Performance indicators for public transit connectivity in multi-modal transportation networks

Sabyasachee Mishra [a*], Timothy F. Welch [a], Manoj K. Jha [b]

[a] National Center for Smart Growth Research and Education, University of Maryland, College Park, MD 20742, United States
[b] Center for Advanced Transportation and Infrastructure Engineering Research, Department of Civil Engineering, Morgan State University, 1700 East Cold Spring Lane, Baltimore, MD 21251, United States

Abstract

Connectivity plays a crucial role as agencies at the federal and state level focus on expanding the public transit system to meet the demands of a multimodal transportation system. Transit agencies have a need to explore mechanisms to improve connectivity by improving transit service. This requires a systemic approach to develop measures that can prioritize the allocation of funding to locations that provide greater connectivity, or in some cases direct funding towards underperforming areas. The concept of connectivity is well documented in social network literature and to some extent, transportation engineering literature. However, connectivity measures have limited capability to analyze multi-modal public transportation systems which are much more complex in nature than highway networks.

In this paper, we propose measures to determine connectivity from a graph theoretical approach for all levels of transit service coverage integrating routes, schedules, socio-economic, demographic and spatial activity patterns. The objective of using connectivity as an indicator is to quantify and evaluate transit service in terms of prioritizing transit locations for funding; providing service delivery strategies, especially for areas with large multi-jurisdictional, multi-

[*] Corresponding author.: Tel.: +1 301 405 9424, fax.: +1 301 314 5639
E-mail address: mishra@umd.edu (Sabyasachee Mishra), twelch@umd.edu (Timothy F. Welch), manoj.jha@morgan.edu (Manoj K. Jha)

modal transit networks; providing an indicator of multi-level transit capacity for planning purposes; assessing the effectiveness and efficiency for node/stop prioritization; and making a user friendly tool to determine locations with highest connectivity while choosing transit as a mode of travel. An example problem shows how the graph theoretical approach can be used as a tool to incorporate transit specific variables in the indicator formulations and compares the advantage of the proposed approach compared to its previous counterparts. Then the proposed framework is applied to the comprehensive transit network in the Washington-Baltimore region. The proposed analysis offers reliable indicators that can be used as tools for determining the transit connectivity of a multimodal transportation network.

Key Words: public transportation, connectivity, graph theory, multimodal transit network

1. Introduction

Transit networks consist of nodes and lines to represent their layout. The nodes are called stops and the lines are called links or route segments. Links in a multimodal transit network have different characteristics from those in a road network. While link in a road network is a physical segment that connects one node to another, link of a multi-modal transit network is part of transit line that serves a sequence of transit stops (nodes). Since a stop can be served by different transit lines, multiple transit links may exist between nodes in a multi-modal transit network. But in the case of a highway network only one link exists between two nodes. Headway, frequency, speed, and capacity are critical terms that define the characteristics of a route for a transit link. Similarly, transit nodes are composed of a different set of characteristics than highway nodes. The nodes and links of the transit system are synonymous with the analysis of connectivity in graph theory (Harary 1971). Graphs more or less connected are determined from two invariants such as node and line connectivity.

Determining the level of service of a transit network is a difficult task. There are two principal reasons. First, the number of factors related to service quality, such as walking distance, in-vehicle travel time, waiting time, number of destinations served and number of transfers needed to reach destinations makes transit connectivity a multidimensional problem. Second, the transit system consists of many different routes. Determining the extent to which the routes are integrated and coordinated so that the transit system is connected, is another complex task (Lam and Schuler 1982). The structure of the public transit network is critical in determining performance, coverage, and service of the network. Network connectivity can be used as a measure to study the performance of the transit system which will assist decision makers in prioritizing transit investment and deciding which stops/lines need immediate attention in regard to operation and maintenance (Hadas and Ceder 2010). In this context, connectivity is one of the index measures that can be used to quantify and evaluate transit performance (Borgatti 2005).

Measures of transit connectivity can be used for a number of purposes. First, in a public or quasi-public agency, connectivity can be used as a measure in public spending to quantify transit stop and route performance and to evaluate the overall system performance. Second, in a rural or suburban area where exact information on transit ridership, boardings, and alightings are not available (which are generally obtained from a comprehensive and well-designed transit assignment in a travel demand model or from an advanced transit system where smart cards are used to keep track of revenues) to obtain a measure of performance for developing service delivery strategies. Third, to serve as a performance measure in a large scale urban multi-modal transit network containing local buses, express buses, metro, local light rail, regional light rail, bus rapid transit, and other transit services which serve both urban and rural areas, where transit services are provided by different public and private agencies with little coordination (an example being Washington DC-Baltimore area, with more than 18 agencies providing services). Fourth, to provide an assessment of effectiveness and efficiency of a transit system with quantifiable measures that can be used to prioritize the nodes/links in a transit system, particularly in terms of emergency evacuation. Fifth, to assist transit agencies with the development of a set of tools for the potential transit users to assess the level and quality of transit service at their place of residence or work.

This paper proposes a unique approach to measuring transit connectivity, particularly for applications where the use of transit assignment models or ridership tracking tools is not available. This method incorporates a graph theoretic approach to determine the performance of large-scale multimodal transit networks to quantify the measures of connectivity at the node, line, transfer center, and regional level. This is achieved through an assessment of connectivity that incorporates unique qualities of each transit line and measures of accessibility. By combining these criteria in a single connectivity index, a quantitative measure of transit performance is developed that goes beyond the traditional measure of centrality. The new connectivity index significantly extends the set of performance assessment tools decision makers can utilize to assess the quality of a transit system.

The next section presents the literature review indicating the use of connectivity in past research, followed by the objective of research showing the scope of improvement in existing literature. The methodology section describes a step by step process of obtaining transit connectivity. An example problem is then presented demonstrating various connectivity indexes. A case-study shows how the concept can be applied in real world applications. The next section shows results of the case study. Finally, findings of the study are discussed in the conclusion section.

2. Literature Review

Centrality measures are well studied in the literature. However, their application to public transit is rare. Table 1 represents a summary of connectivity index measures (or derivatives thereof) found in the literature. The first measure in Table 1 is degree of centrality. The total number of direct connections a node has to other system nodes is defined as the degree centrality. Equation (1) suggest that the degree of centrality of a node $D_c(n)$ in a larger network "N" is the sum of the number of links originated from "p" number of nodes crosses through node "n" (δ_{np}), where p represents all nodes except n (i.e. $p \in (N - n)$). This measure is then normalized by dividing by the total number of system nodes N minus 1. Equation (2) represents a conditional statement to support the degree centrality, where δ_{np} represents a binary indicator variable which takes the value 1, if node "p" is incident upon node "n", and 0 otherwise. Degree centrality is the most widely used measure of connectivity in the literature which ranges from transportation to computer science to epidemiology (Martínez et al. 2003; Liu et al. 2005; Bell, Atkinson, and Carlson 1999; Junker, Koschutzki, and Schreiber 2006; Guimerà et al. 2005).

<<Table 1 about here>>

The degree centrality $D_c(n)$ simply counts the number of direct connections a node has to other nodes in the network, but does not account for the quality of the connection or indirect accessibility to other nodes. Eigenvector centrality acknowledges that not all connections are equal. It assigns relative 'scores' to all nodes in the network based on the principle that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. The eigenvector centrality $D_e(n)$ of node n, in the network N (n, l), is defined in equation (3), which is the multiplication of degree centrality to δ_{np}, and scaled by the eigenvalue λ. Degree centrality ($D_c(n)$) is the eigenvector in equation (3). The eigenvector centrality succeeded the development of degree centrality and is used for a number of studies.

As defined by Freeman (1979), a node's closeness centrality is the sum of graph-theoretic distances from all other nodes, where the distance from a node to another is defined as the length (in links) of the shortest path from one to the other. Equation (4) shows the formulation for closeness centrality. Nodes with low closeness scores have short distances from others, and will tend to be more accessible. In topology and related areas in mathematics, closeness is one of the basic concepts in a topological space.

Betweenness centrality is defined as the share of times that a node n_1 needs a node n (whose centrality is being measured) in order to reach a node n_2 via the shortest path. Equation (5) shows the formulation for betweenness centrality. Alternatively, betweenness centrality basically counts the number of geodesic paths that pass through a node n. The denominator exists to address the case where there are multiple geodesics between n_1 and n_2, and node n is only along some of them. Hence, betweenness is essentially n's share of all paths between pairs that utilize node n—the exclusivity of n's position.

Previous node indexes did not take into account transit characteristics. Park and Kang (2011) introduced the transit characteristics into the node centrality measures and proposed the connectivity index as a true measure of a transit node. The connectivity index of a node can be defined as the sum of connecting powers[2] of all lines crossing through a node n. The connectivity index is shown in equation (6). The total connecting power of a node is the multiple of connecting power of a line at node n ($P_{l.n}^{t}$). The conditional value of presence of a line is represented by a binary indicator variable ($\mu_{l,n}$), which takes the value 1 if line l contributes to the connectivity at node n, and 0 otherwise. The characteristics of a link contain the performance of a series of nodes in that link. A link is a part of the transit route, which in turn is a function of the speed, distance, frequency, headway, capacity, acceleration, deceleration, and other factors. Since a route will contain both in-bound and out-bound, the line performance will in part depend upon the directionality of the transit route, that is, whether the line is circular or bidirectional.

The total connecting power of line l at node n is the average of outbound and inbound connecting power and can be defined as

[2] Please refer to equation (9) in Table 1 for the formulation of connecting power.

$$P_{l,n}^{t} = \frac{P_{l,n}^{o} + P_{l,n}^{i}}{2} \tag{6.1}$$

The outbound connecting power of a line *l*, at node n can be defined as (Park and Kang 2011)

$$P_{l,n}^{o} = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^{o} \tag{6.2}$$

where, C_l is the average vehicle capacity of line *l*, F_l is the frequency on line *l* (60 is divided by F_l to determine the number of operation per hour), H is the daily hours of operation of line *l*, V_l is the speed of line *l*, and $D_{l,n}^{o}$ is the distance of line *l* from node *n* to the destination. The parameter α is the scaling factor coefficient for capacity which is the reciprocal of the average capacity of the system multiplied by the average number of daily operations of each line, β is the scaling factor coefficient for speed represented by the reciprocal of the average speed on each line, and γ is the scaling factor coefficient for distance which is the reciprocal of the average network route distance. Similarly, the inbound connecting power of line *l* can be defined as

$$P_{l.n}^{i} = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^{i} \tag{6.3}$$

where, $P_{l.n}^{i}$ is the inbound connecting power of line *l* at node *n*. While the outbound connecting power of a transit line at a certain transit stop represents connectivity from the stop to the downstream stops of the transit line, the inbound connecting power measures connectivity from the upstream stops of the transit line to the stop under consideration.

Analyzing connectivity of transfer centers is critical to exploring the performance of a combination of several transit stops through which passengers change their mode of transportation. Level of service is one of the critical measures that determine the performance of the transfer center. Equation (7) represents the connectivity index of a transfer center. Where, $\rho_{n_1,n}$ is the passenger acceptance rate and is defined as

$$\rho_{n_1,n} = a \times exp^{-bt_{n_1,n}} \qquad (7.1)$$

where, a and b are the parameters of passenger acceptance rate, and $t_{n_1,n}$ is the transfer time to travel from node n_1 to n. The parameters for a and b are assumed from Kim and Kwon (2005) and estimated based on model estimation which found that walk time provided an R-square value of .9846; that is, walk time alone explained 98.46 percent of the passenger transfer acceptance rates.

Similarly, the connectivity index of a region (equation 8) can be defined as the sum of connectivity indexes of all nodes and scaled by the density measure, where ρ_R is a density measure of region R. The density could be a measure in population, employment, and household in the region. The line connecting power and connectivity indexes are shown in equation (9), and (10).

