Research Article

Performance Evaluation Criterion at Equal Pumping Power for Enhanced Performance Heat Transfer Surfaces

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The existing equations for the thermal performance evaluation, at equal pumping power for the artificially roughened and smooth surfaced multitube and rectangular duct heat exchangers, have been critically reviewed because the literature survey indicates that a large number of researchers have not interpreted these equations correctly. Three of the most widely used equations have been restated with clearly defined constraints and conditions for their application. Two new equations have been developed for the design constraints not covered earlier.

1. Introduction

Artificial roughness in the form of ribs or baffles has been employed to enhance heat transfer in conventional heat exchangers, gas turbine blade cooling channels, and solar air heaters. However, the enhancement in heat transfer is also associated with significant increase in pressure drop and pumping power. Various thermal performance evaluation criteria, which take account of both heat transfer enhancement and pumping power, have been suggested for these enhanced performance surfaces based on different design constraints and objectives [1–6]. However, the criterion based on equal pumping power for the roughened and smooth surfaces suggested by Webb and Eckert [2] has been most widely used [7–41].

The performance ratios used by different researchers for the multitube heat exchangers are

$$R_1 = \frac{(\mathrm{St}/\mathrm{St}_s)}{(f/f_s)^{1/3}}, \tag{1}$$

$$R_2 = \frac{q}{q_s} = \frac{h}{h_s}, \tag{2}$$

$$R_3 = \frac{(\mathrm{Nu}/\mathrm{Nu}_s)}{(f/f_s)^{1/3}}. \tag{3}$$

The Stanton number, St, Nusselt number, Nu, and the friction factor, f, without subscript refer to the rough surface and with the subscript, s, refer to the smooth surface. The ratio of heat transfer coefficients for the rough and smooth surfaces, h/h_s, in (2) is replaced by the ratio of the overall heat transfer coefficients, U/U_s, when heat transfer between two fluids is considered. The heat transfer coefficient h_s or U_s in (2) is calculated at the Reynolds number Re_s for the smooth surfaced tubes, which is defined by

$$\mathrm{Re}_s = \left(\frac{f}{f_s}\right)^{1/3} \mathrm{Re}. \tag{4}$$

One of the old references where an equation similar to (1) has been given is that of Walker and Wilkie [1] for flow along the axis of a cluster of heated roughened rods in a smooth channel for the fixed mass flow rate and pumping power as constraints and the flow area as variable. Webb and Eckert [2], while discussing three different design objectives for multitube heat exchangers, proposed (1) for the design objective of enhanced heat transfer rate at equal pumping power for the roughened and smooth tubes. However, they

developed this equation for a particular set of constraints and this equation is applicable only when a condition of specified ratio of mass velocities through the smooth and rough tubes is fulfilled. These constraints and conditions are discussed later. Many investigators [8–19], while using (1), did not take notice of the condition specified by Webb and Eckert [2] or Walker and Wilkie [1], and some of them [11–19] have used the performance ratio R_1 for the square and rectangular ducts without mentioning the modification of the original constraints and condition specified for the multitube exchangers.

The performance ratio given by (2) has been suggested for enhanced surfaces by Bergles et al. [3] when the basic geometry of the exchanger is kept fixed and the mass flow rate is variable for fixed pumping power. These constraints are different from those specified by Webb and Eckert for (1). However, the researchers have, generally, regarded the two as equivalent.

When Re = Re_s and Pr = Pr_s, St/St_s = Nu/Nu_s and (1) will transform to (3). Such transformation is justified only when Re = Re_s for the constraints and condition of (1), but some of the investigators while using (3) failed to satisfy this condition [20–30]. However, it is worth investigating whether such equation can be developed from fundamentals for design constraints different from those specified by Webb and Eckert for (1) and Bergles et al. for (2).

The performance ratios R_1 and R_3 have been presented in the form of plots of these ratios versus the relative mass velocity [2], the roughness parameters [8–10, 12], and the friction factor [10] and versus the roughness Reynolds number [11, 33]. A number of researchers [13–23, 25–31] have used Reynolds number as the abscissa parameter while Sara et al. [32] preferred Re_s as the abscissa parameter for the plots of performance ratios. Bergles et al. [3] advocate abscissa Re_s as a convenient reference for plot of ratio R_2 and the same has been used by Sethumadhavan and Raja Rao [34, 35]. However, such choice must be based on some logical ground and not on convenience.

