*Progress In Electromagnetics Research M, Vol. 39, 115–122, 2014*

# Performance Analysis of Rain Rate Models for Microwave Propagation Designs over Tropical Climate

**Obiseye O. Obiyemi[1, *], Joseph S. Ojo[2], and Tunji S. Ibiyemi[3]**

**Abstract**—Rain attenuation is a major source of impairment to signal propagation at microwave and millimeter wave bands. The procedures for the estimation of rain attenuation values as regard to microwave signals however rely mainly on 1-minute rain rate statistics, particularly those obtained locally from experimental measurement campaigns over a given location. In this paper, we present recent results on 1-minute rain rate statistics required for satellite and terrestrial link designs, as obtained from a 2-year measurement over Akure, Nigeria. The performance of the selected rain rate models; Rice-Holmberg (RH) model, the Kitami model, Moupfouma model and the global ITU rain rate model were tested based on four metrics namely: Prediction error, Root Mean Square Error (RMSE), Spread-Corrected Root Mean Square Error (SC-RMSE) and the Spearman's rank correlation. Results indicate that no single model completely outperforms all others. Interestingly, the RH model is particularly best behaved over the distribution, while the Moupfouma model performs suitably well. Others seem to vary largely from the measured rain rate distribution. Results for the rain rate exceeded for 0.01% of the time agrees with earlier estimates for the cumulative rain rate distribution derived from higher integration-time statistics over this tropical site.

## 1. INTRODUCTION

Precipitation due to rain is the most challenging source of impairment to signal propagation in the microwave and millimeter wave bands. This remains a major problem in the deployment of terrestrial and satellite communication systems, particularly for operations above 10 GHz [1–5].

Several efforts have however led to the development of rain rate and rain attenuation models. Invariably, such efforts have facilitated attenuation mitigation at various stages of communication link design and deployment. Interestingly, the prediction of the attenuation induced over a link is mainly dependent on the precipitation data, preferably those locally measured over such a location. However, the dearth of such local precipitation data over Nigeria has been a major challenge over the years and this affects the accuracy of predicted fade margins as required for the planning of satellite and terrestrial communication links over Nigeria. Hence, the attainment of designed availability objectives on microwave and millimeter wave links strongly depends on the local rainfall data for a particular location.

Although, global digital maps have been developed [6, 7] to cater for locations where local data do not exist, precipitation estimates from these maps do not agree with those obtained over Nigeria [8, 9] and other tropical climates. Hence, this has led to the generation of contour maps for Nigeria based on RH model [10, 11] and recently based on Moupfouma model [12–14]. However, all these results are based on the existing models using hourly rain rate and daily rain intensity.

In this paper, the analysis of 1-minute rain rate statistics observed from a 2-year measurement in Akure, South Western Nigeria is presented, while some vital precipitation characteristics required

---

*Received 30 August 2014, Accepted 22 October 2014, Scheduled 27 October 2014*

[*] Corresponding author: Obiseye Oluwaniyi Obiyemi (obiseye.obiyemi@uniosun.edu.ng).

[1] Department of Electrical and Electronic Engineering, Osun State University, Nigeria. [2] Department of Physics, Federal University of Technology, Akure, Nigeria. [3] Department of Electrical & Electronics Engineering, University of Ilorin, Nigeria.

for propagation planning is quantified. The degree of performance for selected rain rate models is also evaluated based on the rain rate predicted over this location.

## 2. REVIEW OF PRECIPITATION ESTIMATES FOR MICROWAVE APPLICATIONS IN NIGERIA

Due to the dearth of appropriate precipitation data for estimating rain induced attenuation on satellite and terrestrial links in the tropical climates such as Nigeria, rain rate data of longer integration time has been broadly useful for the planning of microwave and millimeter wave links. For instance, the development of contour maps in [12, 13] is actually based on a hybrid rainfall data, which comprises of a 30-year local rainfall data from Nigerian Meteorological Agency (NIMET) and a complementing satellite data from the Tropical Rainfall Measurement Mission (TRMM). In their studies, the Moupfouma-Martin distribution function and the Rice-Holmberg models were used to obtain the 1-minute equivalent statistics.