3. Problem Statement and Objectives

Many measures of transit service and accessibility have been put forth in the literature, but few offer a metric to measure the quality of service and performance of a large multi-modal regional transit system. The literature that does purport to offer such insight requires significant amounts of data not only about the transit system, but also of the complete demographics of the service area (Beimborn et al. 2004, Modarres 2003). Other methods require a full transportation demand and transit assignment models, tools that are prohibitively expensive for many localities (Lam and Schuler 1982).

Measuring transit system performance and the level of service at many different levels is vital to funding decisions (Dajani and Gilbert 1978). Agencies with the objective to improve the transit system using external funds must make the case that the project will be a worthwhile improvement to the system. At the same time, agencies interested in investigating the potential effect of removing a stop, group of stops or transit line from service must know the potential effect it will have on the performance of the system. In the absence of complex transportation demand models, this information is nearly impossible to obtain (Baughan et al. 2009). A methodology that reduces the need for large amounts of data, yet provides important information on system performance is critical to the decision making process. Transit planning agencies may also be interested in applying such an index to determine the best use of land surrounding well connected transit nodes. Beyond Transit Oriented Development (TOD) style plans, the connectivity index provides a way for planners to measure passenger acceptance rates and accessibility for a single node based on its access within an entire multi-modal regional transportation network.

The objectives of this paper are several fold, with the overall goal of providing a strong measure of system performance with the lowest possible data requirements. This paper will first seek to construct a list of node and link based commonly encountered flow processes and define them in terms of a few underlying characteristics; second, to determine and propose the best suited measures in terms of transit connectivity; third, to examine these measures by running simulations of flow processes and comparing the results in a real world case study; and fourth, to suggest the best practices which can be adopted for decision making. All the aforementioned problems require the development of a tool to quantify connectivity of a public transportation system. The proposed methodology is presented in the next section and the notations are shown in Appendix-I.

4. Methodology

The methodology presented in this paper is for transit systems at different levels. As the very nature of nodes, lines, transfer centers and regions, each require a unique formulation. The description below explains the mathematical construct of these transit levels in a step-by-step manner.

4.1 Node Connectivity

The proposed methodology consists of better representations of transit node index measures. In the proposed formulation we consider the congestion effects achieved because of lane sharing of transit lines of buses, light rail, bus rapid transit, and other similar transit facilities. We have redefined the connecting power of a transit line. The other measures have not incorporated the transit attractiveness as it relates to land use and transportation characteristics of the area the associated with the transit line. As discussed previously, the connecting power of a transit line is a function of the inbound and outbound powers, as the connecting power may vary depending on the direction of travel. The inbound and outbound connecting power of a transit line can be redefined as follows.

$$P_{l,n}^{o} = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^{o} \times \vartheta A_{l,n} \qquad (11)$$

$$P_{l,n}^{i} = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^{o} \times \vartheta A_{l,n} \qquad (12)$$

The addition in equation (11) is a term for activity density of transit line "l" at node "n", and ϑ is the scaling factor for the variable. The density measurement represents the development pattern based on both land use and transportation characteristics. The literature defines the level of development a number of ways, but for simplification purposes we have considered it to be the ratio of households and employment in a zone to the unit area. Mathematically, activity density (equation (13)) is defined as:

$$A_{l,n} = \frac{H_{l,n}^{z} + E_{l,n}^{z}}{\Theta_{l,n}^{z}} \tag{13}$$

The connectivity index measures aggregate connecting power of all lines that are accessible to a given node. However not all lines are equal; nodes with access to many low quality routes may attain a connectivity index score equal to a node with only a couple very high quality transit lines. This means that while both nodes are able to provide good access, the node with the fewest lines provides the most access with the lowest need to transfer. To scale the index scores based on the quality of individual lines, that is, scaling for the least number of transfers needed to reach the highest number and quality of destinations, the node scores are adjusted by the number of transit lines incident upon the node. The inbound and outbound connecting power of a transit line can be further refined as:

$$P_{l,n}^{o} = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^{o} \times \vartheta A_{l,n} \times \varphi T_{l,n} \tag{14}$$

$$P_{l,n}^{i} = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^{o} \times \vartheta A_{l,n} \times \varphi T_{l,n} \tag{15}$$

This equation adds the number to transit lines "l" at node "n", and φ is the scaling factor for the number of transit lines. The transfer scale is simply the sum of the connectivity index scores for each of the transit lines that cross a node divided by the count of the number of lines that are incident upon the node. The transfer scaled index (equation (13)) is defined as:

$$T_{l,n} = \frac{\sum P_{l,n}^{t}}{\Theta_{l}^{n}} \tag{16}$$

4.2 Line Connectivity

The total connecting power of a line is the sum of the averages of inbound and outbound connecting powers for all transit nodes on the line as presented in equation (6.1) scaled by the number of stops on each line. The scaling measure is used to reduce the connecting score of lines with many stops like bus lines to properly compare to lines with only a few stops like rail. The line connectivity can be defined as following:

$$\theta_l = (|S_l| - 1)^{-1} \sum P_{l,n}^{t} \qquad (17)$$

4.3 Transfer Center

The concept of a connectivity index of a transfer center is different from the connectivity measure of a conventional node. Transfer centers are groups of nodes that are defined by the ease of transfer between transit lines and modes based on a coordinated schedule of connections at a single node or the availability of connections at a group of nodes within a given distance or walk time. This paper defines a transfer center as the group of nodes within half mile of any rail station in the transit network. The sum of the connecting power of each node in the transfer center is scaled by the number of nodes on the transfer center. Thus, a node in a heavily dense area is made comparable to the transfer center in a less dense area. This scaling procedure is particularly important when comparing transfer centers in a multimodal network where one transfer center may be primarily served by a well-connected commuter rail line and other may have many bus lines and rail lines connecting to the center. The following equation shows the connectivity index of a transfer center.

$$\theta_{tc} = (|S_\omega| - 1)^{-1} \sum P_{l.n}^{t} \left(\rho_{n_1,n}\right) \qquad (18)$$

4.4 Regional (large area) Connectivity

The connecting power of a Region or any other large area has several important implications. The performance of a given area is the sum of the connectivity of all nodes within that area scaled by the number of nodes. This scaling makes it possible to compare the quality of connectivity between areas of differing density. The regional connectivity index equation is shown below.

$$\theta_R = (|S_R| - 1)^{-1} \sum P_{l,n}^{t} \qquad (19)$$

5. Example Problem

In this section, the methodology is described with the help of an example problem. The example network shown in Fig. 1(a) consists of (1) ten stops, (2) five lines, (3) two transfer centers, and (4) five Traffic Analysis Zones (TAZs). Three characteristics of each line such as operating speed, capacity and number of operations are given. Each TAZ is attributed with a density measure which is the ratio between the population and corresponding area. Table 2 shows the comparison of the approaches reported in the literature, and the new approach, at the node, line, transfer center, and region level. In the comparison only degree centrality is obtained from the literature, which is presented at the node level. For the line, node and transfer center connectivity, the related measures from the literature are analyzed in the example problem.

<<Fig. 1(a) about here>>

<<Fig. 1(b) about here>>

5.1 Node and Line Level

The node level measures are presented in Table 2. Degree centrality of node-1 is 0.222 (i.e. 2/9), as there are two nodes incident upon node-1 (node 2, and node 5), and there are nine remaining nodes in the system (please refer to equation (1) for the formula). Similarly degree centrality for all the nodes can be determined. The connectivity index of node-1 is 1.543. This number is derived from three steps:

<<Table 2 about here>>

A zoomed version of node-1 is shown in Fig. 1(b). There are two lines crossing through node-1 (Line-1 and Line-3). The inbound connecting power of line-1 at node-1 ($P_{1.1}^{i}$) is calculated from equation 6.2. ,

$$P_{1.1}^{i} = \left(\frac{1}{468.457} \times 400\right) \times \left(\frac{1}{20.062} \times 20\right) \times \left(\frac{1}{8.437} \times 10\right) = 1.008$$

where, α = 468.457, β =20.062, and γ = 8.437. The capacity of line-1 is 400 (i.e. 8x50). These parameters represent the average of all the corresponding characteristics. Similarly, the outbound connecting power of line-1 at node-1 ($P_{1.1}^{o}$) is 1.008. So the total connecting power of line-1 at node-1 ($P_{1.1}^{t}$) is 1.008 (i.e. (1.008+1.008)/2).

The inbound connecting power of line-3 at node-1 is

$$P_{3.1}^{i} = \left(\frac{1}{468.457} \times 500\right) \times \left(\frac{1}{20.062} \times 17\right) \times \left(\frac{1}{8.437} \times 5\right) = 0.535$$

The outbound connecting power ($P_{3.1}^{i}$) of line-3 at node-1 is 0.535, and the total connecting power ($P_{3.1}^{t}$; is the average of inbound and outbound connecting powers, i.e. (0.535+0.535)/2 = 0.535.

The connectivity index of node 1 is

$$P_{l.n}^{t} = P_{1.1}^{t} + P_{3.1}^{t} = 1.008 + 0.535 = 1.543$$

Similarly the connecting powers of all the nodes can be determined.

Now the connecting power can be further improved using the extended the methodology. Please see equation (11), and (12) for formulation on connecting power for nodes. The inbound connecting power of line-1 at node-1 using activity scale can be determined as the ratio of density of node 1, which resides in TAZ-1 (density =4), and the average system density (density=3). Alternatively, 1.008*(4/3) = 1.344. Similarly the outbound connecting power for line-1 at node-1 is 1.344. The total connecting power of line-1 at node-1 is the sum of inbound and outbound connecting powers, i.e. 1.344. The inbound, outbound, and total connecting power of line-3 at node-1 is 0.535*(4/3) = 0.714. So the total connecting power of node-1 using activity scaling is 1.344+0.714=2.060. It should be noted that the connecting power of node-1 has increased from 1.543 (without using activity scaling) to 2.060 with using activity scaling.

Following similar convention, the connecting power using transfer scale can be used. The connecting power is related to the incidence of lines passing through a node. Please see equation (14), (15), and (16) for formulation on connecting power for nodes. There are two transit lines crossing through node-1. So the connecting power using transfer center scale for node-1 is 1.545/2, i.e. 0.772. The next step is to determine the combined connectivity index using both activity scale and transfer scale. For node-1 the revised connectivity index of node 1 is 2.060/2 = 1.030. Similarly connectivity indexes for all nodes can be determined.

5.2 Transfer Center Level

There are two transfer centers in the example problem (Fig. 1 (a)). Transfer center-1 connects through three nodes (4, 5, and 6). The connectivity index of these three nodes using the old method is 1.021, 0.536, and 1.557. The connectivity index of the transfer center-1 is 1.763, i.e. (1.021+0.536+1.557) / (3-1). But with the proposed method, the connectivity index of transfer center-1 is 2.938, i.e. (1.702+0.893+1.298) / (3-1). Similarly the connectivity index of transfer center-2 is 2.195 and 0.732 using the extended and existing methods respectively. The difference in the score of the transfer centers is the deference in the access each stop theoretically provides. Node 6 is in a dense area with many activities and node 10 is in a rural location with few activities; thus method two provides a measure of how well connected a center is to the system and to the underlying area that creates demand for transit trips.

5.3 Region Level

There are five zones (or regions) in the example problem. The connectivity of each zone is the sum of the connectivity index for all nodes in the zone scaled by the population of the zone relative to average zonal population. Zone-1 contains one node (1) and a population of 25. The connectivity of Zone-1 under the old method is 1.839 (i.e. 1.545*(25/21)); with the proposed method the zone has a connectivity score of 2.452 (i.e. 1.030*(25/21). While the top two zones are in the same order, there are differences between the rank of the zones and the scale of the index for each zone. The primary difference between each zone with the proposed method is the quality of each line in terms of the number of transfers it requires to each a given score.

