are equally probable from a constellation set composed of QAM symbols or PSK symbols with size M, and the pilot symbols are selected from a binary phase-shift keying (BPSK) constellation.

With the above assumptions, the received signals corresponding to the kth transmitted symbol at the destination terminal and the relay terminal during the first time slot are given by

$$r_{SD}(k) = \sqrt{P_S} h_{SD}(k) x(k) + n_{SD}(k), \tag{1}$$

$$r_{SR}(k) = \sqrt{P_S} h_{SR}(k) x(k) + n_{SR}(k), \tag{2}$$

respectively, where P_S is the average power of the transmitted signal at the source terminal; $h_{SD}(k)$ and $h_{SR}(k)$ are the channel coefficients from the source terminal to the destination terminal with distribution $\mathcal{CN}(0, \sigma_{SD}^2)$ and from the source terminal to the relay terminal with distribution $\mathcal{CN}(0, \sigma_{SR}^2)$, respectively; $x(k)$ is the kth transmitted symbol from the source terminal, and $n_{SD}(k)$ and $n_{SR}(k)$ are the additive receiver noises at the destination terminal and the relay terminal, respectively, with the same distribution $\mathcal{CN}(0, N_0)$. Throughout this paper, we assume that the transmitted symbols have an average energy of 1, i.e., $\mathrm{E}[|x(k)|^2] = 1$. According to Protocol II, the relay terminal will first normalize the received signal by a factor of $\sqrt{\mathrm{E}(|r_{SR}(k)|^2)}$ (to ensure the unity of average energy). Then, the normalized signal will be amplified and forwarded to the destination terminal during the second time slot. Therefore, the received signal of the kth symbol at the destination terminal within the second time slot is given by

$$r_{RD}(k) = \frac{\sqrt{P_R}}{\sqrt{P_S|h_{SR}(k)|^2 + N_0}} h_{RD}(k) r_{SR}(k) + n_{RD}(k), \tag{3}$$

where P_R is the average power of the transmitted signal at the relay terminal, $h_{RD}(k)$ is the channel coefficient from the relay terminal to the destination terminal with distribution $\mathcal{CN}(0, \sigma_{RD}^2)$, and $n_{RD}(k)$ is the additive receiver noise at the destination terminal with distribution $\mathcal{CN}(0, N_0)$. Using (2), we can rewrite (3) as

$$r_{RD}(k) = \frac{\sqrt{P_S P_R}}{\sqrt{P_S|h_{SR}(k)|^2 + N_0}} h_{SR}(k) h_{RD}(k) x(k) + n'_{RD}(k), \tag{4}$$

where

$$n'_{RD}(k) = \frac{\sqrt{P_R}}{\sqrt{P_S|h_{SR}(k)|^2 + N_0}} h_{RD}(k) n_{SR}(k) + n_{RD}(k). \tag{5}$$

Assuming that $n_{SR}(k)$ and $n_{RD}(k)$ are independent, it can be shown that the noise term $n'_{RD}(k)$ is a complex Gaussian random process with distribution $\mathcal{CN}(0, (\sqrt{P_R}/\sqrt{P_S|h_{SR}(k)|^2 + N_0} + 1)N_0)$.

Since the PSAM scheme is used for the channel estimation, the packed transmission can be divided into blocks by pilot symbols. In each block, there are L symbols where the first symbol is assigned to a pilot symbol and the remaining $L-1$ symbols are assigned to data symbols. The channel estimation at each symbol position in a block is obtained using N_1 pilot symbols on the left of the symbol position and N_2 pilot symbols on the right of the symbol position. Therefore, $N = N_1 + N_2$ pilot symbols are employed to estimate the channel coefficient at the desired symbol position.