For fixed geometry rectangular ducts, Liou and Hwang [36] presented the following performance ratio:

$$R_4 = \frac{\text{Nu}}{\text{Nu}_s^*}, \tag{5}$$

where Nu_s^* is the Nusselt number for the smooth duct at the Reynolds number Re_s defined by (4). The performance ratio R_4 has been plotted by researchers against parameter $f^{1/3}$ Re [36–40]. However, Ichimiya [41] plotted Nu/Nu_s versus the parameter $f^{1/3}$ Re.

Looking to the above discussion, the following objectives have been outlined for this paper: (i) to critically review the above-mentioned performance ratio equations, and to clearly restate and differentiate between the conditions and constraints for application of (1), (2), and (5); (ii) to specify the conditions and constraints for the application of (3); and (iii) to develop a performance ratio for the rectangular ducts when mass flow rate is kept fixed. To achieve the outlined objectives, the authors have thought that it is necessary to critically review the development of these equations from

fundamentals, especially looking to the errors or omissions in application of different performance ratios.

For the development of the performance ratios in the present work, the fluid properties, and the temperature difference between the heat transferring surface and the bulk fluid have been assumed to be the same for both the rough and smooth surfaces. The error due to these assumptions is small for exchangers operating with low temperature differences. The analyses and results are based on the comparison of thermal performances of roughened and smooth channels of equal heat exchange surface area at equal pumping power.

2. Performance Ratio for Multitube Heat Exchangers

The relative heat transfer rate of the rough and smooth surfaces having surface areas A and A_s, respectively, is given by Webb and Eckert [2] and Bergles et al. [3] as

$$\frac{q}{q_s} = \frac{hA\,(\Delta T)}{[hA\,(\Delta T)]_s}, \tag{6}$$

where ΔT is the temperature difference between the heat transferring surface and bulk fluid.

In case of heat exchange between fluids on the two sides of a surface, the thermal resistance external to the surface under consideration (including scale, tube wall, and secondary fluid resistances) has been assumed to be the same for the rough and smooth heat exchangers.

In general, ΔT in (6) for the roughened surface will be somewhat smaller than the smooth surface due to the enhancement in the heat transfer coefficient. For equal values of temperature differences between heat transferring surface and bulk fluid for the rough and smooth surfaces, that is, $\Delta T = \Delta T_s$, as assumed by the previous investigators [2, 3], the relative heat transfer rate equals the relative heat conductance, that is,

$$\frac{K}{K_s} = \frac{(hA)}{(h_s A_s)}. \tag{7}$$

Knowing that $h \propto \text{St}\,G$, this equation transforms to

$$\frac{K}{K_s} = \left(\frac{\text{St}}{\text{St}_s}\right)\left(\frac{A}{A_s}\right)\left(\frac{G}{G_s}\right), \tag{8}$$

where G is the mass velocity through a tube.

For a multitube heat exchanger with N parallel tubes of diameter D and length L, the mass flow rate $m = N(\pi/4)D^2 G$ and pressure drop $\Delta p = 4fLG^2/(2\rho D)$. Using these relations the pumping power equation can be transformed as follows:

$$P = \left(\frac{m}{\rho}\right)\Delta p = \frac{f\,(\pi DNL)\,G^3}{(2\rho^2)} \propto fAG^3, \tag{9}$$

where A (= πDNL) is the area of the surface exchanging heat. Hence, the relative pumping power relation for the rough and smooth surfaces, from (9), is

$$\frac{P}{P_s} = \left(\frac{f}{f_s}\right)\left(\frac{A}{A_s}\right)\left(\frac{G}{G_s}\right)^3. \tag{10}$$

The ratio of mass velocities for the smooth and rough tubes (G_s/G) has been termed as relative mass velocity G^* by Webb and Eckert [2]. Equations (8) and (10) in terms of G^* are

$$\frac{K}{K_s} = \left(\frac{St}{St_s}\right)\left(\frac{A}{A_s}\right)\left(\frac{1}{G^*}\right), \tag{11}$$

$$\frac{P}{P_s} = \left(\frac{f}{f_s}\right)\left(\frac{A}{A_s}\right)\left(\frac{1}{G^*}\right)^3. \tag{12}$$

For different constraints and mass flow rate conditions, the transformations of the above equations yield the equations of thermal performance evaluation as explained below.

Case (A): Flow Rate and Tube Diameter Fixed, Heat Exchanger Length, and Number of Tubes Variable. Webb and Eckert [2] gave a single expression for the performance evaluation based on the assumptions that the heat exchanger mass flow rate, m, and tube diameter, D, are held constant, and G^* is changed by varying number of tubes. For flow through tubes, $G^* = Re_s/Re$.

