Performance-Fairness Trade-off for Wi-Fi and LTE-LAA Coexistence

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ABSTRACT Long Term Evolution (LTE)-License Assisted Access (LAA), which leverages unlicensed resource sharing with the Wi-Fi network, is a promising technique to address the spectrum scarcity issue in present and future wireless networks. However, unlicensed spectrum sharing between Wi-Fi and LAA requires fair resource allocation with specific performance guarantees for both sets of Wi-Fi and LAA stations. In this paper, an optimal communication policy is devised for LAA stations coexisting on a single unlicensed channel with Wi-Fi stations. The inter-network collisions are avoided through non-overlapping transmission phases for Wi-Fi and LAA networks. The throughput performance of LAA network is maximized while guaranteeing a proportionally fair performance among LAA stations and a fair share for Wi-Fi stations. The proposed scheme, unlike the state-of-the-art coexisting mechanism, jointly optimizes the transmission probability and the transmission rate for each LAA station. The formulated optimization problem to maximize network throughput is solved analytically. The numerical results demonstrate a significant improvement in the LAA throughput, more than 75 %, as compared to the case when transmission probabilities are not optimized. Moreover, a notable gain of $8-9$ % in the fairness index reflects the intra-network fairness of the proposed LAA network over the conventional LAA network.

INDEX TERMS Licensed assisted access (LAA), Wi-Fi, proportional fairness, random channel access, transmit power control.

I. INTRODUCTION

Recently wireless communication industry has turned its attention towards utilizing the unlicensed spectrum as an efficient means to address the spectrum scarcity and rapidly growing demand for data traffic by users [1]. In this regard, different variants of the fourth generation Long Term Evolution (LTE) have been proposed to leverage the unlicensed 5 GHz band which is mainly used by the Wi-Fi network, e.g., LTE unlicensed (LTE-U), LTE licensed assisted access (LAA) and MulteFire [2]. LTE-U emerged as the first standard for unlicensed sharing presented by LTE-U forum on the basis of LTE specifications in release 12 [3]. In release 13, the 3rd generation partnership project (3GPP) issued LTE-LAA as a global standard to coexist with the unlicensed band [4]. Multefire was proposed in 2017 as a radio technology to self deploy LTE in the unlicensed band without the need of an anchor in licensed band [5]. LTE-U

and LAA are based on carrier aggregation while MulteFire is based on standalone operation.

The overall aim of the above mentioned variants is the fair coexistence among different radio technologies, i.e., an LTE network should not impact on the Wi-Fi network more than an additional Wi-Fi network in terms of throughput [6].The idea of unlicensed spectrum sharing is extended to become the part of fifth generation new radio unlicensed (NR-U) standard [7]. LAA has been proposed as the basis for the channel access mechanism for 5G New Radio-Unlicensed (NR-U) by industry and academia [8]–[10]. In this work, we focus on LTE-LAA/ Wi-Fi coexistence.

The standardization for the LTE-LAA and Wi-Fi coexistence in the unlicensed 5 GHz band is still in under development. In this regard, the latest specifications for tuning the transmission opportunity, which is considered in this work, were discussed in the 3GPP working group meeting in Jan 2020 [11]. Moreover, the recent performance analysis studies such as the one done in [12] have identified many issues with the current coexistence specification. Multiple problems

with the current existing releases were reported in [13]. The solution to these problems are not only crucial for 5 GHz unlicensed band, but the lessons learned from the 5 GHz coexistence are vital for fair and efficient coexistence deployments in 6 GHz and all future bands.

LTE can adversely affect the Wi-Fi throughput performance without a fair coexistence scenario. This is because of the difference in access mechanisms for Wi-Fi and LTE networks. A Wi-Fi network follows IEEE 802.11a Distributed Coordination Function (DCF) based on the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) mechanism to coexists with one another [14]. In order to maintain fair spectrum sharing with Wi-Fi, the conventional LTE-LAA adopts a contention based channel accessing protocol. It is equipped with Listen before talk (LBT) mechanism which uses the clear channel assessment (CCA) to access the unlicensed channel and considers equal channel access probability for each LAA station [6]. The standard LTE-LAA mechanism has been shown to provide better data rates and higher airtime efficiency than the standalone Wi-Fi network operating in the unlicensed spectrum [15]. However, equal access probability does not take into the near-far effect of the LAA stations and their individual throughput is affected. *Therefore, it is important to design a coexistence scheme which addresses both the inter-network fairness as well as the intra-network fairness in a LTE-LAA/Wi-Fi network.*

A. RELATED WORK

The goal of Wi-Fi and LTE-LAA coexistence is to increase the throughput of LAA network but not to degrade the performance of the Wi-Fi network. Some papers have looked at the performance analysis of standard or modified listen before talk (LBT) mechanism for LTE-LAA/Wi-Fi networks [16]–[21], without necessarily focusing on the fairness issue. Other works have addressed spatial reuse [22], [23], energy consumption [24], traffic offloading [25], power allocation [26] and resource allocation [27] issues. In this work we are interested in the fairness issue. Two main approaches have been proposed in the literature to handle the fairness issue: (i) varying the transmission or channel occupancy time (COT) and (ii) varying the idle time. Transmission time of LAA station is the time for which it keeps the channel occupied and idle time is defined as the time it keeps the channel vacant. A LAA station switches from the transmitting mode and rests in the idle mode when it has no data to transmit or it is undergoing initial contention phase.

The first approach deals with transmission time modification. The percentage of time a LAA or Wi-Fi station occupies a channel accounts for overall channel efficiency and fairness between the two networks. Varying the COT under different load conditions is a coexistence solution. The work in [28] proposed adjusting COT for LAA stations from a range of values depending upon Wi-Fi load. Another COT modification of appending Clear-To-Send frame for reserving the channel to make LAA less intrusive was studied in [29]. Recently, some papers have used machine learning

algorithms to optimize Wi-Fi and LTE-LAA coexistence. A Q-learning technique for estimating the channel occupancy time of LAA under different Wi-Fi traffic conditions was proposed in [30]. The learning outcome was then used to adapt COT and interference power constraint. A channel sharing scheme was proposed in [31] where LAA stations monitor the Wi-Fi activity for adaptive duty cycling. A Q-learning based approach was presented to intelligently select an optimal combination of transmission and mute time for LTE-U and Wi-Fi network in [32]. A deep reinforcement learning based approach was adopted to optimize transmission time of LAA in [33].

The second approach deals with idle time modification. Under this approach, a key strategy is dynamically adapting the wait time to inculcate fairness among coexisting nodes. A LAA station undergoes a backoff phase where it waits for its turn to transmit relying upon the CW size. The CW size adaptation has been presented to get the most out of available resources for both the LAA and Wi-Fi network in [34]. A Q-learning based solutions to cope up with the challenge of fair coexistence for LAA and Wi-Fi were proposed in [35], [36]. The focus was on controlling the contention window (CW) size for every LAA eNB in accordance with channel state and traffic load information. Another supervised machine learning based scheme was proposed in [37] to learn from past collisions and predict the CW size on the basis of negative acknowledgements of packet. All the stations contending on a channel had to sense it idle for a continuous duration. Making this contention time adaptive reduces the intra network collisions and increases the system fairness. Initial sensing time optimization schemes were introduced in [38], [39]. Initial CW size and sensing time based adaptation was proposed in [20] to achieve proportional fairness for LTE-LAA and Wi-Fi network.

The work in [40] considers LTE supplemental downlink (SDL) methodology which is used only in the downlink to support higher downlink support for LTE stations. The work in [41], on the other hand, employs an alternating slot assignment model which assigns different time slots to LTE-U BS or Wi-Fi stations. Whereas, the system model we consider in this work is based on the uplink communication and employs a time division multiplexing based solution. Hence, the perspective of resource sharing is different in comparison to [40] and [41]. Note that we do not advocate that one model is better than the other and believe that different access schemes are designed to serve different purposes.

Some works have considered both idle time and transmission time modification. A fairness evaluation for Wi-Fi and LTE in unlicensed spectrum for three different coexistence procedures: continuous transmission, discontinuous transmission and LBT was addressed in [42]. The work in [43] presented a dynamic transmission time and a fixed waiting time configuration for LTE-LAA and Wi-Fi coexistence.

With 5G NR-U advancement, enhanced LAA models are required to provide efficient unlicensed coexistence. Prior works on LTE-LAA are mainly focused on inter-network

fairness, i.e., fairness between Wi-Fi and LTE-LAA networks. Whereas, little attention is given to resource allocation based intra-network fairness among LTE-U and Wi-Fi network. The work in [44] highlighted orthogonal channel allocation scheme based on wireless conditions of the unlicensed channel being shared by LTE-U and Wi-Fi. Proportional fairness for LTE-U/ Wi-Fi coexistence network was proposed for a resource allocation problem in [45]. However, the throughput based intra-network fairness for a LAA network was ignored in literature. This critical improvement is addressed in our work to provide a throughput trade-off to an enhanced level of fairness.

B. CONTRIBUTIONS

The main contributions of this work are given as follows:

- In this paper, we model different transmission probability for each LAA station dependent upon the wireless channel conditions and fair resource sharing. This ensures a proportionally fair resource utilization among the LAA stations. To the best of our knowledge, all the aforementioned literature for Wi-Fi and LAA coexistence has assumed equal and independent packet transmission probability for Wi-Fi and LAA stations.
- We propose an optimal access scheme for LAA, rather than the conventional LAA-LBT policy, to coexist with the Wi-Fi network over the unlicensed spectrum. We divide the channel utilization into two non-overlapping phases for Wi-Fi and LAA. Although, prior work in [33] has presented the idea of inter network collision avoidance through a similar strategy, they do not take into the account the near-far effect of the different stations and the physical channel conditions.
- We formulate the mathematical model for the average throughput per LAA station. In addition, we formulate the optimization problem in order to simultaneously maximize the total throughput of LAA network with performance guarantee for Wi-Fi network in terms of fair throughput share. The formulated optimization problem is analytically solved and optimal design parameters are obtained.
- The proposed scheme provides more that 75 % throughput gain as compared to the benchmark scheme in which transmission probability is uniform for all LAA stations. The performance gain is profound when LAA stations are far from the eNB. In addition, a notable gain of $8 - 9$ % in the fairness index is observed. This reflects the improved intra-network fairness of the proposed LAA network over the conventional LAA network.

