Performance Modeling and Evaluation of IEEE 802.11 IBSS Power Save Mode

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Abstract

The IEEE 802.11 standard defines a power management algorithm for wireless LAN. In the power management for Independent Basic Service Set (IBSS), time is divided into Beacon Intervals (BIs) and each BI is divided into an Announcement Traffic Indication Message (ATIM) window and a data window. The stations that have successfully transmitted an ATIM frame within the ATIM window compete to transmit data frames in the rest of the BI. This paper analyzes the performance of the IEEE 802.11 Power Save Mode (PSM) in single hop ad hoc networks using a discrete-time Markov chain for a data frame transmission together with the corresponding ATIM frame transmission. The paper presents an analytical model to compute the throughput, average delay and power consumption in IEEE 802.11 IBSS in PSM under ideal channel and saturation conditions. The impact of network size on the throughput, delay and power consumption of the IEEE 802.11 DCF in Power Save Mode is also analyzed. This can be used to find an efficient scheme that can maximize the network throughput while saving power consumption for resource constrained ad-hoc wireless networks. The analytical work is validated with simulation results obtained from Qualnet 5.0.1 network simulator.

Keywords: IEEE 802.11 standards, Markov model, Power Save Mode, ATIM frame, power consumption.

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1. Introduction

The IEEE 802.11 architecture uses basic service set (BSS) as the building block of the network. There are two types of BSS - *Infrastructured BSS* and *Independent BSS*, termed as IBSS. In infrastructured BSS, the wireless stations communicate through a central coordinator, the access point (AP). The APs are connected to the Internet through a wired distributed system. In IBSS, the wireless stations can communicate directly without any central coordinator or APs. IBSS is also known as ad hoc network. It has several applications in vehicular communication, mobile networks and sensor networks.

The IEEE 802.11 [1] standard for wireless LAN presents contention and polling based medium access protocols known as distributed co-ordination function (DCF) and point co-ordination function (PCF) respectively, of which the former is a promising and cost effective channel access protocol for ad hoc wireless networks. The DCF is a carrier sense multiple access/collision avoidance (CSMA/CA) based protocol and uses the binary exponential backoff (BEB) algorithm to access the channel. If the medium is sensed idle for an interval larger than distributed interframe space (DIFS) period then a station starts to transmit frames; otherwise it defers the transmis-

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sion until the medium is free. The station generates a backoff time given by:

$$\text{Backoff time} = \text{Random()} \times \text{Slot time}$$

The random value is uniformly distributed over $[0, CW - 1]$, where $CW_{\min} \leq CW \leq CW_{\max}$, where $CW_{\min}$ and $CW_{\max}$ are the minimum and maximum contention window sizes, respectively. These values are based on the physical modulation. As long as the channel is sensed idle the backoff counter is decreased and the backoff value is frozen when the channel is sensed busy. After each unsuccessful transmission the value of CW is doubled up to $CW_{\max} = 2^m(CW_{\min})$. The constant m is called maximum backoff stage. For a successful transmission the CW is reset to $CW_{\min}$. Several analytical models are presented for analysis of the IEEE 802.11 DCF. Bianchi [2] presents a two dimensional Markov chain model at the MAC layer to analyze the saturation throughput of the IEEE 802.11 DCF. In [3], the authors present a modified version of Bianchi's model with a fixed retry limit. A number of papers [4, 5, 6, 7, 8, 9, 10] are built upon the modeling of IEEE 802.11 DCF for handling error-prone channels, non-ideal transmission channels, capture effects and QoS. However, all these analytical models do not consider IEEE 802.11 DCF with power save mode (PSM).

There are some works that analyze the throughput and delay of IEEE 802.11 DCF using the Bianchi model [2] with some modifications. In [11], the authors present delay analysis of IEEE 802.11 protocol with no hidden terminals and fixed retry limit. The paper [12] considers the busy medium condition and how it affects the backoff mechanism. Wang *et al* [13] presents the access delay of DCF with constant contention window size. Xiao [14] presents the saturation throughput, delay and frame dropping probabilities for IEEE 802.11e. In [15], the authors define different types of delays and the relations among these delays. However, the above works do not consider modeling the power consumption for IEEE 802.11 DCF.

