Performance Analysis of LMMSE Receivers for M-ary QAM in Rayleigh Faded CDMA Channels

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***Abstract*—In this paper, we develop approximations for the symbol error rate in a wireless code-division multiple-access channel. We assume that each user employs spectrally efficient M-ary quadrature amplitude modulation and undergoes independent Rayleigh fading. We study the performance of linear minimum mean-squared error receivers in situations where: i) the channels of all users are known perfectly; ii) the receiver knows only the average powers of the interferers but the channel of the user of interest is still assumed to be perfectly known; and iii) the channels of the interferers are unknown and there is estimation error in the channel estimate of the user of interest. In the last of these cases, the symbol error rate is a function of the variance of the channel estimation error. We also determine an expression for this error variance when the channel estimate is obtained from optimal linear smoothing of a sequence of pilot symbols. Recent results on the performance of linear receivers in large systems with random spreading play an important role in our developments.**

***Index Terms*—Channel estimation, code-division multiple access (CDMA), linear receivers, multiuser detection, pilot symbols, random spreading, Rayleigh fading.**

I. Introduction

THE linear minimum mean-squared error (LMMSE) multiuser receiver has received considerable attention in the past decade especially in the context of code-division multiple-access (CDMA) networks ([1, ch. 6] and references therein). This popularity can be attributed to the excellent tradeoff between performance and complexity that is offered by the LMMSE receiver along with the fact that adaptive implementations of the receiver require very little in the way of side information [2]–[5].

Most of the multiuser detection literature has focused on situations where each user employs simple modulation formats such as binary or quadrature phase-shift keying. In this paper, we consider the performance of a number of different LMMSE receivers for a CDMA channel with each user employing spectrally efficient M-ary quadrature amplitude modulation (QAM). For related work on CDMA systems employing spectrally efficient modulation, see [6]–[8].

We model the uplink path gains as independent Rayleigh faded channels and assume rapid variation of these channels with time. The performance measure of interest is the symbol error rate averaged over the Rayleigh distributed path gains. We obtain approximate expressions for this average symbol error rate for a number of cases.

1) We first assume that the channels of all users are known perfectly at the receiver (Section III).
2) We next drop the assumption that the channels of interfering users are known and assume that the receiver knows only the average powers of the interferers. The channel of the user of interest is still assumed to be perfectly known (Section IV).
3) Finally, we consider the more general case where we also allow for an estimation error in the channel estimate of the user of interest (Section V).

In the last (and most practical) of these cases, the symbol error rate is a function of the variance of the channel estimation error. We move on in Section VI to determine an expression for this error variance when the channel estimate is obtained from optimal linear smoothing of a sequence of pilot symbols that are periodically inserted into the stream of data symbols. The use of pilot symbols has been considered in single-user Rayleigh faded channels in the seminal work [9]. This paper extends the approach of [9] to CDMA systems employing LMMSE receivers. Other work considering performance analysis of pilot symbol assisted receivers for CDMA channels can be found in [10]–[13].

In all of the above cases, our final performance measures are simplified using large system approximations that involve modeling the signature sequences as random quantities and looking at the limit of large spreading gain and a large number of users [14]–[16].

Throughout this paper, the symbol error rate approximations are compared with simulated values for practical sized systems to verify their accuracy.

II. Rayleigh Fading CDMA Channels

The model for the received signal after down conversion and chip-matched filtering is given by

$$\mathbf{r}(m) = \sqrt{P}a_1(m)b_1(m)\mathbf{s}_1 + \sum_{k=2}^{K}\sqrt{P}a_k(m)b_k(m)\mathbf{s}_k + \mathbf{w}(m) \quad (1)$$

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where $a_k(m)$ is the channel of user k in symbol m, $b_k(m)$ is the data symbol for user k in symbol m, $\mathbf{w}(m)$ is a white Gaussian noise vector, and $\mathbf{s}_k$ is the signature sequence of user k, which we assume is repeated from symbol to symbol. Each signature sequence is a column vector of length N (the processing gain), which is assumed known at the receiver. For performance analysis, we will assume that the entries (chips) of $\mathbf{s}_k$ are independent and identically distributed random variables with mean zero and variance $1/N$. Write $\mathbf{S} = [\mathbf{s}_1\, \mathbf{s}_2 \ldots \mathbf{s}_K]$ for the $N \times K$ matrix of signature sequences and $\mathbf{S}_1 = [\mathbf{s}_2\, \mathbf{s}_3 \ldots \mathbf{s}_K]$ for the $N \times (K-1)$ matrix with the signature sequence of user one removed.

A more compact representation for the signal model is

$$\begin{aligned}\mathbf{r}(m) &= \sqrt{P}\mathbf{S}\mathbf{A}(m)\mathbf{b}(m) + \mathbf{w}(m)\\ &= \sqrt{P}a_1(m)b_1(m)\mathbf{s}_1 + \sqrt{P}\mathbf{S}_1\mathbf{A}_1(m)\mathbf{b}_1(m) + \mathbf{w}(m)\end{aligned}$$

where $\mathbf{A}(m) = \mathrm{diag}(a_1(m), a_2(m), \ldots, a_K(m))$, $\mathbf{b}(m) = [b_1(m)b_2(m)\ldots b_K(m)]'$ and $\mathbf{A}_1(m)$ and $\mathbf{b}_1(m)$ are similarly defined but with $a_1(m)$ and $b_1(m)$ removed.

We further assume the following.

1) The channel process $\{a_k\}$ is a stationary, circularly symmetric, complex Gaussian random process with $E[a_k(m)] = 0$ and $E[a_k(m+n)a_k^*(m)] = R_a(n)$ with $R_a(0) = 1$.
2) The vector noise process $\{\mathbf{w}(m)\}$ is a stationary, circularly symmetric, complex Gaussian random process with $E[\mathbf{w}(m)] = 0$ and $E[\mathbf{w}(m+n)\mathbf{w}^*(m)] = \sigma^2\mathbf{I}\delta(n)$.
3) The data process $\{b_k\}$ is a white random process with each $b_k(m)$ selected from an M-ary QAM alphabet with $E[b_k(m)] = 0$ and $E[|b_k(m)|^2] = 1$.
4) The signature sequences, data, noise, and channel processes are independent.

One assumption that we have made above is that the average (received) power of each user is the same (since $E[|a_k(m)|^2] = 1$ for all k). This might correspond to a situation where there is power control at the central receiver: assume that the power control operates on a time scale that is slow compared with the Rayleigh fading but fast compared to the changes in average power due to distance-based path loss and shadowing. Having said that, the assumption is made only for ease of exposition, and all results can be readily extended to the more general situation of unequal average received powers. The term signal-to-noise ratio (SNR) is reserved for the quantity P/σ^2.

Throughout, we denote vectors and matrices (random and deterministic) by boldface characters. The transpose of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}'$ and the conjugate transpose by $\mathbf{A}^*$.

III. Slowly Fading Channels

A. The Linear MMSE Detector

The linear multiuser receiver for a particular user (say, User 1) takes the received vector in symbol m, $\mathbf{r}(m)$, and produces a scalar decision statistic $z(m) = \mathbf{c}^*(m)\mathbf{r}(m)$, where the linear receiver $\mathbf{c}(m)$ is a (complex) column vector of length N. The LMMSE receiver is chosen so as to minimize the mean-squared error between the data symbol of the user of interest $b_1(m)$ and the output of the linear receiver $z(m)$: $\Delta = E[|b_1(m) - \mathbf{c}^*(m)\mathbf{r}(m)|^2]$. The well-known solution [1, ch. 6] is to choose

$$\mathbf{c}(m) = \left(E\left[\mathbf{r}(m)\mathbf{r}^*(m)\right]\right)^{-1} E\left[b_1^*(m)\mathbf{r}(m)\right]$$

where we note that the presence of the background noise guarantees the existence of the matrix inverse. It is worth remarking that the above expectations should be seen as conditioned on whatever information is assumed known by the receiver. This information might consist of the signature sequences of all users and perhaps the channels of some or all of the users.