C. NOTATION AND PAPER ORGANIZATION

A list of the important variables and parameters is given in Table 1.

The following notation is used in this paper. $p(\cdot)$ and $F(\cdot)$ represent the probability and the cumulative distribution function (CDF), respectively. $\mathcal{L}(\cdot)$ denotes the Lagrangian

TABLE 1. List of important parameters and variables.

Parameter Symbol	Description
W	Number of Wi-Fi stations.
w	Index of Wi-Fi station, where $w = 1, 2, ..., W$.
L	Number of LAA stations.
ℓ	Index of the LAA station, where $\ell = 1, 2, ..., L$.
τ_0	Fraction of time sharing between Wi-Fi and LAA.
θ	Idle slot time duration for Wi-Fi and LAA.
m	Backoff stage of Wi-Fi DCF.
$\text{CW}_{\min}$	Initial contention window size of Wi-Fi DCF.
τ_{w}	Transmission probability of Wi-Fi station in a slot.
$p_{\text{c,w}}$	Collision probability of Wi-Fi station in a slot.
$R_w^{\min}$	Minimum average throughput per Wi-Fi station.
$R_w^{\max}$	Maximum average throughput per Wi-Fi station.
$\tilde{R}_w$	Achievable average throughput per Wi-Fi station.
τ_ℓ	The channel access probability of the LAA station ℓ in a slot.
$p_{\text{s},\ell}$	The probability of successfully accessing the channel by LAA station ℓ.
$R_{\text{s},\ell}$	The average rate of ℓth LAA station during the successful channel access.
R_ℓ	The fixed rate of ℓth LAA station during the transmission phase.
$\tilde{R}_\ell$	The average throughput per LAA station.
$\mathcal{U}$	A set of all LAA stations excluding the station ℓ.
P_ℓ	The transmitted power for the LAA station ℓ.
$P_{\max}$	The available average power allocated per LAA station.
d_ℓ	The distance between the LAA eNB and the ℓth station.
h_ℓ	The fading channel gain of ℓth station.
γ_ℓ	The signal to noise ratio (SNR) of ℓth station.
α	The path loss exponent.
κ	The path loss factor.
σ^2	The additive white Gaussian noise (AWGN) power.

function. $\nabla(\cdot)$ is used for the gradient and $[\cdot]^\top$ is used for the transpose operator. The rest of this paper is organized as follows. The system model is described in Section II. The optimization problem and its derived solution is presented in Section III. The numerical results are discussed in Section IV.

Lastly, Section V provides the concluding remarks of the paper.

II. SYSTEM MODEL

A. WI-FI AND LAA COEXISTENCE MODEL

We consider a Wi-Fi and LTE-LAA network coexisting on a single channel of 20 MHz in the 5 GHz unlicensed band as illustrated in the Fig. 1. In this work, our goal is to advocate for a cooperative LTE-LAA and Wi-Fi communication policy for a fair coexistence. Therefore, we limit our system model to the fundamental single 20 MHz (non-aggregate) channel scenario which serves the purpose. Considering a multi-channel scenario with 40 MHz, 80 MHz, or 160 MHz aggregated channels gives rise to interference issues and other challenges which are outside the scope of this work [12]. Note that typically the LAA deployments are for the outdoors and Wi-Fi deployments are for the indoors [46]–[48]. The signal strength of both LAA and Wi-Fi devices operating outdoors is comparable. This leads to increased coexistence and hidden node problem. However, in last few years the proprietary Wi-Fi networks are increasingly being deployed in urban areas to support Wi-Fi in the outdoors [7]. This new rapidly spreading scenario is one of the most critical deployments in regards to the coexistence issues of LAA and Wi-Fi. Therefore, we consider the outdoors deployment setting. Nevertheless, the mathematical model in this work is valid for any channel size as long as both LAA and Wi-Fi operate on the same channel.

The network in Fig. 1a consists of a LTE-LAA eNB with L LAA stations and a Wi-Fi Access Point (AP) with W Wi-Fi stations. The macro eNB controls the unlicensed channel assignment to Wi-Fi and the LAA eNB in its transmission range. Our focus is coexistence of Wi-Fi and LAA after the channel has been assigned. We assume both Wi-Fi and LAA stations are in saturated mode, i.e., the nodes always have a packet to transmit after a successful transmission. Moreover, we model the total transmission time over the channel as N slots each of duration T. Each time slot T is further divided into t equal mini-slots of length θ. In order to support the coexistence between Wi-Fi and LAA networks, each time slot T is divided into two phases: the Wi-Fi transmission phase with duration $\tau_0 T$ and the LAA transmission phase with duration $(1 - \tau_0)T$ as shown in the Fig. 1b. Here, $0 < \tau_0 < 1$ is used to control the fraction of time assigned to Wi-Fi and LAA. During the Wi-Fi transmission phase, the Wi-Fi stations follow the standard DCF protocol [45] to access the channel with uniform access probabilities. During the LAA transmission phase, the LAA stations contend among each other for the remaining $(1 - \tau_0)T$ time based on their channel access probabilities. Generally, in a Wi-Fi and LAA coexistence network, a collision occurs when two or more nodes transmit on a given channel at the same time, while an intended receiver is in their transmission range. This introduces two types of collisions: inter-network collisions and intra-network collisions. In our system model, Wi-Fi and
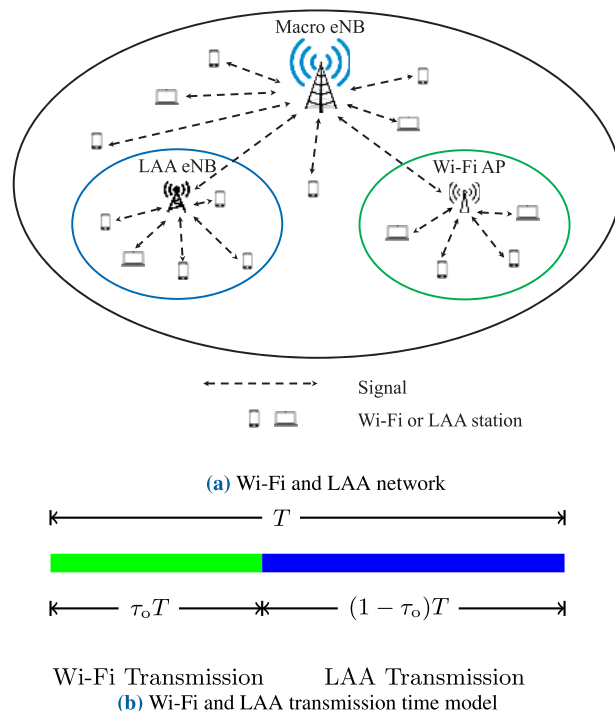

Signal

Wi-Fi or LAA station

(a) Wi-Fi and LAA network

(b) Wi-Fi and LAA transmission time model

FIGURE 1. Illustration of system model: Wi-Fi and LAA coexisting on a single unlicensed channel.

LAA stations transmit on their non-overlapping transmission slots, thus, there will be no collisions between Wi-Fi and LAA stations. However, the probability of intra-network collisions is non-zero.

We analyze the performance of the proposed network setting in terms of the channel utilization for successful transmission of data. This is referred to the average throughput per station for the Wi-Fi and LAA. Firstly, for the proposed coexistence scenario, we obtain an achievable average throughput for a Wi-Fi station. Secondly, we calculate the average throughput of each LAA station based on the wireless channel conditions.

For a multi-channel channel scenario, there are two operations for LBT [49], i.e., Type-A and Type-B. For Type-A, our proposed problem setting can straightforwardly be extended. This will require disjoint rate adaptation for each of the aggregate channels. On the other hand, Type-B will require joint rate adaptation as well as optimization of the short clear channel assessment parameter used in Type-B. Nevertheless, multi-channel scenario specifically for LBT Type-B operation poses an important challenge which is a promising future work direction.

B. AVERAGE THROUGHPUT FOR WI-FI STATIONS

IEEE 802.11 standard specifies Wi-Fi stations to adopt CSMA/CA [14]. Each station employs the binary exponential backoff (BEB) mechanism to transmit packets. It senses the channel for a distributed interframe space (DIFS) period to check its availability. If the channel is found idle the backoff counter is decremented. The station attempts once backoff

counter is zero and channel if found idle. After the packet transmission, the node waits for a period of short interframe space (SIFS) to receive an acknowledgement (ACK) from the AP. After a successful packet delivery, the station switches to the initial contention window (CW) size, CW_{min}. Under collision, the Wi-Fi station doubles the CW until it reaches the maximum contention window size, CW_{max}.