Let us denote the pilot symbols employed to estimate the channel gain $h_{SD}(k)$ of the desired data symbol $x(k)$ as an $N \times 1$ vector $\mathbf{p}_{SD} = [x(k - L(N_1 - 1) - l), ..., x(k - l), x(k + L - l), ..., x(k + LN_2 - l)]^T$, where $l = 1, 2, ..., L-1$ is the

$$\mathbf{C}_{\mathbf{r}_{SD}} = \begin{bmatrix} P_S R_{SD}(0)+N_0 & P_S R_{SD}(L) & \cdots & P_S R_{SD}((N-1)L) \\ P_S R_{SD}(L) & P_S R_{SD}(0)+N_0 & \cdots & P_S R_{SD}((N-2)L) \\ \vdots & \vdots & \ddots & \vdots \\ P_S R_{SD}((N-1)L) & P_S R_{SD}((N-2)L) & \cdots & P_S R_{SD}(0)+N_0 \end{bmatrix} \tag{6}$$

$$\mathbf{c}_{h_{SD},\mathbf{r}_{SD}}(l) = [\sqrt{P_S} R_{SD}(-L(N_1-1)-l), \cdots, \sqrt{P_S} R_{SD}(L-l), \cdots \sqrt{P_S} R_{SD}(L(N_2-1)-l)] \tag{7}$$

offset of the desired data symbol $x(k)$ to the first pilot symbol on its left side. Using (1), we obtain the received signal vector $\mathbf{r}_{SD}$, corresponding to the transmitted pilot vector $\mathbf{p}_{SD}$ at the destination terminal as

$$\mathbf{r}_{SD} = \sqrt{P_S}\ \text{diag}(\mathbf{p}_{SD})\mathbf{h}_{SD} + \mathbf{n}_{SD}, \tag{8}$$

where $\mathbf{h}_{SD} = [h_{SD}(k-L(N_1-1)-l), ..., h_{SD}(k-l), h_{SD}(k+L-l), ..., h_{SD}(k+LN_2-l)]^T$ and $\mathbf{n}_{SD} = [n_{SD}(k-L(N_1-1)-l), ..., n_{SD}(k-l), n_{SD}(k+L-l), ..., n_{SD}(k+LN_2-l)]^T$ are the channel gain and noise components at the pilot symbols' positions, respectively.

Without loss of generality, we assume that positive unit energy symbols are transmitted as pilot symbols, i.e., $\mathbf{p}_{SD}$ is an all-one vector. Then, (8) simplifies to

$$\mathbf{r}_{SD} = \sqrt{P_S}\mathbf{h}_{SD} + \mathbf{n}_{SD}. \tag{9}$$

With these observations, the channel estimate for $h_{SD}(k)$ can be obtained by the LMMSE as [4]

$$\hat{h}_{SD}(k) = \mathbf{w}_{SD}\mathbf{r}_{SD}. \tag{10}$$

where $\mathbf{w}_{SD} = \mathbf{c}_{h_{SD},\mathbf{r}_{SD}}(l)\mathbf{C}^{-1}_{\mathbf{r}_{SD}}$ is an $1 \times N$ LMMSE filter vector, $\mathbf{C}_{\mathbf{r}_{SD}} = \text{E}[\mathbf{r}_{SD}\mathbf{r}^H_{SD}]$ and $\mathbf{c}_{h_{SD},\mathbf{r}_{SD}}(l) = \text{E}[h^*_{SD}(k)\mathbf{r}_{SD}]$ are the autocorrelation matrix of $\mathbf{r}_{SD}$ and cross-correlation vector of $h_{SD}(k)$ and $\mathbf{r}_{SD}$, respectively. From the LMMSE theory [4], we know that $\hat{h}_{SD}(k)$ is distributed as $\mathcal{CN}(0, \mathbf{c}_{h_{SD},\mathbf{r}_{SD}}(l)(\mathbf{C}^{-1}_{\mathbf{r}_{SD}})^H\mathbf{c}^H_{h_{SD},\mathbf{r}_{SD}}(l))$. Let us define the discrete autocorrelation function of $h_{SD}(k)$ as $R_{SD}(\kappa) = \text{E}[h_{SD}(k)h_{SD}(k+\kappa)^*]$. Then, using the system model and channel properties described above, we finally obtain $\mathbf{C}_{\mathbf{r}_{SD}}$ and $\mathbf{c}_{h_{SD},\mathbf{r}_{SD}}(l)$ as shown at the top of this page. From the LMMSE filter vector $\mathbf{w}_{SD}$, we can see that each data symbol in a block will have a different estimator. Therefore, we need totaly $L-1$ different estimators for $L-1$ data symbols in a block. However, due to the periodic pilot insertion, an identical estimator will be adopted for the data symbol in the same positions across all blocks in a packet. Therefore, without loss of generality, we will henceforth only consider $L-1$ different estimators for the data symbols in one particular block and employ the index l instead of k to distinguish them. With this in mind, we can express the estimation error of the lth estimator as