5.4 Synthesis of the Example Problem

For a comparison of the previous and proposed transit network measures, a summary of the results is shown in Table 3.

<<Table 3 about here>>

The degree of centrality is a simple method that provides only an indication of the best connected nodes in the simplest of terms, the normalized number of connections, but goes no further. The existing connectivity index attempts to refine the centrality measure by including transit characteristics but fails to account for productions, attractions and required transfers that can alter the real connectivity of a system. The extended method adds context to the existing method by scaling for the opportunities in area surrounding the node and the number of transfers required to get to those opportunities.

Under the existing connectivity index method node-3, just as with degree centrality, is the highest ranked node; this is because the sum of the characteristics of all the lines connecting to the node is higher than all other nodes. Under the extended connecting power method, the connecting power of each line is scaled by the activity density and normalized by the number of lines (or transfers). As a result, node-4 which is situated in a high density location, close to other high density nodes and incident upon a single line with the highest connecting power is the top ranked node.

The existing and extended methodologies of transfer centers come to opposite conclusions about transfer center rankings. While both account for the quality of service at each center, that is, route speed, capacity and operations, the connecting power method prioritizes transfer center-1 due to the density of its location and level of connection it has to other high quality areas. It ranks transfer center-2 lower because the transfer center nodes are in low density areas and the connecting lines provide direct connections only to relatively low quality nodes.

6. Case Study

The proposed framework is applied to a comprehensive transit network in the Washington-Baltimore region. The complete transit network is adapted from Maryland State Highway Administration data. The transit database consists of two largest transit systems namely, Washington Metropolitan Area Transit Authority (WMATA), and Maryland Transit Administration (MTA). WMATA is a tri-jurisdictional government agency that operates transit service in the Washington, D.C. metropolitan area, including the Metrorail (rapid transit), Metrobus (fixed bus route) and MetroAccess (paratransit), and is jointly funded by the District of Columbia, together with jurisdictions in suburban Maryland and northern Virginia. There is approximately $300 million spent in the WMATA capital, operating and maintenance cost of which $150 million per year of Federal funds available that are required to be matched by $50 million in annual contributions from DC, Northern Virginia and suburban Maryland, each for ten years.

WMATA has the second highest rail ridership in the US with over 950,000 passengers per day. This is second only to New York. The WMATA Metro provides an extensive heavy rail system with 106.3 route miles. The WMATA bus system also serves an extensive ridership of over 418,000 unlinked daily trips. Fig. 2(a) shows the WMATA network at Union Station.

<<Fig. 2(a) about here>>

<<Fig. 2(b) about here>>

On the other hand, MTA is a state-operated mass transit administration in Maryland. MTA operates a comprehensive transit system throughout the Baltimore-Washington Metropolitan Area. There are 77 bus lines serving Baltimore's public transportation needs. The system has a daily ridership of nearly 300,000 passengers along with other services that include the Light Rail, Metro Subway, and MARC Train. The Baltimore Metro subway is the 11th most heavily used system in the US with nearly 56,000 daily riders. Nearly half the population of Baltimore lack access to a car, thus the MTA is an important part of the regional transit picture. The system has many connections to other transit agencies of Central Maryland: WMATA, Charm City Circulator, Howard Transit, Connect-A-Ride, Annapolis Transit, Rabbit Transit, Ride-On, and TransIT. Fig. 2(b) shows MTA network around Camden in station downtown Baltimore. Both the WMATA Metro rail system and the Baltimore transit system are connected by the MARC commuter rail system. This system has a daily ridership of over 31,000. In the next section, results of the proposed methodology are discussed (APTA 2011). The complete methodology is integrated in a Geographic Information System (GIS) user interface using ArcInfo (ESRI 2010).

7. Results

The results reported in the following sections are based on the application of methods developed in this paper on a large-scale multi-modal network of Washington DC and Baltimore region. Table 4 provides a summary of Baltimore/Washington regional transit system. The system represents one of the largest and most heavily patronized transit systems in the county. The application of the methodology to this complex network provides a demonstration of public transit performance in the Baltimore/Washington Region.

<<Table 4 about here>>

7.1 Node level

The Washington/Baltimore region has a significant number of transit nodes, each of which provide a varying degree of connectivity to the network. Determining network connectivity and funding prioritization is a highly complex task in a multi-modal network. Funding prioritization is additionally aided by the connectivity index by providing decision makers with a tool to measure network resilience. As with any network, transit systems are designed to interact with many different nodes, while remaining functional in the event that a particular node becomes inaccessible. Additionally, resiliency tests based on connectivity can reveal if there is an over concentration of connections which rely on a given node, line, or region.

<<Table 5 about here>>

Table 5 shows the top ten nodes in the network. This table presents a potential problem for the regional transportation network. The results show that nine of the ten most connected nodes are located in the same zone. These zones are less than a few blocks from each other, thus it is feasible that an event could occur that would remove these nodes from service. If all ten of the nodes were to be removed from service, regional network connectivity would be reduced from a score of 4,283 to 4,083 or by about 5%. This is remarkable in that these nodes represent less than 5% of connectivity and less than 0.1% of the total system nodes, yet the system connectivity is heavily reliant of these few connections. A similar comparison can be made for all the nodes in the network.

Fig. 3 is a three-dimensional graph of node connectivity in the Baltimore/Washington region. The map shows the extent of connectivity for the three major transit areas, Washington DC, Baltimore and Silver Spring. The figure also illustrates the location of zone 64 which has the highest concentration of well-connected transit nodes.

<<Fig. 4 about here>>

Fig. 4 plots the lines (rail in green and bus in orange) that the best connected node (node number 5841 and zone number 64) in the region can reach within a single transfer. While other nodes in the system provide access to as many locations and lines as possible, this node is able to move riders from the origin to all of the locations shown in Fig. 4 with the fewest resources and lowest transfer times. Additionally, a review of this site shows that land use can be improved to capitalize on the regional connectivity of this node. To the north of the node is the Baltimore City Hall and the US Post Office and Court house. To the south of the node is a parking structure and a surface parking lot. Since this node can be reached from most of Baltimore in a single transfer and much of Washington DC in two transfers, the city could opt to zone the area for higher density and encourage development. This would likely not significantly increase congestion around the site if transit usage could be encouraged.

7.2 Line Level

The quality of connectivity for a transit line is determined by several factors. First the line needs to provide access to at least some dense development, second the line should provide access to desirable locations with the fewest number of transfers; third the line must connect to other modes to maximize connectivity. The line connectivity index is applied to the Baltimore/Washington regional transportation network. The region provides both rail and bus services. The rail services analyzed in this paper include WMATA's Metro, Baltimore's light rail, metro system and the regional MARC commuter lines jointly operated by AMTRAK and CSX. All significant local and regional bus services were included in the analysis. Not included in the study were national bus and rail services like Greyhound and AMTRAK. While these services do provide a level of connectivity, the primary concern of this paper is how local and regional systems work to create regional connectivity that local and state decision makers can influence.

Fig. 5 shows the line connectivity index for the federal triangle area and vicinity of Washington DC. The map clearly shows that there is a concentration of highly connected lines that are near the Farragut, McPherson Square and Metro North transfer centers. On the other hand, there are very few lines with a high connectivity index that are in close proximity to Union Station.

<<Fig. 5 about here>>

Fig. 6 shows the connectivity of transit lines in downtown Baltimore. There are several very linear transit lines which provide a high degree of connectivity. These lines serve as the backbone for transit service, enhancing connectivity for all lines that intersect them. This configuration results in a high level of connectivity with fewer resources.

<<Fig. 6 about here>>

7.3 Rail service

The analysis shows, somewhat intuitively, that the two metro systems, one by WMATA the other by MTA, provide the highest level of service in terms of line connectivity, followed by Baltimore's light rail system. WMATA provides the highest level of line connectivity along its red and blue lines. The Yellow and Orange lines have the lowest level of connectivity (Fig. 7). In Baltimore, the MARC (commuter rail) line has the highest level of connectivity, followed by the Metro (subway) line then the yellow line (light rail). Not surprisingly, commuter rail lines that pass through mainly suburban and rural areas provide the least amount of connectivity, as they typically only connect with bus nodes at the beginning and end of the lines.

<<Fig. 7 about here>>

The results provide some insight on how future investments in rail and bus should be prioritized. Heavy rail systems that provide a backbone service for bus connections have the highest potential for regional connectivity. Commuter rail systems provide connectivity for moving passengers between metropolitan areas, but provide a lower level of connectivity overall. When bus and heavy rail service is coordinated with commuter rail service, line and system connectivity are enhanced.

7.4 Bus service

Fig. 8 shows the regional connectivity bus index. The line with the highest connecting power is MTA's route 5 which passes through the node with the highest connectivity score in the system. The line has a combination of local and express buses that run through Downtown Baltimore and connects to many other major bus routes as well as all three rail modes. The bus line with the second best connectivity is MTA's express route 150 which has rail connections and provides a transfer point to Howard County bus service.

<<Fig. 8 about here>>

Surprisingly, the best connected bus lines in the Baltimore/Washington region are in Baltimore, despite the fact the WMATA has an extensive route system that relies on the metro service to serve as the backbone of the bus service. Perhaps it is the reliance on a second mode which limits the number of direct bus routes WMATA offers that reduces the connectivity of its bus lines. Most of the major routes on Washington are radial, in that they are meant to feed a central rail station. The suburban lines that are less centralized suffer from low connectivity because they typically serve residential areas and provide access to a Metro stations rather than dense employment and shopping areas. Baltimore's bus lines offer more connectivity with fewer resources by structuring their bus (and rail) service as a network rather than radial system.

7.5 Transfer Center

For this case study, transfer centers are defined as rail station locations for heavy rail, commuter rail and light rail in Washington DC and Baltimore. A transfer center consists of all transit nodes (rail or bus) within one half mile of a rail station. A half mile radius was selected as a reasonable distance for most transit riders to walk from one mode to another. A typical person walking five miles per hour can thus reach the rail station from any other transit node in the transfer center area within 10 minutes.

A half-mile catchment area is typical distance that is assumed in transportation planning to be the service area of Local Street Transit according the American Public Transportation Association (APTA) Standards Development Program, best practices for Defining Transit Areas of Influence (APTA 2009). It is the common distance when measuring transit catchment areas in the literature. Please see Zhao et al (2003) for regression analysis determining that half a mile is the appropriate upper limit for catchment and service areas. Similar findings also appear in Guerra et al (2011) as half a mile of catchment area to be an appropriate measure.

The results of this study show that the connectivity of a given transfer center does not strictly correlate with the total daily passengers that pass through that center. In Washington DC, WMATA states that Union Station has the highest level of daily activity in the system. While the number of passengers that pass through Union Station is high, it is not the center of the METRO system and is not well connected to the bus network. While Union Station is an excellent transfer station at a larger scale, it is not well connected in terms of local transfer centers.

Table 6 lists the top 20 transfer centers based on total connectivity index score. The results indicate that transfer centers located in the WMATA Metro system provide the greatest level of connectivity. Aside from Union Station, the top stations also have the highest number of boardings in the Metrorail system. The one surprisingly well connected transfer center is Shot Tower Station in Baltimore. This transfer center is a transit station along the Green line, but is in close proximity to MTA's route 5 and 150 bus lines noted earlier for their high level of connectivity.

<<Table 6 about here>>

Table 7 lists the 20 least connected transfer centers in the Washington/Baltimore region. The worst centers in terms of connectivity tend to suffer from low levels of development near the station. For instance, several stations in WMATA's Green line have very low connectivity scores. The line has been a controversial one from the beginning of WMATA Metro planning in 1955; when the line was not part of the original plan. Throughout the Green line construction phase, decision makers largely favored funding construction and extensions of the Red line. As a result of funding and planning arguments, the connectivity (i.e. performance) of the Green line has lagged significantly behind that of the well-funded Red line. This connectivity index provides decision makers with a new tool to determine which locations to prioritize for future funding when concerned about transit service planning.