Similarly, the cumulative rain rate estimates in [11] covers 37 different locations across Nigeria. These estimates are also based on a 9-year rainfall data obtained from the TRMM. Here, monthly precipitation data was obtained from two operational products of the TRMM namely 3A12 V6 and 3B43 V6.

Other recent and useful experimental campaigns are those of Nigeria Environmental and Climatic Observatory Programme (NECOP) now referred to as Tropospheric Data Acquisition Network (TRODAN). This campaign features among other stations the installation of three experimental sites at the Federal University of Technology, Akure (FUTA), Ondo State, University of Lagos (UNILAG), Lagos State and University of Yola (UNIYOLA), Adamawa State. These in-situ precipitation measurement setups are calibrated to log at 5 minutes [15], thus requiring other forms of conversion to lower integration time, such as the 1-minute recommended for rain attenuation prediction over a point location. Measurement is ongoing at these stations and it generates data bank on certain measured parameters for observatory and radio-climatological research purposes. Precipitation data from this measurement setup has been useful for planning terrestrial and satellite communication networks [9, 15, 16] over Nigeria.

The measurement campaign reported in [8] also presents the cumulative estimates of the rainfall rates over Akure. In this case however, the experimental estimate is based on the precipitation data available over this site, which was measured across the vertical profile using the Micro Rain Radar (MRR) situated at the Department of Physics, Federal University of Technology, Akure (FUTA). Although the data used here was logged in the required 1-minute integration time, it is important to have additional ground based (tipping bucket rain gauge or ordinary rain gauge) measurement setup in order to compensate for measurement outage due to the breakdown of this equipment to incessant power outage.

Other notable experimental rain rate estimates in South-Western Nigeria include the works of Ajayi and Ofoche [17], Semire et al. [18, 19], Obiyemi et al. [16], and that of Ojo and Olurotimi [20]. In their studies, characteristics of the rainfall rate logged at different time were used to develop conversion factors for Ile-Ife, Ogbomoso and Akure respectively. The conversion factors are mainly useful for estimating the equivalent 1-minute rain rate statistics, especially for locations where rain rate statistics exist in other integration time.

## 3. EXPERIMENTAL SETUP

The measurement was conducted at FUT Akure (7.17°N, 5.18°E), which is in the South-Western part of Nigeria. Akure has an average annual rainfall of 1485.57 mm and it belongs to the P-zone of the ITU-R rain climatic zoning [7, 15]. The in-situ precipitation measurement commenced in July 2012 and is ongoing. However, the data used covers an observation period of two years (July, 2012 to June, 2014).

The measurement was conducted using an electronic weather station (Davis 6250 Vantage Vue). The precipitation data was collected using a self-emptying tipping spoon (with a resolution of 0.2 mm per tip), which is part of the Integrated Sensor Suit (ISS). The accuracy of the gauge is +1% at 1 litter/h with a measuring range of a minimum of 2 mm/h to a 400 mm/h. The gauge is accurate to within 2% up to 250 mm/h. Rainfall up to 400 mm/h is measured with the resolution of 0.2 mm. The data logger

**Figure 1.** The outdoor ISS and the indoor console units of the Davis Vantage Vue electronic weather station.

scans the data at an interval of one second, integrated over 1 minute. The availability of the gauge is about 99.2%. The 0.8% unavailability is due to system maintenance. The ISS also houses other sensors for monitoring parameters such as relative humidity, wind speed, wind direction, temperature among others. It serves as the outdoor unit, which employs frequency-spread spectrum technique to wirelessly connect with the indoor console. Figure 1 shows the outdoor ISS unit and the indoor console for the measurement.

The data logger (Davis instrument WeatherLink) employed for capturing and harvesting the data is connected to a personal computer (PC) for data harvesting. The logged files contain the log date and log time for reference. Hence, the clock of the data logger was synchronized with that of the PC during calibration.