In resource constrained wireless networks like mobile ad hoc networks, sensor networks, vehicular networks, etc., power is an important resource to be managed. The design of energy efficient protocols for such networks is an important research area. The IEEE 802.11 standard defines power save algorithm for both infrastructured BSS and IBSS, where a wireless station goes to sleep mode when no data communication takes place. However, the power save algorithm for infrastructured BSS and IBSS are different in nature. In infrastructured BSS, the AP acts as the central coordinator, and uses polling based functionality to instruct the wireless stations to go to sleep mode when there is no data communication. However, as there is no central coordinator in IBSS, the wireless stations should be synchronized for sleep-wake up cycle. In IEEE 802.11 DCF PSM for IBSS, time is divided into beacon intervals, and each beacon interval is divided into an announcement traffic indication message (ATIM) window and a data window. If a station successfully transmits an ATIM frame in the ATIM window, then it is allowed to transmit a data frame in the data window. Otherwise it goes to sleep mode in the data window. This paper analyzes the performance of the IEEE 802.11 Power Save Mode (PSM) for IBSS.

The IEEE 802.11 standard [1] defines the PSM scheme to manage power using the ATIM-BI cycle. However, several medium access control (MAC) protocols are designed for wireless LANs to further improve the power consumption over standard algorithms. Miller *et al* [16] propose a scheme based on carrier sensing window which is shorter than the ATIM window. In [17], the authors introduce a MAC protocol to improve power save in wireless LANs. The idea behind this protocol is that different nodes use different ATIM window sizes, and an adaptable ATIM window size is chosen dynamically. In [18], the authors propose to send a time synchronization function (TSF) beacon at the end of each ATIM window and add certain scheduling information in the beacon. This information ensures the data packet transmission to be contention free, which can help to achieve higher throughput and low energy consumption. Carvalho *et al.* [19] propose an analytical study of the IEEE 802.11 ad hoc networks, only considering the active state where a station may be in transmit, receive or idle state. The analytical model assumes a station is always in the active state and not in sleep state. In [20], the authors derive a formula to calculate the energy consumption of a station. It divides the total energy into six different parts: successful transmission, successful reception, unsuccessful transmissions because of collision, over hearing, idle listening and reception of collision. But they do not consider the power save or sleep state. Zheng *et al.* [21] propose an analytical study of the IEEE 802.11 power save mode using the transient analysis techniques and analyze the delay. The ATIM frame and data frame transmission depend on the CSMA/CA mechanism specified in the IEEE 802.11 DCF [1]. However, the analysis of [21] depends on the assumption of packet arrival rate, which is highly dynamic in real environments.

Recent papers have analyzed the performance of the IEEE 802.11 Power Save Mode in infrastructured BSS [22, 23, 24]. However to the best of our knowledge no one has modeled the performance of IEEE 802.11 power save mode in IBSS using ATIM frame transmission. The probability of successful transmission of an ATIM frame has a great impact on the data frame transmission of a node in IBSS PSM. In [25], a discrete time Markov model is introduced to calculate the probability that an ATIM frame is transmitted successfully. The throughput of the IEEE 802.11 PSM can be calculated using the ATIM frame transmission success probability. In [26], the throughput obtained in IEEE 802.11 DCF in PSM is analyzed using a Discrete time Markov model of the ATIM frame and data frame transmission. This paper extends these two previous models for analysis of delay and power consumption of a data frame transmission in IEEE 802.11 IBSS power save mode. Furthermore, the effect of power save algorithm on network throughput and delay is analyzed both analytically and using simulation, and the throughput-power tradeoff in IEEE 802.11 DCF is discussed in more detail. The effect of beacon interval size on network performance is also analyzed. This analysis gives the direction for providing an efficient power saving algorithm by dynamic tuning of beacon interval size. Such an algorithm would provide maximum power saving with minimum loss in throughput. This paper is the full and extended version of the previous works reported in [25] and [26].

The outline of rest of the paper is as follows. Section 2 presents a brief overview of the IEEE 802.11 PSM in IBSS. A discrete time Markov model is proposed in Section 3 to calculate the throughput using the probability that an ATIM frame is transmitted successfully. Sections 4 and 5 present an analytical model for delay analysis and for power consumption, respectively. In Section 6, simulation results are reported to validate the proposed theoretical model. This section also gives a detailed analysis of the performance of IEEE 802.11 DCF in PSM for IBSS. Finally Section 7 concludes the paper.