<<Table 7 about here>>

7.6 Region (large Area)

We divided the Washington/Baltimore region into 1,609 analysis zones to measure large area connectivity. Each zone represents an aggregate of several traffic analysis zones. These zones follow the shape of the transportation network so that no major highway is bisected by the zone boundary. The zone structure also attempts to have an equal number of households in several income categories and an equal amount of employment in several job types across all zones.

This level of analysis is beneficial in two ways. First, it can help to point out areas that are under-served by transit. This can aid the prioritization of funding and other planning activities to provide equal connectivity across the region. Secondly, the identification of highly connected links and poorly connected links can aid in helping decision makers plan for service that better connects lower connectivity areas with lower connectivity resources without the cost of restructuring the system or providing new routes.

Fig. 9 shows the results of the large area connectivity analysis. It shows that zones with rail and bus connections have the highest level of connectivity while zones that have a single rail connection or just one bus route servicing the area have very little connectivity. In some cases a zone has a low connectivity score even though a highly connected route passes through the zone. These are the zones that this index is designed to capture. By simply adding an additional stop in the area that has no other connections, yet a route running through it, residents and businesses can have regional transit connectivity with little additional expense.

<<Fig. 9 about here>>

8. Conclusion

The objective of this paper is to develop connectivity indicators to represent the potential ability of a transit system encompassing comprehensive clustered development in a multimodal transportation network. Connectivity defines the level of coordination of the transit routes, coverage, schedule, speed, operational capacity, urban form characteristics, and is an influential element of the image of any transit network. Though the concept of connectivity is used in social networks and partly in transportation engineering, its application in transit analysis has been limited. The difficulty for development of connectivity indicators lies in the complex interacting factors embedded in a multimodal transit network encompassing various public transportation modes with different characteristics, such as buses, express buses, subways, light rail, metro rail, commuter and regional rail. In addition, multimodal transit networks, like road networks consist of nodes and links. However, links in a multimodal transit network have different characteristics from those in a road network as link in a multimodal transit network are part of a transit line that serve a sequence of transit stops (nodes) and a stop can be served by different transit lines; multiple links may exist between nodes in a multimodal transit network. The indicator development process is further complicated as connectivity varies by urban form with differences among geographical, land use, highway and trip pattern characteristics between regions. The performance indicator should include all the aforementioned complexities and should be quantified to portray connectivity of the multimodal transportation network.

In this paper first the connectivity indexes used for different purposes in the social networks are reviewed. Then a new set of indicators are developed to reflect the transit mode, network, and zonal characteristics. A set of connectivity indexes is developed for (1) node, (2) link, (3) transfer center, and (4) region. The node connectivity index includes the transit lines passing through it, their characteristics such as speed, capacity, frequency, distance to destination, activity density of the location, and degree centrality. The link connectivity index is the sum of connectivity indexes of all stops it passes through and normalized to the number of stops. The concept of a connectivity index of a transfer center is different from the connectivity measure of a conventional node. Transfer centers are groups of nodes that are defined by the ease of transfer between transit lines and modes based on a coordinated schedule of connections at a single node or the availability of connections at a group of nodes within a given distance or walk time. The sum of the connecting power of each node in the transfer center is scaled by the number of nodes in the transfer center. Thus, a node in a heavily dense area is made comparable to a transfer center located in a less dense area. Lastly, the connecting power of a region is defined by the urban form, and the characteristics of nodes, lines, and transfer centers.

The connectivity index proposed in the paper is presented with the help of an example network. The example problem shows the distinction between proposed extended connectivity indexes, and existing formulations found in the literature in a step-by-step manner. The results of the example problem identifies measures in terms of nodes located in rural areas, lines with lower speed, lack of frequency, lower capacity, and missed transfers. The lack of transfer between nodes that occurs in multi-legged trips is a major contributor to the calculation of the connectivity index. The proposed methodology is also applied to a comprehensive multimodal transit network in the Washington-Baltimore region. The network resiliency is examined at node, link, transfer center, and regional level. Highly connected transfer centers and regions are identified.

The major contributions of the paper include (1) extending the graph theoretic approach to determine the performance of the multimodal transit network; (2) quantifying the measures of connectivity at the node, line, transfer center, and regional level; (3) applying the methodology to demonstrate the proposed approach in a simplified example problem; (4) examining the transit network performance of Washington-Baltimore region; (5) providing a comprehensive framework for analyzing connectivity, and efficiency of transit networks for agencies that do not have access to well-developed travel demand and transit assignment models, and (6) integrating the complete methodology in a GIS user interface to enhance visualization, and interpretation of the results. Further this study can be extended to analyze changes in the performance measure with changes to the transit network as a sensitivity analysis; incorporating other attributes to the current formulation, and extending the proposed research for prioritizing locations in the case of transit emergency evacuation.

9. Acknowledgement

The authors are thankful to Dr. Seong-Cheol Kang, Associate Research Fellow at The Korea Transport Institute, for sharing his paper and initial ideas on transit connectivity.

10. Reference

Ahmed, Adel, Tim Dwyer, Michael Forster, Xiaoyan Fu, Joshua Ho, Seok-Hee Hong, Dirk Koschützki, et al. 2006. GEOMI: GEOmetry for Maximum Insight. In *Graph Drawing*, ed. Patrick Healy and Nikola S. Nikolov, 3843:468-479. Berlin, Heidelberg: Springer Berlin Heidelberg.

Aittokallio, Tero, and Benno Schwikowski. 2006. "Graph-based methods for analysing networks in cell biology." *Briefings in Bioinformatics* 7 (3): 243 -255. Araujo, O., and J. A. de la

Peña. 1998. “The connectivity index of a weighted graph.” *Linear Algebra and its Applications* 283 (1-3; November 1): 171-177.

APTA. 2009. Defining Transit Areas of Influence. American Public Transportation Association, Document Number: APTA SUDS-UD-RP-001-09, available at

http://www.aptastandards.com/portals/0/SUDS/SUDSPublished/APTA%20SUDS-UD-009-01_areas_of_infl.pdf, Accessed, February 4, 2012.

APTA. 2011. Public Transit Ridership Report, Third Quarter 2011 (www.apta.com); at

http://www.apta.com/resources/statistics/Documents/Ridership/2011-q3-ridership-APTA.pdf

Bader, David A., and Kamesh Madduri. 2006. Parallel Algorithms for Evaluating Centrality Indices in Real-world Networks. In *Parallel Processing, International Conference on*, 0:539-550. Los Alamitos, CA, USA: IEEE Computer Society.

Barthlemy, M. 2004. “Betweenness centrality in large complex networks.” *The European Physical Journal B - Condensed Matter* 38 (2; March): 163-168.

Basak, Subhash C., Sharon Bertelsen, and Gregory D. Grunwald. 1994. “Application of graph theoretical parameters in quantifying molecular similarity and structure-activity relationships.” *Journal of Chemical Information and Computer Sciences* 34 (2; March 1): 270-276.

Baughan, K. J., Constantinou, C. C., and Stepanenko, A. S. (2009). "Understanding Resiliency and Vulnerability in Transport Network Design," Proceedings of European Transport Conference.

Bell, David C., John S. Atkinson, and Jerry W. Carlson. 1999. "Centrality measures for disease transmission networks." *Social Networks* 21 (1; January): 1-21. Bonacich, Phillip. 2007. "Some unique properties of eigenvector centrality." *Social Networks* 29 (4; October): 555-564.

Bonacich, Phillip, and Paulette Lloyd. 2001. "Eigenvector-like measures of centrality for asymmetric relations." *Social Networks* 23 (3; July): 191-201.

Borgatti, Stephen P. 2005. "Centrality and network flow." *Social Networks* 27 (1; January): 55-71.

Brandes, Ulrik. 2001. "A faster algorithm for betweenness centrality." *The Journal of Mathematical Sociology* 25 (2; June): 163-177.

Caporossi, Gilles, Ivan Gutman, and Pierre Hansen. 1999. "Variable neighborhood search for extremal graphs: IV: Chemical trees with extremal connectivity index." *Computers & Chemistry* 23 (5; September 1): 469-477.

Caporossi, Gilles, Ivan Gutman, Pierre Hansen, and Ljiljana Pavlovic. 2003. "Graphs with maximum connectivity index." *Computational Biology and Chemistry* 27 (1; February): 85-90.

Carrington, Peter J., John Scott, and Stanley Wasserman. 2005. *Models and methods in social network analysis*. Cambridge University Press.

Costenbader, Elizabeth, and Thomas W. Valente. 2003. “The stability of centrality measures when networks are sampled.” *Social Networks* 25 (4; October): 283-307.

Crucitti, Paolo, Vito Latora, and Sergio Porta. 2006. “Centrality in networks of urban streets.” *Chaos: An Interdisciplinary Journal of Nonlinear Science* 16 (1): 015113.

Dajani J.S., and Gilbert, G. (1978). “Measuring the performance of transit systems.” Transportation Planning and Technology, vol. 4(2), pp. 97-103.

Derrible, Sybil, and Christopher Kennedy. 2009. “Network Analysis of World Subway Systems Using Updated Graph Theory.” *Transportation Research Record: Journal of the Transportation Research Board* 2112 (-1; December): 17-25.

ESRI. (2010). “ArcInfo 10.0 for Desktop”, Developed by ESRI, Redlands, CA, USA.

Estrada, Ernesto, and Juan A. Rodríguez-Velázquez. 2005. “Subgraph centrality in complex networks.” *Physical Review E* 71 (5; May 6): 056103.

Frank, Lawrence D., James F. Sallis, Terry L. Conway, James E. Chapman, Brian E. Saelens, and William Bachman. 2006. “Many Pathways from Land Use to Health: Associations between Neighborhood Walkability and Active Transportation, Body Mass Index, and Air Quality.” *Journal of the American Planning Association* 72 (1; March): 75-87.

Freeman, Linton C. 1978. “Centrality in social networks conceptual clarification.” *Social Networks* 1 (3): 215-239.

Garroway, Colin J, Jeff Bowman, Denis Carr, and Paul J Wilson. 2008. “Applications of graph theory to landscape genetics.” *Evolutionary Applications* 1 (4; November 1): 620-630.

Gauthier, H. L. 1968. “Transportation and The Growth of the Sao Paulo.” *Journal of Regional Science* 8 (1): 77–94.Goh, K.-I., E. Oh, B. Kahng, and D. Kim. 2003. “Betweenness centrality correlation in social networks.” *Physical Review E* 67 (1; January 13): 017101.

Guerra, E., Cervero, R., & Tischler, D. (2011). The Half-Mile Circle: Does It Best Represent Transit Station Catchments? *University of California, Berkeley*.

Guimerà, R., S. Mossa, A. Turtschi, and L. A. N. Amaral. 2005. “The worldwide air transportation network: Anomalous centrality, community structure, and cities’ global roles.” *Proceedings of the National Academy of Sciences of the United States of America* 102 (22; May 31): 7794 -7799.

Hadas, Yuval, and Avishai (Avi) Ceder. 2010. “Public Transit Network Connectivity.” *Transportation Research Record: Journal of the Transportation Research Board* 2143 (-1; December): 1-8.

Harary, F. (1971). “Graph Theory.”Addison-Wesley Publishing Company, Massachusetts, USA.

Hilgetag, Claus C., and Marcus Kaiser. 2004. “Clustered Organization of Cortical Connectivity.” *Neuroinformatics* 2 (3): 353-360.

Jiang, B., and C. Claramunt. 2004. “A Structural Approach to the Model Generalization of an Urban Street Network*.” *GeoInformatica* 8 (2; June): 157-171.

