

Performance Analysis of Optical Spectral-Amplitude-Coding CDMA Systems Using a Super-Fluorescent Fiber Source

Zou Wei, H. Ghafouri-Shiraz, and H. M. H. Shalaby

Abstract—It has been proposed that the arrived photon count obeys negative binomial (NB) distribution when a thermal optical source is used. In this letter, we have analyzed the bit-error rate (BER) of an optical spectral-amplitude-coding CDMA system using NB distribution. It has been shown that when the number of simultaneous users is small, the resultant BER is much lower than that calculated using Gaussian approximation. When the number of simultaneous users is large (for example $N > 19$ when the effective power is -27 dBm), both NB distribution and Gaussian approximation give almost the same value for the BER of the system.

Index Terms—Negative binomial (NB) distribution, optical CDMA, spectral-amplitude-coding, super-fluorescent fiber source.

I. INTRODUCTION

THERMAL broad-band optical sources, such as erbium-doped super-fluorescent fiber sources (SFSs) have been used in spectral amplitude-coding optical code-division multiple-access (CDMA) system for a long time [1]. These sources generate light by spontaneous emission due to which an excess noise results. This excess noise is also referred to as phase-induced intensity noise (PIIN) and its effect on the system performance has been analyzed based on Gaussian approximation [2]. However, Gaussian approximation is accurate only when the simultaneous user number is large and, hence, the central limit theorem is valid to be applied. When there are only a few simultaneous users, the system performance will depend on the exact probability distribution of PIIN.

When both the random PIIN and the Poisson-distributed photon arrival process are taken into account, the photon count arrived at the receiver from a SFS obeys negative binomial (NB) distribution [3]. It has been proposed that this NB-distributed photon count lead to a lower bit-error rate (BER) in wavelength-division multiplexing systems when OOK modulation is used in each channel because NB distribution has lower probability density function tails than Gaussian distribution [4]. However, to our knowledge, the effect of NB-distributed photon count has never been studied precisely in optical spectral-amplitude-coding CDMA systems. All previously

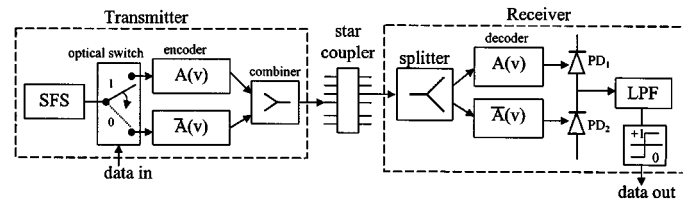


Fig. 1. Block diagram of a spectral amplitude-coding optical CDMA system.

published relevant papers have used signal-to-noise ratio and Gaussian approximation to analyze the BER.

In this letter, we have analyzed the BER of an optical spectral-amplitude-coding CDMA system using NB distribution directly. We have also compared the results obtained using this approach with that of Gaussian approximation. Hadamard code and complementary encoding/decoding have been used in our analyzed system.

II. SYSTEM DESCRIPTION

The block diagram of the spectral amplitude-coding optical CDMA system is shown in Fig. 1. When the data bit is "1," the emitted broad-band pulse from the SFS launches into the upper optical spectral encoder where the power spectral distribution (PSD) of the pulse follows the shape of $A(v)$ (see Fig. 1). When the data bit is "0," the broad-band pulse is switched into the complementary encoder and a PSD as $\bar{A}(v)$ results. Two examples of encoders/decoders are explained in details in [1] and [2]. The encoded optical pulses are combined and then sent to a star coupler where all optical signals from users are superimposed. At the desired receiver, this superimposed signal are split into two equal parts and both parts are input into decoders separately. The two decoders here are also complementary of each other and they have the same functions as encoders in the transmitter. Then after balanced detection and low-pass filter (LPF), the original data can be recovered.