In this section, we consider a situation where the channels of all users fade slowly enough so that it is reasonable to assume that the channels of all users are perfectly known. With this assumption, we have from (1)

$$E\left[\mathbf{r}(m)\mathbf{r}^*(m)\right] = P\mathbf{S}\mathbf{D}(m)\mathbf{S}^* + \sigma^2\mathbf{I}$$

where

$$\mathbf{D}(m) = \mathrm{diag}\left\{|a_1(m)|^2, |a_2(m)|^2, \ldots, |a_K(m)|^2\right\}$$

and

$$E\left[b_1^*(m)\mathbf{r}(m)\right] = \sqrt{P}a_1(m)\mathbf{s}_1.$$

The LMMSE receiver for User 1 is thus given by

$$\mathbf{c}(m) = \sqrt{P}a_1(m)\left(P\mathbf{S}\mathbf{D}(m)\mathbf{S}^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1.$$

Making use of the matrix inversion lemma, we have alternatively

$$\mathbf{c}(m) = \kappa\sqrt{P}a_1(m)\left(P\mathbf{S}_1\mathbf{D}_1(m)\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1$$

where $\mathbf{D}_1(m) = \mathrm{diag}\{|a_2(m)|^2, \ldots, |a_K(m)|^2\}$ and

$$\kappa = \left[1 + P\,|a_1(m)|^2\,\mathbf{s}_1^*\left(P\mathbf{S}_1\mathbf{D}_1(m)\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1\right]^{-1}$$

is a real scalar. The decision statistic produced at the output of the receiver is

$$z(m) = \kappa\sqrt{P}a_1^*(m)\mathbf{s}_1^*\left(P\mathbf{S}_1\mathbf{D}_1(m)\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{r}(m).$$

Writing

$$\beta_s = P\mathbf{s}_1^*\left(P\mathbf{S}_1\mathbf{D}_1(m)\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1 \tag{2}$$

and substituting for $\mathbf{r}(m)$ from (1), we see that

$$z(m) = \kappa\,|a_1(m)|^2\,\beta_s b_1(m) + \kappa v(m) \tag{3}$$

where $v(m)$ is the sum of the interference and noise terms

$$\begin{aligned}v(m) = \sqrt{P}a_1^*(m)\mathbf{s}_1^*&\left(P\mathbf{S}_1\mathbf{D}_1(m)\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\\ &\times\left(\sum_{k=2}^{K}\sqrt{P}a_k(m)b_k(m)\mathbf{s}_k + \mathbf{w}(m)\right).\end{aligned}$$

Using the statistical properties of the data symbols and the noise, we see that

$$E\left[v(m)\right] = 0, \quad E\left[|v(m)|^2\right] = |a_1(m)|^2\,\beta_s \tag{4}$$

and that $v(m)$ is a circularly symmetric (complex) random variable.

We assume that the output of the LMMSE receiver $z(m)$ is passed to a decision device. This decision device forms

$$\hat{b}_1(m) = \frac{z(m)}{\kappa \left|a_1(m)\right|^2 \beta_s} = b_1(m) + \frac{v(m)}{\left|a_1(m)\right|^2 \beta_s} \quad (5)$$

and chooses the constellation point closest to $\hat{b}_1(m)$. This simple minimum distance detector would in fact be the maximum likelihood detector (based on observation of $z(m)$) if the interference plus noise term $v(m)$ was a complex Gaussian random variable. Indeed there is a lot of evidence to suggest that this Gaussian approximation is accurate for a wide range of system parameters [17], [18]. Next, we will evaluate the symbol error rate under this Gaussian approximation.

Before proceeding, we note that it should be clear from (3) and (4) that the quantity $|a_1(m)|^2\beta_s$ is the signal-to-interference ratio (SIR) of the LMMSE receiver.

B. Symbol Error Rate

Our starting point in this section is the simple model of (5), i.e.,

$$\hat{b}_1(m) = b_1(m) + u(m)$$

where (under our approximation) $u(m)$ is a circularly symmetric complex Gaussian random variable with zero mean and variance $1/(|a_1(m)|^2\beta_s)$.

Using the (single-user) results presented in Appendix A, we see immediately that the symbol error rate for square QAM, conditioned on β_s, is given by

$$P_M = \left(1 - \frac{1}{M}\right) - 2\rho\left(1 - \frac{1}{\sqrt{M}}\right) \times \left[\frac{1}{\sqrt{M}} + \frac{2}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\tan^{-1}\rho\right] \quad (6)$$

where

$$\rho = \sqrt{\frac{\beta_s}{\frac{2}{3}(M-1) + \beta_s}}.$$

An upper bound on the symbol error probability is

$$P_M < 2\left(1 - \frac{1}{\sqrt{M}}\right)[1-\rho] < 2(1-\rho). \quad (7)$$

It should be noted that (22) is only an upper bound when the noise term is Gaussian. In our model, it should be considered as an approximation only—not as an upper bound.

C. Large System Approximation

The performance measures derived in the previous section were left as functions of the average SIR β_s. Referring back to (2), we see that β_s is a function of the signature sequences of all users and the channels of all users except User 1. The average symbol error rates we obtained were averaged over the channel of User 1 but were not averaged over β_s.

We could attempt to determine the distribution of β_s based on knowledge of distributions of the signature sequences and channels; however, this is a very difficult problem. Alternatively, we can call on some powerful new results that look at the behavior of β_s when the system is large [15], [16], [19]. We have the following result, which follows directly from [16, Theorem 3.1].

Result 1: As $N \to \infty$ with $K = \alpha N$ and α constant, and under some mild assumptions on the signature sequences (in addition to those given in Section II), the random variable

$$\beta_s = P\mathbf{s}_1^*\left(P\mathbf{S}_1\mathbf{D}_1(m)\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1$$

converges in probability to the deterministic $\beta_s^\dagger$, which is the unique positive solution to the fixed-point equation

$$\beta_s^\dagger = \left[\frac{\sigma^2}{P} + \alpha\int_0^\infty \frac{x}{1 + x\beta_s^\dagger}e^{-x}dx\right]^{-1}. \quad (8)$$

Remark 1: In the above, we have used the fact that the probability density function of the squared magnitude of the channel gains, (i.e., $|a_k(m)|^2$) is given by

$$f(x) = \begin{cases} 0, & x < 0 \\ e^{-x}, & x \geq 0. \end{cases}$$

Remark 2: Equation (8) is easily solved using the simple iteration

$$\beta_s^{(i+1)} = \left[\frac{\sigma^2}{P} + \alpha\int_0^\infty \frac{x}{1 + x\beta_s^{(i)}}e^{-x}dx\right]^{-1}$$

starting with any initial condition $\beta_s^{(0)} \geq 0$. The integral in the above iteration can be expressed in terms of an exponential integral function.

The limiting value of β_s is a function of the average SNR (P/σ^2) and the system loading (α). We have essentially obtained our averaging by looking at a large system. In the limit, the dependence on the realizations of the signature sequences and the channel gains disappears and we are left with a deterministic quantity.

We can substitute $\beta_s^\dagger$ for β_s in (21) or (22) to get our final approximations for the average symbol error probability.