Eliminating G^* from (11) and (12), they arrived at the following relation:

$$\frac{(K/K_s)}{\left[(P/P_s)^{1/3}\ (A/A_s)^{2/3}\right]} = \frac{(St/St_s)}{(f/f_s)^{1/3}}. \tag{13}$$

For the constraints of equal pumping power and surface area, that is, $P/P_s = A/A_s = 1$, Webb and Eckert transformed the above equation to

$$\frac{K}{K_s} = \frac{(St/St_s)}{(f/f_s)^{1/3}} \tag{14}$$

$$\text{for } G_s = \left(\frac{f}{f_s}\right)^{1/3} G. \tag{15}$$

Equation (15) has been obtained from (12) and is the same as (4) when put in the terms of the Reynolds number. It means that the values of the Stanton number and friction factor for the smooth duct must be calculated at Re_s corresponding to G_s. To avoid the confusion, the present authors suggest that the smooth duct variables be written with asterisk (*) to indicate that their values are to be evaluated at different Reynolds numbers.

Webb and Eckert arrived at (14) with the assumption of $D_s = D$. Hence, the equal surface area constraint yields $N_s L_s = NL$. It implies that the length of the equivalent smooth surfaced exchanger will be $L_s = (N/N_s)L$.

Alternative Form of Equation for the Case (A). Equation (7) for the relative heat conductance can be written in terms of Nusselt number ratio as

$$\frac{K}{K_s} = \left(\frac{Nu}{Nu_s}\right)\left(\frac{A}{A_s}\right)\left(\frac{D_s}{D}\right). \tag{16}$$

For the constraints of $D_s = D$ and $A_s = A$, this equation transforms to

$$\frac{K}{K_s} = \frac{Nu}{Nu_s^*}, \tag{17}$$

where Nu_s^* refers to the smooth tube Nusselt number at the Reynolds number Re_s defined either by $G_s/G = Re_s/Re$ or (4). For $D_s = D$, the performance ratio given by (17) equals h/h_s^*.

Case (B): Flow Rate and Heat Exchanger Length Fixed, Tube Diameter, and Number of Tubes Variable. If the constraints of Case (A) are modified to $m_s = m$, $L_s = L$, $A_s = A$, and $Re_s = Re$, the area and mass flow constraints give

$$D_s N_s = DN, \tag{18a}$$

$$G_s D_s = GD. \tag{18b}$$

Combining these equations,

$$G_s = G\left(\frac{N_s}{N}\right). \tag{19}$$

The performance ratio equation for this case will be

$$\frac{K}{K_s} = \frac{(St/St_s)}{(f/f_s)^{1/3}}, \tag{20}$$

$$\text{for } \frac{G_s}{G} = \frac{N_s}{N} = \frac{D}{D_s} = \left(\frac{f}{f_s}\right)^{1/3}, \tag{21}$$

that is, the number and diameter of the tubes are changed from N to N_s and from D to D_s, respectively, to satisfy the condition of $N_s/N = (D/D_s) = G_s/G = (f/f_s)^{1/3}$.

For the given condition of $Re_s = Re$, the ratio St/St_s in (20) can be replaced by Nu/Nu_s, which will give the performance ratio of (3). Since $(f/f_s)^{1/3} = D/D_s$, the ratio $(Nu/Nu_s)/(f/f_s)^{1/3}$ of (3) equals h/h_s.

It must be noted that (20) of performance ratio is of the same form as (14), but the values of Stanton number and friction factor for the rough and smooth tubes in (20) are to be evaluated at the same Reynolds number.

Case (C): Basic Geometry Fixed and Flow Rate Variable. The performance ratio equation of Bergles et al. [3], given by (2), can be deduced from (7). They assumed a fixed geometry condition ($D_s = D$, $L_s = L$, $N_s = N$), which gives $A_s = A$. Equation (4) is satisfied when mass velocity G_s through the smooth tubes is greater than the mass velocity G through the roughened tubes. Therefore, for the fixed geometry constraint, the mass flow rate for the smooth surfaced exchanger will be higher than the mass flow rate for the roughened one. The ratio h/h_s^* can be replaced by Nu/Nu_s^* since $D_s = D$.