III. ANALYSIS OF THE SYSTEM PERFORMANCE

The light from a broad-band SFS is generated by spontaneous emission. Hence, all phase components exist at the photodetectors. Therefore, SFS is a thermal source with its light intensity having a negative exponential distribution [3]. When both this random intensity corrugation and Poisson photon arrival process

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The authors are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798.

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are taken into account, the photon count arrived at either photodetector obeys NB distribution whose probability mass function is given by [3]

$$P[\Lambda = k] = \frac{\Gamma(k+M)}{\Gamma(k+1)\Gamma(M)} \left(\frac{E(\Lambda)}{M+E(\Lambda)} \right)^k \left(\frac{M}{M+E(\Lambda)} \right)^M \quad (1)$$

where

- Λ random variable representing the arrived photon count (from zero to infinity);
- M degree of freedom (DOF) of the broad-band thermal light;
- $\Gamma(k)$ gamma function of integer k given by $\Gamma(k) = (k-1)!$;
- $E(\Lambda)$ mean of arrived photon number.

The coherence time of a thermal source τ_c is given by [2]

$$\tau_c = \frac{\int_0^\infty G^2(v) dv}{\left[\int_0^\infty G(v) dv \right]^2} \quad (2)$$

where $G(v)$ is the PSD of the thermal source. The variance of Λ of this distribution is given by [3]

$$\sigma_\Lambda^2 = E(\Lambda) + E^2(\Lambda)/M \quad (3)$$

when the data bit period $T \gg \tau_c$, the DOF M is equal to U for a polarized thermal source and is equal to $2U$ for an ideally unpolarized thermal source, where U is given by $U \cong T/\tau_c$. In the analysis, we have assumed the following.

- 1) Each light source is ideally unpolarized and its spectrum is flat over the bandwidth $[v_0 - \Delta v/2, v_0 + \Delta v/2]$, where v_0 is the central optical frequency and Δv is the optical source bandwidth in Hertz.
- 2) Each spectral component has identical spectral width.
- 3) Each user has equal power at the receiver.
- 4) Each bit-stream from each user is synchronized.

Based on the above assumptions, the PSD at the input of photodetectors PD₁ and PD₂ (see Fig. 1) of the l th receiver over one data bit period can be expressed as

$$G_1(v) = \frac{P_s}{\Delta v} \sum_{k=1}^K \sum_{i=1}^N \{ [c_k(i)d_k + \bar{c}_k(i)\bar{d}_k] \cdot c_l(i) \cdot \text{rect}(i) \} \quad (4)$$

$$G_2(v) = \frac{P_s}{\Delta v} \sum_{k=1}^K \sum_{i=1}^N \{ [c_k(i)d_k + \bar{c}_k(i)\bar{d}_k] \cdot \bar{c}_l(i) \cdot \text{rect}(i) \} \quad (5)$$

where

- K number of simultaneous users;
- d_k data bit ("0" or "1") sent by the k th user and \bar{d}_k is its complement;
- P_s received effective power of each user after the 1×2 splitter.

N is the length of Hadamard sequences. $C_k^{(i)}$ is the i th element of the k th Hadamard address sequence with $\bar{C}_k^{(i)}$ as its complement. Let $u(v)$ be the unit step function [2], the $\text{rect}(i)$ function

in (4) and (5) is given by

$$\text{rect}(i) = u \left[v - v_0 - \frac{\Delta v}{2N}(-N + 2i - 2) \right] - u \left[v - v_0 - \frac{\Delta v}{2N}(-N + 2i) \right]. \quad (6)$$

Based on the correlation properties of Hadamard code [2], using the approximation $\sum_{m=1}^k d_m c_m^{(i)} \approx K/2$, when simultaneous user number $K > 1$, we can, respectively, express the light coherence times at the input of PD₁ and PD₂ as

$$\tau_{c1} = \frac{2K}{(K-1+2d_l)\Delta v} \quad (7)$$

$$\tau_{c2} = \frac{2K}{(K-1+2\bar{d}_l)\Delta v}. \quad (8)$$

Let us denote the number of arrived photons at photodetectors PD₁ and PD₂ as k_1 and k_2 . Both of them have NB distribution profiles with the mean values as $E(k_1)$, $E(k_2)$ and DOFs as M_1 , M_2 . When the received data bit is "1," using the correlation property of Hadamard code the means of k_1 and k_2 can be expressed as

$$E[k_1] = (K+1)P_s T/4hf \quad (9)$$

$$E[k_2] = (K-1)P_s T/4hf \quad (10)$$

where

- h Planck's constant;
- f optical source frequency;
- T data bit period.