A homogeneous Wi-Fi network when it fully utilizes the channel (the case when $\tau_0 = 1$), provides a maximum throughput/rate to the stations. For a total number of W Wi-Fi stations, let R_w^{max} be the maximum average rate per station, where $w = 1, 2, \ldots, W$. Whereas, for a coexistence network, when the channel is being shared by both Wi-Fi and LAA (the case when $0 < \tau_0 < 1$), then performance is affected. According to the 3GPP standard [6], ideally a single LAA station coexisting with Wi-Fi on the unlicensed channel must not degrade the Wi-Fi performance more than an additional Wi-Fi station. Therefore, for a maximum allowable performance degradation we consider that each LAA station acts just like a Wi-Fi station, then the minimum average rate for each station is calculated as [50].

$$R_w^{min} = \frac{p_{t,w} p_{s,w} D}{(1 - p_{t,w})\theta + p_{t,w} p_{s,w} T_s + p_{t,w}(1 - p_{s,w})T_c}, \quad (1)$$

where D is the average packet size and

$$p_{t,w} = 1 - (1 - \tau_w)^n \quad (2)$$

is the transmission probability of at least one station in a mini-slot t and $n = W + L$ denotes the total number of stations (actual Wi-Fi stations plus LAA stations behaving like Wi-Fi stations). In (2), τ_w is the stationary transmission probability of each station which is calculated by solving [50]

$$\tau_w = \frac{2(1 - 2p_{c,w})}{(1 - 2p_{c,w})(CW_{min} + 1) + p_{c,w}(1 - (2p_{c,w})^m)}, \quad (3)$$

where

$$p_{c,w} = 1 - (1 - \tau_w)^{n-1} \quad (4)$$

is the stationary probability of collision in a single slot, CW_{min}, m are the initial window size and maximum backoff stage, respectively, based on the DCF mechanism. We assume that all the Wi-Fi stations have equal payload size and τ_w. In (1),

$$p_{s,w} = \frac{n\tau_w(1 - \tau_w)^{n-1}}{p_{t,w}} \quad (5)$$

is the probability of any station to successfully transmit in the slot, given the fact that there is at least one transmission. Let T_s be the average period for the successful transmission and T_c be the average period for a collision. From [50]

$$T_s = \frac{(H + D + ACK)}{R_b} + SIFS + 2\delta + DIFS \quad (6)$$

and

$$T_c = \frac{(H + D)}{R_b} + \delta + DIFS, \quad (7)$$

where H is the header size, ACK is the acknowledge size, δ is the propagation delay, R_b is the channel bitrate, SIFS is the short interframe space and DIFS is the distributed interframe space for Wi-Fi.

Since, we have assumed Wi-Fi and LAA network to have non-overlapping transmission phases, therefore the conventional Wi-Fi activity is not being altered during its own transmission phase.

C. AVERAGE THROUGHPUT FOR LAA STATIONS

During the LAA transmission phase, all the L stations contend to access the channel with their fixed probabilities. We model the channel between LAA eNB and ℓth station over the mini-slot t, as a quasi static block fading channel which follows a Rayleigh distribution. The fading power gain of the channel from the eNB to the station ℓ or vice versa is denoted as h_ℓ.

In a mini-slot t, the channel access probability for the ℓth station is denoted as τ_ℓ where, $\ell = 1, 2, \ldots, L$. The probability $p_{s,\ell}$ is defined as the probability of the ℓth station to successfully access the channel when only the ℓth station accesses the channel, given the condition that at least one station has accessed the channel. This is written as

$$p_{s,\ell} = \frac{L\tau_\ell \prod_{i \in \mathcal{U}} (1 - \tau_i)}{\left(1 - \prod_{k=1}^{L} (1 - \tau_k)\right)}. \quad (8)$$

where, $\mathcal{U}$ is a set of all LAA stations excluding the station ℓ. The average throughput for the ℓth station for a given slot is calculated as

$$\tilde{R}_\ell = R_{s,\ell} \, p_{s,\ell}, \quad (9)$$

where, $R_{s,\ell}$ is the average rate of the station ℓ during the successful channel access and $p_{s,\ell}$ is the probability of successfully accessing the channel from (8). We assume that the station ℓ transmits at a fixed transmission rate $(1 - \tau_0)R_\ell$ during the whole transmission phase. This yields the average rate $R_{s,\ell}$ as a product of the fixed rate and the probability of successful transmission/non-outage, $p(\text{non-outage})$ and is given by

$$R_{s,\ell} = R_\ell(1 - \tau_0) \, p(\text{non-outage})$$
$$= R_\ell(1 - \tau_0) \, p\left(\log_2(1 + \gamma_\ell) \geq R_\ell\right), \quad (10)$$

where γ_ℓ is the Signal-to-Noise-Ratio (SNR). It is obtained as

$$\gamma_\ell = \frac{\kappa P_\ell |h_\ell|^2}{d_\ell^\alpha \sigma^2}, \quad (11)$$

where $\kappa = (c/4\pi f)^2$ is the pathloss factor, c is the speed of light and f is the carrier frequency, P_ℓ is the transmit power for station ℓ, d_ℓ is the distance between the eNB and station ℓ, α is the path loss exponent, $|h_\ell|^2$ is the fading channel gain

and σ^2 is the additive white Gaussian noise (AWGN) power. Substituting (11) into (10) and solving, we get

$$R_{s,\,\ell} = R_\ell (1 - \tau_0) \left(1 - F_{h_\ell} \left(\frac{d_\ell^\alpha \sigma^2 (2^{R_\ell} - 1)}{\kappa P_\ell} \right) \right), \quad (12)$$

where, $F_{h_\ell}(\cdot)$ represents the CDF for the fading channel gain $|h_\ell|^2$. Transforming (12) in terms of Rayleigh fading channel it becomes an exponential which is expressed as

$$R_{s,\,\ell} = R_\ell (1 - \tau_0) \exp \left(- \frac{\lambda d_\ell^\alpha \sigma^2 (2^{R_\ell} - 1)}{\kappa P_\ell} \right), \quad (13)$$

where, λ is the fading parameter.

III. PROBLEM FORMULATION AND SOLUTION

In order to develop a fair utilization of the unlicensed channel for Wi-Fi and LAA network, we consider two levels of fairness: *(a) the fairness between Wi-Fi and LAA networks*, and *(b) the fairness among LAA stations*.

A. FAIRNESS BETWEEN WI-FI AND LAA

Fairness between both the networks is maintained by sharing the channel access opportunity between them. During Wi-Fi and LAA coexistence scenario ($0 < \tau_0 < 1$), we observe that Wi-Fi stations will have a very low data rate if LAA stations aggressively use the unlicensed channel. This requires appropriate resource sharing opportunity such that Wi-Fi stations' performance is not compromised more than a minimum threshold and LAA stations get the chance to make the most out of the allotted time. We capture this by finding the favourable value of the sharing parameter, τ_0, which ensures that Wi-Fi stations can maintain an achievable average throughput per station, $\tilde{R}_w$, calculated as [44]

$$\tilde{R}_w = R_w(\tau_0) = \tau_0 \cdot R_w^{\max}, \quad (14)$$

and varies between

$$R_w^{\min} \leqslant R_w(\tau_0) \leqslant R_w^{\max} \quad (15)$$

during the time $\tau_0 T$. Here, $R_w^{\max}$ is calculated from (1) when $n = W$, i.e., the maximum achievable rate for a Wi-Fi only system when the channel is not being shared by LAA. When Wi-Fi shares the channel, it gives a fraction of opportunity to LAA stations, thus, it is expected to maintain a minimum throughput of $R_w^{\min}$.

B. PROPORTIONAL FAIRNESS AMONG LAA STATIONS

The time division access network setting for Wi-Fi and LAA allows disjoint design for both kinds of stations. This opportunity is exploited in our work. In particular an optimal transmission strategy is designed for the LAA stations. Once the Wi-Fi performance thresholds are met, the objective is to enhance the performance of LAA by devising a transmission control policy for the LAA transmission period $(1 - \tau_0)T$. It is easy to notice that considering equal access probability, τ_ℓ, for all the LAA stations, the stations at greater distance from the LAA eNB will have more outage probability. Consequently, their average throughput will be low as compared

to those at a smaller distance from eNB. Therefore, an equal access probability design results in unfairness among LAA stations.

We consider distinct access probability for each LAA station in order to incorporate the effect of wireless channel conditions. We also consider that, a station ℓ can transmit at an average power $P_\ell(1 - \tau_0)$ which cannot be greater than the maximum transmit power allocated per station, $P_{\max}$. This can be given by the following inequality

$$P_\ell \tau_\ell (1 - \tau_0) \leqslant P_{\max}. \quad (16)$$

The overall goal is to improve the LAA network throughput while encouraging fairness among its stations. Unfair distribution of the resources will result in degraded performance of individual stations. The parameters which affect a LAA station performance are: the fraction of channel utilization time, channel state, channel access probability, data rate and transmitted power. Hence, an optimal transmission strategy based on the aforementioned parameters is required to enhance individual LAA station performance (throughput) with fairness consideration, which eventually will contribute towards the overall network performance. In particular, the system objective is to be modelled in a particular fashion by considering the trade-off between the overall system performance and the level of fairness in terms of individual station performance.

In the literature, there are three popular system objectives for throughput/rate maximization, which differ in terms of the overall system performance and fairness among the stations [51]. These system objectives are (i) sum throughput maximization, (ii) min-max throughput maximization, and (iii) proportionally-fairness throughput maximization.

The sum throughput maximization objective prioritizes the stations with better signal strength, thereby allocating more system resources to boost their throughput. As a result, the overall system throughput performance increases at the cost of throughput-unfairness among the stations. On the other hand, the max-min throughput maximization objective targets strict throughput fairness at the cost of reduced overall system throughput performance. The motivation behind proportionally-fair throughput maximization objective is to strike a balance between the system throughput and fairness among stations. This objective achieves some level of fairness among stations by providing each station with a performance that is proportional to its signal strength. This is achieved by reducing the opportunity of the stations with strong signal strength, getting larger share of system resources compared to the weak stations. More system resources are allocated to the stations when their instantaneous channel condition is better relative to their channel statistics. Thereby, proportional-fairness is achieved without compromising much throughput efficiency performance. Since the signal strength fluctuates independently for different stations, this strategy effectively exploits multi-user diversity. This is achieved by maximizing the sum of logarithmic throughput cost function of the

individual stations [51]–[53], i.e., $\sum_{\ell=1}^{L} \log \tilde{R}_\ell$, where $\tilde{R}_\ell$ is defined in (9).