D. Accuracy of Approximation

In this section, we will give an indication of the accuracy of the Gaussian and the large system approximations for a finite sized system. Fig. 1 compares results obtained from (6) (with $\beta_s = \beta_s^\dagger$) with results obtained from simulation. The horizontal axis is the average SNR per bit $P/(\sigma^2\log_2 M)$. Results are shown for constellations with $M = 4$, $M = 16$, and $M = 64$. The simulations have spreading gain $N = 31$ and have $K = 20$ users; hence we have $\alpha = 20/31$. In each trial, new signature sequences and new channels are randomly generated. For each combination of parameters, trials are conducted until 1000 symbol errors are observed. While the system simulated has only a moderate processing gain, we observe that the large system approximation to the symbol error rate is quite accurate over the range of parameters tested.

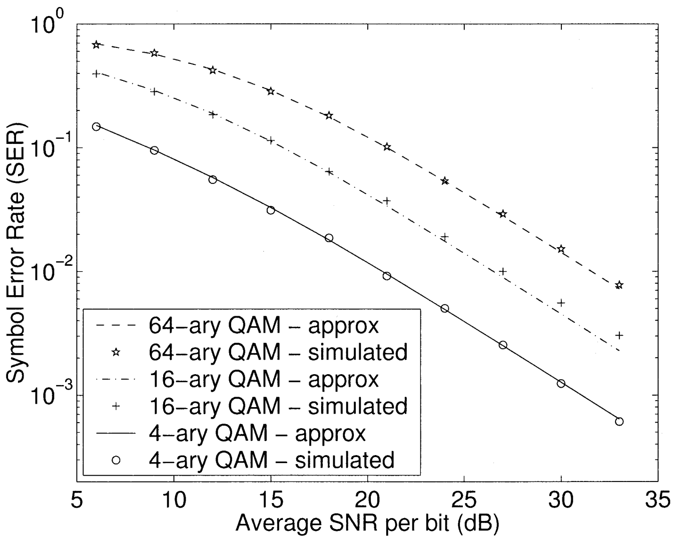

Fig. 1. Comparison of SER approximation wth simulation $N = 31,\ K = 20$.

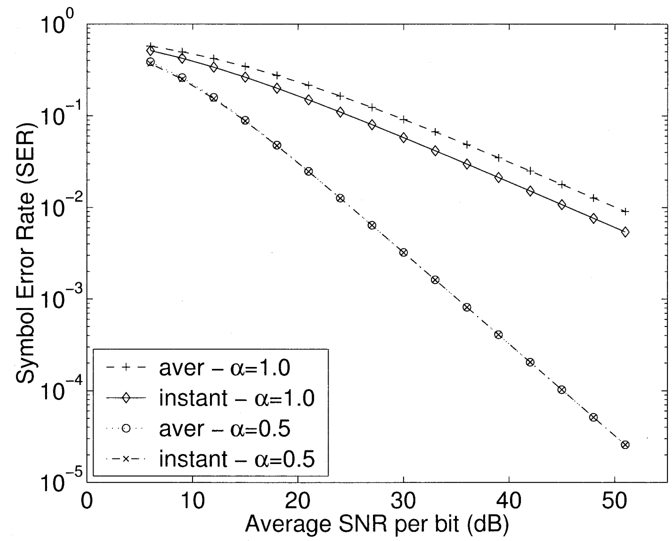

Fig. 2. Performance of MMSE receivers with average and instantaneous powers.

IV. Fast Fading Channels With Perfect Channel Estimation

In this section, we will drop the assumption that the channels of the interfering users are known and assume only that the receiver has knowledge of the average powers of the interferers. For the moment we will stick with the assumption that the receiver has a perfect estimate for the channel of User 1—the user of interest. With these assumptions, and referring to (1), we have

$$E\left[\mathbf{r}(m)\mathbf{r}^*(m)\right] = P\left|a_1(m)\right|^2 \mathbf{s}_1\mathbf{s}_1^* + P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}$$

where we have used the fact that $E[|a_k(m)|^2] = 1$ for all users, and

$$E\left[b_1^*(m)\mathbf{r}(m)\right] = \sqrt{P}a_1(m)\mathbf{s}_1.$$

In this case the expectations are not conditioned on the channels of the interfering users $a_2(m), \ldots, a_K(m)$. This means that the diagonal matrix

$$\mathbf{D}(m) = \mathrm{diag}\left\{\left|a_1(m)\right|^2, \left|a_2(m)\right|^2, \ldots, \left|a_K(m)\right|^2\right\}$$

from the previous section is replaced by the matrix $\mathrm{diag}\{|a_1(m)|^2, 1, \ldots, 1\}$. The LMMSE receiver for User 1 is thus given by

$$\mathbf{c}(m) = \kappa\sqrt{P}a_1(m)\left(P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1 \tag{9}$$

where

$$\kappa = \left[1 + P\left|a_1(m)\right|^2 \mathbf{s}_1^*\left(P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1\right]^{-1} \tag{10}$$

and we have again made use of the matrix inversion lemma.

We can proceed from this point exactly as in Section III but with $\mathbf{D}_1(m)$ replaced by $\mathbf{I}$ throughout. In particular, β_s will be replaced by β_f where

$$\beta_f = P\mathbf{s}_1^*\left(P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1. \tag{11}$$

We have the following result concerning the large system limit of β_f.

Result 2: As $N \to \infty$ with $K = \alpha N$ and α constant, and under some mild assumptions on the signature sequences (in addition to those given in Section II), the random variable β_f converges in probability to the deterministic $\beta_f^\dagger$, which is the unique positive solution to the fixed-point equation

$$\beta_f^\dagger = \left[\frac{\sigma^2}{P} + \alpha\frac{1}{1+\beta_f^\dagger}\right]^{-1}.$$

In this case, the fixed-point equation is a quadratic equation with desired solution

$$\beta_f^\dagger = \frac{1}{2}\left((1-\alpha)^2\left(\frac{P}{\sigma^2}\right)^2 + 2(1+\alpha)\left(\frac{P}{\sigma^2}\right) + 1\right)^{\frac{1}{2}} + \frac{1}{2}\left((1-\alpha)\left(\frac{P}{\sigma^2}\right) - 1\right). \tag{12}$$

Final approximate expressions for the average symbol error rate when each user employs M-ary QAM are obtained by replacing β_s with $\beta_f^\dagger$ in (6) and (7).

The linear MMSE receiver we consider in this section required less information than the receiver considered in Section III. In particular, we assume in this section that the average power of the interferers was available but not the (instantaneous) channel gains of the interferers. It is of interest to examine the loss in performance that results from going without knowledge of the interferers channels (similar comparisons are discussed in [19] and [20]).

Fig. 2 compares the approximate symbol error rate of both receivers for 16-ary QAM. These curves are generated from (6) with β_s replaced by $\beta_s^\dagger$ or $\beta_f^\dagger$. Plots are shown for system loadings of $\alpha = 0.5$ and $\alpha = 1$. The performance loss from using average powers is negligible for $\alpha = 0.5$ and still quite small for $\alpha = 1$.

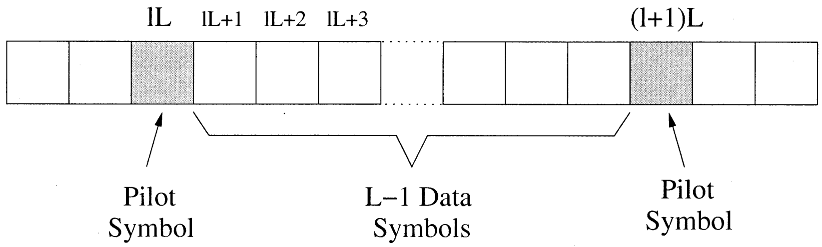

Fig. 3. Frame structure.

V. Fast Fading Channels With Channel Estimation Error

In this section, we will consider the more general case where we have only an imperfect estimate of the channel of User 1 at the receiver.