Junker, Bjorn, Dirk Koschutzki, and Falk Schreiber. 2006. “Exploration of biological network centralities with CentiBiN.” *BMC Bioinformatics* 7 (1): 219.

KIM, J., J. KIM, M. JUN, and S. KHO. 2005. “Determination of a Bus Service Coverage Area Reflecting Passenger Attributes.” *Journal of the Eastern Asia Society for Transportation Studies* 6: 529–543.

Lam, T. N., and Schuler, H. J.. 1982. “Connectivity index for systemwide transit route and schedule performance.” *Transportation Research Record*, vol. 854, pp.17-23.

Latora, V, and M Marchiori. 2007. “A measure of centrality based on network efficiency.” *New Journal of Physics* 9 (6; June): 188-188.

Leydesdorff, Loet. 2007. “Betweenness centrality as an indicator of the interdisciplinarity of scientific journals.” *Journal of the American Society for Information Science and Technology* 58 (9; July 1): 1303-1319.

Liu, Xiaoming, Johan Bollen, Michael L. Nelson, and Herbert Van de Sompel. 2005. “Co-authorship networks in the digital library research community.” *Information Processing & Management* 41 (6; December): 1462-1480.

Martínez, A., Y. Dimitriadis, B. Rubia, E. Gómez, and P. de la Fuente. 2003. “Combining qualitative evaluation and social network analysis for the study of classroom social interactions.” *Computers & Education* 41 (4; December): 353-368.

Modarres, A. (2003). “Polycentricity and transit service.” Transportation Research Part-A:Policy and Practice, vol. 137(10), pp. 841-864.

Moore, Spencer, Eugenia Eng, and Mark Daniel. 2003. "International NGOs and the Role of Network Centrality in Humanitarian Aid Operations: A Case Study of Coordination During the 2000 Mozambique Floods." *Disasters* 27 (4; December 1): 305-318.

Newman, M. E. J. 2004. "Analysis of weighted networks." *Physical Review E* 70 (5; November 24): 056131.

Newman, M.E. J. 2005. "A measure of betweenness centrality based on random walks." *Social Networks* 27 (1; January): 39-54.

Opsahl, Tore, Filip Agneessens, and John Skvoretz. 2010. "Node centrality in weighted networks: Generalizing degree and shortest paths." *Social Networks* 32 (3; July): 245-251.

Otte, Evelien, and Ronald Rousseau. 2002. "Social network analysis: a powerful strategy, also for the information sciences." *Journal of Information Science* 28 (6; December 1): 441 -453.

Özgür, Arzucan, Thuy Vu, Güneş Erkan, and Dragomir R. Radev. 2008. "Identifying gene-disease associations using centrality on a literature mined gene-interaction network." *Bioinformatics* 24 (13; July 1): i277 -i285.

Park, J., and S. C Kang. 2011. A Model for Evaluating the Connectivity of Multimodal Transit Networks. In *Transportation Research Board 90th Annual Meeting*.

Randic, Milan. 2001. "The connectivity index 25 years after." *Journal of Molecular Graphics and Modelling* 20 (1; December): 19-35.

Ruhnau, Britta. 2000. "Eigenvector-centrality -- a node-centrality?" *Social Networks* 22 (4; October): 357-365.

Sabljic, Aleksandar, and Davor Horvatic. 1993. "GRAPH III: A computer program for calculating molecular connectivity indices on microcomputers." *Journal of Chemical Information and Computer Sciences* 33 (3; May 1): 292-295.

Scott, Darren M., David C. Novak, Lisa Aultman-Hall, and Feng Guo. 2006. "Network Robustness Index: A new method for identifying critical links and evaluating the performance of transportation networks." *Journal of Transport Geography* 14 (3; May): 215-227.

Sun, Li-Xian, and Klaus Danzer. 1996. "Fuzzy cluster analysis by simulated annealing." *Journal of Chemometrics* 10 (4; July 1): 325-342.

White, Douglas R., and Stephen P. Borgatti. 1994. "Betweenness centrality measures for directed graphs." *Social Networks* 16 (4; October): 335-346.

White, Howard D. 2003. "Pathfinder networks and author cocitation analysis: A remapping of paradigmatic information scientists." *Journal of the American Society for Information Science and Technology* 54 (5; March 1): 423-434.

Yang, Xiaoguang, Chao Zhang, and Bin Zhuang. 2007. Evaluation Model for the Urban Public Transit Network Connectivity Based on Graph Theory. In *Plan, Build, and Manage Transportation Infrastructures in China*, 35-35. Shanghai, China. Zhao, F., Chow, L. F., Li, M. T., Ubaka, I., & Gan, A. (2003). Forecasting transit walk accessibility: regression model alternative to buffer method. *Transportation Research Record: Journal of the Transportation Research Board*, *1835*(-1), 34–41.

Table 1

Literature on centrality and connectivity measures in social networks and transportation

Measure	Mathematical Construct	Eq. No.	Definition	Application
Node-Measure: Degree Centrality	$D_c(n) = \frac{\sum_{p\in N}\delta_{np}}{n-1}$, where, $\delta_{np} = \begin{cases} 1 \; if \; p \; is \; dependet \; on \; n, \forall p \in (N-n) \\ 0 \qquad Otherwise \end{cases}$	(1) (2)	Normalized score based on total number of direct connections to other network nodes	***Network and Graph Theory*** (Borgatti 2005; Freeman 1978; Latora and Marchiori 2007; Costenbader and Valente 2003; Martínez et al. 2003); ***computer and information science*** (Liu et al. 2005; H. D. White 2003; Bell, Atkinson, and Carlson 1999; Bader and Madduri 2006); gene-disease (Özgür et al. 2008; Junker, Koschutzki, and Schreiber 2006; Aittokallio and Schwikowski 2006); shortest path (Borgatti 2005; Opsahl, Agneessens, and Skvoretz 2010; Ahmed et al. 2006); ***transportation*** (Jiang and Claramunt 2004; Guimerà et al. 2005; Derrible and Kennedy 2009)
Node-Measure: Eigenvector Centrality	$D_e(n) = \frac{\sum_{p\in N}\delta_{np} \times D_c(n)}{\lambda}$	(3)	Assigns relative 'scores' to all nodes in the network based on the principle on connections	***Network and Graph Theory*** (Bonacich 2007; Bonacich and Lloyd 2001; Ruhnau 2000;Bonacich and Lloyd 2001); ***Social Science*** (Ahmed et al. 2006; Estrada and Rodríguez-Velázquez 2005; Newman 2004; Garroway et al. 2008; Moore, Eng, and Daniel 2003; Carrington, Scott, and Wasserman 2005)
Node-Measure: Closeness Centrality	$D_{cc}(n) = \frac{\sum_{n_1\in N} L_{n,n_1}}{N-1}, \forall N > 2$	(4)	Sum of graph-theoretic distances from all other nodes	***Network and Graph Theory; Shortest path*** (Ahmed et al. 2006; Leydesdorff 2007; Crucitti, Latora, and Porta 2006) ; ***Computer science*** (Otte and Rousseau 2002; Liu et al. 2005; Bell, Atkinson, and Carlson 1999)

Node-Measure: Betweeness Centrality	$D_b(n) = \sum_{n_1} \sum_{n_2} \frac{\delta_{n_1,n_2}(n)}{\delta_{n_1,n_2}},\ n_1 \neq n \neq n_2$	(5)	Sum of the number of geodesic paths that pass through a node *n*	***Network and Graph Theory*** (Otte and Rousseau 2002; Newman 2005; D. R. White and Borgatti 1994; Crucitti, Latora, and Porta 2006) ; ***computer and information science*** (Liu et al. 2005; Bell, Atkinson, and Carlson 1999; Barthlemy 2004; Goh et al. 2003); shortest path (Ahmed et al. 2006; Brandes 2001)
Node-Measure: Connectivity Index	$\theta_n = \sum_{l \in L} P^t_{l,n}\, \mu_{l,n}$	(6)	Sum of connecting powers all lines crossing through a node *n*	***Transportation*** (Lam and Schuler 1982; Hadas and Ceder 2010; Yang, Zhang, and Zhuang 2007; D. M. Scott et al. 2006; Park and Kang 2011) ***Network and Graph Theory*** (Caporossi, Gutman, and Hansen 1999; Randic 2001; Caporossi et al. 2003; Araujo and de la Peña 1998; Gauthier 1968; Frank et al. 2006)
Node-Measure: Transfer Center (Cluster): Connectivity Index	$\theta_\omega = \frac{1}{\lvert S_\omega \rvert - 1} \sum_{n_1 \in S_\omega} \sum_{n \in S_\omega, n_1 \neq n} \theta_n\, \rho_{n_1,n}$	(7)	Sum of connecting powers all lines crossing through a transfer center	***Transportation and Other applications*** (Ahmed et al. 2006; Leydesdorff 2007; Park and Kang 2011; Basak, Bertelsen, and Grunwald 1994;Sabljic and Horvatic 1993; Hilgetag and Kaiser 2004; Sun and Danzer 1996)
Node-Measure: Region Connectivity Index	$\theta_R = \frac{1}{\lvert S_\sigma \rvert - 1} \sum_{n \in S_R} \theta_n$	(8)	Sum of connecting powers all nodes in a region	***Transportation and Other applications*** (Ahmed et al. 2006; Leydesdorff 2007; D. R. White and Borgatti 1994; Crucitti, Latora, and Porta 2006; Yang, Zhang, and Zhuang 2007; Park and Kang 2011)
Line-Measure: Connecting Power	$P^t_{l.n} = \frac{P^o_{l.n} + P^i_{l.n}}{2}$	(9)	Connectivity power of a line which is a function of transit characteristics	***Transportation and Other applications*** (Ahmed et al. 2006; Leydesdorff 2007; Yang, Zhang, and Zhuang 2007; Park and Kang 2011)
Line-Measure: Connectivity Index	$\theta_l = \frac{1}{\lvert S_l \rvert - 1} \sum_{n \in S_l, n \neq n_0} \theta_n$	(10)	Sum of connecting powers all nodes in a line	***Transportation and Other applications*** (Ahmed et al. 2006; Leydesdorff 2007; D. R. White and Borgatti 1994; Crucitti, Latora, and Porta 2006; Park and Kang 2011)

Table 2

Measures of connectivity

Feature	Method	Measure	1	2	3	4	5	6	7	8	9	10	SCORE
Node	Old	Degree Centrality	0.222	0.222	0.333	0.222	0.222	0.333	0.222	0.222	0.222	0.222	N/A
		CI	1.543	1.009	2.976	1.021	0.536	1.557	2.015	0.993	0.946	1.939	N/A
	New	Connecting Power – Activity Scale	2.060	1.009	2.976	1.702	0.893	2.596	1.343	0.331	0.315	0.646	N/A
		Connecting Power – Transfer Scale	0.772	1.009	0.992	1.021	0.536	0.779	1.007	0.993	0.946	0.969	N/A
		Connecting Power – *(Combined)*	1.030	1.009	0.992	1.702	0.893	1.298	0.672	0.331	0.315	0.323	N/A
Line	Old	CI	2.212	2.163	1.455	1.979	2.344						N/A
	New	CI	2.418	2.462	2.219	0.928	1.575						N/A
Transfer Center	Old	Transfer Center 1- CI				1.021	0.536	1.557					1.763
		Transfer Center 2- CI								0.993	0.946	1.939	2.195
	New	Transfer Center 1- CI				1.702	0.893	1.298					2.938
		Transfer Center 2- CI								0.331	0.315	0.323	0.732
Region	Old	CI	1.839	3.795	1.847	1.439	5.191						N/A
	New	CI	1.226	1.906	0.462	0.480	6.489						N/A