To address the above problem, we propose a transmission policy for LAA stations which maximizes their average throughput in a proportionally fair manner, when LAA and Wi-Fi stations coexist. This proposed transmission policy is given by the solution of the following optimization problem

$$\underset{\tau_0, \tau_\ell, P_\ell, R_\ell}{\text{maximize}} \quad \sum_{\ell=1}^{L} \log \tilde{R}_\ell$$

subject to

$$C_1 : 0 < \tau_0 < 1,$$
$$C_2 : 0 < \tau_\ell < 1, \quad \forall \ell$$
$$C_3 : P_\ell \tau_\ell (1 - \tau_0) \leqslant P_{\max}, \quad \forall \ell$$
$$C_4 : R_w^{\min} \leqslant R_w(\tau_0) \leqslant R_w^{\max}, \quad \forall w. \quad (17)$$

where $\tau_0, \tau_\ell, P_\ell$ and R_ℓ are the design variables. Here, the constraints C_1 and C_2 provide the range for the channel sharing parameter τ_0 (between LAA and Wi-Fi) and the probability of channel access for the LAA stations, respectively. The power allocation for each LAA station based on its channel access probability is ensured by condition C_3. Lastly, constraint C_4 accounts for the condition that LAA's coexistence with Wi-Fi does not compromise Wi-Fi's performance more than a Wi-Fi only system. This establishes inter-network fairness as well. The $\tilde{R}_\ell$ in the objective function is defined in (9). Using the basic calculus and algebraic calculations it can be shown that the problem in (17) is a non-convex optimization problem.

In order to solve the maximization problem in (17), we first present Lemma 1.

Lemma 1: The optimal P_ℓ to maximize the objective function in problem (17) while satisfying the constraints is given as

$$P_\ell = \frac{P_{\max}}{\tau_\ell(1 - \tau_0)}. \quad (18)$$

Proof: Every station can transmit less than or equal to the maximum available power, $P_{\max}$. In order to maximize the objective function in problem (17), the constraint C_3 must attain the maximum available value of the average power, i.e., $P_\ell \tau_\ell (1 - \tau_0) = P_{\max}$ which yields (18). ∎
Based on Lemma 1, we remove P_ℓ as a design variable from (17) and plug in its value from (18) in the objective function of (17). The equivalent problem is then given by

$$\underset{\tau_0, \tau_\ell, R_\ell}{\text{maximize}} \sum_{\ell=1}^{L} \log \left\{ \frac{L \tau_\ell \prod_{i \in \mathcal{U}} (1 - \tau_i)}{\left(1 - \prod_{k=1}^{L} (1 - \tau_k)\right)} R_\ell (1 - \tau_0) \right.$$
$$\left. \times \exp \left(-\frac{\lambda d_\ell^\alpha \sigma^2 \tau_\ell (2^{R_\ell} - 1)(1 - \tau_0)}{\kappa P_{\text{av}}} \right) \right\}$$

subject to

$$C_1 \; C_2 \text{ and } C_4. \quad (19)$$

Now we only have τ_0, τ_ℓ and R_ℓ as the design variables. Using the basic calculus and algebraic calculations it can be shown that the problem in (19) is a non-convex optimization problem. The solution to the optimization problem in (19) is given by the following theorem.

Theorem 1: The optimal channel sharing parameter between Wi-Fi and LAA is given as

$$\tau_0^* = \frac{R_w^{\min}}{R_w^{\max}}, \quad (20)$$

and the optimal rate of LAA station ℓ is given as

$$R_\ell^* = \hat{R}_\ell, \quad (21)$$

and the optimal channel access probability of the LAA station ℓ is given as

$$\tau_\ell^* = \hat{\tau}_\ell, \quad (22)$$

where $\hat{R}_\ell$ and $\hat{\tau}_\ell$ are obtained by simultaneously solving the following set of equations for all the LAA stations

$$\hat{R}_\ell = \frac{W_0\left(\frac{P_{\max}}{\lambda D_\ell \hat{\tau}_\ell (1 - \frac{R_w^{\min}}{R_w^{\max}})} \right)}{\log(2)}, \quad \forall \ell, \quad (23)$$

$$\frac{\hat{\tau}_\ell (L - 1) \left(1 - (1 - \hat{\tau}_\ell) \prod_{k \in \mathcal{U}} (1 - \tau_k) \right)}{\hat{\tau}_\ell (1 - \hat{\tau}_\ell) \left(1 - (1 - \hat{\tau}_\ell) \prod_{k \in \mathcal{U}} (1 - \tau_k) \right)}$$
$$- \frac{(1 - \hat{\tau}_\ell) \left(1 - \prod_{k \in \mathcal{U}} (1 - \tau_k) \right)}{\hat{\tau}_\ell (1 - \hat{\tau}_\ell) \left(1 - (1 - \hat{\tau}_\ell) \prod_{k \in \mathcal{U}} (1 - \tau_k) \right)}$$
$$= -\frac{\lambda D_\ell (2^{\hat{R}_\ell} - 1)(1 - \frac{R_w^{\min}}{R_w^{\max}})}{P_{\max}}, \quad \forall \ell, \quad (24)$$

where $W_0(\cdot)$ is the principal branch of the Lambert-W function and

$$D_\ell = \frac{d_\ell^\alpha \sigma^2}{\kappa}.$$

Proof: The proof is given in Appendix A. ∎
The insights from Theorem 1 are discussed in the following remarks.

Remark 1: In order to account for the Wi-Fi performance posed by the constraint C4 in optimization problem in (19), it is observed that the optimum value of time sharing variable between Wi-Fi and LAA in (20) is keeping the Wi-Fi throughput to the minimum threshold. Accordingly, (20) ensures that each Wi-Fi station achieves at least the minimum average throughput and this will then allow the system to maximize the throughput of the LAA stations. As the ratio $\frac{R_w^{\min}}{R_w^{\max}}$ is always less than 1, therefore a fair chance of channel utilization is

provided to LAA as well. Note that we are not prioritizing Wi-Fi's performance improvement as our goal lies in the overall LAA throughput improvement without significantly impacting Wi-Fi's performance.

Remark 2: The solution of $\hat{R}_\ell$ and $\hat{\tau}_\ell$ (by solving (23) and (24)) exists, when the constraint for τ_ℓ in the optimization problem (26) is a strict inequality. It is interpreted as $0 < \hat{\tau}_\ell < 1$ for each ℓ station of LAA. This is a practical condition for a coexistence scenario.

The proposed scheme for WiFi and LAA coexistence is summarized in the Algorithm 1.

Algorithm 1 *Proposed System Operation*

1: **Inputs:** System parameters given in Table 2.
2: **Initialize:** τ_0 following (20), P_l following (18), and compute LAA station parameters for the given channel realization following Theorem 1.
3: **Repeat** for each transmission slot T.
4: Each Wi-Fi station follows the standard CSMA/CA with BEB within Wi-Fi transmission duration, $\tau_0 T$.
5: After $\tau_0 T$, each LAA station l attempts channel access with probability τ_l and transmits its packet at a transmission rate R_l with transmit power level P_l within the LAA transmission duration $(1 - \tau_0)T$.
6: Update LAA station parameters for the given channel realization following Theorem 1.

IV. NUMERICAL RESULTS

In this section, we evaluate the solution proposed in Section III using numerical simulations. Without loss of generality, we consider $L = 2, 6, 10, 14, 18$ LAA stations coexisting with $W = 5, 10$ Wi-Fi stations. According to the 3GPP standard modelling [49], a LAA eNB model allows the distribution of the stations within a radius of 40 m. Therefore, among the L stations of LAA, we populate one half at the radius of $r_1 = 5$ m and the other half at the radius of $r_2 = 30$ m from the eNB in order to include the near-far effect. The other parameters values are summarized in Table 2.

The optimal solution for above simulation values is obtained after solving (20), (23) and (24) in MATLAB. The optimum numerical values of τ_0^*, R_ℓ^* and τ_ℓ^* are then used to calculate the individual average throughput and sum average throughput of the LAA network.

A. SETTINGS FOR THE SCHEMES COMPARED
For comparative performance analysis with the proposed scheme we consider multiple existing schemes. Note that the proposed scheme is novel and its comparison with the existing schemes in their original form would not be fair. Hence, we adapt some closely related existing schemes in the following to compare them with the proposed scheme.

1) LAA-LBT MECHANISM
We adapt the analytical model of LAA-LBT mechanism for the heterogeneous LAA/Wi-Fi coexistence network scenario

TABLE 2. Simulation parameters and their values.

Parameter	Value	Parameter	Value
θ	$9\,\mu s$	f	5 GHz
m	6	λ	1
$CW_{\min}$	16	α	4
R_b	1 Mbps	σ^2	-90 dBm
SIFS	$16\,\mu s$	$P_{\max}$	30 dBm
DIFS	$34\,\mu s$	D	8184 bits
δ	$14\,\mu s$	ACK + H	240 + 400 bits

presented in [16] to our scenario. Similar to the proposed scheme, this scheme considers Wi-Fi and LAA stations to coexist over a single channel under saturated traffic. Different from the proposed scheme, this scheme does not consider non-overlapping contention times for Wi-Fi and LAA, thus it may experience both intra-network and inter-network collisions. Moreover, unlike the proposed scheme, the individual sum throughput for the LAA and Wi-Fi networks are calculated under ideal channel conditions.

2) FAIRNESS-CONSTRAINED COEXISTENCE SCHEME
In [21], a fairness-constrained LAA/Wi-Fi coexistence scheme based on optimal tuning of the initial window sizes and LAA transmission opportunity was proposed. Similar to the proposed scheme, this scheme considers Wi-Fi and LAA stations to coexist over a single channel under saturated traffic and maintains the 3GPP notion of fairness for the Wi-Fi stations to not be affected more than an additional Wi-Fi. In contrast to our proposed scheme which considers a more realistic scenario based on channel fading and outage, the scheme in [21] does not consider any packet loss and assumes ideal channel conditions. Another differentiating factor from the proposed scheme is that this scheme does not consider non-overlapping contention times for Wi-Fi and LAA, thus it may experience both intra-network and inter-network collisions. We implement the fairness scheme considering case 2 of Theorem 1 in [21] to calculate the maximum sum rate for LAA stations.