The receiver will be as in (9) but with $a_1(m)$ replaced by an estimate $\hat{a}_1(m)$

$$\mathbf{c}(m) = \sqrt{P}\hat{a}_1(m)\left(P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1.$$

Note that the real scalar κ in (9) does not actually impact the results and has thus been omitted. We can break this receiver into two parts: the first is the multiuser receiver whose job is solely to suppress interference; the second is the single-user receiver responsible for rotating and scaling the scalar output of the multiuser receiver.

The first part can be made totally independent of $a_1(m)$ (and of m)

$$\bar{\mathbf{c}} = \sqrt{P}\left(P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1 \tag{13}$$

and produces as output

$$r(m) = \bar{\mathbf{c}}^*\mathbf{r}(m) = \beta_f a_1(m)b_1(m) + v(m)$$

where

$$v(m) = \bar{\mathbf{c}}^*\left[\sum_{k=2}^{K}\sqrt{P}a_k(m)b_k(m)\mathbf{s}_k + \mathbf{w}(m)\right]$$

is a circularly symmetric complex random variable with zero mean and variance β_f that is independent of $a_1(m)$ and $b_1(m)$. Note that we will come across the time-invariant $\bar{\mathbf{c}}$ again when we look at channel estimation.

At this point we make the following approximations.

A1) As in the previous sections we approximate the noise plus interference term as a complex Gaussian random variable.

A2) We also approximate the channel estimation error $a_1(m) - \hat{a}_1(m)$ as a circularly symmetric complex Gaussian random variable with mean zero and variance Δ that is independent of $\hat{a}_1(m)$, $v(m)$, and $b_1(m)$.

Under these approximations the (single-user) results from Appendix A on the symbol error rate for square QAM with channel estimation errors can be directly applied. In particular, we can approximate the symbol error rate by (24) where

$$\rho(x) = \sqrt{\frac{\beta_f(1-\Delta)}{\frac{2}{3}(M-1) + \beta_f\left[1 + \left(\frac{2}{3}(M-1)x^2 - 1\right)\Delta\right]}}. \tag{14}$$

In a large system we replace β_f by $\beta_f^\dagger$ to get our final approximation.

VI. Pilot-Based LMMSE Channel Estimation

In the previous section we derived an upper bound on the symbol error probability for our multiuser M-ary QAM system in the presence of channel estimation error. The resulting error bound is a function of the variance of the channel estimation error, and it is valuable to relate this variance to key physical parameters such as the channel fading rate and the quantity of resources dedicated to channel estimation. This section provides a summary of the relevant results from [13], which we need in this paper to proceed with our proposed coupled estimator/detector structure and subsequent analysis.

In this section, we consider a particular channel estimation structure where pilot symbols are periodically inserted into the sequence of data symbols of user one. In particular, suppose that a pilot symbol is inserted after every block of $L-1$ data symbols at locations $\ldots, -2L, -L, 0, L, 2L, \ldots$, as illustrated in Fig. 3.

We will assume for simplicity that $b_1(lL) = 1$. Define the sampled output sequence

$$\begin{aligned}\mathbf{y}(l) &= \mathbf{r}(lL)\\ &= \sqrt{P}x(l)\mathbf{s}_1 + \sum_{k=2}^{K}\sqrt{P}a_k(lL)b_k(lL)\mathbf{s}_k + \mathbf{w}(lL)\end{aligned} \tag{15}$$

where we have defined $x(l) = a_1(lL)$.

A. Optimal Linear Smoothing

We wish to make use of the entire sequence of pilot symbols to obtain a channel estimate for user one at any time. To begin, observe that we must handle the general situation of estimating the channel during a symbol that does not necessarily coincide with a pilot point. Define the shifted and sampled channel process for User 1

$$x_p(l) = a_1(lL+p), \quad 0 \le p \le L-1$$

and note that $x(l) = x_0(l)$. We wish to estimate $x_p(l)$ by linearly smoothing the (pilot) observation process $\{\mathbf{y}(l)\}$. The optimal linear smoother will not depend on l but will vary with p.

To begin we define, for the sampled channel process, the autocorrelation function

$$R_{x_p}(n) = E\left[x_p(l+n)x_p^*(l)\right] = R_a(nL)$$

and the power spectral density

$$S_{x_p}(w) = \sum_{n=-\infty}^{\infty} R_{x_p}(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} R_a(nL)e^{-j\omega n}.$$

The above expressions do not depend on p, so we will write $R_{x_p}(n) = R_x(n)$ and $S_{x_p}(\omega) = S_x(\omega)$. We will also need to deal with the correlation function

$$R_{x_p x}(n) = E\left[x_p(l+n)x^*(l)\right] = R_a(nL+p)$$

and corresponding spectral density function

$$S_{x_p x} = \sum_{n=-\infty}^{\infty} R_{x_p x}(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} R_a(nL+p)e^{-j\omega n}.$$

We wish to form the estimate

$$\hat{x}_p(l) = \sum_{n=-\infty}^{\infty} \mathbf{h}_p(n)\mathbf{y}(l-n)$$

where $\mathbf{h}_p(n)$ is a row vector of length N. We have the following theorem [13].

Theorem 1: The optimal (MMSE) linear smoother is

$$\mathbf{h}_p(n) = h_p(n)\sqrt{P}\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}$$

where

$$h_p(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{S_{x_p x}(\omega)}{1+\beta_f S_x(\omega)}e^{j\omega n}d\omega \tag{16}$$

and β_f is given in (11). The corresponding MMSE is

$$\Delta_p = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\beta_f\left|S_{x_p x}(\omega)\right|^2}{1+\beta_f S_x(\omega)}d\omega. \tag{17}$$

The optimal channel estimate is thus

$$\hat{x}_p(l) = \sum_{n=-\infty}^{\infty} h_p(n)\bar{\mathbf{c}}^*\mathbf{y}(l-n)$$

where $\bar{\mathbf{c}}$ was defined in the context of data estimation in (13). This optimal linear smoother first projects the received vector at each pilot point to a scalar and then the time sequence of scalar values is processed by a single input, single output (SISO) linear smoother to produce the final channel estimates. At this point it is worth highlighting that the multiuser aspect of both data and channel estimation problems is taken care of by the same *front-end* linear multiuser receiver $\bar{\mathbf{c}}$.

Remark 3: In a large system with random spreading, β_f, as given in (11), converges to $\beta_f^\dagger$ defined in (12) and all instances of β_f above can be replaced by $\beta_f^\dagger$. The MSE Δ_p and the SISO linear smoother $h_p(n)$ are thus independent of the signature sequences in a large system.

B. Spectral Density of Sampled Channel Process

The performance of the optimal linear smoother as given in (17) depends on the spectral densities $S_x(\omega)$ and $S_{x_p x}(\omega)$. It is of interest to relate these to the power spectral density of the original channel process $\{a_1(m)\}$.

Recall that $R_x(n) = R_a(nL)$ so that the autocorrelation function of $\{x(m)\}$ results from decimating or downsampling the autocorrelation function of $\{a_1(m)\}$. The resultant power spectral density is

$$S_x(\omega) = \frac{1}{L}\sum_{m=0}^{L-1} S_a\left(\frac{\omega - m2\pi}{L}\right). \tag{18}$$

Similarly, we have

$$S_{x_p x}(\omega) = \frac{1}{L}\sum_{m=0}^{L-1} \exp\left(jp\frac{\omega - m2\pi}{L}\right)S_a\left(\frac{\omega - m2\pi}{L}\right). \tag{19}$$

These expressions can be substituted into (17) to make explicit the dependence of the MSE on the spectral density of the channel process $S_a(\omega)$, the rate of insertion of pilot symbols L, and the time shift p.