3. Performance Ratio for Rectangular Ducts

The relative heat transfer rate equation in terms of the Nusselt number ratio, for this case, is

$$\frac{q}{q_s} = \frac{K}{K_s} = \left(\frac{Nu}{Nu_s}\right)\left(\frac{A}{A_s}\right)\left(\frac{D_{hs}}{D_h}\right). \tag{22}$$

The pumping power equation for a rectangular duct of cross-section ($W \times H$) can be transformed using mass

flow rate, $m = G(WH)$, and hydraulic diameter, $D_h = 4WH/[2(W + H)]$:

$$P = \left(\frac{GWH}{\rho}\right)\left(\frac{4fLG^2}{2\rho}\right)\left[\frac{2(W + H)}{4WH}\right] \tag{23}$$
$$\propto fG^3 [2L(W + H)] = f(P'L)G^3,$$

where P' is the perimeter of the duct.

The equal pumping power condition for the square or rectangular ducts can be achieved either by keeping duct geometry fixed and changing the mass flow rate or by keeping mass flow rate fixed and changing mass velocity by varying the duct cross-section.

Case (A): Basic Geometry Fixed and Flow Rate Variable. For the fixed geometry constraint of the duct, that is, $W_s = W$, $H_s = H$, and $L_s = L$,

$$A_s = A, \qquad D_{hs} = D_h, \qquad \frac{G_s}{G} = \frac{\text{Re}_s}{\text{Re}}. \tag{24}$$

Equation (22) will transform to

$$\frac{q}{q_s} = \frac{\text{Nu}}{\text{Nu}_s^*}, \tag{25}$$

where Nu_s^* is the Nusselt number for the smooth duct at Re_s defined by (4), which is the condition of equal pumping power.

For a duct with heat transfer from all the four walls, the areas for heat transfer and pumping power equations will be equal. While for an asymmetrically heated duct, the two will have different values. However, for a fixed geometry duct, the above given results will be applicable without any change.

Case (B): High Aspect Ratio Duct and Flow Rate Fixed. When the mass flow rate, m, is fixed, the condition of equal pumping power can be achieved by increasing the mass velocity G_s by decreasing the smooth duct cross-section or vice versa. For a duct of high aspect ratio ($W \gg H$), whose width W and length L are fixed for the equal area constraint, hydraulic diameter $D_h \approx 2H$ and the Reynolds number can be expressed as

$$\text{Re} \approx \frac{2m}{(W\mu)}, \tag{26}$$

that is, $\text{Re} = \text{Re}_s$ if mass flow rate, m, and duct width, W, are fixed.

The pumping power equation for this case will be

$$P \approx \frac{f(WL)G^3}{\rho^2}. \tag{27}$$

Thus, the condition of equal pumping power is achieved by increasing the mass velocity G_s in the smooth duct to satisfy the condition $G_s = G(f/f_s)^{1/3}$. This can be affected by decreasing the height of the smooth duct to $H_s = m/(G_s W)$. Hence, for $m = m_s$, $H_s G_s = HG$. The performance ratio will be given by the heat transfer enhancement ratio, that is,

$$\frac{q}{q_s} = \frac{h}{h_s}. \tag{28}$$

Equation (28) can be transformed by using the relations $h = \text{Nu}k/D_h \approx \text{Nu}k/(2H)$ for high aspect ratio ducts and $H_s G_s = HG$:

$$\frac{h}{h_s} = \frac{[\text{Nu}k/(2H)]}{[\text{Nu}_s k/(2H_s)]}$$
$$= \frac{(\text{Nu}/\text{Nu}_s)}{(H/H_s)}$$
$$= \frac{(\text{Nu}/\text{Nu}_s)}{(G_s/G)} \tag{29a}$$
$$= \frac{(\text{Nu}/\text{Nu}_s)}{(f/f_s)^{1/3}}$$
$$= \frac{(\text{St}/\text{St}_s)}{(f/f_s)^{1/3}} \quad \text{for } \text{Re} = \text{Re}_s. \tag{29b}$$

For the duct with heat transfer from all the four walls, the surface area $A = 2(W + L) \approx 2WL$, and for an asymmetrically heated duct (with broader wall heated), $A = WL$.

The boundary condition of asymmetric heating of a high aspect ratio duct is encountered in solar air heaters. Gas turbine blade cooling channels are modeled as rectangular ducts of various aspect ratios with heat flux at all the four walls.

4. Presentation of Results

Various performance ratio equations discussed and developed in this paper have been listed in Table 1 along with the constraints and the conditions to be fulfilled. All the geometric and flow parameters have been fully defined.