Table 3

Comparison of methods

Feature	Method	Measure	Highest Ranked 1	Highest Ranked 2	Lowest Ranked 1	Lowest Ranked 2
Node	Old	Degree Centrality	3, 6	NA*	NA	NA
		Connectivity Index	3	7	5	9
	New	Connecting Power	4	6	9	10
Line	Old	Connectivity Index	5	1	3	2
	New	Connectivity Index	1	2	4	5
Transfer Center	Old	Transfer Center - CI	2	1	NA	NA
	New	Transfer Center - CI	1	2	NA	NA
Region	Old	Connectivity Index	5	2	4	1
	New	Connectivity Index	5	2	4	3

Note: The C.I of all lower order nodes is 0.222 therefore no further ranking is possible

Table 4

Summary of transit System

Attribute	Bus	Rail
Number of Lines	949	33
Route Miles	11,827	1,121
Nodes	7,713	208
Average Speed (Free Flow)	22	47

Table 5

Network resiliency

Rank/*Attribute*	**NODES**	**SMZ**	**SCORE**
1	5841	64	28.50
2	5857	64	25.52
3	5853	64	20.86
4	5840	64	20.23
4	5854	64	20.23
6	5846	64	18.33
7	5845	64	17.99
8	6865	64	17.00
9	5849	64	16.84
10	48658	1188	14.17
Top 10 Connectivity			199.67
Total Regional Connectivity			4283.46
Connectivity without top nodes			4083.79
Reduction in regional connectivity			-4.89%
Representation of total nodes			0.14%

Table 6

Top 20 transfer centers, by connectivity score

Station	System	Rail Lines Serving	Score
Farragut West	WMATA	Blue, Orange Lines	7.262
Farragut North	WMATA	Red Line	6.933
Foggy Bottom-Gwu	WMATA	Blue, Orange Lines	6.633
Mcpherson Square	WMATA	Blue, Orange Lines	6.604
Metro North	WMATA	Blue, Orange, Red Lines	5.559
Dupont Circle	WMATA	Red Line	5.462
Federal Triangle	WMATA	Blue, Orange Lines	5.207
Gallery Pi - Chinatown	WMATA	Green, Red, Yellow Lines	4.472
Archives-Navy Memorial	WMATA	Green, Yellow Lines	4.352
Shot Tower/Market Place	MTA	Baltimore Metro	4.295
Convention Center	MTA	Blue, Yellow Lines	4.160
Charles Center	MTA	Baltimore Metro	4.131
University Center/Baltimore St	MTA	Blue, Yellow Lines	4.120
Lexington Market	MTA	Baltimore Metro	3.842
Camden Yards	MTA	Marc Camden Line, Light Rail Blue, Yellow Lines	3.658
Smithsonian	WMATA	Blue, Orange Lines	3.479
Judiciary Square	WMATA	Red Line	3.286
Mount Vernon Square-Udc	WMATA	Green, Yellow Lines	3.239
L'enfant Plaza	WMATA	Blue, Green, Orange, Yellow Lines	2.745
Centre St	MTA	Blue, Yellow Lines	2.654

Table 7

20 lowest transfer centers, by connectivity score

Station	System	Rail Lines Serving	Score
Branch Avenue	WMATA	Green Line	0.009
Linthicum	MTA	Blue, Yellow Lines	0.027
Falls Road	MTA	Blue Line	0.027
North Linthicum	MTA	Blue, Yellow Lines	0.027
Nursery Road		Blue, Yellow Lines	0.030
College Park-U Of Maryland	WMATA	Green Line	0.031
Greenbelt	WMATA	Green Line	0.034
Baltimore Highlands	MTA	Blue, Yellow Lines	0.040
Bwi Business District	MTA	Yellow Line	0.045
Ferndale	MTA	Blue Line	0.047
Suitland	WMATA	Green Line	0.050
Westport	MTA	Blue, Yellow Lines	0.055
BWI Airport	MTA	Yellow Line	0.056
Addison Road-Seat Pleasant	WMATA	Blue Line	0.058
Mount Washington	WMATA	Blue Line	0.059
Old Court	MTA	Baltimore Metro	0.059
Cromwell Station/Glen Burnie	MTA	Blue Line	0.061
New Carrollton	WMATA	Orange Line	0.062
Patapsco	MTA	Blue, Yellow Lines	0.067
Cherry Hill	MTA	Blue, Yellow Lines	0.071