## 4. RESULTS AND ANALYSIS

In order to analyze the rainfall data, the useful rainfall rate was sorted and classified into the four rainfall types using recent ranges in [21], namely: the drizzle rain type (below 5 mm/h); the widespread rain type (between 5 mm/h and 10 mm/h); the shower rain type (between 10 and 40 mm/h); and the thunderstorm rain type (above 40 mm/h). Table 1 shows detailed statistics of the 2-year rainfall data based on the different rain rate classification. Others were discarded and these consist of the clear sky conditions where the rainfall rate is 0 mm/h. From the characteristics shown in Table 1, only 5.3% of the total precipitation statistics is thunderstorm. 73.5% is recorded for drizzle, 9.73% for widespread and 11.5% for shower rainfall regimes.

**Table 1.** Summary of rain rate events recorded over the observation period.

| **Rain regime** | **Number of events** | **Total rainfall intensity (mm/h)** | **Rain (Minutes)** | **Percentage (%) Rain Minutes** |
|---|---|---|---|---|
| Drizzle ($0 < R \leq 5$) | 135 | 11643.4 | 6054 | 73.5 |
| Widespread ($5 > R \leq 10$) | 77 | 5596.2 | 801 | 9.73 |
| Showers ($10 > R \leq 40$) | 68 | 20303.2 | 945 | 11.5 |
| Thunderstorm ($40 > R$) | 47 | 32967.2 | 436 | 5.3 |

Figure 2 shows the cumulative rain rate distribution as obtained for Akure at 7.17°N, 5.18°E over the years, based on different in-situ measurement setups. The estimate for the rainfall rate exceeded at 0.01% of the time is 120 mm/h. This is as obtained from the 2-year 1-minute measurement using the electronic Davis Vantage Vue weather station. This is not in sharp contrast with earlier conversion-based estimates [11, 13, 15] and the estimate from the 1-minute MRR measurement [8] for the same location.

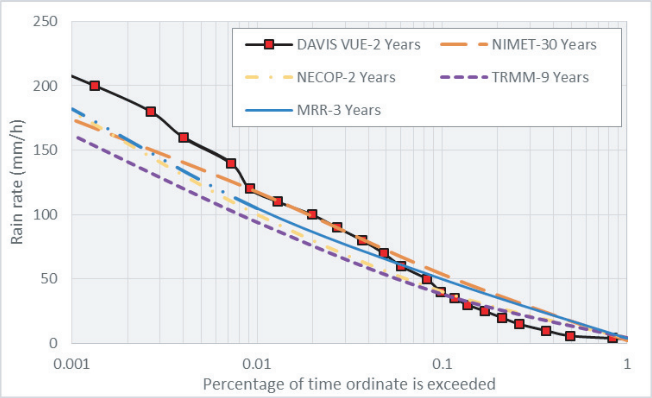

**Figure 2.** Cumulative rain rate distribution estimated over Akure, South Western Nigeria based on different in-situ measurement setups.

## 5. CUMULATIVE DISTRIBUTION OF RAIN RATE MODELS OVER AKURE

A number of rain rate climatological models for short integration time statistics exist. Four different models were selected and they are the rain rate models developed by Rice-Holmberg (RH) [22], Kitami [23], Moupfouma [24] and the global ITU recommendation as detailed in the radiowave propagation series [6]. These models were selected based on the assumed applicability to the tropical locations [8, 9, 15, 16].

The RH model depends on the average annual precipitation and the thunderstorm ratio as the two main parameters for estimating the rain rate from the local climatological data available. These parameters can either be extrapolated from the maps presented in [22] or estimated from the long-term average annual precipitation, the number of thunderstorm days and the maximum monthly precipitation in 30 years.

The Kitami model was proposed by Ito and Hosoya [23] and it is based on two regional climatic parameters namely; the thunderstorm ratio and the average annual precipitation. The Moupfouma's model, developed by Moupfouma and Martin is as detailed in [24, 25]. This model offers a simple approach to the prediction of the rain rate distribution for both temperate and tropical climates. The widely used ITU model is as detailed in the ITU P 837-6 recommendation. The prediction approach here is based on the annual rainfall amount of convective-type rain $M_C$ (mm), the annual rainfall amount of stratiform-type rain $M_S$ (mm) and the probability of rainy 6-hour periods $P_{r6}$ (%), all available on ITU's 3M Group website [6]. The comparison of the predicted and measured rain rate distribution is shown Figure 3.