$$e_{SD}(l) = h_{SD}(l) - \hat{h}_{SD}(l). \tag{11}$$

Furthermore, the estimation error $e_{SD}(l)$ is distributed as $\mathcal{CN}(0, \sigma^2_{e,SD}(l))$, where $\sigma^2_{e,SD}(l)) = \sigma^2_{SD} - \mathbf{c}_{h_{SD},\mathbf{r}_{SD}}(l)(\mathbf{C}^{-1}_{\mathbf{r}_{SD}})^H\mathbf{c}^H_{h_{SD},\mathbf{r}_{SD}}(l)$. From (11) it follows that we can model the channel gain $h_{SD}(l)$ as the sum of the channel estimate $\hat{h}_{SD}(l)$ and the estimation error $e_{SD}(l)$, i.e.,

$$h_{SD}(l) = \hat{h}_{SD}(l) + e_{SD}(l). \tag{12}$$

Similarly, we can model the channel gain from the source terminal to the relay terminal $h_{SR}(l)$ and the channel gain from the relay terminal to the source terminal $h_{RD}(l)$ as

$$h_{SR}(l) = \hat{h}_{SR}(l) + e_{SR}(l), \tag{13}$$

$$h_{RD}(l) = \hat{h}_{RD}(l) + e_{RD}(l), \tag{14}$$

where

$$\begin{aligned} \hat{h}_{SR}(l) &= \mathbf{c}_{h_{SR},\mathbf{r}_{SR}}(l)\mathbf{C}^{-1}_{\mathbf{r}_{SR}}\mathbf{r}_{SR}, \\ \hat{h}_{SR}(l) &\sim \mathcal{CN}(0, \mathbf{c}_{h_{SR},\mathbf{r}_{SR}}(l)(\mathbf{C}^{-1}_{\mathbf{r}_{SR}})^H\mathbf{c}^H_{h_{SR},\mathbf{r}_{SR}}(l)), \\ e_{SR}(l) &\sim \mathcal{CN}(0, \sigma^2_{e,SR}(l)), \\ \sigma^2_{e,SR}(l) &= \sigma^2_{SR} - \mathbf{c}_{h_{SR},\mathbf{r}_{SR}}(l)(\mathbf{C}^{-1}_{\mathbf{r}_{SR}})^H\mathbf{c}^H_{h_{SR},\mathbf{r}_{SR}}(l), \\ \hat{h}_{RD}(l) &= \mathbf{c}_{h_{RD},\mathbf{r}_{RD}}(l)\mathbf{C}^{-1}_{\mathbf{r}_{RD}}\mathbf{r}_{RD}, \\ \hat{h}_{RD}(l) &\sim \mathcal{CN}(0, \mathbf{c}_{h_{RD},\mathbf{r}_{RD}}(l)(\mathbf{C}^{-1}_{\mathbf{r}_{RD}})^H\mathbf{c}^H_{h_{RD},\mathbf{r}_{RD}}(l)), \\ e_{RD}(l) &\sim \mathcal{CN}(0, \sigma^2_{e,RD}(l)), \\ \sigma^2_{e,RD}(l) &= \sigma^2_{RD} - \mathbf{c}_{h_{RD},\mathbf{r}_{RD}}(l)(\mathbf{C}^{-1}_{\mathbf{r}_{RD}})^H\mathbf{c}^H_{h_{RD},\mathbf{r}_{RD}}(l). \end{aligned}$$