Basically, all the tabulated performance ratios are heat transfer enhancement ratios. They are of two forms: either h/h_s when D_{hs} (or D) $\neq D_h$ (or D) or h/h_s^* ($= \text{Nu}/\text{Nu}_s^*$) when D_{hs} (or D) $= D_h$ (or D). However, the ratio U/U_s^* for heat transfer between two fluids cannot be replaced by Nu/Nu_s^*. It is to note that the choice of the performance ratio depends on the constraints of the application. It is also desirable that an investigator using any of the performance ratio equations should explicitly mention the design constraints associated with that equation and also the conditions to be fulfilled.

For the two out of the six performance ratios given in Table 1, the Reynolds numbers for the rough and smooth surfaces are equal, that is, $\text{Re} = \text{Re}_s$. Therefore, Re or Re_s is a logical parameter for the plots of the performance results when these criteria are used. For the remaining four performance ratios, values of the Stanton number and friction factor for the roughened and smooth surfaces are calculated at different Reynolds numbers. Further, the roughness parameters, such as the relative roughness height, pitch, and orientation of the roughness elements with respect to the flow, are not having any meaning for a smooth surface. Therefore, neither the Reynolds number nor the roughness parameters can be the logical abscissa parameter for presentation of the performance of the roughened surfaces versus a smooth surface when these four criteria are applied. In these cases,

TABLE 1: Performance ratio equations.

Investigators	Performance ratio	Constraints	Conditions to be fulfilled
Multi-tube Exchangers			
Webb and Eckert [2]	$(\mathrm{St}/\mathrm{St}_s^*)/(f/f_s^*)^{1/3}$	$m_s = m$, $D_s = D$, and $A_s = A$, $N_s \neq N$, and $L_s \neq L$	$G_s > G$, $L_s > L$, $N_s < N$, $L_s = (N/N_s)L$, $G_s/G = N/N_s = \mathrm{Re}_s/\mathrm{Re} = (f/f_s)^{1/3}$, $\mathrm{Re}_s = (f/f_s^*)^{1/3}\,\mathrm{Re}$
Present authors	$\mathrm{Nu}/\mathrm{Nu}_s^*\ (= h/h_s^*)$	$m_s = m$, $D_s = D$, and $A_s = A$, $N_s \neq N$, and $L_s \neq L$	$G_s > G$, $L_s > L$, $N_s < N$, $L_s = (N/N_s)L$, $G_s/G = N/N_s = \mathrm{Re}_s/\mathrm{Re} = (f/f_s)^{1/3}$, $\mathrm{Re}_s = (f/f_s^*)^{1/3}\,\mathrm{Re}$
Present authors	$(\mathrm{St}/\mathrm{St}_s)/(f/f_s)^{1/3}$, $(\mathrm{Nu}/\mathrm{Nu}_s)/(f/f_s)^{1/3}$, or h/h_s	$m_s = m$, $L_s = L$, $A_s = A$, and $\mathrm{Re}_s = \mathrm{Re}$, $D_s \neq D$, $N_s \neq N$	$G_s > G$, $D_s < D$, $N_s > N$, $G_s/G = (f/f_s)^{1/3} = N_s/N = D/D_s$
Bergles et al. [3]	h/h_s^* or U/U_s^* $(h/h_s^* = \mathrm{Nu}/\mathrm{Nu}_s^*)$	$D_s = D$, $L_s = L$, $N_s = N$, and $A_s = A$, $m_s \neq m$	$m_s > m$, $G_s > G$, $G_s/G = \mathrm{Re}_s/\mathrm{Re}$, $\mathrm{Re}_s = (f/f_s^*)^{1/3}\,\mathrm{Re}$
Rectangular Duct			
Liou and Hwang [36]	$\mathrm{Nu}/\mathrm{Nu}_s^* = h/h_s^*$	$m_s \neq m$, $W_s = W$, $H_s = H$, $L_s = L$, and $A_s = A$	$m_s > m$, $G_s > G$, $G_s/G = \mathrm{Re}_s/\mathrm{Re}$, $\mathrm{Re}_s = (f/f_s^*)^{1/3}\,\mathrm{Re}$
Present authors	h/h_s, $(\mathrm{Nu}/\mathrm{Nu}_s)/(f/f_s)^{1/3}$, or $(\mathrm{St}/\mathrm{St}_s)/(f/f_s)^{1/3}$	High aspect ratio duct $(W \gg H)$: $W_s = W$, $L_s = L$, $H_s \neq H$, and $m_s = m$	$G_s > G$, $H_s < H$, $H_s G_s = HG$, $G_s = G(f/f_s)^{1/3}$, $\mathrm{Re} = \mathrm{Re}_s \approx 2m/(W\mu)$

*The parameters with asterisk (*) are to be evaluated at the Reynolds number Re_s defined in column 4.

the common parameter for the smooth and rough surfaces is $(f_s^{*1/3}\mathrm{Re}_s)$ or $(f^{1/3}\mathrm{Re})$. Some researchers have used this abscissa parameter for enhanced performance rectangular ducts as mentioned earlier.