We can simplify things greatly by making some assumptions about $S_a(\omega)$ and L. In particular, suppose that

$$S_a(\omega) = 0, \quad \omega_{ND} < |\omega| \le \pi$$

and that $L\omega_{ND} < \pi$. These conditions imply that there is no aliasing so that

$$S_x(\omega) = \frac{1}{L}S_a\left(\frac{\omega}{L}\right), \quad |\omega| \le \pi$$

and

$$S_{x_p x}(\omega) = \frac{1}{L}\exp\left(jp\frac{\omega}{L}\right)S_a\left(\frac{\omega}{L}\right), \quad |\omega| \le \pi.$$

This allows us to write

$$h_p(n) = \frac{1}{2\pi}\int_{-\omega_{ND}}^{\omega_{ND}} \frac{S_a(\omega)}{1+\frac{\beta_f}{L}S_a(\omega)}e^{j\omega(p+nL)}d\omega$$

and

$$\Delta = \frac{1}{2\pi}\int_{-\omega_{ND}}^{\omega_{ND}} \frac{S_a(\omega)}{1+\frac{\beta_f}{L}S_a(\omega)}d\omega.$$

The impact of varying L on the MSE is clearly isolated in this expression. Also observe that the MSE does not depend on p, which means that the channel estimate in the middle of a frame is just as good as an estimate at a pilot symbol.

C. Special Case: Ideal Low-Pass Channel

Before proceeding, we consider the special case where the channel is bandlimited and has a flat spectrum up to the normalized Doppler frequency ω_{ND}:

$$S_a(\omega) = \begin{cases} \frac{\pi}{\omega_{ND}}, & \omega < \omega_{ND} \\ 0, & \text{otherwise.} \end{cases}$$

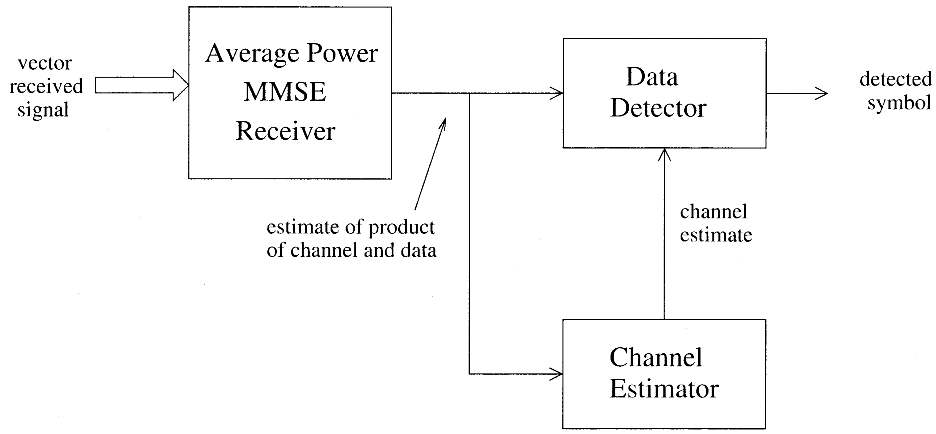

Fig. 4. Structure of receiver.

If $L\omega_{ND} < \pi$, then we have

$$h_p(n) = \left(1 + \frac{\pi\beta_f}{L\omega_{ND}}\right)^{-1} \left[\frac{\sin \omega_{ND}(p+nL)}{\omega_{ND}(p+nL)}\right]$$

and

$$\Delta = \left(1 + \frac{\pi\beta_f}{L\omega_{ND}}\right)^{-1}.$$

VII. Coupled Estimator/Detector Structure

The results of Sections V and VI can now be combined to give a coupled data and channel estimator along with an approximate expression for the performance, as measured by symbol error rate. The receiver is illustrated in Fig. 4.

The first stage of LMMSE channel estimation and of LMMSE data estimation (assuming only the average power of the interferers is known) involves forming

$$r(m) = \bar{\mathbf{c}}^* \mathbf{r}(m) = \beta_f a_1(m) b_1(m) + v(m)$$

where

$$\bar{\mathbf{c}} = \sqrt{P}\left(P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_1$$

is given in (13) and repeated here for convenience.

In order to obtain channel estimates for times $\ldots, -L+p, p, L+p, \ldots$, the sequence of outputs at the pilot points $m = \ldots, -L, 0, L, \ldots$ is passed through a smoother with impulse response given by $\ldots, h_p(-1), h_p(0), h_p(1), \ldots$, where $h_p(n)$ is given in (16). The channel estimates are then fed to a data detector, which forms

$$\hat{b}_1(m) = \frac{\hat{a}_1^*(m) r(m)}{\beta_f \left|\hat{a}_1^*(m)\right|^2}$$

and then passes this soft data estimate to a minimum distance detector.

The front-end linear multiuser receiver is time-invariant, depending only on the average powers of the interfering users and the signature sequences of all users, which we assume are repeated from symbol to symbol. This front-end is thus appropriate for situations when the channels of all users are changing quickly. The output of the multiuser receiver is a scaled estimate of the product $a_1(m)b_1(m)$ and the remainder of the receiver is simply a pilot symbol aided single-user receiver.

An approximate expression for the symbol error rate of this multiuser receiver can be obtained from (24) and (14), remembering that the channel estimation error is in general dependent on the position of the symbol relative to the pilots. We have

$$P_M = \frac{1}{L-1} \sum_{p=1}^{L-1} P_M^{(p)}$$

where

$$P_M^{(p)} \approx \frac{2}{M} \sum_{i \in S_1} \left[1 - \rho_p\left(|b_i|\right)\right] + \frac{3}{2M} \sum_{j \in S_2} \left[1 - \rho_p\left(|b_j|\right)\right] + \frac{1}{M} \sum_{k \in S_3} \left[1 - \rho_p\left(|b_k|\right)\right]$$

with

$$\rho_p(x) = \sqrt{\frac{\beta_f^\dagger (1 - \Delta_p)}{\frac{2}{3}(M-1) + \beta_f^\dagger \left[1 + \left(\frac{2}{3}(M-1)x^2 - 1\right)\Delta_p\right]}}$$

and where Δ_p is given by (17) (with $\beta_f = \beta_f^\dagger$). As defined in the Appendix, $\{b_1, b_2, \ldots, b_M\}$ are the points in the M-ary QAM constellation normalized such that the average energy per symbol is one and S_1, S_2, and S_3 are index sets for interior, edge, and corner symbols, respectively.

If the channel is bandlimited and pilots are inserted frequently enough to avoid aliasing, then as mentioned in Section VI, the channel estimation error variance Δ_p does not depend on the position within a frame p, and a simpler error expression results.

The final symbol error rate approximation is a function of

- the number of symbols in the QAM constellation M;
- the pilot insertion period L;
- the average signal-to-noise ratio P/σ^2;
- the system loading $\alpha = K/N$;
- the (common) power spectral density of the channels $S_a(\omega)$.

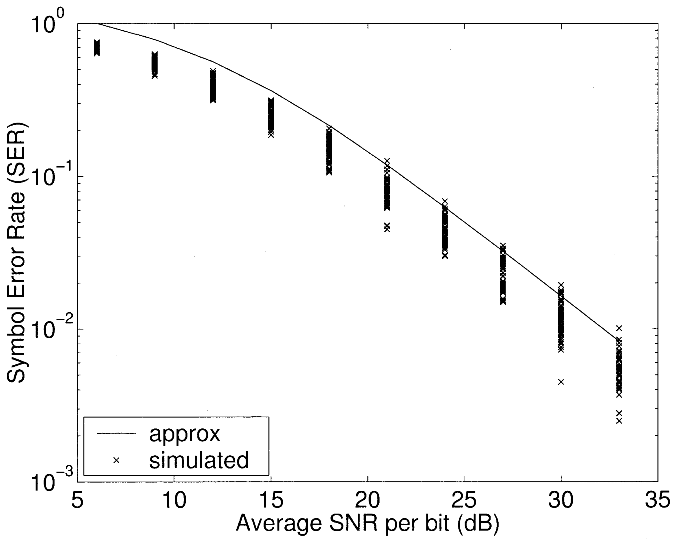

Fig. 5. Comparison of SER approximation with simulation.