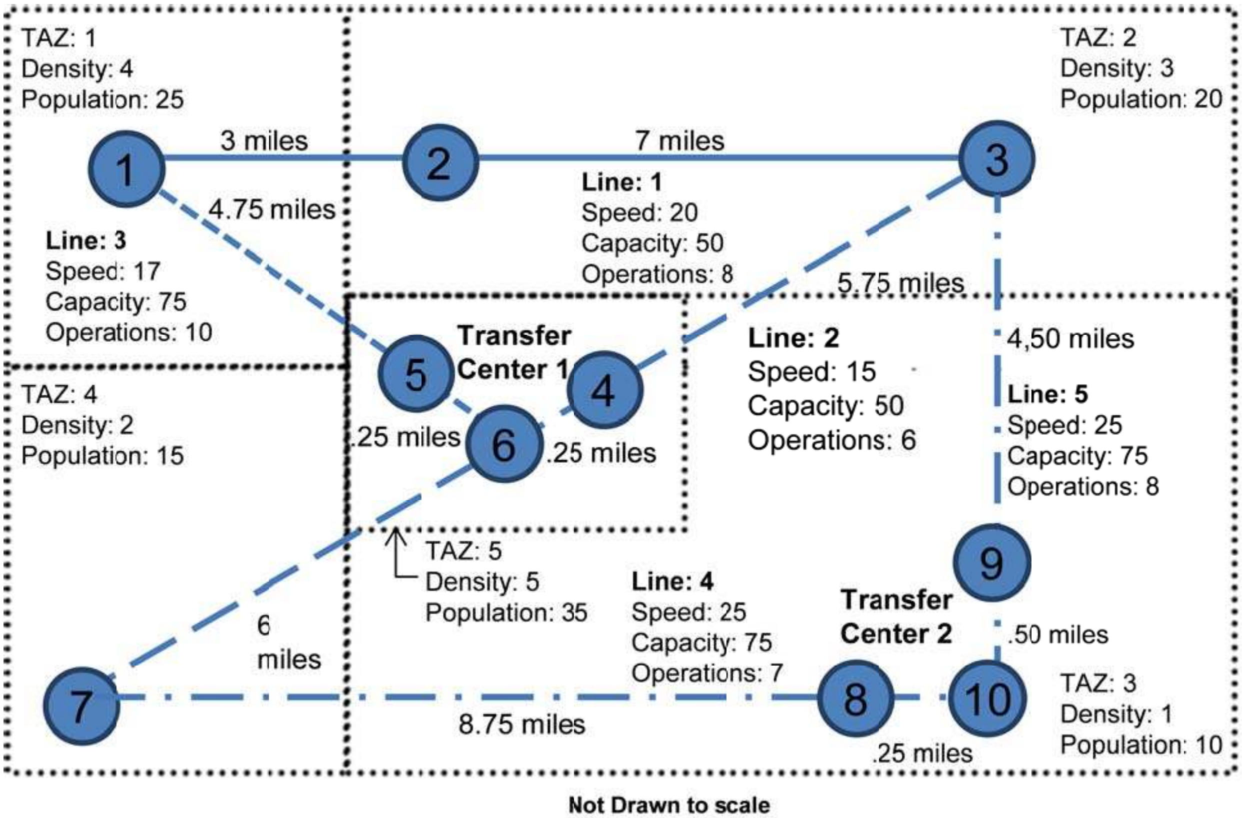

Fig. 1(a). Example of transit system

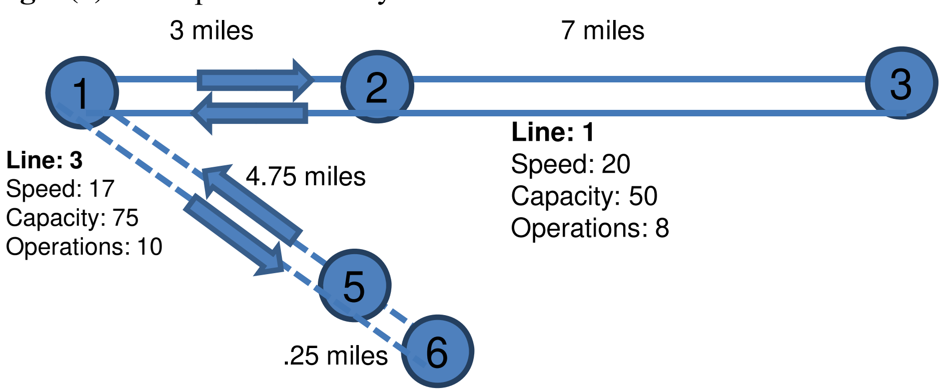

Fig. 1(b). Zoomed version of Node-1

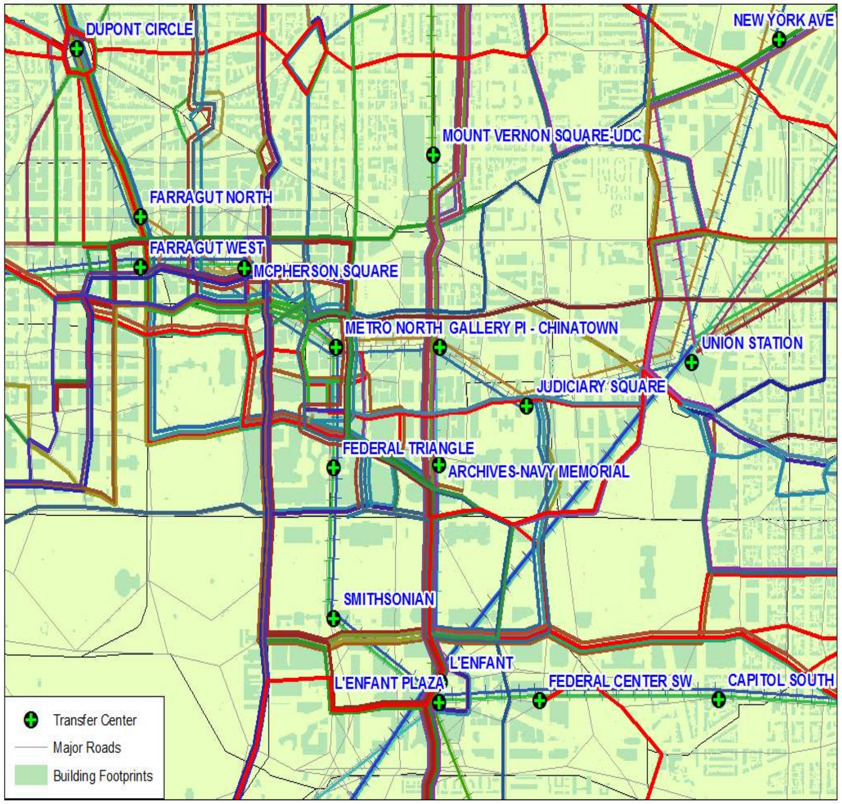

Fig. 2(a). Thematic of the transit lines in Washington DC

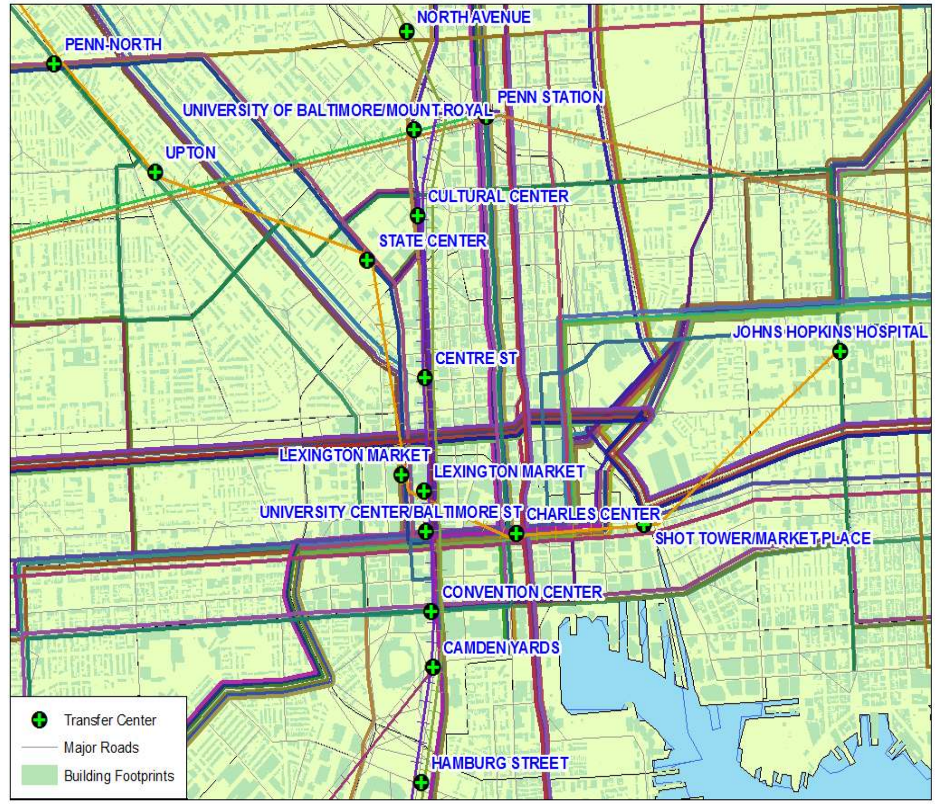

Fig. 2(b). Thematic of the transit lines in Baltimore

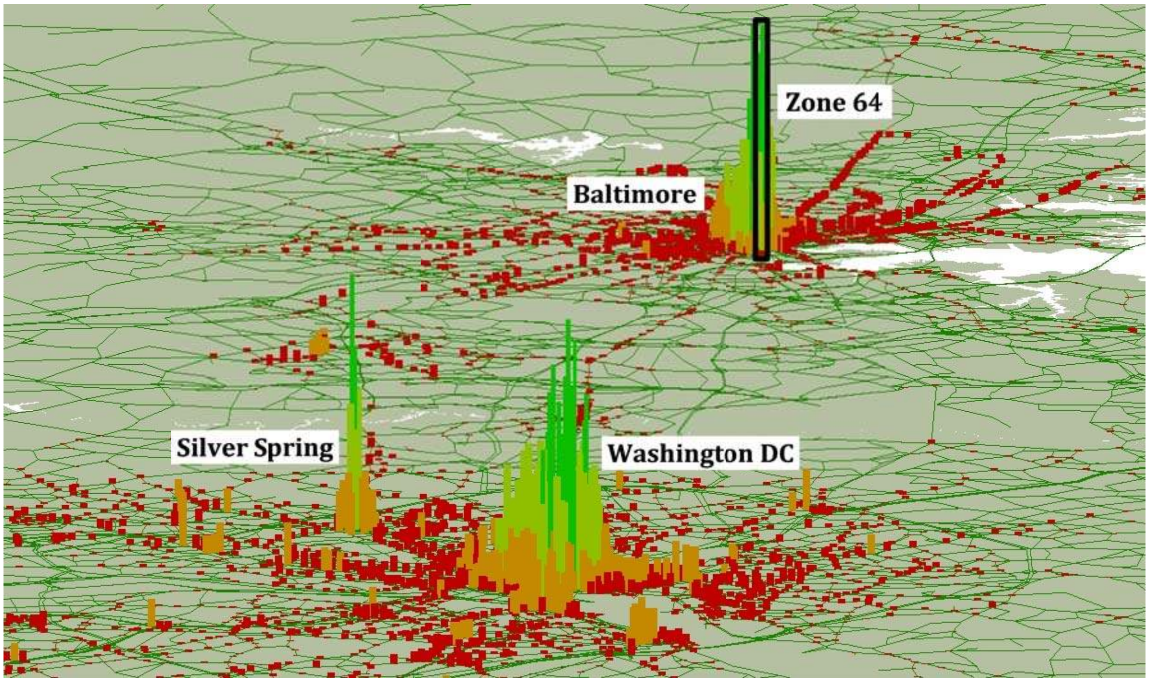

Fig. 3. Regional Node Connectivity

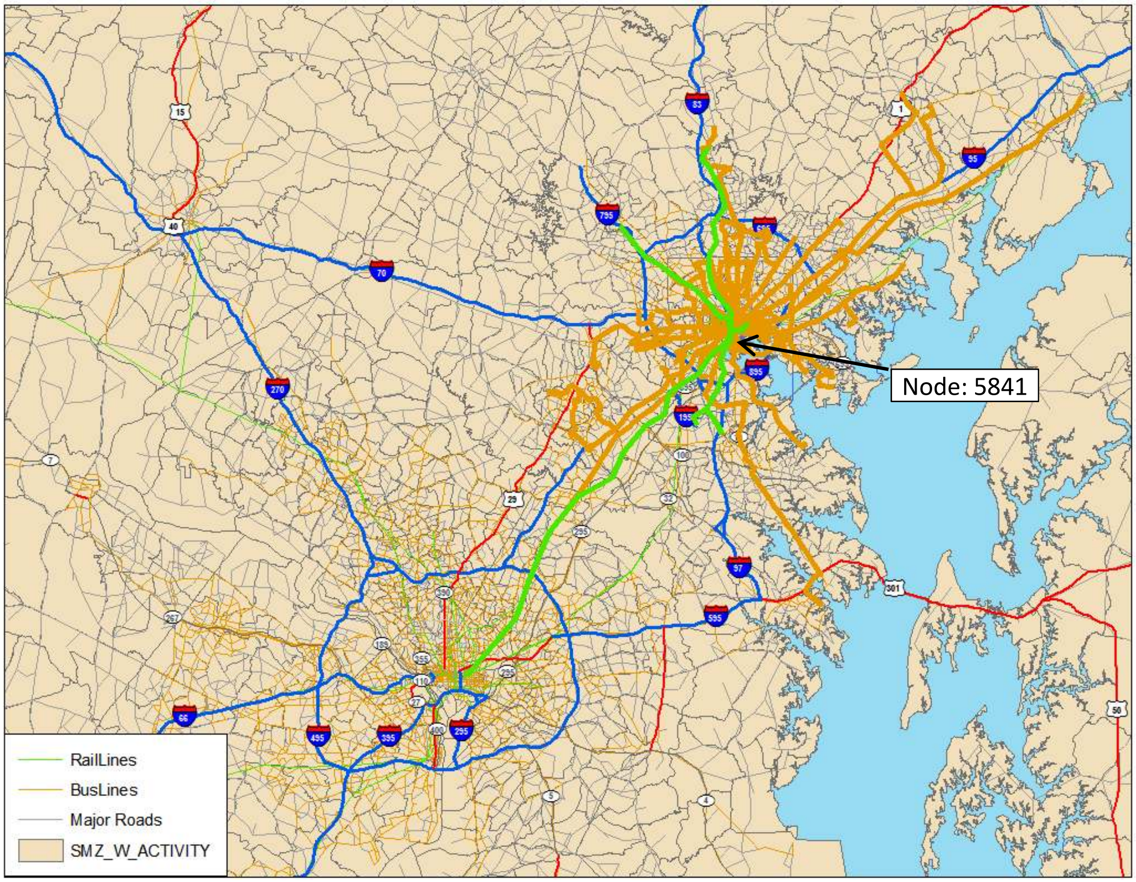

Fig. 4. Node Connectivity

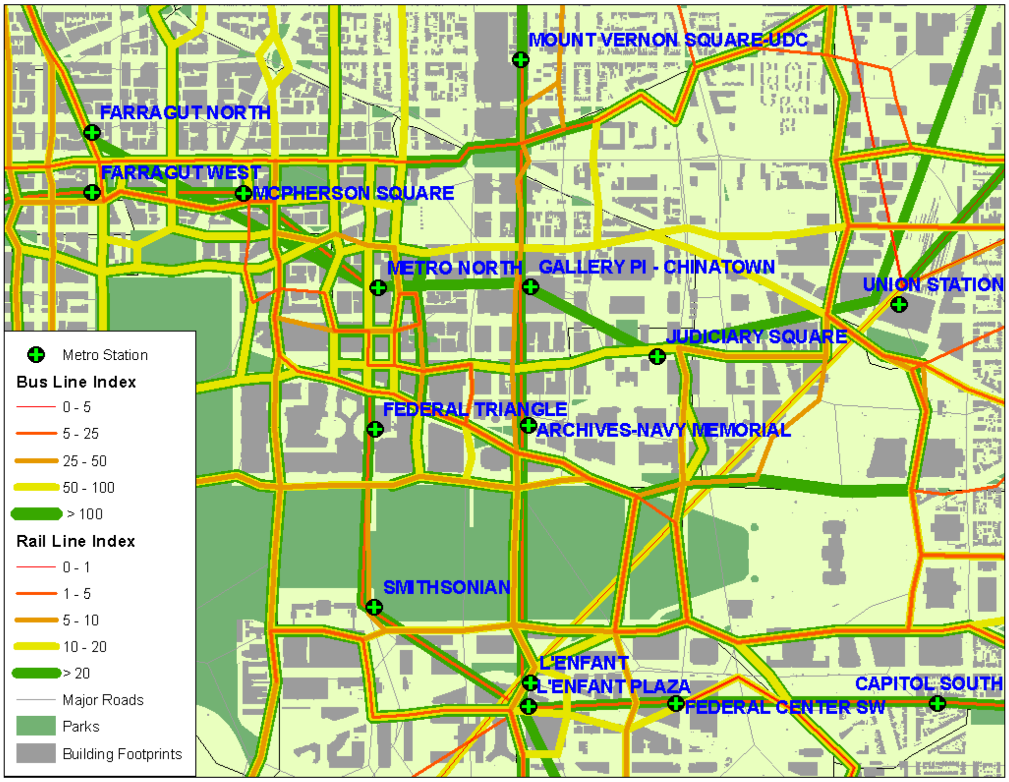

Fig. 5. Washington transit line index

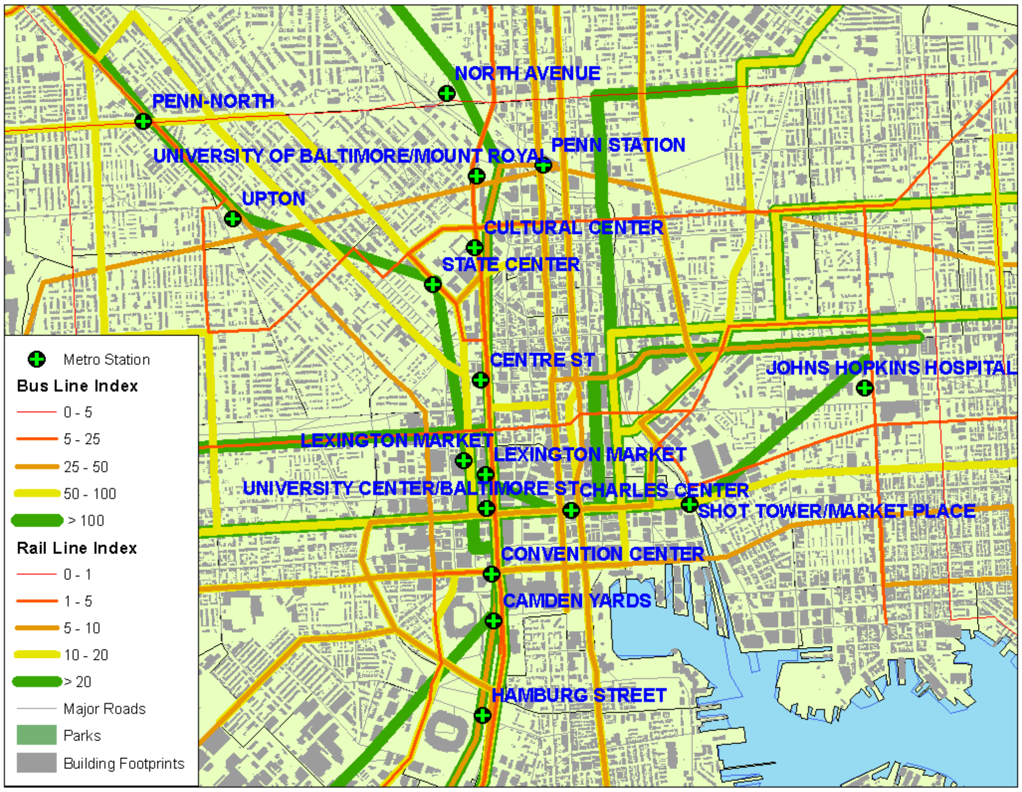

Fig. 6. Baltimore transit line index

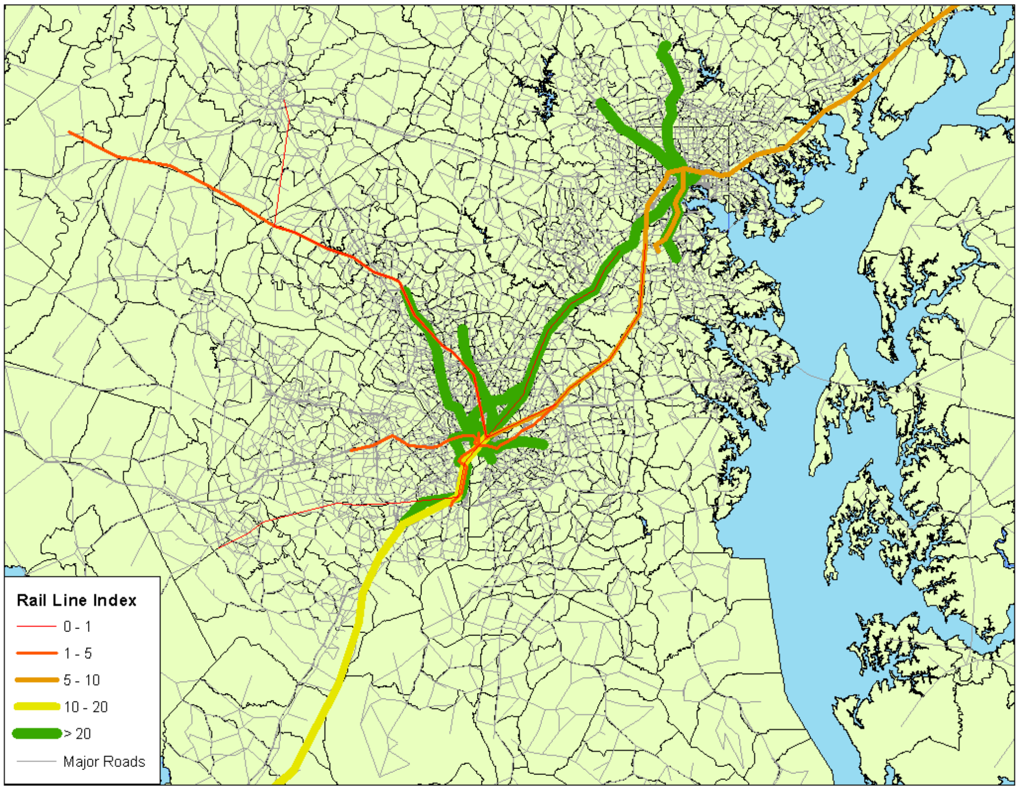

Fig. 7. Regional rail index

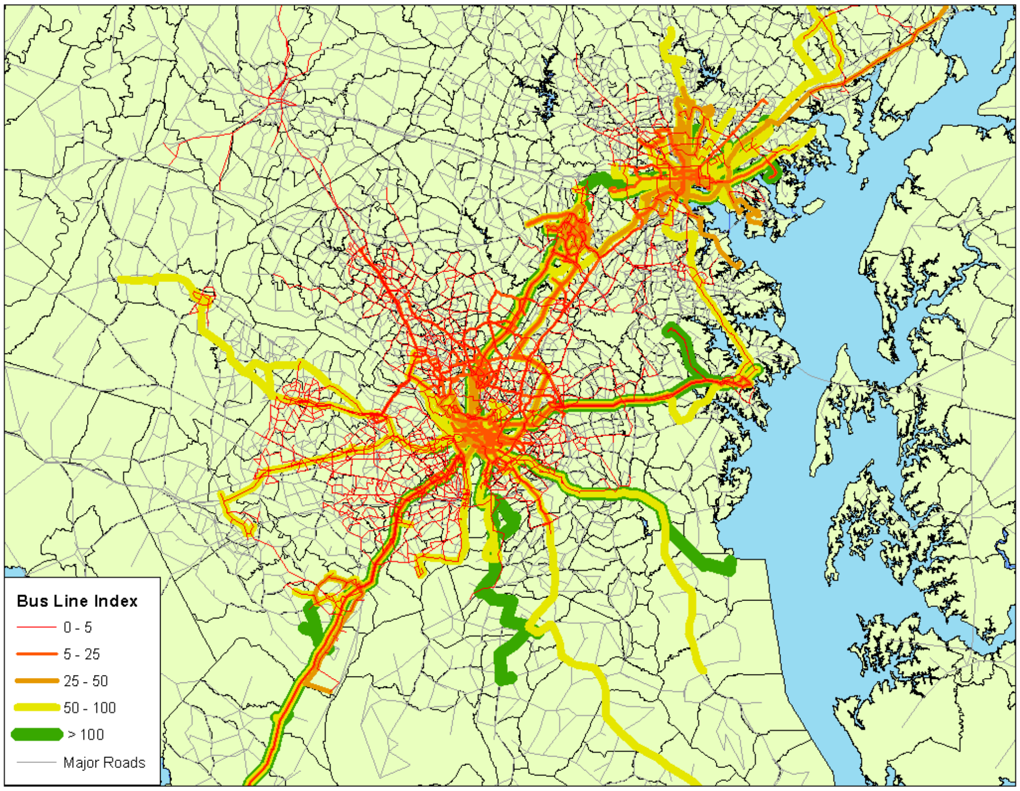

Fig. 8. Regional bus index

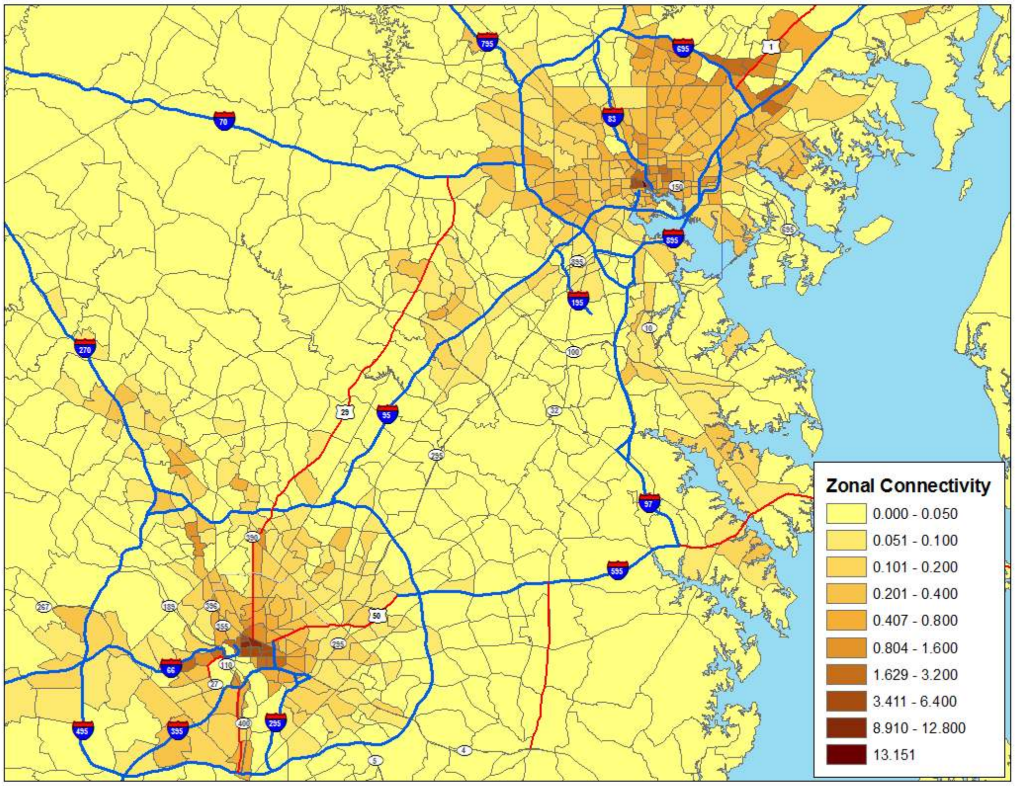

Fig. 9. Regional connectivity index

Appendix-I: Notations for Transit Connectivity

Notation		Explanation
$D_c(n)$	:	Degree of centrality of node *n*
$D_{cc}(n)$	:	Closeness Centrality
$D_e(n)$	:	Eigenvector centrality of node *n*
D_l^i	:	Inbound distance of link *l*
D_l^o	:	Outbound distance of link *l* from node *n* to destination
L_{n,n_1}	:	Shortest distance between node n_1 to *n*
$P_{l,n}^i$		Inbound connecting power of link *l*
$P_{l.n}^o$	:	Outbound connecting power of link *l*
$P_{l,n}^t$	:	Total connecting power of line *l* at node n
S_R	:	Set of stops in region *R*
S_l	:	Set of stops in line *l*