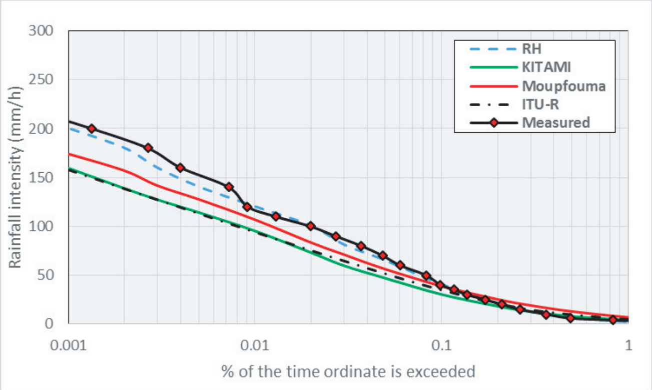

**Figure 3.** Comparison of measured and predicted rain rate distribution over Akure.

The measured rain rate for 0.01% is 120 mm/h, while equivalent estimates for the same time percentage are 120.42, 95.3, 106.81 and 94.4 mm/h for RH, KITAMI, Moupfouma and ITU-R models respectively.

The point rain rate estimate for Akure is not in discord with those obtained over other tropical locations. For instance, the point rainfall rates observed over 2 years using a network of rain gauges in Singapore are 104, 118, 115, 111 mm/h for installations at Nanyang Technological University (NTU), Shunfu, Paya Lebar and Tampines respectively [26]. A similar result is presented in [27] for Brazil where the rainfall rates exceeded for 0.01% of the total time at Rio de Janeiro, Belem and Manaus are 118, 123 and 126 mm/h respectively. Estimates over locations in Malaysia also indicate that the 0.01 % rain rate is around 120 mm/h [28].

## 6. PERFORMANCE EVALUATION OF SELECTED RAIN RATE MODELS

The performance of the selected rain rate distribution models was evaluated using four metrics namely; Prediction error, Root Mean Square Error (RMSE), Spread-Corrected Root Mean Square Error (SC-RMSE) and the rank correlation.

Prediction error $\varepsilon_p$ is the first error metric considered for assessing the performance of the rain rate distribution models. It is also known to be the difference between the predicted value and the median measured value and it is given as;

$$\varepsilon_p = R_p - R_{p,m} \tag{1}$$

where $R_p$ and $R_{p,m}$ are the predicted and measured rain rate estimates for $0.001\% < P < 1\%$.

The maximum and mean error was estimated using Equations (2) and (3) respectively.

$$MaxError = \max(\varepsilon_p) \tag{2}$$

$$MeanError = \frac{1}{n}\sum_{i}^{n} \varepsilon_p \tag{3}$$

The other metric employed here is the RMSE, which is a frequently used measure of the difference for numerical prediction. It is also known as the Root Mean Square Deviation (RMSD) and it is mostly used between values predicted by a model and the values actually observed from the environment that is being modelled.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} |\varepsilon_p|^2} \tag{4}$$

The SC-RMSE for a given model and percentage of the time is the absolute value of the error, reduced by the standard deviation of the estimates. This is as given in Equation (5). SC-RMSE gives an idea of error in excess of expected variance due to temporal variation [29, 30].

$$\varepsilon_p^! = |\varepsilon_p| - \sigma_p \tag{5}$$

$$SC - RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} |\varepsilon_p^!|^2} \tag{6}$$

The Spearman's correlation coefficient is the fourth error metric used in this work. It is a statistical measure of the strength of a monotonic relationship between paired data. It is a non-parametric measure of statistical dependence between the measured and the predicted rainfall rate. It is denoted by $\rho$ and is design constrained as;

$$-1 < \rho < 1 \tag{7}$$

The standard Spearman's rank correlation coefficient is defined as;

$$\rho = \frac{\sum_p \left(R_p - \overline{R_p}\right)\left(R_{p,m} - \overline{R_{p,m}}\right)}{\sqrt{\sum_p \left(R_p - \overline{R_p}\right)^2 \sum_p \left(R_{p,m} - \overline{R_{p,m}}\right)^2}} \tag{8}$$

where $R_p$ and $R_{p,m}$ are the predicted and measured rain rate estimates while $\overline{R_{p,m}}$ and $\overline{R_p}$ are the mean measured and predicted rain rate respectively for $0.001\% < P < 1\%$.