Note that $\mathbf{r}_{SR}$, $\mathbf{c}_{h_{SR},\mathbf{r}_{SR}}(l)$, $\mathbf{C}_{\mathbf{r}_{SR}}$, $\mathbf{r}_{RD}$, $\mathbf{c}_{h_{RD},\mathbf{r}_{RD}}(l)$, and $\mathbf{C}_{\mathbf{r}_{RD}}$ can be determined similarly as $\mathbf{r}_{SD}$, $\mathbf{c}_{h_{SD},\mathbf{r}_{SD}}(l)$ and $\mathbf{C}_{\mathbf{r}_{SD}}$.

Throughout this paper, we assume that $\hat{h}_{SR}(l)$ is known to the relay terminal, while $\hat{h}_{SD}(l)$, $\hat{h}_{SR}(l)$, and $\hat{h}_{RD}(l)$ are known to the destination terminal. Note that this assumption will increase the load for the transmission between relay and destination terminals. However, from a theoretical point of view, it can serve as a benchmark for the evaluation of other practical schemes. With these assumptions, we can rewrite the received signals at the destination terminal during two time slots as

$$r_{SD}(l) = \sqrt{P_S}\hat{h}_{SD}(l)x(l) + n_1(l), \tag{15}$$

$$r_{RD}(l) = A(l)\sqrt{P_S P_R}\hat{h}_{SR}(l)\hat{h}_{RD}(l)x + n_2(l), \tag{16}$$

where

$$n_1(l) = \sqrt{P_S} e_{SD}(l)x(l) + n_{SD}(l), \quad (17)$$

$$n_2(l) = A(l)\sqrt{P_S P_R}\Big(e_{RD}(l)e_{SR}(l) + \hat{h}_{SR}(l)e_{RD}(l) + \hat{h}_{RD}(l)e_{SR}(l)\Big)x(l) + n_{RD}(l) \quad (18)$$
$$+ A(l)\sqrt{P_R}(\hat{h}_{RD}(l) + e_{RD}(l))n_{SR}(l),$$

$$A(l) = \frac{1}{\sqrt{P_S|\hat{h}_{SR}(l)|^2 + P_S\sigma^2_{e,SR}(l) + N_0}}. \quad (19)$$

It turns out that both the effective noise terms $n_1(l)$ and $n_2(l)$ have zero mean. After some manipulations, it can also be shown that the variance of $n_1(l)$ and $n_2(l)$ equals $\sigma^2_{n1}(l) = P_S\sigma^2_{e,SR}(l) + N_0$ and $\sigma^2_{n2}(l) = A^2(l)P_S P_R(|\hat{h}_{RD}(l)|^2\sigma^2_{e,SR}(l) + |\hat{h}_{SR}(l)|^2\sigma^2_{e,RD}(l) + \sigma^2_{e,RD}(l)\sigma^2_{e,SR}(l)) + N_0(A^2(l)P_R|\hat{h}_{RD}(l)|^2 + \sigma^2_{e,RD}(l) + N_0)$, respectively. Note that the effective noise term $n_1(l)$ is Gaussian distributed, while the effective noise term $n_2(l)$ is non-Gaussian distributed due to the presence of the product terms of two independent Gaussian variables in (18). However, our simulations have shown that the PDF of $n_2(l)$ is almost Gaussian distributed when the variance of the channel estimation errors is small. Note that with a properly selected value for L, the variance of the channel estimation errors of cooperative communication systems using the PSAM scheme should in general be very small. To simplify the following analysis, we will treat $n_2(l)$ as Gaussian distributed, i.e., $n_2(l) \sim \mathcal{CN}(0, \sigma^2_{n2}(l))$. As we can see from the simulation results in Section IV, this approximation is reasonable.