3) BENCHMARK SCHEME
We also consider a more comparable scheme and refer to it as the benchmark scheme. This benchmark scheme is inspired from the idea presented in [6], [49] for Wi-Fi and LAA coexistence in which all the LAA stations have equal probability of accessing the channel in a given time slot [6], [49], i.e., for each station ℓ, $\tau_\ell = 1/L$. We also set a fixed transmission rate for all the LAA stations over the whole transmission period, i.e., $R_\ell = R_m$, $\forall \ell$, where the transmission rate R_m is the one that maximizes the average throughput per station at the average distance of two different radii from the eNB, i.e., $\frac{r_1+r_2}{2}$. In addition, the benchmark scheme uses the same transmission time distribution between Wi-Fi and LAA in terms of the time sharing fraction τ_0. Thus, it maintains the fair share between Wi-Fi and LAA, but does not consider the
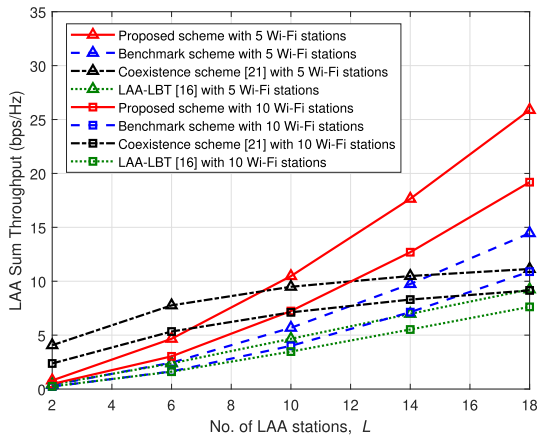
FIGURE 2. Sum throughput for LAA stations under the proposed and existing schemes.

fairness among LAA stations. This is the differentiating factor between the benchmark and the proposed scheme.

Moreover, for the brevity of analysis we calculate the benchmark optimal values of τ_0, τ_ℓ and R_m similarly as Theorem 1. Later we plug in these values in (9) to calculate the benchmark value for average throughput per LAA station.

B. THROUGHPUT COMPARISON OF THE PROPOSED SCHEME WITH OTHER EXISTING SCHEMES

We compare the LAA sum throughput of the network for the proposed scheme with the other state-of-the-art schemes, i.e., the benchmark scheme, the fairness constrained coexistence scheme in [21] and the conventional LAA-LBT coexistence mechanism in [16]. The LAA sum throughput is calculated as $\sum_{\ell=1}^{L} \tilde{R}_\ell$.

Fig. 2 plots the LAA sum throughput in bps/Hz versus the number of LAA stations, L, for $W = 5$ and 10 Wi-Fi stations for the four considered schemes. We can see that for all the schemes, the sum throughput increases as the number of LAA station increases, which is to be expected.

First we compare the proposed scheme with the benchmark scheme as both follow no inter-network collision mechanism. From the Fig. 2, we can see that the proposed scheme provides higher sum throughput compared to benchmark scheme. More importantly, the results show that for a given number of Wi-Fi stations, as the number of LAA stations decreases, the relative gap between proposed and benchmark scheme increases. We quantify this gap in terms of the throughput gain of the proposed scheme over the benchmark scheme, defined as a ratio of (LAA sum throughput of proposed scheme - LAA sum throughput of benchmark scheme)/(LAA sum throughput of benchmark scheme), expressed as a percentage. For example, when $W = 5$ and $L = 14$, the proposed scheme provides a throughput gain of approximately 81% over the benchmark scheme. As the number of Wi-Fi stations increases this gain does not decrease significantly. For instance, for $L = 14$, the throughput gain only decreases slightly from 81% for $W = 5$ to 79% for $W = 10$.
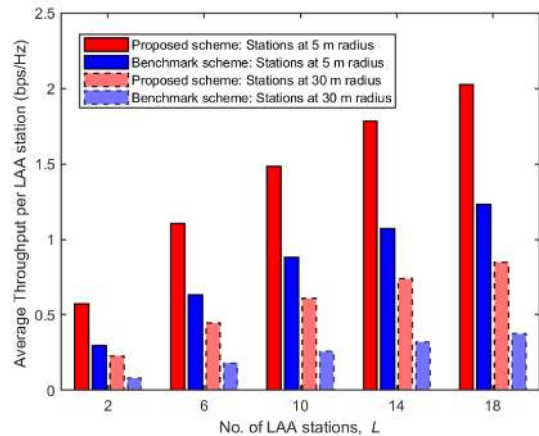
**(a)** Coexistence with 5 Wi-Fi stations.
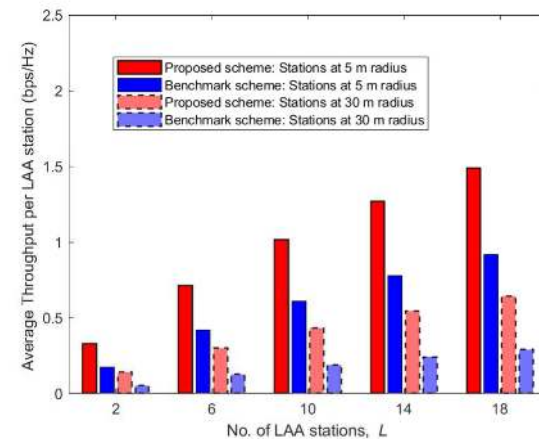

(b) Coexistence with 10 Wi-Fi stations.

FIGURE 3. Average throughput per LAA station located at 5 m and 30 m radius from the eNB.

Secondly, we compare the proposed scheme to the LAA-LBT scheme [16] and the fairness constrained coexistence scheme [21]. These schemes suggest operating LAA and Wi-Fi simultaneously which may cause inter-network and intra-network collisions. In Fig. 2 we see that the proposed scheme outperforms the LAA-LBT coexistence mechanism. The Fig. 2 shows that the sum throughput for the fairness constrained scheme is greater than our proposed scheme when LAA stations are lesser in number. However, when the number of LAA stations increase, our proposed scheme provides a better throughput performance. This is because when the number of LAA stations coexisting with the Wi-Fi increases, the inter-network and intra-network collisions increases for the model in [21] and the sum throughput saturates. Thus, our proposed scheme with zero inter-network collisions provides a scalable solution under practical conditions of higher number of coexisting users.

Next we look at the average throughput per LAA station for the proposed and the benchmark scheme. Fig. 3 plots the average throughput per LAA station versus number of LAA stations, L, coexisting with (a) $W = 5$ Wi-Fi stations and (b) $W = 5$ Wi-Fi stations. For each value of the number
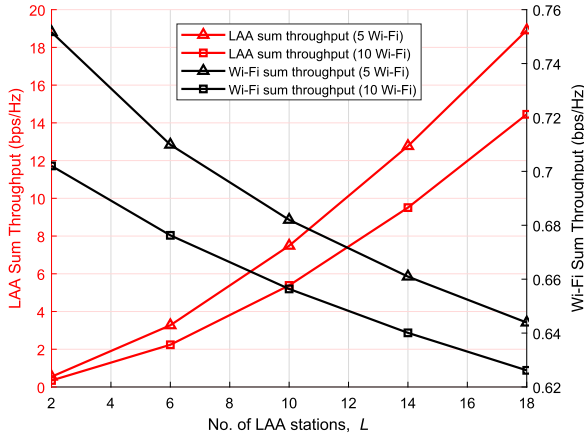
(a) Performance for the proposed scheme.
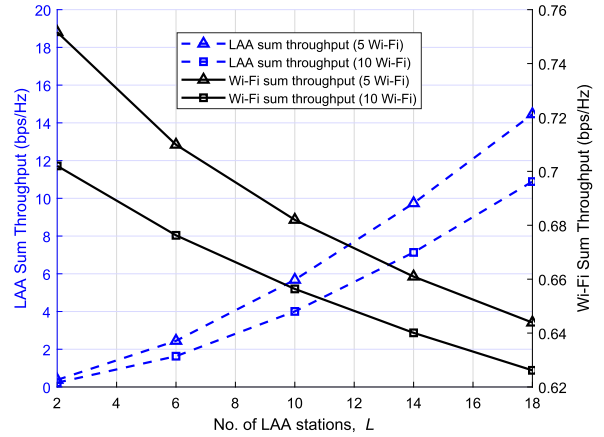
(b) Performance for the benchmark scheme.

FIGURE 4. Sum throughput for Wi-Fi and LAA coexisting on a single unlicensed channel.
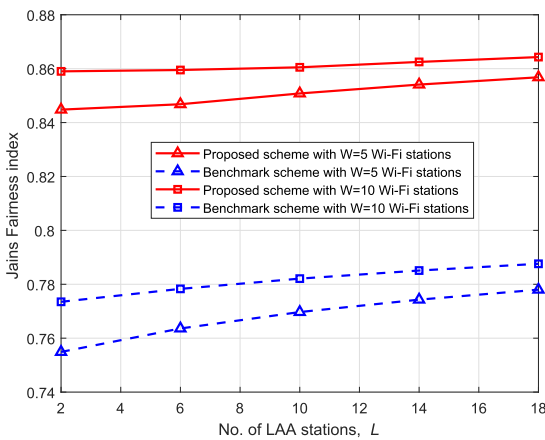

FIGURE 5. Jain's fairness index for the LAA under the proposed and benchmark scheme.

of LAA stations, we assume that half are located randomly at a radius of 5 m and other half are randomly located at a radius of 30 m. We can see that the average throughput per LAA station is higher for the proposed scheme, compared to the benchmark scheme. For the number of LAA stations in the range 2-18, the increase in the average throughput of a LAA station for the proposed scheme compared to the benchmark scheme is in the range of 88% to 180% for $W = 5$ (Fig. 3a) and 87% to 167% for $W = 10$ (Fig. 3b). In addition, the LAA stations at the greater distance (30 m) show more throughput improvement compared to the ones at the closer distance (5 m). This indicates that the proportional fairness is achieved among the LAA stations. The fairness is examined in more detail in the next section.