2. The IEEE 802.11 DCF in power save mode

The IEEE 802.11 standard [1] has two different power modes, *power on* and *power save*. In power on mode a station transmits or receives frames at any time, whereas in power save mode (PSM) a station goes to sleep state periodically to save battery power. The stations in PSM wake up to listen to beacon messages and stay awake for an ATIM window period. When the stations are in PSM, the transmitter buffers all the frames and announces them in the ATIM window through an ATIM frame. The ATIM frame is a control frame which is exchanged by the stations to determine whether to go for sleep mode or stay awake for data transmission after the end of the ATIM window. The transmission of ATIM frame and data frame are according to CSMA/CA DCF specified in the IEEE 802.11 [1]. If a station is unable to transmit an ATIM frame during the ATIM window, e.g., due to contention with another station or ending of the ATIM window, the data frame is rebuffered and an attempt is made to transmit an ATIM frame during the next ATIM window. A station may enter the power save state at the end of the ATIM window if it does not transmit or receive an ATIM frame successfully. The power save mode is illustrated through an example. In Fig. 1, station A announces a frame destined for station B by transmitting an ATIM frame during the ATIM window. Station B sends ATIM-ACK to station A and remains awake for the rest of the beacon interval. Station C goes to *power save* state at the end of the ATIM window, thus saving energy.

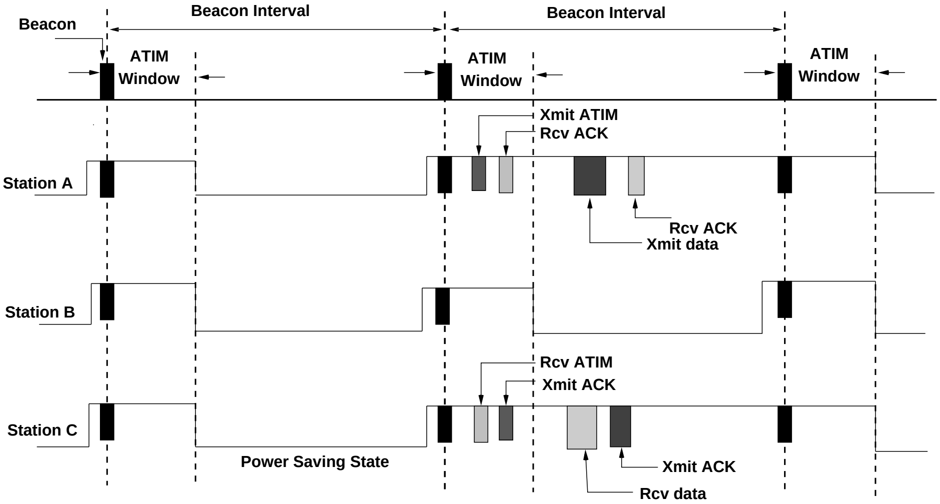

Figure 1: Power save mode in IBSS [1]

The stations that have successfully transmitted an ATIM frame within the ATIM window compete to transmit a data frame in the rest of the beacon interval. If the station is unable to transmit the data frame in the beacon interval in which it was announced, e.g., due to contention with other stations or ending of the data window, the data frame is rebuffered and the station again transmits an ATIM frame during the next ATIM window. A station may discard data frames which are buffered for an excessive amount of time. It may be noted that in the IEEE 802.11 standard [1] neither the retry limit nor the condition for discarding the ATIM frame have been specified. However, the paper [17] defined the retry limit of three for an ATIM frame transmission within an ATIM window and up to three BIs.