The DOFs are given by $M_1 = 2T/\tau_{c1}$ for k_1 and $M_2 = 2T/\tau_{c2}$ for k_2 where τ_{c1} and τ_{c2} are given by (7) and (8), respectively.

The number of generated electrons k_{e1} after balanced detection is given by $k_{e1} = \eta(k_1 - k_2) = \eta k_{d1}$, where η is the photodetector quantum efficiency and $k_{d1} = k_1 - k_2$. Because the used threshold is zero, the effect of quantum efficiency η can be neglected in the analysis of BER. In the following, we assume that $\eta = 1$ and only need to consider the random variable k_{d1} . The probability of k_{d1} can be expressed as

$$P[k_{d1} = k] = \sum_{i=-\infty}^{\infty} \{ P[k_1 = i+k] \cdot P[k_2 = i] \} \quad (11)$$

where $P[k_1]$ and $P[k_2]$ can be obtained using (1) from the calculated values of M_1 , M_2 , $E(k_1)$, and $E(k_2)$. To calculate BER using (11), we need the following definition. Given a random variable ξ , we define its probability vector \bar{P}_ξ as $\bar{P}_\xi = \{\dots a_{i-1}, a_i, a_{i+1}, \dots\}$, where each element a_i is the probability of $\xi = i$, i.e., $a_i = P[k = i]$; and assume that the probability vectors of k_1 , k_2 and k_{d1} are denoted as \bar{P}_{k1} , \bar{P}_{k2} and \bar{P}_{d1} , respectively. Also, we denote the inverse sequence of \bar{P}_{k2} as $\text{inv}(\bar{P}_{k2})$, i.e., $\text{inv}(\bar{P}_{k2}) = \{\dots a_{k2, i+1}, a_{k2, i}, a_{k2, i-1}, \dots\}$ where $a_{k2, i} = P[k_2 = i]$. Let $\text{conv}(x, y)$ represents convolution of sequences x and y , (11) can be transformed into the following expression:

$$\bar{P}_{d1} = \text{conv}[\bar{P}_{k1}, \text{inv}(\bar{P}_{k2})]. \quad (12)$$

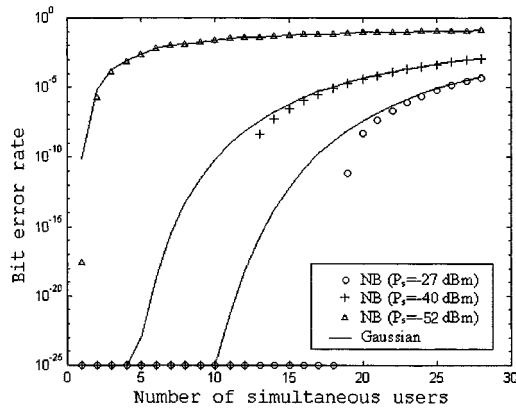


Fig. 2. Variations of the BER with number of simultaneous users using NB-distribution (dots) and Gaussian approximation (lines).

Therefore, the BER when the data bit is “1” can be written as

$$P_{e1} = \left\{ \sum_{i=-\infty}^{-1} P[k_{d1} = i] \right\} + P[k_{d1} = 0]/2. \quad (13)$$

When data bit is “0,” we can calculate BER P_{e2} by swapping \bar{P}_{k1} and \bar{P}_{k2} in (12) which results the same BER. Assumed that the probabilities of sending data bit “1” and “0” are equal, the total BER is expressed as $P_e = (P_{e1} + P_{e2})/2$. Because $P_{e1} = P_{e2}$, therefore, $P_e = P_{e1}$. This means that the total BER can be obtained by using (13).