It must be noted that the value of the friction factor f_s^* for the smooth channel in the equation $\mathrm{Re}_s = [\mathrm{Re}\,(f/f_s^*)^{1/3}]$ corresponds to the Reynolds number Re_s and, therefore, this equation must be converted to an explicit form by substitution of friction correlation of smooth duct and rearranging the terms. In case of a heat exchanger with heat transfer between two fluids, the present criteria represent upper limits to heat transfer enhancement [3]. Once the preferred type of roughness is selected, the thermal resistances external to the enhanced surface and effects of variable fluid properties may be considered.

4.1. Application to Solar Air Heater Ducts with Roughened Absorber Plate. In order to illustrate the application of the performance evaluation criteria presented above for high aspect ratio rectangular ducts with asymmetric heating, data from an earlier experimental study of the first author [42] have been used. The flow and thermal boundary conditions of the roughened duct of their study correspond closely to those of solar air heater ducts. The discrete and discontinuous repeated rectangular cross-section rib roughness arrangements employed in the study are shown in Figure 1. The ribs have been arranged in both v-up and v-down patterns. They compared the thermal performance of different rib arrangements based on equal pumping power for roughened

and smooth ducts using performance parameter $\mathrm{Nu}/\mathrm{Nu}_s^*$ defined by (25) and presented the performance plots of this parameter versus the parameter $f^{1/3}\mathrm{Re}$ as shown in Figure 2. The constraints and conditions for the application of this criterion, as detailed in Table 1, are basically fixed geometry and reduced flow rate for the roughened ducts. It is an appropriate criterion when the smooth surfaced absorber plate of an existing solar air heater is replaced by a roughened absorber plate.

Alternatively, the thermal performance at equal pumping power has been presented in Figure 3 as plots of $(\mathrm{Nu}/\mathrm{Nu}_s)/(f/f_s)^{1/3}$ [or $(\mathrm{St}/\mathrm{St}_s)/(f/f_s)^{1/3}$] versus the Reynolds number. The basic conditions and constraints, as outlined in Table 1, are those of equal mass flow rates for smooth and roughened ducts and an increased height of the roughened duct as compared to the smooth duct.

It is interesting to note that the ordinate values of the above discussed performance plots and hence the conclusion drawn from these plots regarding the selection of the preferred roughness type are the same. But the first criterion specifies the operational condition of reduced flow rate for the existing solar air heater when the smooth surfaced absorber plate is replaced by a roughened one, while the second criterion gives design recommendation of increased duct height for the roughened duct solar air heater.