S_ω	:	Set of stops in transfer center ω
S_σ	:	Set of stops in region center σ
V_l	:	Average Speed of link *l*
n_0	:	Initial stop
$t_{n_1,n}$	:	Transfer time from n_1 to *n*
δ_{n_1,n_2}	:	Total number of paths between n_1 and n_2
$\delta_{n_1,n_2}(n)$	:	Number of paths exist between n_1 and n_2 those pass through *n*
δ_{np}	:	A binary indicator variable for determining the degree centrality, which takes the value of 1 when node *p* is dependent on *n*, and 0 otherwise
θ_R	:	Connectivity index for region *R*
θ_l	:	Connectivity index for line *l*
θ_n	:	Connectivity index for node *n*
θ_{tc}	:	Connectivity index for transfer center ω
$\rho_{n_1,n}$	:	Passenger acceptance rate from node n_1 to *n*
ρ_R	:	Density measure for region *R*
a	:	Parameter for passenger acceptance rate
b	:	Parameter for passenger acceptance which is sensitive to travel time
L	:	link
N	:	Node
N	:	Network system
P	:	Node dependent on *n*
α	:	Scaling factor coefficient for Capacity of line *l*
β	:	Scaling factor coefficient for Speed of line *l*
γ	:	Scaling factor coefficient for distance of line *l*
λ	:	Eigenvalue
$A_{l,n}$	:	Activity density of line *l*, at node *n*
ϑ	:	Scaling factor for activity density
$H_{l,n}^z$	:	Number of households in zone *z* containing line *l* and node *n*
$E_{l,n}^z$	:	Employment for zone *z* containing line *l* and node *n*
$\Theta_{l,n}^z$	:	Area of *z* containing line *l* and node
Θ_l^n	:	Number of lines *l* at node *n*