III. SER Analysis for AF cooperative systems

With the above assumption and the estimated channel coefficients, maximum ratio combining (MRC) can be applied at the destination terminal to minimize the SER of the system. The combined signal from the MRC detector at the destination terminal can be written as [15]

$$r(l) = c_1(l)r_{SD}(l) + c_2(l)r_{RD}(l), \quad (20)$$

where the combing factors $c_1(l)$ and $c_2(l)$ are given by

$$c_1(l) = \frac{\sqrt{P_S}\hat{h}^*_{SD}(l)}{P_S\sigma^2_{e,SD}(l) + N_0},$$

$$c_2(l) = \frac{A(l)\sqrt{P_S P_R}\hat{h}^*_{SR}(l)\hat{h}^*_{RD}(l)}{N_0\Delta_0(l) + A^2(l)P_S P_R\Delta_1(l)},$$

respectively, with

$$\Delta_0(l) = A^2(l)P_R|\hat{h}_{RD}(l)|^2 + N_0 + \sigma^2_{e,RD}(l),$$

$$\Delta_1(l) = |\hat{h}_{RD}(l)|^2\sigma^2_{e,SR}(l) + |\hat{h}_{SR}(l)|^2\sigma^2_{e,RD}(l) + \sigma^2_{e,RD}(l)\sigma^2_{e,SR}(l).$$

Using (1), (4), and (20), we can present the instantaneous SNR $\gamma(l)$ of the output signal from the MRC detector as [15]

$$\gamma(l) = \gamma_1(l) + \gamma_2(l)$$

where

$$\gamma_1(l) = \frac{P_S|\hat{h}_{SD}(l)|^2}{P_S\sigma^2_{e,SD}(l) + N_0}, \quad (21)$$

$$\gamma_2(l) = \frac{P_S P_R|\hat{h}_{SR}(l)|^2|\hat{h}_{RD}(l)|^2}{P_S P_R\Delta_1(l) + N_0\Delta_2(l)}, \quad (22)$$

$$\Delta_2(l) = \Big(P_S|\hat{h}_{SR}(l)|^2 + N_0\Big)\left(1 + \sigma^2_{e,RD}(l)\right) + P_R|\hat{h}_{RD}(l)|^2. \quad (23)$$

Now, the SER of the system conditioned on the estimated channel coefficients with M-QAM and M-PSK can be expressed in the following form [16]

$$P_{QAM}(l|\widehat{h}_{SD}(l), \widehat{h}_{SR}(l), \widehat{h}_{RD}(l)) = \frac{4B}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{K_Q\gamma(l)}{2\sin^2\theta}\right)d\theta - \frac{4B^2}{\pi}\int_0^{\frac{\pi}{4}} \exp\left(-\frac{K_Q\gamma(l)}{2\sin^2\theta}\right)d\theta, \quad (24)$$

$$P_{PSK}(l|\widehat{h}_{SD}(l), \widehat{h}_{SR}(l), \widehat{h}_{RD}(l)) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{K_P\gamma(l)}{\sin^2\theta}\right)d\theta, \quad (25)$$

respectively, where $B = 1 - 1/\sqrt{M}$, $K_Q = 3/(M-1)$, and $K_P = \sin^2(\pi/M)$. Recall that the MGF $\Phi_Z(s)$ of a random variable Z can be expressed in terms of its PDF $p_Z(z)$ as [16]

$$\Phi_Z(s) = \int_{-\infty}^{\infty} \exp(-sz)p_Z(z)dz \quad (26)$$

for any real number s. Averaging the conditional SER in (24) and (25) over the Gaussian distributed channel estimates $\widehat{h}_{SD}(l)$, $\widehat{h}_{SR}(l)$, and $\widehat{h}_{RD}(l)$, we obtain the SER of the system with M-QAM and M-PSK as follows:

$$P_{QAM}(l) = \frac{4B}{\pi}\int_0^{\frac{\pi}{2}} \Phi_{\gamma_1}\left(l, \frac{K_Q}{2\sin^2\theta}\right)\Phi_{\gamma_2}\left(l, \frac{K_Q}{2\sin^2\theta}\right)d\theta - \frac{4B^2}{\pi}\int_0^{\frac{\pi}{4}} \Phi_{\gamma_1}\left(l, \frac{K_Q}{2\sin^2\theta}\right)\Phi_{\gamma_2}\left(l, \frac{K_Q}{2\sin^2\theta}\right)d\theta, \quad (27)$$

$$P_{PSK}(l) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \Phi_{\gamma_1}\left(l, \frac{K_P}{\sin^2\theta}\right)\Phi_{\gamma_2}\left(l, \frac{K_P}{\sin^2\theta}\right)d\theta, \quad (28)$$

where $\Phi_{\gamma_1}(l,s)$ and $\Phi_{\gamma_2}(l,s)$ are the MGF of $\gamma_1(l)$ and $\gamma_2(l)$, respectively.