Figure 4 provides the graphical depictions of the performance of the selected models based on their respective error estimates using different metrics. The composite comparison statistics for the selected models is as shown in Table 2.

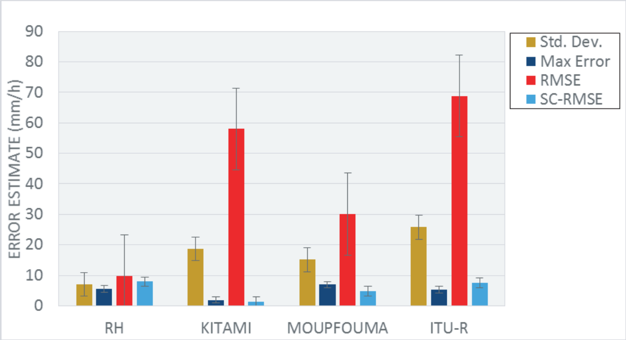

**Figure 4.** Performance evaluation using selected error metrics.

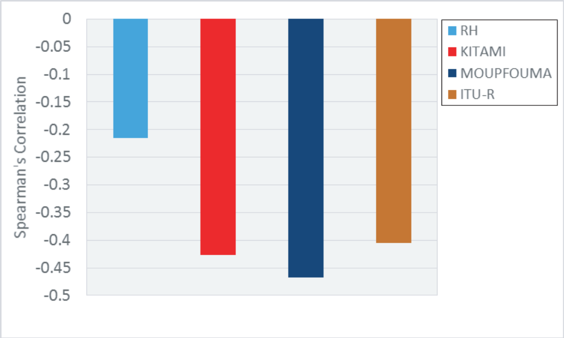

**Figure 5.** Spearman's rank correlation over the distribution.

**Table 2.** The composite comparison statistics for the models selected for prediction over Akure.

| Performance criteria | RH (mm/h) | KITAMI (mm/h) | MOUPFOUMA (mm/h) | ITU-R (mm/h) |
|---|---|---|---|---|
| **Mean Prediction Error** | −3.2533 | −19.3520 | −10.0231 | −22.9505 |
| **Standard deviation** | 7.0077 | 18.6385 | 15.1293 | 25.7919 |
| **Max Error** | 5.5000 | 1.9145 | 6.9418 | 5.2500 |
| **RMSE** | 9.7600 | 58.0561 | 30.0694 | 68.8514 |
| **SC-RMSE** | 7.9251 | 1.2778 | 4.8173 | 7.5646 |
| **Spearman Correlation** | −0.2149 | −0.4265 | −0.4681 | −0.4052 |

Considering the variability of all the rain rate predictions, the RH model does much better than others; presenting a standard deviation of 7.01 mm/h and mean prediction error of −3.25 mm/h. Results indicate that predictions by Moupfouma exhibit fair variability. Kitami and ITU-R models provide high prediction variability with standard deviation of 18.64 mm/h and 25.79 mm/h respectively, while estimates of the mean prediction errors are −19.35 mm/h and −22.95 mm/h respectively.

Assessing the performances of the rain rate models based on the RMSE estimates, 9.76 mm/h was obtained for the RH model. Equivalent estimates for the Moupfouma, ITU-R and Kitami models are 30.07, 68.85 and 58.06 mm/h respectively.

Looking at performance cum the SC-RMSE, all the models perform fairly well over the distribution, although the 7.93 mm/h estimate for RH appears to be the highest. However, Kitami model is well behaved for this metric, producing a value of 1.28 mm/h, while the Moupfouma and ITU-R models record 4.82 mm/h and 7.56 mm/h respectively.

Estimates from the final metric, Spearman's rank correlation actually indicate that there is a moderate correlation between the predicted and measured rank orderings. Considering the −0.22 estimate recorded for RH model, it is clear that this model performs particularly poorly by this metric. The correlation coefficient is less than 0.5 at $P < 0.001$. Figure 5 shows Spearman's rank correlation between the predicted and measured rain rate distribution.