C. INTER-NETWORK AND INTRA-NETWORK FAIRNESS

First we look at the fairness between Wi-Fi and LAA networks. Fig. 4 plots the sum throughput of LAA stations and the sum throughput of the Wi-Fi stations versus the number of coexisting LAA stations, L, for (a) the proposed scheme and (b) the benchmark scheme. We can see that as the number of LAA stations increases, the sum throughput of the LAA stations increases and the sum throughput of Wi-Fi

stations decreases for both the schemes. This is because the transmission time is being shared among the Wi-Fi and LAA stations. Although both the benchmark and proposed schemes maintain a fair coexistence with Wi-Fi by not sabotaging Wi-Fi throughput more than the minimum threshold, the proposed scheme provides enhanced throughput performance for the LAA, in Fig. 4a as compared to the benchmark scheme in Fig. 4b.

Next we examine the intra-network fairness among the LAA stations. The fairness of the LAA system is determined in terms of Jain's fairness index (J_I) [54] as

$$J_I = \frac{\left(\sum\limits_{\ell=1}^{L} \tilde{R}_\ell \right)^2}{L \sum\limits_{\ell=1}^{L} \tilde{R}_\ell^2}. \tag{25}$$

Fig. 5 plots the Jain's fairness index in (25) versus the number of LAA stations, L, for the proposed and benchmark schemes. The four different curves represents the two cases of coexistence scenarios for the proposed scheme and the benchmark scheme with $W = 5$ and $W = 10$ Wi-Fi stations. The results show that for both cases, there is a notable gain of $8 - 9\%$ in the fairness index for the proposed scheme as compared to the benchmark scheme. In addition, the proposed system is more fairer when the number of Wi-Fi stations increases from 5 to 10. This can be explained as follows. When the number of Wi-Fi stations is increased, the average throughput per LAA station reduces resulting in a decreased value of $L \sum\limits_{\ell=1}^{L} \tilde{R}_\ell^2$, hence an increased value of J_I. In other words, the more number of stations coexisting, the more fairly resources are distributed among them. This demonstrates the advantage of the proposed scheme.

D. OPTIMAL CHANNEL SHARING PARAMETER: IMPACT OF LAA ON WI-FI

We can see the impact of number of LAA stations on the Wi-Fi network performance in terms of the channel sharing
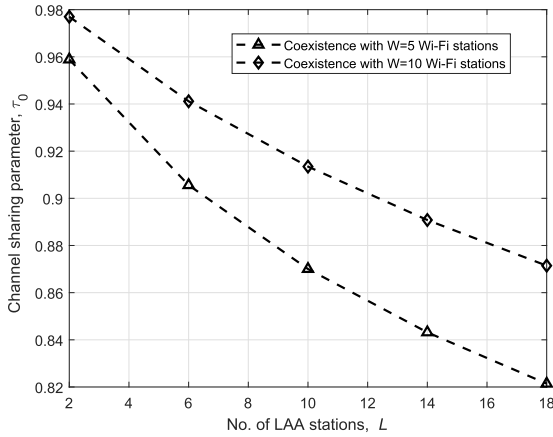
FIGURE 6. Channel sharing parameter, τ_0 for LAA and Wi-Fi coexistence network.

parameter. Fig. 6 plots the optimal channel sharing parameter, τ_0 in (20) versus the number of LAA stations, L, coexisting with $W = 5$ and $W = 10$ Wi-Fi stations. The results show that for a fixed number of LAA stations when the number of Wi-Fi stations increases, τ_0 increases. A larger fraction of time is provided for the Wi-Fi network to accommodate increased number of Wi-Fi stations. In addition, it can be also seen that the percentage drop in the value of τ_0 for increasing number of LAA stations is more when LAA is coexisting with $W = 5$ Wi-Fi stations as compared to the case when it is coexisting with $W = 10$ Wi-Fi stations. The lesser congested Wi-Fi network has larger capacity to accommodate LAA stations while maintaining its own network throughput to a minimum bearable threshold.

V. CONCLUSION

In this paper, a fair coexistence scheme is proposed for the Wi-Fi and LAA network. The proposed mechanism incorporates the inter-network fairness between the Wi-Fi and LAA networks, and the intra-network fairness among LAA system. The core new idea of the proposed scheme is to devise different transmission probability for each LAA station dependent upon the wireless channel conditions and fair resource utilization, rather than the conventional coexistence approach based on uniform transmission probabilities. We design a non-overlapping transmission policy in which Wi-Fi throughput tolerance is tuned through the optimum time sharing fraction between Wi-Fi and LAA. In addition, we formulate a joint optimization problem in order to maximize the sum throughput of LAA network with performance guarantee for Wi-Fi network in terms of fair throughput share. The analytical solution provides the optimal design parameters which are validated through a comparison of proposed scheme with the state-of-the-art coexistence scheme. Our results show that, the proposed scheme ensures a significantly high LAA sum throughput while maintaining notable proportional fairness among its stations. The throughput performance gain is profound for the far LAA stations. On the other hand,

Wi-Fi network throughput is also maintained to the minimum threshold.

This work can easily be extended for alternative approaches, for example when Wi-Fi is prioritized over LAA. Moreover, access probabilities of both Wi-Fi and LAA can also simultaneously be optimized to increase overall throughput of the systems.

APPENDIX A
PROOF OF THEOREM 1
The optimization problem in (19) can be transformed into an equivalent problem written in the standard form as follows

$$
\underset{\tau_0, \tau_\ell, R_\ell}{\text{minimize}} - \sum_{\ell=1}^{L} \log \left\{ \frac{L\tau_\ell \prod_{i \in \mathcal{U}} (1 - \tau_i)}{\left(1 - \prod_{k=1}^{L} (1 - \tau_k) \right)} R_\ell \right.
$$
$$
\left. \times (1 - \tau_0) \exp\left(-\frac{\lambda D_\ell \tau_\ell (2^{R_\ell} - 1)(1 - \tau_0)}{P_{\max}} \right) \right\}
$$

subject to

$$
0 < \tau_0, \quad \tau_0 < 1,
$$
$$
0 < \tau_\ell, \quad \tau_\ell < 1, \quad \forall \ell
$$
$$
R_w^{\min} \leqslant \tau_0 \cdot R_w^{\max}, \quad \tau_0 \cdot R_w^{\max} \leqslant R_w^{\max}, \quad \forall w. \tag{26}
$$

where,

$$
D_\ell = \frac{d_\ell^\alpha \sigma^2}{\kappa}. \tag{27}
$$

Using the basic calculus and algebraic calculations it can be shown that the problem in (26) is a convex optimization problem.

The Lagrangian function for (26) can be given as
$$
\mathcal{L}(\tau_0, \tau_\ell, R_\ell, \boldsymbol{\mu})
$$
$$
= - \sum_{\ell=1}^{L} \log \left\{ \frac{L\tau_\ell \prod_{i \in \mathcal{U}} (1 - \tau_i)}{\left(1 - \prod_{k=1}^{L} (1 - \tau_k) \right)} R_\ell \right.
$$
$$
\left. \times (1 - \tau_0) \exp\left(-\frac{\lambda D_\ell \tau_\ell (2^{R_\ell} - 1)(1 - \tau_0)}{P_{\max}} \right) \right\}
$$
$$
- \mu_1 \tau_0 + \mu_2 (\tau_0 - 1) - \mu_3 \tau_\ell + \mu_4 (\tau_\ell - 1)
$$
$$
+ \mu_5 (R_w^{\min} - R_w^{\max} \tau_0) + \mu_6 R_w^{\max} (\tau_0 - 1), \tag{28}
$$

where $\mu_i \in \boldsymbol{\mu} = \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$ is the Lagrange multiplier corresponding to the ith constraint.

The Karush-Kuhn-Tucker (KKT) conditions for (26) are
$$
-\tau_0 < 0, \quad \tau_0 - 1 < 0, \quad -\tau_\ell < 0, \quad \tau_\ell - 1 < 0,
$$
$$
R_w^{\min} - R_w^{\max} \tau_0 < 0, \quad R_w^{\max} (\tau_0 - 1) < 0, \tag{29a}
$$
$$
\mu_1 \geqslant 0, \quad \mu_2 \geqslant 0, \quad \mu_3 \geqslant 0, \quad \mu_4 \geqslant 0, \quad \mu_5 \geqslant 0, \quad \mu_6 \geqslant 0, \tag{29b}
$$

$$
-\mu_1 \tau_0 = 0, \quad \mu_2 (\tau_0 - 1) = 0, \quad -\mu_3 \tau_\ell = 0,
$$
$$
\mu_4 (\tau_\ell - 1) = 0, \quad \mu_5 (R_w^{\min} - R_w^{\max} \tau_0) = 0,
$$

$$\mu_6 R_w^{\max}(\tau_0 - 1) = 0, \tag{29c}$$

$$\nabla_{\tau_0, \tau_\ell, R_\ell} \mathcal{L}(\tau_0, \tau_\ell, R_\ell, \boldsymbol{\mu}) = \left[\frac{\partial \mathcal{L}}{\partial \tau_0} \ \frac{\partial \mathcal{L}}{\partial \tau_\ell} \ \frac{\partial \mathcal{L}}{\partial R_\ell} \right]^\top$$

$$= [\,0\ 0\ 0\,]^\top. \tag{29d}$$

Here, ∇ is the gradient operator and $[\cdot]^\top$ represents the transpose of the matrix.