From (21), we can easily see that $\gamma_1(l)$ has an exponential distribution with the parameter $\alpha_1(l) = [P_S\sigma^2_{e,SD}(l) + N_0]/[P_S\sigma^2_{\hat{h},SD}(l)]$. Therefore, the MGF of $\gamma_1(l)$ can be written as

$$\Phi_{\gamma_1}(l,s) = \frac{1}{1 + \frac{s}{\alpha_1(l)}}. \quad (29)$$

To attain the MGF $\Phi_{\gamma_2}(l,s)$, we first need to derive the PDF of $\gamma_2(l)$. Applying the same procedure as in the proof of

Therorem 1 in [13], we can finally express the PDF of the instantaneous SNR $\gamma_2(l)$ as

$$p_{\gamma_2}(l,z) = \int_0^1 \frac{\alpha_2(l)\alpha_3(l)[\beta(l)x+z]}{x^2(1-x)^2} \cdot \exp\left[-\frac{\alpha_2(l)z}{x} - \frac{\alpha_3(l)(\beta(l)x+z)}{1-x}\right] dx,\ z \geq 0, \quad (30)$$

where

$$\alpha_2(l) = \frac{(P_R+N_0)\sigma^2_{e,RD}(l)+N_0}{P_R\sigma^2_{\widehat{h},RD}(l)},$$
$$\alpha_3(l) = \frac{P_S\sigma^2_{e,SR}(l)+N_0}{P_S\sigma^2_{\widehat{h},SR}(l)},$$
$$\beta(l) = \frac{P_SP_R\sigma^2_{e,SR}(l)\sigma^2_{e,RD}(l)+(1+\sigma^2_{e,RD}(l))N_0^2}{(P_S\sigma^2_{e,SR}(l)+N_0)[(P_R+N_0)\sigma^2_{e,RD}(l)+N_0]}.$$

Then, the MGF $\Phi_{\gamma_2}(l,s)$ is given by

$$\begin{aligned}\Phi_{\gamma_2}(l,s) &= \int_{-\infty}^{\infty} \exp(-sz)p_{\gamma_2}(l,z)dz \\ &= \int_0^{\infty} \exp(-sz)\int_0^1 \frac{\alpha_2(l)\alpha_3(l)[\beta(l)x+z]}{x^2(1-x)^2} \\ &\quad \times \exp\left\{-\frac{\alpha_2(l)z}{x} - \frac{\alpha_3(l)[\beta(l)x+z]}{1-x}\right\} dxdz \\ &= \int_0^1 \alpha_2(l)\alpha_3(l)\exp\left\{-\frac{\alpha_3(l)\beta(l)x}{1-x}\right\} \\ &\quad \times [a(l,x,s)+b(l,x,s)]\,dx, \end{aligned} \quad (31)$$

where

$$a(l,x,s) = \frac{\beta(l)}{\alpha_2(l)+(s-2\alpha_2(l)+\alpha_3(l))x+\upsilon(l,x,s)},$$
$$\upsilon(l,x,s) = (-2s+\alpha_2(l)-\alpha_3(l))x^2+sx^3,$$
$$b(l,x,s) = \frac{1}{(\alpha_2(l)+[s-\alpha_2(l)+\alpha_3(l)]x-sx^2)^2}.$$