This paper follows Algorithm 1 for the transmission of an ATIM frame and data frame in IBSS PSM. In Algorithm 1, the variable $\mathrm{BeaconNum}_{\mathrm{ATIM}}$ represents the number of beacon intervals for ATIM frame. This algorithm is derived from the idea proposed in [17]. A station may be unable to transmit an ATIM frame due to either contention with other stations or reaching the end of the ATIM window at the time of ATIM frame transmission. Similarly, an unsuccessful transmission of data frame can occur either due to contention with other stations or reaching the end of the data window, before the ACK is received successfully. According to algorithm 1, the station sets the value of contention window (CW_{ATIM}) to $\mathrm{CW}_{\mathrm{min}}$ for ATIM, where $\mathrm{CW}_{\mathrm{min}}$ is the minimum contention window size. CW_{ATIM} is doubled up to $\mathrm{CW}^{\mathrm{a}}_{\mathrm{max}}$ for an unsuccessful transmission of an ATIM frame, where $\mathrm{CW}^{\mathrm{a}}_{\mathrm{max}}$ is the maximum contention window size for an ATIM frame transmission, $\mathrm{CW}^{\mathrm{a}}_{\mathrm{max}} = 2^2 \times (\mathrm{CW}_{\mathrm{min}})$. An ATIM frame may collide with another ATIM frame. In this case the station will retransmit the ATIM frame with a retry limit of three within one ATIM window. If an ATIM-ACK is not received within the same ATIM window, then the corresponding data is rebuffered for another try in the next ATIM window. An attempt is made to transmit the ATIM frame up to three ATIM windows. After three ATIM windows if the ATIM frame is not transmitted successfully then the data frame is dropped.

Algorithm 2 is the procedure for data frame transmission after successful transmission of an ATIM frame. Here $\mathrm{CW}^{\mathrm{d}}_{\mathrm{max}}$ is the maximum contention window size for a data frame transmission. Initially, the station sets the value of contention window $\mathrm{CW}_{\mathrm{data}}$ to $\mathrm{CW}_{\mathrm{min}}$. $\mathrm{CW}_{\mathrm{data}}$ is doubled up to $\mathrm{CW}^{\mathrm{d}}_{\mathrm{max}}$ for each

Algorithm 1 Transmission of a data frame with ATIM frame in power save mode

```
 1: BeaconNum$_{\mathrm{ATIM}} \leftarrow 0$
 2: $CW_{ATIM} \leftarrow CW_{\min}$
 3: $W \leftarrow$ random integer from an uniform distribution over the interval $[0, CW_{ATIM} - 1]$
 4: while $W > 0$ do
 5:   if Channel = Idle then
 6:     $W \leftarrow W - 1$
 7:   end if
 8: end while
 9: Transmit ATIM frame.
10: if ATIM window ends before ATIM-ACK is received then
11:   BeaconNum$_{\mathrm{ATIM}} \leftarrow$ BeaconNum$_{\mathrm{ATIM}} + 1$
12:   if BeaconNum$_{\mathrm{ATIM}} \leq 2$ then
13:     GOTO 2
14:   else
15:     DROP the ATIM frame.
16:   end if
17: else
18:   if ATIM-ACK is not received successfully then
19:     $CW_{ATIM} \leftarrow 2 \times CW_{ATIM}$
20:     if $CW_{ATIM} \leq CW^{a}_{\max}$ then
21:       GOTO 3
22:     else
23:       BeaconNum$_{\mathrm{ATIM}} \leftarrow$ BeaconNum$_{\mathrm{ATIM}} + 1$
24:       if BeaconNum$_{\mathrm{ATIM}} \leq 2$ then
25:         GOTO 2
26:       else
27:         DROP the ATIM frame.
28:       end if
29:     end if
30:   else
31:     Use Algorithm 2 to transmit the DATA frame
32:   end if
33: end if
```

Algorithm 2 Data frame transmission in power save mode

```
 1: $CW_{data} \leftarrow CW_{\min}$
 2: $W \leftarrow$ random integer from an uniform distribution over the interval $[0, CW_{data} - 1]$
 3: while $W > 0$ do
 4:   if Channel = Idle then
 5:     $W \leftarrow W - 1$
 6:   end if
 7: end while
 8: Transmit DATA frame.
 9: if data window ends before ACK is received then
10:   DROP the data frame.
11: else
12:   if ACK is not received after ACK time out then
13:     $CW_{data} \leftarrow 2 \times CW_{data}$
14:     if $CW_{data} \leq CW^{d}_{\max}$ then
15:       GOTO 2
16:     else
17:       DROP the DATA frame.
18:     end if
19:   else
20:     Success of data frame transmission
21:     GOTO 1
22:   end if
23: end if
```

unsuccessful transmission of a data frame. The standard does not specify the number of beacon intervals for data frame transmission. In the paper [21] the authors have explained by theoretical analysis and simulation results that a single data window is sufficient to successfully transmit a data frame after transmitting an ATIM frame successfully in the ATIM window. The same assumption is made in this paper for data frame transmission.