5. Conclusions

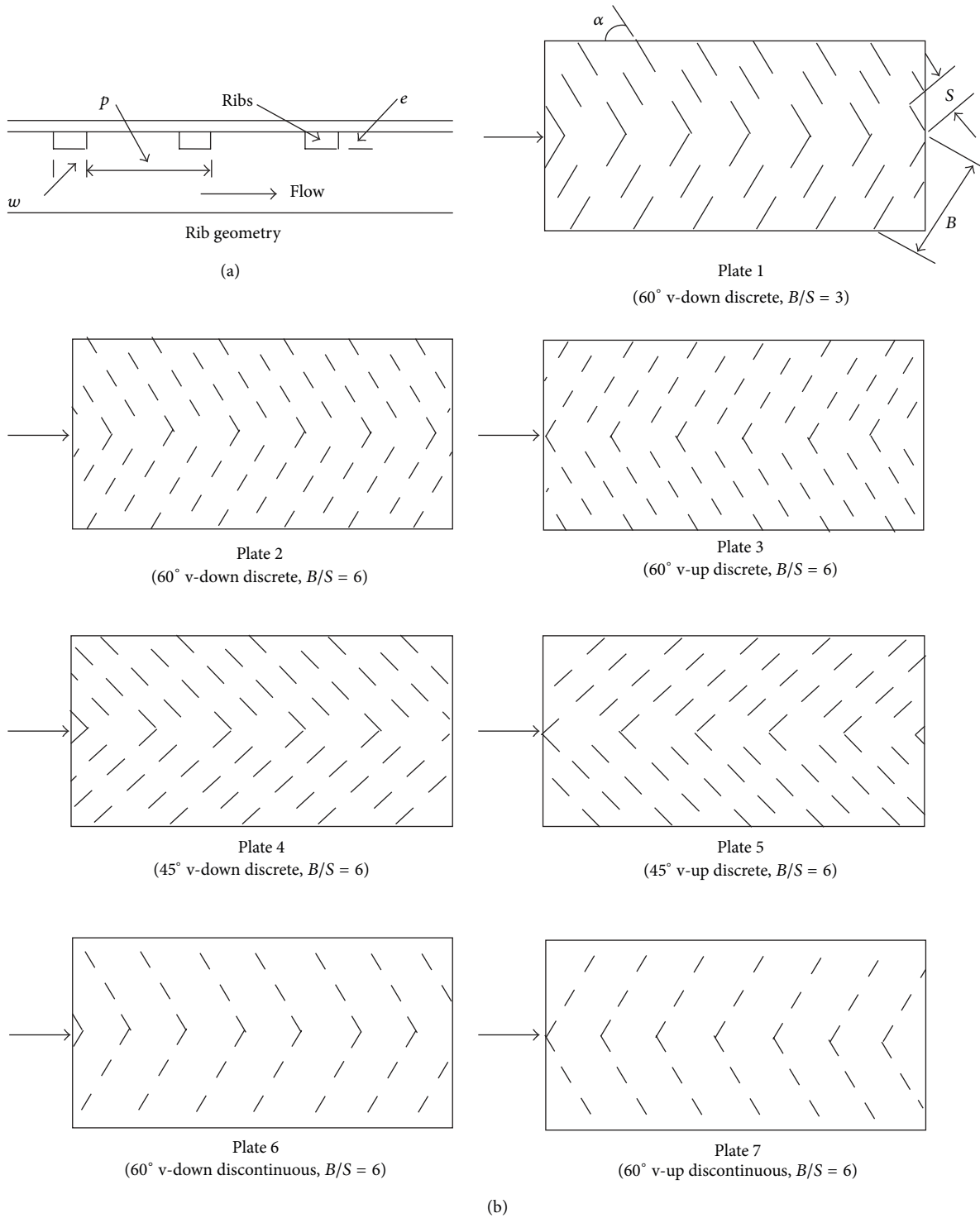Various existing equations for the thermal performance evaluation at equal pumping power for the roughened and

Figure 1: (a) Longitudinal section of the rectangular duct with 60° v-down discrete rib roughness ($e/D_h = 0.0469$, $p/e = 10.63$, $w/e = 2.06$). (b) Rib arrangements [42].

smooth surfaced multitube and rectangular duct heat exchangers (such as solar air heater and gas turbine blade cooling channels) have been critically reviewed. The design constraints and conditions for their application have been explicitly stated. Two new performance ratio equations have been developed for the constraints not covered earlier. Finally the logical abscissa parameters for the plots of performance evaluation results have been suggested.
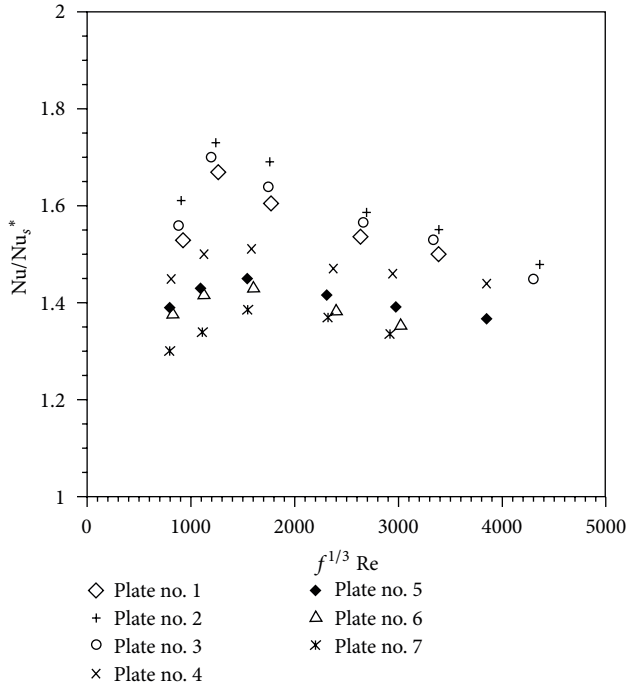
FIGURE 2: Thermal performance at equal pumping power; Basic geometry fixed, $G_s/G = \text{Re}_s/\text{Re} = (f/f_s^*)^{1/3}$ [42].
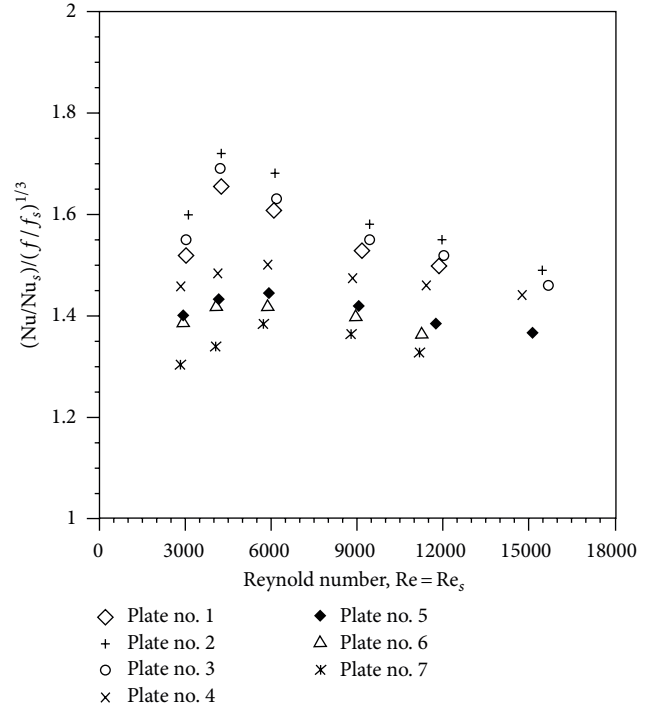