By substituting the derived MGFs $\Phi_{\gamma_1}(l,s)$ and $\Phi_{\gamma_2}(l,s)$ into (27) and (28), we can express the SER at the lth data position with M-QAM and M-PSK as

$$P_{QAM}(l) = \frac{1}{\pi}\int_0^{\pi/2}\int_0^1 \frac{4B\eta(l)}{\alpha_1(l)+\frac{K_Q}{2\sin^2\theta}} f(l,t,\frac{K_Q}{2\sin^2\theta})\ dtd\theta - \frac{1}{\pi}\int_0^{\pi/4}\int_0^1 \frac{4B^2\eta(l)}{\alpha_1(l)+\frac{K_Q}{2\sin^2\theta}} f(l,t,\frac{K_Q}{2\sin^2\theta})\ dtd\theta, \quad (32)$$

$$P_{PSK}(l) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\int_0^1 \frac{\eta(l)}{\alpha_1(l)+\frac{K_P}{\sin^2\theta}} f(l,t,\frac{K_P}{\sin^2\theta})\ dtd\theta, \quad (33)$$

respectively, where

$$f(l,t,s) = \exp\left(-\frac{\alpha_3(l)\beta(l)t}{1-t}\right)[a(l,t,s)+b(l,t,s)],$$
$$\eta(l) = \alpha_1(l)\alpha_2(l)\alpha_3(l)\exp[\alpha_3(l)\beta(l)].$$

Averaging $P_{QAM}(l)$ and $P_{PSK}(l)$ over all $L-1$ data positions in one block, we finally get the accurate SER expression for the considered system with M-QAM and M-PSK signals as

$$\overline{P}_{QAM} = \frac{1}{L-1}\sum_{l=1}^{L-1} P_{QAM}(l),$$
$$\overline{P}_{PSK} = \frac{1}{L-1}\sum_{l=1}^{L-1} P_{PSK}(l).$$

Note that if we assume that there is no channel estimation error at the receiver, i.e., $\sigma^2_{e,SD}(l) = \sigma^2_{e,SR}(l) = \sigma^2_{e,RD}(l) = 0$ for $l = 1, ..., L-1$, the above results lead to the SER of a cooperative system with perfect CSI, which is in agreement with the results in [10].

IV. Numerical Results

We verify the correctness of the theoretical results obtained for the SER of cooperative communication systems by simulations. Exemplarily, we consider a AF cooperative communication system with 4-QAM and 8-PSK modulation formats using the PSAM scheme for the channel estimation. In our performance evaluations, we set $P_S = P_R$ and the variance of the noise was chosen to be $N_0 = 1$. We also assume that the complex channel gains are described by the autocorrelation functions $R_{SD}(\kappa) = R_{SR}(\kappa) = R_{RD}(\kappa) = J_0(2\pi f_{\max}\kappa T_s)$, where $J_0(x)$ is the zeroth order Bessel function of the first kind, $f_{\max}$ is the maximum Doppler frequency, and T_s is the symbol duration. Note that the variances of the complex channel gains are normalized to unity. We further assume that a pilot spacing of $L = 5$ is used in all simulations. This value ensures adequate channel sampling for the system under consideration. The power loss resulting from the pilots is accounted for in all curves. Finally, we assume that the LMMSE is used for the channel estimation and N=4 pilot symbols are employed to estimate the channel coefficients.

Figure 1 shows the theoretical and the Monte Carlo simulation results of the SER for the AF cooperative communication system with 4-QAM. The results are presented for two different values of the normalized maximum Doppler frequency, namely $f_{\max}T_s = 0.01$ and $f_{\max}T_s = 0.05$. The graphs with perfect CSI at the relay and destination terminal are also provided to serve as benchmarks. From Fig. 1, we observe that the theoretical results fit well with the simulated SER for both cases. This justifies the Gaussian approximation assumed for the distribution of $n_2(l)$ in (16). We also find that when the PSAM scheme is used for the channel estimation, the SER performance of the AF cooperative communication system will degrade as the normalized maximum Doppler frequency $f_{\max}T_s$ increases. However, if we assume perfect CSI, the normalized maximum Doppler frequency will not affect