Although no single model distinctly outperforms others, the general observation is that the RH model performs very well from every indication. The RH model is particularly well behaved as evaluated using the chosen metrics.

## 7. CONCLUSION

In this paper, we have estimated the cumulative rain rate distribution from a 1-minute precipitation statistics over Akure, Nigeria. Rain rates exceeded for $0.001\% < P < 1\%$ of the time was compared with earlier estimates over this location, particularly for estimates derived from precipitation statistics with higher integration time. We also evaluated the performance of four prominent rain rate models based on the prediction for this region using different error metrics. The performance criteria are based on the estimated statistics for the Prediction error, Root Mean Square Error (RMSE), Spread-Corrected Root Mean Square Error (SC-RMSE) and the Spearman's rank correlation over the entire rain rate distribution. Results show that no single model provides a good fit and at the same time outperforms all others. However, the RH model is particularly well behaved as evaluated using the chosen metrics. Other models seem to vary largely from the measured rain rate distribution, although the Moupfouma model performs well over some metrics.

## REFERENCES

1. Maruyama, T., Y. Shirato, M. Akimoto, and M. Nakatsugawa, "Service area expansion of quasi-millimeter FWA systems through site diversity based on detailed rainfall intensity data," *IEEE Transactions on Antennas and Propagation*, Vol. 56, No. 10, 3285–3292, 2008.
2. Panagopoulos, A. D., P. D. M. Arapoglou, and P. G. Cottis, "Satellite communications at KU, KA, and V bands: Propagation impairments and mitigation techniques," *IEEE Communications Surveys & Tutorials*, Vol. 6, No. 3, 2–14, 2004.
3. Kanellopoulos, S. A., A. D. Panagopoulos, C. I. Kourogiorgas, and J. D. Kanellopoulos, "Satellite and terrestrial links rain attenuation time series generator for heavy rain climatic regions," *IEEE Transactions on Antennas and Propagation*, Vol. 61, No. 6, 3396–3399, 2013.
4. Andrade, F. J. A. and L. A. da Silva Mello, "Rain attenuation time series synthesizer based on the gamma distribution," *IEEE Antennas and Wireless Propagation Letters*, Vol. 10, 1381–1384, 2011.
5. Kourogiorgas, C. and A. D. Panagopoulos, "New physical-mathematical model for predicting slant-path rain attenuation statistics based on inverse Gaussian distribution," *IET Microwaves, Antennas & Propagation*, Vol. 7, No. 12, 970–975, 2013.
6. Radiowave Propagation Series, I.T.U., "Characteristics of precipitation for propagation modelling," *Recommendation ITU-R P.837-6*, ITU-R, Geneva, 2012.
7. Radiowave Propagation Series, I.T.U., "Characteristics of precipitation for propagation modelling," *Recommendation ITU-R P.837-1*, ITU-R, Geneva, 1994.
8. Ibiyemi, T. S., M. O. Ajewole, J. S. Ojo, and O. O. Obiyemi, "Rain rate and rain attenuation prediction with experimental rain attenuation efforts in south-western Nigeria," *20th Telecommunications Forum (TELFOR)*, Serbia, Belgrade, November 20–22, 2012.
9. Ojo, J. S. and S. E. Falodun, "NECOP propagation experiment: Rain-rate distributions observations and prediction model comparisons," *International Journal of Antennas and Propagation*, 2012.
10. Ajayi, G. O., S. Feng, S. M. Radicella, and B. M. Reddy, "Handbook on radiopropagation related to satellite communications in tropical and subtropical countries," *International Centre for Theoretical Physics*, 1996.
11. Omotosho, T. V. and C. O. Oluwafemi, "One-minute rain rate distribution in Nigeria derived from TRMM satellite data," *Journal of Atmospheric and Solar-Terrestrial Physics*, Vol. 71, No. 1, 625–633, 2009.
12. Ojo, J. S., M. O. Ajewole, and S. K. Sarkar, "Rain rate and rain attenuation prediction for satellite communication in Ku and Ka bands over Nigeria," *Progress In Electromagnetics Research B*, Vol. 5, 207–223, 2008.
13. Ojo, J. S., M. O. Ajewole, and L. D. Emiliani, "One-minute rain-rate contour maps for microwave-communication-system planning in a tropical country: Nigeria," *IEEE Antennas and Propagation Magazine*, Vol. 51, No. 1, 82–89, 2009.