It can be shown that the problem in (26) is convex in τ_0 (there exists a global minima). From (29d) we have $\frac{\partial \mathcal{L}}{\partial \tau_0} = 0$. Taking the first derivative of (28) with respect to τ_0 and setting it equal to zero yields

$$\sum_{\ell=1}^{L} \frac{1}{1-\tau_0} - \sum_{\ell=1}^{L} \frac{\lambda D_\ell \tau_\ell (2^{R_\ell} - 1)}{P_{\max}} - \mu_1 + \mu_2$$
$$- R_w^{\max} \mu_5 + R_w^{\max} \mu_6 = 0. \tag{30}$$

It can be shown that the problem in (26) is convex in τ_ℓ and R_ℓ, $\forall \ell$. It means there lies a global minima for every τ_ℓ and R_ℓ, respectively, $\forall \ell$. Taking the first derivative of (28) with respect to τ_ℓ and setting it equal to zero ($\frac{\partial \mathcal{L}}{\partial \tau_\ell} = 0$) yields

$$\frac{L-1}{1-\tau_\ell} - \frac{1 - \prod_{k \in \mathcal{U}} (1 - \tau_k)}{\tau_\ell \left(1 - (1-\tau_\ell) \prod_{k \in \mathcal{U}} (1-\tau_k) \right)}$$
$$+ \frac{\lambda D_\ell (2^{R_\ell} - 1)(1 - \tau_0)}{P_{\max}} - \mu_3 + \mu_4 = 0, \quad \forall \ell \tag{31}$$

where $\mathcal{U}$ is a set of all LAA stations excluding the station ℓ.

Similarly, taking the first derivative of (28) with respect to R_ℓ and setting it equal to zero ($\frac{\partial \mathcal{L}}{\partial R_\ell} = 0$) yields

$$-\frac{1}{R_\ell} + \frac{\lambda D_\ell \tau_\ell 2^{R_\ell} \log(2)(1 - \tau_0)}{P_{\max}} = 0, \quad \forall \ell. \tag{32}$$

For the complementary slackness conditions (29c) to be satisfied, either the constraints or the corresponding Lagrange multiplier should be zero. Considering the case when the Lagrange multipliers $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_6 = 0$, $\mu_5 \neq 0$, i.e., when $\mu_1, \mu_2, \mu_3, \mu_4, \mu_6$ do not exist and μ_5 exists. It implies that the constraint $R_w^{\min} - R_w^{\max} \tau_0$ must follow the equality and be set to zero. This yields

$$\hat{\tau}_0 = \frac{R_w^{\min}}{R_w^{\max}}. \tag{33}$$

Substituting the values of the lagrange multipliers $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_6 = 0$, $\mu_5 \neq 0$, the expressions (30), (31) and (32) can be rewritten as

$$\mu_5 = \frac{L}{R_w^{\max}(1 - \tau_0)} - \frac{1}{R_w^{\max}} \sum_{\ell=1}^{L} \frac{\lambda D_\ell \tau_\ell (2^{R_\ell} - 1)}{P_{\max}}, \quad \forall \ell, \tag{34}$$

$$\frac{\tau_\ell (L-1) \left(1 - (1-\tau_\ell) \prod_{k \in \mathcal{U}} (1-\tau_k) \right)}{\tau_\ell (1-\tau_\ell) \left(1 - (1-\tau_\ell) \prod_{k \in \mathcal{U}} (1-\tau_k) \right)}$$

$$- \frac{(1-\tau_\ell) \left(1 - \prod_{k \in \mathcal{U}} (1-\tau_k) \right)}{\tau_\ell (1-\tau_\ell) \left(1 - (1-\tau_\ell) \prod_{k \in \mathcal{U}} (1-\tau_k) \right)}$$
$$= -\frac{\lambda D_\ell (2^{R_\ell} - 1)(1 - \tau_0)}{P_{\max}}, \quad \forall \ell, \tag{35}$$

and

$$R_\ell = \frac{W_0 \left(\frac{P_{\max}}{\lambda D_\ell \tau_\ell (1-\tau_0)} \right)}{\log(2)}, \quad \forall \ell, \tag{36}$$

respectively. Here, $W_0(\cdot)$ is the principal branch of the Lambert-W function. By setting $\tau_0 = \hat{\tau}_0$ and numerically solving (35) and (36) for R_ℓ and τ_ℓ, we get their values $\hat{R}_\ell$ and $\hat{\tau}_\ell$ as in (23) and (24), respectively. Plugging $\hat{R}_\ell$, $\hat{\tau}_\ell$ and $\hat{\tau}_0$ in (34) gives a positive solution of μ_5. It can be shown that $\hat{R}_\ell$, $\hat{\tau}_\ell$ and $\hat{\tau}_0$ satisfy all the KKT conditions when the Lagrange multiplier μ_5 is positive and $\mu_1, \mu_2, \mu_3, \mu_4, \mu_6$ are zero. Therefore, it is the optimal solution for the problem in (26).

For all other cases, similar steps can be followed and it can be shown that those cases violate one or more KKT conditions. Hence, for all other cases, the corresponding solution becomes invalid.

REFERENCES

[1] J. Zhang, M. Wang, M. Hua, T. Xia, W. Yang, and X. You, "LTE on license-exempt spectrum," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 1, pp. 647–673, 1st Quart., 2018.

[2] M. Labib, V. Marojevic, J. H. Reed, and A. I. Zaghloul, "Extending LTE into the unlicensed spectrum: Technical analysis of the proposed variants," *IEEE Commun. Standards Mag.*, vol. 1, no. 4, pp. 31–39, Dec. 2017.

[3] H. Cui, V. C. M. Leung, S. Li, and X. Wang, "LTE in the unlicensed band: Overview, challenges, and opportunities," *IEEE Wireless Commun.*, vol. 24, no. 4, pp. 99–105, Aug. 2017.

[4] Qualcomm Incorporated. *Progress on LAA and its Relationship to LTE-U and MulteFire.* Accessed: Nov. 20, 2018. [Online]. Available: https://www.qualcomm.com/media/documents/files/laa-webinar-feb-2016.pdf

[5] C. Rosa, M. Kuusela, F. Frederiksen, and K. I. Pedersen, "Standalone LTE in unlicensed spectrum: Radio challenges, solutions, and performance of MulteFire," *IEEE Commun. Mag.*, vol. 56, no. 10, pp. 170–177, Oct. 2018.

[6] *Study Licensed-Assisted Access to Unlicensed Spectrum*, Standard 3GPP 36.889 V13.0.0, Jun. 2015.

[7] S. Lagen, L. Giupponi, S. Goyal, N. Patriciello, B. Bojovic, A. Demir, and M. Beluri, "New radio beam-based access to unlicensed spectrum: Design challenges and solutions," *IEEE Commun. Surveys Tuts.*, vol. 22, no. 1, pp. 8–37, 1st Quart., 2020.

[8] N. Patriciello, S. Lagen, B. Bojovic, and L. Giupponi, "N-U and IEEE 802.11 technologies coexistence in unlicensed mmWave spectrum: Models and evaluation," *IEEE Access*, vol. 8, pp. 71254–71271, 2020.

[9] M. Hirzallah, M. Krunz, B. Kecicioglu, and B. Hamzeh, "5G new radio unlicensed: Challenges and evaluation," *IEEE Trans. Cognit. Commun. Netw.*, early access, Dec. 2, 2020, doi: 10.1109/TCCN.2020.3041851.

[10] Q. Incorporated. *How Does Unlicensed Spectrum With NR-U Transform What 5G Can do for You.* [Online]. Available: https://www.qualcomm.com/media/documents/files/presentation-how-nru-can-tran%sform-what-5g-can-do-for-you.pdf

[11] *3GPP RAN1 Meeting Standardization NR-Unlicensed*, document 802.11-20/0195r0, 3GPP, Jan. 2020.

[12] V. Sathya, M. I. Rochman, and M. Ghosh, "Measurement-based coexistence studies of LAA Wi-Fi deployments in chicago," *IEEE Trans. Wireless Commun.*, vol. 28, no. 1, pp. 1–8, Nov. 2020.

[13] I. Latif. (Jul. 2019). *Efficient and Fair Medium Sharing Enabled by a Common Preamble*. [Online]. Available: http://grouper.ieee.org/groups/802/11/Workshops/2019-July-Coex/Quantenn%a_Contribution.pdf

[14] *IEEE Standard for Information Technology-Telecommunications and Information Exchange Between Systems Local and Metropolitan Area Networks-Specific Requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, Standard 802.11-2016 (Revision of IEEE Std. 802.11-2012), Dec. 2016.

[15] B. Chen, J. Chen, Y. Gao, and J. Zhang, "Coexistence of LTE-LAA and Wi-Fi on 5 GHz with corresponding deployment scenarios: A survey," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 1, pp. 7–32, 1st Quart., 2017.

[16] N. Bitar, M. O. Al Kalaa, S. J. Seidman, and H. H. Refai, "On the coexistence of LTE-LAA in the unlicensed band: Modeling and performance analysis," *IEEE Access*, vol. 6, pp. 52668–52681, 2018.

[17] E. Pei and J. Jiang, "Performance analysis of licensed-assisted access to unlicensed spectrum in LTE release 13," *IEEE Trans. Veh. Technol.*, vol. 68, no. 2, pp. 1446–1458, Feb. 2019.

[18] Y. Gao, X. Chu, and J. Zhang, "Performance analysis of LAA and WiFi coexistence in unlicensed spectrum based on Markov chain," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2016, pp. 1–6.

[19] J. Xiao, J. Zheng, L. Chu, and Q. Ren, "Performance modeling and analysis of the LAA Category-4 LBT procedure," *IEEE Trans. Veh. Technol.*, vol. 68, no. 10, pp. 10045–10055, Oct. 2019.