FIGURE 3: Thermal performance at equal pumping power; Flow rate fixed ($m = m_s$), $H_s/H = G/G_s = (f_s/f)^{1/3}$.

Nomenclature

A: Heat transfer surface area [m^2]
B/S: Relative roughness length of discrete ribs [—]
c_p: Specific heat of air [Jkg^{-1}K^{-1}]
D: Pipe inside diameter [m]
D_h: Hydraulic diameter of duct = $4WH/[2(W + H)]$ [m]
e: Rib height [m]
e/D_h: Relative roughness height [—]
f: Fanning friction factor [—]
G: Mass velocity [kgs^{-1}m^{-2}]
G^*: Relative mass velocity = G_s/G [—]
h: Heat transfer coefficient [Wm^{-2}K^{-1}]
H: Duct height [m]
k: Thermal conductivity of fluid [Wm^{-1}K^{-1}]
K: Overall heat conductance = hA [WK^{-1}]
L: Length of the flow passage [m]
m: Mass flow rate [kg/s]
N: Number of parallel tubes in flow passage [—]
Nu: Nusselt number [—]
p: Rib pitch [m]
P: Pumping power [W]
P': Perimeter of rectangular section duct [m]
Pr: Prandtl number [—]
p/e: Relative roughness pitch [—]
q: Heat transfer rate [W]
Re: Reynolds number, DG/μ or D_hG/μ [—]
St: Stanton number, $h/(Gc_p)$ [—]
ΔT: Temperature difference [°C]
U: Overall heat transfer coefficient [Wm^{-2}K^{-1}]
w: Width of rib [m]
W: Width of the duct [m].

Greek Symbols

Δp: Pressure drop [Pa]
μ: Viscosity of fluid [Pa s]
ρ: Density of fluid [kg/m^3].

Subscript

s: Smooth surface—
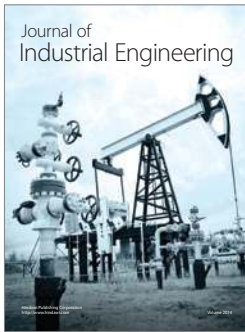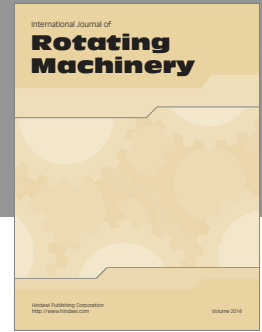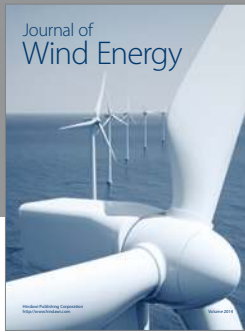   unsubscripted variables refer
   to rough surfaces.

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