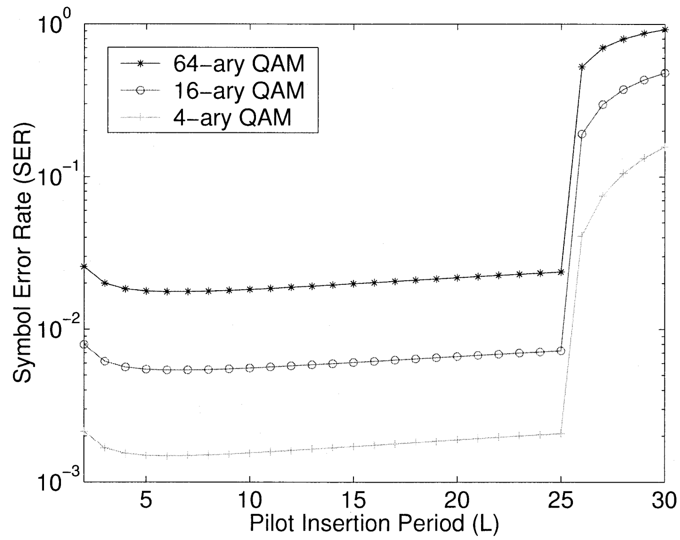

Fig. 6. Variation of SER approximation with pilot insertion period for different constellation sizes $\alpha = 0.5$.

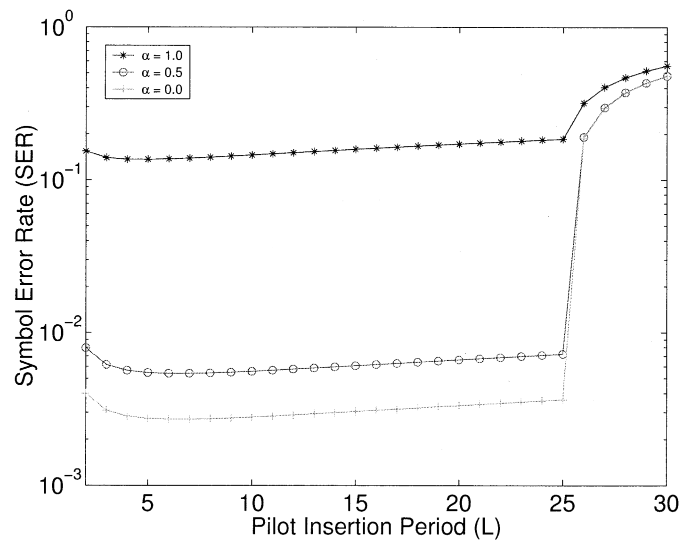

Fig. 7. Variation of SER approximation with pilot insertion period for different system loadings $M = 16$.

VIII. Numerical Studies

In this section, we present some simulation results to explore the accuracy of our analytical performance measures and then study the performance of the coupled receiver as the rate of inserting pilot symbols is varied.

A. Comparison With Simulation

We begin by listing all the parameters for the system under study.

1) We consider a synchronous CDMA system with 25 users and a spreading gain of 50 leading to a system loading of $\alpha = K/N = 0.5$.
2) For each of 50 simulation runs, the spreading sequences are randomly chosen at the beginning and then fixed for the duration of the run. Each simulation point thus corresponds to a different realization of the spreading sequences—there is no averaging over the sequences.
3) Each user employs 64-QAM and sends symbols at a rate of 64 ksps so that the bit rate is 384 kbps.
4) The channel of each user is assumed to have a flat spectrum up to a cutoff frequency of $\omega_{ND} = 0.04\pi$, which corresponds to a normalized Doppler frequency of 0.02. In simulations, this channel was approximated using a fifth-order Chebyshev filter.
5) Pilot symbols are inserted after every nine data symbols so that $L = 10$. For the simulated system, channel estimates are obtained using a 33-tap smoother formed by truncating the optimal smoother given in Section VI-C.

Fig. 5 shows the symbol error rate versus average SNR obtained from simulation along with the analytical approximation. Results from all 50 simulation runs are plotted for each value of average SNR per bit.

The spread in SER values due to choice of signature sequences clearly increases as the average SNR per bit increases. When the SNR is low, the performance is limited by the background noise and thus the impact of randomly chosen spreading sequences is not great. When the SNR is high, however, the performance is limited by the multiple access interference and the system is significantly more sensitive to the choice of spreading sequences. It is important to note that, for all SNR values, this spread will decrease as the size of the system increases (as N and K get larger) in line with the large system results presented earlier.

Our approximate expression for the symbol error rate is seen to be larger than the values obtained by simulation for almost all realizations of the signature sequences. This makes our approximations fairly robust to the choice of signature sequences—even for moderate size systems such as the one simulated.

B. Variation of Performance With Pilot Insertion Period

We now examine how the symbol error rate varies with L in a number of settings. Figs. 6 and 7 plot the approximate symbol error rate versus L for $2 \le L \le 30$. In both plots, we have assumed that the channel spectrum is flat with $\omega_{ND} = 0.04\pi$.

In this case $L = 25$ is the largest value of L that satisfies the antialiasing condition $L\omega_{ND} \leq \pi$.

Both plots have the average SNR per data bit set to 30 dB. Note that the average SNR per data bit is given by $((L/L - 1)(1/\log_2 M))P/\sigma^2$. As L and M are varied, we are thus required to vary P/σ^2 so as to maintain the average SNR per data bit at a constant value.

Fig. 6 looks at the variation in performance for $\alpha = 0.5$ and $M = 4, 16$, and 64. Fig. 7 looks at the variation in performance for $\alpha = 0, 0.5$, and 1.0, with M fixed at 16. Observe that in both cases, the performance is quite insensitive to L for $L < 25$ and degrades very quickly as soon as the antialiasing condition is broken.

IX. Conclusion

In this paper, we have derived approximate expressions for the symbol error rate of users in a wireless CDMA channel. Each user sends symbols from an M-ary QAM constellation and suffers fast Rayleigh fading. At the receiver we assume that the received signal first passes through a linear MMSE receiver and then to a minimum distance detector. We derived approximate expressions for the symbol error rate in three different scenarios.

1) The channels of all users are known perfectly at the receiver.
2) The receiver knows only the average powers of the interferers but the channel of the user of interest is still assumed to be perfectly known.
3) The receiver knows only the average powers of the interferers and there is an estimation error in the channel estimate of the user of interest.

In the last of these cases, the symbol error rate was shown to be a function of the variance of the channel estimation error. We also determined an expression for this error variance when the channel estimate is obtained from optimal linear smoothing of a sequence of pilot symbols.

While simplifying assumptions were made in this paper in order to improve the clarity of the presentation, the results obtained can be extended in a number of directions. In particular, it is trivial to modify the results for the case when the users have unequal average powers or, more generally, different channel statistics. It is also possible to handle frequency-selective fading channels within our framework.