[20] Y. Gao and S. Roy, "Achieving proportional fairness for LTE-LAA and Wi-Fi coexistence in unlicensed spectrum," *IEEE Trans. Wireless Commun.*, vol. 19, no. 5, pp. 3390–3404, May 2020.

[21] Y. Gao, "LTE-LAA and WiFi in 5G NR unlicensed: Fairness, optimization and win-win solution," in *Proc. IEEE SmartWorld, Ubiquitous Intell. Comput., Adv. Trusted Comput., Scalable Comput. Commun., Cloud Big Data Comput., Internet People Smart City Innov. (SmartWorld/SCALCOM/UIC/ATC/CBDCom/IOP/SCI)*, Aug. 2019, pp. 1638–1643.

[22] R. Yin, G. Y. Li, and A. Maaref, "Spatial reuse for coexisting LTE and Wi-Fi systems in unlicensed spectrum," *IEEE Trans. Wireless Commun.*, vol. 17, no. 2, pp. 1187–1198, Feb. 2018.

[23] C. S. Yang, C. K. Kim, J.-M. Moon, S.-H. Park, and C. G. Kang, "Channel access scheme with alignment reference interval adaptation (ARIA) for frequency reuse in unlicensed band LTE: Fuzzy Q-learning approach," *IEEE Access*, vol. 6, pp. 26438–26451, 2018.

[24] M. K. Maheshwari, A. Roy, and N. Saxena, "DRX over LAA-LTE—A new design and analysis based on semi-Markov model," *IEEE Trans. Mobile Comput.*, vol. 18, no. 2, pp. 276–289, Feb. 2019.

[25] Q. Chen, G. Yu, H. Shan, A. Maaref, G. Y. Li, and A. Huang, "Cellular meets WiFi: Traffic offloading or resource sharing?" *IEEE Trans. Wireless Commun.*, vol. 15, no. 5, pp. 3354–3367, May 2016.

[26] H. Lee and H. J. Yang, "Downlink MU-MIMO LTE-LAA for coexistence with asymmetric hidden Wi-Fi APs," *IEEE Trans. Mobile Comput.*, early access, Jun. 18, 2020, doi: 10.1109/TMC.2020.3003314.

[27] U. Challita, L. Dong, and W. Saad, "Proactive resource management for LTE in unlicensed spectrum: A deep learning perspective," *IEEE Trans. Wireless Commun.*, vol. 17, no. 7, pp. 4674–4689, Jul. 2018.

[28] C. K. Kim, C. S. Yang, and C. G. Kang, "Adaptive Listen-Before-Talk (LBT) scheme for LTE and Wi-Fi systems coexisting in unlicensed band," in *Proc. 13th IEEE Annu. Consum. Commun. Netw. Conf. (CCNC)*, Jan. 2016, pp. 589–594.

[29] Q. Chen, G. Yu, and Z. Ding, "Enhanced LAA for unlicensed LTE deployment based on TXOP contention," *IEEE Trans. Commun.*, vol. 67, no. 1, pp. 417–429, Jan. 2019.

[30] A. Galanopoulos, F. Foukalas, and T. A. Tsiftsis, "Efficient coexistence of LTE with WiFi in the licensed and unlicensed spectrum aggregation," *IEEE Trans. Cognit. Commun. Netw.*, vol. 2, no. 2, pp. 129–140, Jun. 2016.

[31] M. Maule, D. Moltchanov, P. Kustarev, M. Komarov, S. Andreev, and Y. Koucheryavy, "Delivering fairness and QoS guarantees for LTE/Wi-Fi coexistence under LAA operation," *IEEE Access*, vol. 6, pp. 7359–7373, 2018.

[32] V. Maglogiannis, D. Naudts, A. Shahid, and I. Moerman, "A Q-learning scheme for fair coexistence between LTE and Wi-Fi in unlicensed spectrum," *IEEE Access*, vol. 6, pp. 27278–27293, 2018.

[33] J. Tan, L. Zhang, Y.-C. Liang, and D. Niyato, "Deep reinforcement learning for the coexistence of LAA-LTE and WiFi systems," in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2019, pp. 1–6.

[34] F. Hao, C. Yongyu, H. Li, J. Zhang, and W. Quan, "Contention window size adaptation algorithm for LAA-LTE in unlicensed band," in *Proc. Int. Symp. Wireless Commun. Syst. (ISWCS)*, Sep. 2016, pp. 476–480.

[35] E. Pei, J. Jiang, L. Liu, Y. Li, and Z. Zhang, "A chaotic Q-learning-based licensed assisted access scheme over the unlicensed spectrum," *IEEE Trans. Veh. Technol.*, vol. 68, no. 10, pp. 9951–9962, Oct. 2019.

[36] Y. Li, T. Zhou, Y. Yang, H. Hu, and M. Hamalainen, "Fair downlink traffic management for hybrid LAA-LTE/Wi-Fi networks," *IEEE Access*, vol. 5, pp. 7031–7041, 2017.

[37] Z. Ali, L. Giupponi, J. Mangues-Bafalluy, and B. Bojovic, "Machine learning based scheme for contention window size adaptation in LTE-LAA," in *Proc. IEEE 28th Annu. Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, Oct. 2017, pp. 1–7.

[38] A. Garcia-Saavedra, P. Patras, V. Valls, X. Costa-Perez, and D. J. Leith, "ORLA/OLAA: Orthogonal coexistence of LAA and WiFi in unlicensed spectrum," *IEEE/ACM Trans. Netw.*, vol. 26, no. 6, pp. 2665–2678, Dec. 2018.

[39] M. Mehrnoush, S. Roy, V. Sathya, and M. Ghosh, "On the fairness of Wi-Fi and LTE-LAA coexistence," *IEEE Trans. Cognit. Commun. Netw.*, vol. 4, no. 4, pp. 735–748, Dec. 2018.

[40] A. K. Bairagi, S. F. Abedin, N. H. Tran, D. Niyato, and C. S. Hong, "QoE-enabled unlicensed spectrum sharing in 5G: A game-theoretic approach," *IEEE Access*, vol. 6, pp. 50538–50554, 2018.

[41] M. Alsenwi, I. Yaqoob, S. R. Pandey, Y. K. Tun, A. K. Bairagi, L.-W. Kim, and C. S. Hong, "Towards coexistence of cellular and WiFi networks in unlicensed spectrum: A neural networks based approach," *IEEE Access*, vol. 7, pp. 110023–110034, 2019.

[42] Y. Li, F. Baccelli, J. G. Andrews, T. D. Novlan, and J. C. Zhang, "Modeling and analyzing the coexistence of Wi-Fi and LTE in unlicensed spectrum," *IEEE Trans. Wireless Commun.*, vol. 15, no. 9, pp. 6310–6326, Sep. 2016.

[43] M. Alhulayil and M. Lopez-Benitez, "Novel LAA waiting and transmission time configuration methods for improved LTE-LAA/Wi-Fi coexistence over unlicensed bands," *IEEE Access*, vol. 8, pp. 162373–162393, 2020.

[44] A. K. Bairagi, N. H. Tran, W. Saad, Z. Han, and C. S. Hong, "A game-theoretic approach for fair coexistence between LTE-U and Wi-Fi systems," *IEEE Trans. Veh. Technol.*, vol. 68, no. 1, pp. 442–455, Jan. 2019.

[45] H. He, H. Shan, A. Huang, L. X. Cai, and T. Q. S. Quek, "Proportional fairness-based resource allocation for LTE-U coexisting with Wi-Fi," *IEEE Access*, vol. 5, pp. 4720–4731, 2017.

[46] V. Sathya, S. M. Kala, M. I. Rochman, M. Ghosh, and S. Roy, "Standardization advances for cellular and Wi-Fi coexistence in the unlicensed 5 and 6 GHz bands," *GetMobile, Mobile Comput. Commun.*, vol. 24, no. 1, pp. 5–15, Aug. 2020.

[47] M. Mehrnoush, V. Sathya, S. Roy, and M. Ghosh, "Analytical modeling of Wi-Fi and LTE-LAA coexistence: Throughput and impact of energy detection threshold," *IEEE/ACM Trans. Netw.*, vol. 26, no. 4, pp. 1990–2003, Aug. 2018.

[48] S. M. Kala, V. Sathya, E. Yamatsuta, H. Yamaguchi, and T. Higashino, "Operator data driven cell-selection in LTE-LAA coexistence networks," in *Proc. Int. Conf. Distrib. Comput. Netw.*, Jan. 2021, pp. 206–214.

[49] *Evolved Universal Terrestrial Radio Access (E-UTRA) Physical Layer Procedures*, Standard 3GPP, 36.213 V13.6.0, Jun. 2017.

[50] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 3, pp. 535–547, Mar. 2000.

[51] S. A. Alvi, X. Zhou, and S. Durrani, "Optimal compression and transmission rate control for node-lifetime maximization," *IEEE Trans. Wireless Commun.*, vol. 17, no. 11, pp. 7774–7788, Nov. 2018.

[52] P. D. Diamantoulakis and G. K. Karagiannidis, "Maximizing proportional fairness in wireless powered communications," *IEEE Wireless Commun. Lett.*, vol. 6, no. 2, pp. 202–205, Apr. 2017.

[53] S. C. Liew and Y. J. Zhang, "Proportional fairness in multi-channel multi-rate wireless networks—Part I: The case of deterministic channels with application to AP association problem in large-scale WLAN," *IEEE Trans. Wireless Commun.*, vol. 7, no. 9, pp. 3446–3456, Sep. 2008.

[54] H. SHI, R. Venkatesha Prasad, E. Onur, and I. G. M. M. Niemegeers, "Fairness in wireless networks: Issues, measures and challenges," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 5–24, 1st Quart., 2014.

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