When the user number $K = 1$, there is no other active user. If data bit is “1,” the coherence time at PD₁ is given by

$$\tau_{c1} = \frac{(P_s/\Delta v)^2(N/2)(\Delta v/N)}{[(P_s/\Delta v)(N/2)(\Delta v/N)]^2} = \frac{2}{\Delta v} \quad (14)$$

and $E[k_1] = P_s T/2hf$. No optical power exists at PD₂. In this case, the BER can be obtained by using the following expression:

$$P_{e1} = \left\{ \sum_{i=-\infty}^{-1} P[k_1 = i] \right\} + P[k_1 = 0]/2. \quad (15)$$

Also, the same BER can be obtained when data bit is “0” and similarly the total BER can be obtained using (15).

IV. NUMERICAL RESULTS

In the BER calculation, we have considered the effects of both PIIN and shot noise. When $|k - E[\Lambda]| \geq 4\sigma_\Lambda$, we found that $P(\Lambda = k)$ is negligibly small. Hence, we assume $P(\Lambda = k) = 0$ in order to simplify the numerical computation. The system operates at 1550-nm wavelength with data bit rate of 622 Mb/s. The encoded bandwidth is 3.75 THz wide (which is equal to 30 nm line-width) and the quantum efficiency is assumed to be one.

Fig. 2 illustrates the BER variations versus the number of simultaneous users for both NB-distribution (dots) and Gaussian approximation (lines) when the effective power is -27 , -40 , and -52 dBm. It has been found that in both cases the BERs increase with the number of simultaneous users K . For a given effective power P_s , when K is small (let us say $K < K_c$ where K_c is the critical user number which depends on P_s), the BER

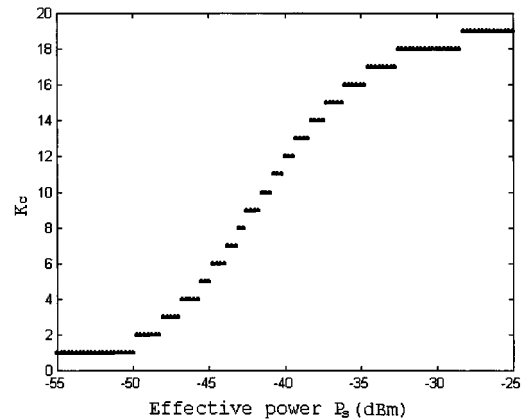


Fig. 3. Variation of critical user number K_c with the received effective power P_s .

calculated by NB distribution is significantly lower than that calculated by Gaussian approximation. However, as the user’s number K increases (i.e., $K > K_c$) the BERs calculated by both methods are very close. For example, when $P_s = -27$ dBm and $K > 19$ (see Fig. 2) both methods almost give the same results.

Fig. 3 shows the critical user number K_c variation versus the received effective power P_s . Here the critical user number K_c is defined as the maximal user number where the BER obtained by NB distribution is 1% less than that obtained using Gaussian approximation (i.e., $[P_e(\text{NB})/P_e(\text{Gaussian})] \leq 1\%$). Fig. 3 clearly shows K_c increases with P_s . This is because the NB distribution becomes so different from Gaussian distribution when the effective power is large that even a large number of users cannot make the two BERs close.

V. CONCLUSION

In this letter, we have analyzed the BER of an optical spectral-amplitude-coding CDMA system using NB distribution directly. We have also compared the results obtained using this approach with that of Gaussian approximation. Hadamard code and complementary encoding/decoding have been used in our analysis. It has been shown that the BER obtained by NB distribution is much lower than that calculated using Gaussian approximation when the simultaneous user number is small. However, when we consider the system capacity, generally there are lots of simultaneous users. In this case, the BERs calculated by both methods are almost the same. Therefore, Gaussian approximation can also provide an accurate estimation for the capacity of optical spectral-amplitude-coding systems.

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