Appendix
Symbol Error Rates for M-ary QAM

In this Appendix, we derive upper bounds on the symbol error probability for a single-user communication system that sends quadrature amplitude modulated data over a frequency-flat Rayleigh fading channel (see also [21]). Results are given first for the case of perfect channel knowledge and second for a model with imperfect channel estimates. Previous work taking into account channel estimation error can be found in [9] and [22]. In these papers, the resultant error expressions are quite involved, especially for high-order signal constellations. In [23], a method was proposed for calculating error expressions when nonperfect channel information is available, but no explicit expressions were given.

In this Appendix, we complement the previous work by focusing on (relatively) simple upper bounds on the symbol error rate of M-ary QAM with channel uncertainty. The simplifications result because we consider symbol as opposed to bit errors, give upper bounds rather than exact results, and assume that the channel estimate is obtained as the MMSE estimate, thereby enforcing a key independence property.

A. System Model

Our starting point in this section is the complex baseband signal after matched filtering, which we model by

$$r(m) = \sqrt{P}a(m)b(m) + w(m)$$

in symbol period m.

1) The data symbols $\{b(m)\}$ form a sequence of independent M-ary QAM symbols. Each of the M symbols is equally likely to occur and the constellation is scaled so that $E[|b(m)|^2] = 1$.
2) The sampled channel process $\{a(m)\}$ is modeled as a stationary sequence of circularly symmetric complex Gaussian random variables. The channel process has zero mean and we assume that $E[|a(m)|^2] = 1$.
3) The filtered and sampled noise sequence $\{w(m)\}$ is a sequence of independent circularly symmetric complex Gaussian random variables with mean zero and variance $E[|w(m)|^2] = \sigma^2$.
4) We further assume that the data, channel, and noise sequences are independent processes.

This model is a single-user equivalent of our main signal model (1).

B. Symbol Error Rate With Known Channel

Suppose initially that the channel process $\{a(m)\}$ is known perfectly at the receiver. In this case, the maximum likelihood receiver forms for each symbol the decision statistic

$$z(m) = \frac{a^*(m)r(m)}{\sqrt{P}\,|a(m)|^2} = b(m) + \frac{a^*(m)w(m)}{\sqrt{P}\,|a(m)|^2}$$

and then passes it to a minimum distance detector.

For a square M-ary QAM constellation, the symbol error probability conditioned on $a(m)$ is exactly [24]

$$P\left(\mathrm{err}|a(m)\right) = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(|a(m)|\sqrt{\frac{3}{M-1}\frac{P}{\sigma^2}}\right) - 4\left[\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(|a(m)|\sqrt{\frac{3}{M-1}\frac{P}{\sigma^2}}\right)\right]^2 \quad (20)$$

which depends on $a(m)$ through only its magnitude. We denote the above conditional error probability by $P_M(|a(m)|)$. Of more interest is the symbol error probability averaged over the Rayleigh distributed $|a(m)|$

$$P_M = \int_0^\infty P_M(u) 2ue^{-u^2} du.$$

To push further, we require expressions for definite integrals of the form

$$I_1 = \int_0^\infty Q(\xi u) u e^{-u^2} du$$

and

$$I_2 = \int_0^\infty Q^2(\xi u) u e^{-u^2} du$$

where ξ is some constant. Fortunately, closed-from expressions are available for these definite integrals; see [25, Sec. 6.287] for I_1 and [25, Sec. 8.258] for I_2. After some manipulation, the end result is

$$P_M = 1 - \frac{1}{M} - 2\rho\left(1 - \frac{1}{\sqrt{M}}\right) \times \left[\frac{1}{\sqrt{M}} + \frac{2}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\tan^{-1}\rho\right] \quad (21)$$

where

$$\rho = \sqrt{\frac{\frac{P}{\sigma^2}}{\frac{2}{3}(M-1) + \frac{P}{\sigma^2}}}$$

is a function of the average signal-to-noise ratio P/σ^2.

An upper bound on the symbol error probability results when the second term in (20) is neglected. The average error probability is then bounded by

$$P_M < 2\left(1 - \frac{1}{\sqrt{M}}\right)(1-\rho) < 2(1-\rho). \quad (22)$$

We reiterate that the above expressions apply to square QAM constellations only. Equivalent expressions could also be developed for general rectangular constellations; however, we do not pursue this here.

C. Symbol Error Rate With Estimated Channel

We now wish to consider the more realistic model where the receiver has access to a channel estimate as opposed to the actual channel. Let the channel estimate be denoted by $\hat{a}(m)$ and rewrite the received signal in the form

$$r(m) = \sqrt{P}\hat{a}(m)b(m) + \sqrt{P}\left(a(m) - \hat{a}(m)\right)b(m) + w(m).$$

The receiver ignores the fact that there is an estimation error and employs the detection rule specified in Section B with the true channel replaced by the available estimate. That is, the receiver forms

$$z(m) = \frac{\hat{a}^*(m) r(m)}{\sqrt{P}\left|\hat{a}(m)\right|^2} = b(m) + v(m)$$

where

$$v(m) = \frac{\hat{a}^*(m)\left(\sqrt{P}\left(a(m) - \hat{a}(m)\right)b(m) + w(m)\right)}{\sqrt{P}\left|\hat{a}(m)\right|^2} \quad (23)$$

and then passes $z(m)$ to a minimum distance detector.

To proceed with our analysis, we need to specify the statistical properties of the channel estimate. We will make the following assumption in this section.

Assumption: The channel estimation error $a(m) - \hat{a}(m)$ is a circularly symmetric complex Gaussian random variable with mean zero and variance Δ and is independent of $\hat{a}(m)$, $w(m)$, and $b(m)$.

This assumption will be true whenever $\hat{a}(m)$ is obtained as the minimum mean-squared error estimate of $a(m)$ based on any collection of random variables that is independent of $w(m)$ and $b(m)$.

Returning to (23) and conditioning on $b(m)$ and $\hat{a}(m)$, we see that $v(m)$ is a circularly symmetric complex Gaussian random variable with mean zero and variance

$$\sigma_v^2 = \frac{\frac{P}{\sigma^2}\left|b(m)\right|^2\Delta + 1}{\frac{P}{\sigma^2}\left|\hat{a}(m)\right|^2}.$$

In this case, the calculation of the symbol error probability is complicated by the fact that the variance of the additive noise depends on the magnitude of the transmitted data symbol.

Consider a data symbol that is not on the edge of the constellation and therefore has four neighbors at the minimum distance. The exact error probability conditioned on this symbol and the channel estimate is

$$P\left(\mathrm{err}|b(m), \hat{a}(m)\right) = 4Q(x)\left(1 - Q(x)\right)$$

where

$$x = \left|\hat{a}(m)\right|\sqrt{\frac{3}{M-1}\frac{\frac{P}{\sigma^2}}{\frac{P}{\sigma^2}\left|b(m)\right|^2\Delta + 1}}.$$

Rather than working with the exact expression, we will use the upper bound

$$P\left(\mathrm{err}|b(m), \hat{a}(m)\right) \le 4Q(x)$$

where x is given above.