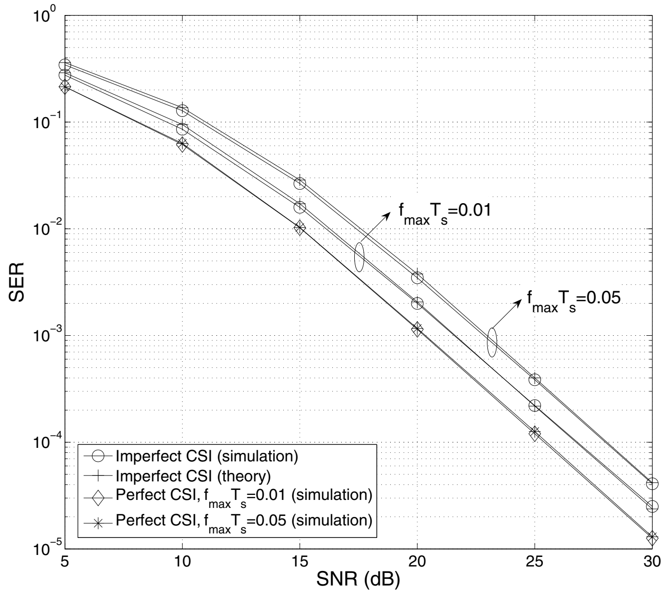

Fig. 1. Comparison of the theoretical and simulation results obtained for the SER of AF cooperative communication systems with 4-QAM signals for various values of the normalized maximum Doppler frequency $f_{\max}T_s$.

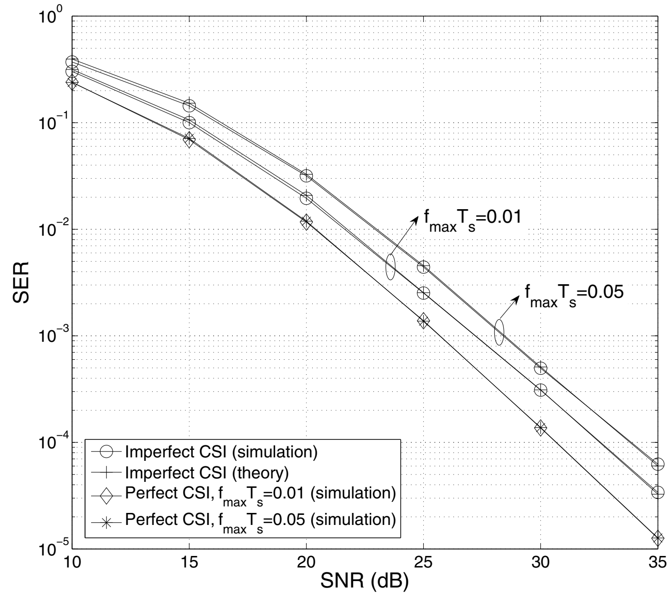

Fig. 2. Comparison of the theoretical and simulation results obtained for the SER of AF cooperative communication systems with 8-PSK signals for various values of the normalized maximum Doppler frequency $f_{\max}T_s$.

the SER performance of the AF cooperative communication system.

Figure 2 shows the theoretical and Monte Carlo simulation results of the SER for the AF cooperative communication system with 8-PSK. Also shown is the SER of the AF cooperative communication system with perfect CSI. Again, we can see that the theoretical results fit well with the simulated SER for both $f_{\max}T_s = 0.01$ and $f_{\max}T_s = 0.05$ cases. The simulation results in Figs. 1 and 2 confirm the validity and accuracy of our SER analysis.

V. Conclusions

We dealt with the problem of performance analysis of AF cooperative communication systems with a PSAM-based LMMSE scheme used for the channel estimation. By deriving the PDF and the MGF of the instantaneous SNR at the destination terminal, we developed an accurate SER formula for AF cooperative communication systems with M-QAM and M-PSK signals. The results encompass the SER performance of AF cooperative communication systems with perfect CSI as a special case. Computer simulation results demonstrated the correctness of our performance analysis. The simulation results also show that the performance of the AF cooperative communication systems are affected by the quality of the channel estimation.

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