14. Ojo, J. S. and T. V. Omotosho, "Comparison of 1-min rain rate derived from TRMM satellite data and raingauge data for microwave applications in Nigeria," *Journal of Atmospheric and Solar-Terrestrial Physics*, Vol. 102, No. 0, 17–25, 2013.
15. Ojo, J. S. and M. O. Ajewole, "Dimensional statistics of rainfall signature and fade duration for microwave propagation in Nigeria," *XXXTH URSI Genaral Assembly and Scientific Symposium*, Istanbul, August 13–20, 2011.
16. Obiyemi, O. O., T. J. Afullo, and T. S. Ibiyemi, "Equivalent 1-minute rain rate statistics and seasonal fade estimates in the microwave band for south-western Nigeria," *Int. Journal of Scientific & Engineering Research*, Vol. 5, No. 1, 239–244, 2014.
17. Ajayi, G. O. and E. B. C. Ofoche, "Some tropical rainfall rate characteristics at Ile-Ife for microwave and millimeter wave applications," *Journal of Applied Meteorology*, Vol. 23, 562–567, 1984.
18. Semire, F. and T. Raji, "Characteristics of measured rainfall rate at Ogbomoso, Nigeria for microwave applications," *Journal of Telecommunications and Information Technology*, No. 2, 85–89, 2011.
19. Semire, F. A., R. Mohd-Mokhtar, T. V. Omotosho, I. Widad, N. Mohamad, and J. S. Mandeep, "Analysis of cumulative distribution function of 2-year rainfall measurements in Ogbomoso, Nigeria," *International Journal of Applied Science and Engineering*, Vol. 10, No. 3, 171–179, 2012.
20. Ojo, J. S. and E. O. Olurotimi, "Tropical rainfall structure characterization over two stations in southwestern Nigeria for radiowave propagation purposes," *Journal of Emerging Trends in Engineering and Applied Sciences*, Vol. 5, No. 2, 116–122, 2014.
21. Alonge, A. A. and T. J. Afullo, "Rainfall rate modeling for various rainfall types in South Africa," *AFRICON, 2011*, IEEE, 2011.
22. Rice, P. and N. Holmberg, "Cumulative time statistics of surface-point rainfall rates," *IEEE Transactions on Communications*, Vol. 21, No. 10, 1131–1136, 1973.
23. Ito, C. and Y. Hosaya, "Worldwide 1-min. Rain rate distribution prediction method which uses thunderstorm ratio as regional climatic parameter," *Electronic Letters*, Vol. 35, No. 18, 1585–1587, 1999.
24. Moupfouma, F. and L. Martin, "Modeling of the rainfall rate cumulative distribution for the design of satellite and terrestrial communication systems," *International Journal of Satellite Communications*, Vol. 13, No. 2, 105–115, 1995.
25. Ojo, J. S. and P. A. Owolawi, "Development of one-minute rain-rate and rain-attenuation contour maps for satellite propagation system planning in a subtropical country: South Africa," *Advances in Space Research*, Vol. 54, No. 8, 1487–1501, 2014.
26. Ong, J.-T. and C.-N. Zhu, "Rain rate measurements by a rain gauge network in Singapore," *Electronics Letters*, Vol. 33, No. 3, 240–242, 1997.
27. Pontes, M. S., C. G. S. Migliora, and L. A. R. Da Silva Mello, "Preliminary results of 12 GHz radlometric measurements in Brazil," *Electronics Letters*, Vol. 25, No. 1, 1989.
28. Abdulrahman, A., et al., "An improved slant path attenuation prediction method in tropical climates," *Radioengineering*, Vol. 22, No. 4, 1083, 2013.
29. Phillips, C., D. Sicker, and D. Grunwald, "Bounding the error of path loss models," *2011 IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, IEEE, 2011.
30. Faruk, N., A. A. Ayeni, and Y. A. Adediran, "On the study of path loss models for accurate prediction of TV signal for secondary users," *Progress In Electromagnetics Research B*, Vol. 49, 155–176, 2013.