While this upper bound is appropriate for all symbols in the constellation, we can tighten it slightly for edge and corner points by changing the constant in front of the Q-function to three or two, respectively. Let the positions of the symbols be listed as $b_1, \ldots, b_M$ and define S_1, S_2, and S_3 as index sets for interior, edge, and corner symbols, respectively. Averaging over the possible transmitted symbols, we arrive at the bound

$$\begin{aligned} P_M\left(\left|\hat{a}(m)\right|\right) \le & \frac{4}{M}\sum_{i\in S_1} Q\left(\left|\hat{a}(m)\right| f\left(\left|b_i\right|\right)\right) \\ & + \frac{3}{M}\sum_{j\in S_2} Q\left(\left|\hat{a}(m)\right| f\left(\left|b_j\right|\right)\right) \\ & + \frac{2}{M}\sum_{k\in S_3} Q\left(\left|\hat{a}(m)\right| f\left(\left|b_k\right|\right)\right) \end{aligned}$$

where

$$f(x) = \sqrt{\frac{3}{M-1}\frac{\frac{P}{\sigma^2}}{\frac{P}{\sigma^2}\Delta x^2 + 1}}.$$

TABLE I
COEFFICIENTS IN THE SER OF 16-QAM WITH CHANNEL ESTIMATION

i	1	2	3
c_i	2	3	1
f_i	1	9	17

TABLE II
COEFFICIENTS IN THE SER OF 64-QAM WITH CHANNEL ESTIMATION

i	1	2	3	4	5	6	7	8	9
c_i	2	4	2	4	4	5	3	3	1
f_i	1	9	17	25	33	49	57	73	97

The final step is to average the above expression over $|\hat{a}(m)|$, which, from our assumption, is Rayleigh distributed with $E[|\hat{a}(m)|^2] = 1 - \Delta$. The result is the following bound:

$$P_M \leq \frac{2}{M} \sum_{i \in S_1} [1 - \rho(|b_i|)] + \frac{3}{2M} \sum_{j \in S_2} [1 - \rho(|b_j|)] + \frac{1}{M} \sum_{k \in S_3} [1 - \rho(|b_k|)] \tag{24}$$

where

$$\rho(x) = \sqrt{\frac{\frac{P}{\sigma^2}(1-\Delta)}{\frac{2}{3}(M-1) + \frac{P}{\sigma^2}\left[1 + \left(\frac{2}{3}(M-1)x^2 - 1\right)\Delta\right]}}.$$

A simpler bound results if we treat every constellation point as an interior point

$$P_M \leq 2\left(1 - \frac{1}{M}\sum_{i=1}^{M} \rho(|b_i|)\right). \tag{25}$$

Observe that when there is no channel estimation error $\Delta = 0$, the above bounds reduce to the bound obtained when the channel is perfectly known (22).

Examples: For 16-QAM, the distance between adjacent symbols is $\sqrt{2/5}$. Equation (24) reduces to

$$P_{16} \leq \frac{3}{2} - \frac{1}{4}\sum_{i=1}^{3} c_i \sqrt{\frac{\frac{P}{\sigma^2}(1-\Delta)}{10 + \frac{P}{\sigma^2}(1 + f_i\Delta)}}$$

where c_i and f_i are listed in Table I.

For 64-QAM, the minimum distance is $\sqrt{2/21}$ and (24) reduces to

$$P_{64} \leq \frac{7}{4} - \frac{1}{16}\sum_{i=1}^{9} c_i \sqrt{\frac{\frac{P}{\sigma^2}(1-\Delta)}{42 + \frac{P}{\sigma^2}(1 + f_i\Delta)}} \tag{26}$$

where c_i and f_i are listed in Table II.

REFERENCES

[1] S. Verdú, *Multiuser Detection*. Cambridge, U.K.: Cambridge Univ. Press, 1998.

[2] P. Rapajic and B. Vucetic, "Adaptive receiver structures for asynchronous CDMA systems," *IEEE J. Select. Areas Commun.*, vol. 12, pp. 685–697, May 1994.

[3] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Commun.*, vol. 42, pp. 3178–3188, Dec. 1994.

[4] S. L. Miller, "An adaptive direct-sequence code-division multiple-access receiver for multiuser interference rejection," *IEEE Trans. Commun.*, vol. 43, pp. 1746–1755, Feb./Mar./Apr. 1995.

[5] M. L. Honig, U. Madhow, and S. Verdú, "Blind adaptive multiuser detection," *IEEE Trans. Inform. Theory*, vol. 41, pp. 944–960, July 1995.

[6] R. R. Gejji, "MQAM/CDMA for increased cellular capacity," in *Proc. IEEE Vehicular Technology Conf.*, Secaucus, NJ, May 1993, pp. 46–48.

[7] A. L. Johansson and A. Svensson, "Successive interference cancellation in multiple data rate DS/CDMA systems," in *Proc. IEEE Vehicular Technology Conf.*, Chicago, IL, July 1995, pp. 704–708.

[8] E. Biglieri, G. Caire, and G. Taricco, "CDMA system design through asymptotic analysis," *IEEE Trans. Commun.*, vol. 48, pp. 1882–1896, Nov. 2000.

[9] J. K. Cavers, "An analysis of pilot symbol assisted modulation for Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 686–693, Nov. 1991.

[10] P. Schramm and R. R. Müller, "Pilot symbol assisted BPSK on Rayleigh fading channels with diversity: Performance analysis and parameter optimization," *IEEE Trans. Commun.*, vol. 46, pp. 1560–1563, Dec. 1998.

[11] F. Ling, "Optimal reception, performance bound, and cut-off rate analysis of reference-assisted coherent CDMA communications with applications," *IEEE Trans. Commun.*, vol. 47, pp. 1583–1592, Oct. 1999.

[12] W. G. Phoel and M. L. Honig, "Performance of coded DS-CDMA with pilot-assisted channel estimation and linear interference suppression," *IEEE Trans. Commun.*, vol. 50, pp. 822–832, May 2002.

[13] J. S. Evans, "Optimal resource allocation for pilot symbol aided multiuser receivers in Rayleigh faded CDMA channels," *IEEE Trans. Commun.*, vol. 50, pp. 1316–1325, Aug. 2002.

[14] A. J. Grant and P. D. Alexander, "Random sequence multisets for synchronous code-division multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 44, pp. 2832–2836, Nov. 1998.

[15] S. Verdú and S. Shamai, "Spectral efficiency of CDMA with random spreading," *IEEE Trans. Inform. Theory*, vol. 45, pp. 622–640, Mar. 1999.

[16] D. N. C. Tse and S. Hanly, "Linear multiuser receivers: effective interference, effective bandwidth and user capacity," *IEEE Trans. Inform. Theory*, vol. 45, pp. 641–657, Mar. 1999.

[17] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Trans. Inform. Theory*, vol. 43, pp. 858–871, May 1997.

[18] J. Zhang, E. K. P. Chong, and D. N. C. Tse, "Output MAI distributions of linear MMSE multiuser receivers in DS-CDMA systems," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1128–1144, Mar. 2001.

[19] J. S. Evans and D. N. C. Tse, "Large system performance of linear multiuser receivers in multipath fading channels," *IEEE Trans. Inform. Theory*, vol. 46, pp. 2059–2078, Sept. 2000.

[20] S. L. Miller, M. L. Honig, and L. B. Milstein, "Performance analysis of MMSE receivers for DS-CDMA in frequency selective fading channels," *IEEE Trans. Commun.*, vol. 48, pp. 1919–1929, Nov. 2000.

[21] K. Yu, J. S. Evans, and I. B. Collings, "Performance analysis of pilot symbol aided QAM for Rayleigh fading channels," in *Proc. Int. Conf. Communications*, New York, Apr. 2002, pp. 1731–1735.

[22] X. Tang, M. S. Alouini, and A. J. Goldsmith, "Effect of channel estimation error on M-QAM BER performance in Rayleigh fading," *IEEE Trans. Commun.*, vol. 47, pp. 1856–1864, Dec. 1999.

[23] M. G. Shayesteh and A. Aghamohammadi, "On the error probability of linearly modulated signals on frequency-flat Ricean, Rayleigh, and AWGN channels," *IEEE Trans. Commun.*, vol. 43, pp. 1454–1466, Feb./Mar./Apr. 1995.

[24] J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2000.

[25] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. New York: Academic, 1994.

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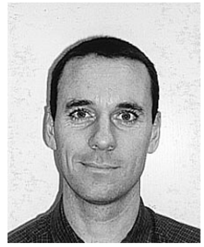

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