3. Modeling and Analysis

3.1. Network Model Assumptions

To model and analyze the Power Save Mode of IEEE 802.11 DCF in IBSS, the following assumptions have been made. A fixed network size of n stations with basic access mechanism is considered. All stations are considered to be in saturation condition, that is at all times each station has data packets to transmit. The ATIM window size is fixed. If a station A successfully transmits an ATIM frame to station B in an ATIM window, then it cannot transmit another ATIM frame to the same station in the same ATIM window. After a successful transmission of an ATIM frame from station A to station B within the ATIM window in a BI, the station A can transmit multiple data frames to station B within the data window of that BI.

3.2. System Model

Consider stochastic processes $s(t)$ representing the backoff stage, $b(t)$ representing the backoff counter and $a(t)$ representing the backoff layer (the beacon interval number counting from 0 to 2) at time t. The backoff stage $s(t)$ represents the retry limit to transmit an ATIM or data frame within one beacon interval. In the paper [4], the Markov chain model for IEEE 802.11 DCF takes freezing of the backoff counter into account by a self loop in each state. However for simplicity as in Bianchi's model [2], in this paper the Backoff counter is decremented by one at the beginning of each slot. The backoff layer $a(t)$ represents the number of beacon intervals used to successfully transmit an ATIM frame. A discrete time Markov model for data frame transmission in PSM is presented in Fig. 2. The following notations are used to represent the transition probabilities, where a single prime and a double prime are used to represent transition probabilities for ATIM and data frame transmission respectively. The state G is a dummy state introduced for ease of presentation and does not have any impact on the solution of the Markov chain model. The following notations are used in presenting the Markov model:

$$P\{(i_1,j_1,k_1)^{'}|(i_0,j_0,k_0)^{'}\} = P\{s(t+1)=i_1, b(t+1)=j_1, a(t+1)=k_1|s(t)=i_0, b(t)=j_0, a(t)=k_0\}.$$

and

$$P\{(i_1,j_1)^{''}|(i_0,j_0)^{''}\} = P\{s(t+1)=i_1, b(t+1)=j_1|s(t)=i_0, b(t)=j_0\}.$$

Note that for data frame transmission there is no third component k, as one data window is used for each successful ATIM frame transmission. In Fig. 2 p_a and p_d are conditional collision probabilities in the ATIM window and data window, respectively, where p_a and p_d are independent of the number of retransmissions and are constant for a fixed network size. Assume that q_a is the probability that the ATIM window ends when a station is attempting to transmit an ATIM frame. Similarly q_d is the probability that the data window ends while transmitting a data frame. The value of q_a depends on the number of competing stations in the ATIM window as well as the ATIM window size. Similarly q_d is proportional to the the number of active stations in the data window. The value of q_d also depends on the data window size. The analysis and estimation of q_a and q_d will be discussed in the section 3.4. The non zero one-step transition probabilities of the Markov chain in Fig. 2 are shown as set of equations in equation (1). W_i is the contention window size at the i^{th} backoff stage and $W_i = 2^i \times W_0$. Here $\mathrm{W}_0 = \mathrm{CW}_{\min} + 1$, the minimum contention window size.

- The first equation indicates that within the ATIM window, the ATIM frame backoff counter decrements with probability $(1 - q_a)$.
- The second equation indicates that at any backoff stage and for any backoff counter value if the ATIM window ends, the protocol tries to retransmit the ATIM frame with backoff stage 0 in the next ATIM window.
- The third equation presents an unsuccessful transmission of an ATIM frame, when the ATIM window ends at the third beacon interval (indicated by $a(t) = 0$).
- The fourth equation indicates a successful transmission of an ATIM frame.
- The fifth equation indicates that at the third ATIM window and at the last retry limit the frame is either successfully transmitted or discarded.
- The sixth equation indicates that there is a collision at the last try within an ATIM window.
- The seventh equation indicates that the station increases the backoff stage and selects the backoff counter uniformly after an unsuccessful transmission of an ATIM frame.
- The eighth equation indicates that within the data window, the data frame backoff counter decrements with probability $(1 - q_d)$.

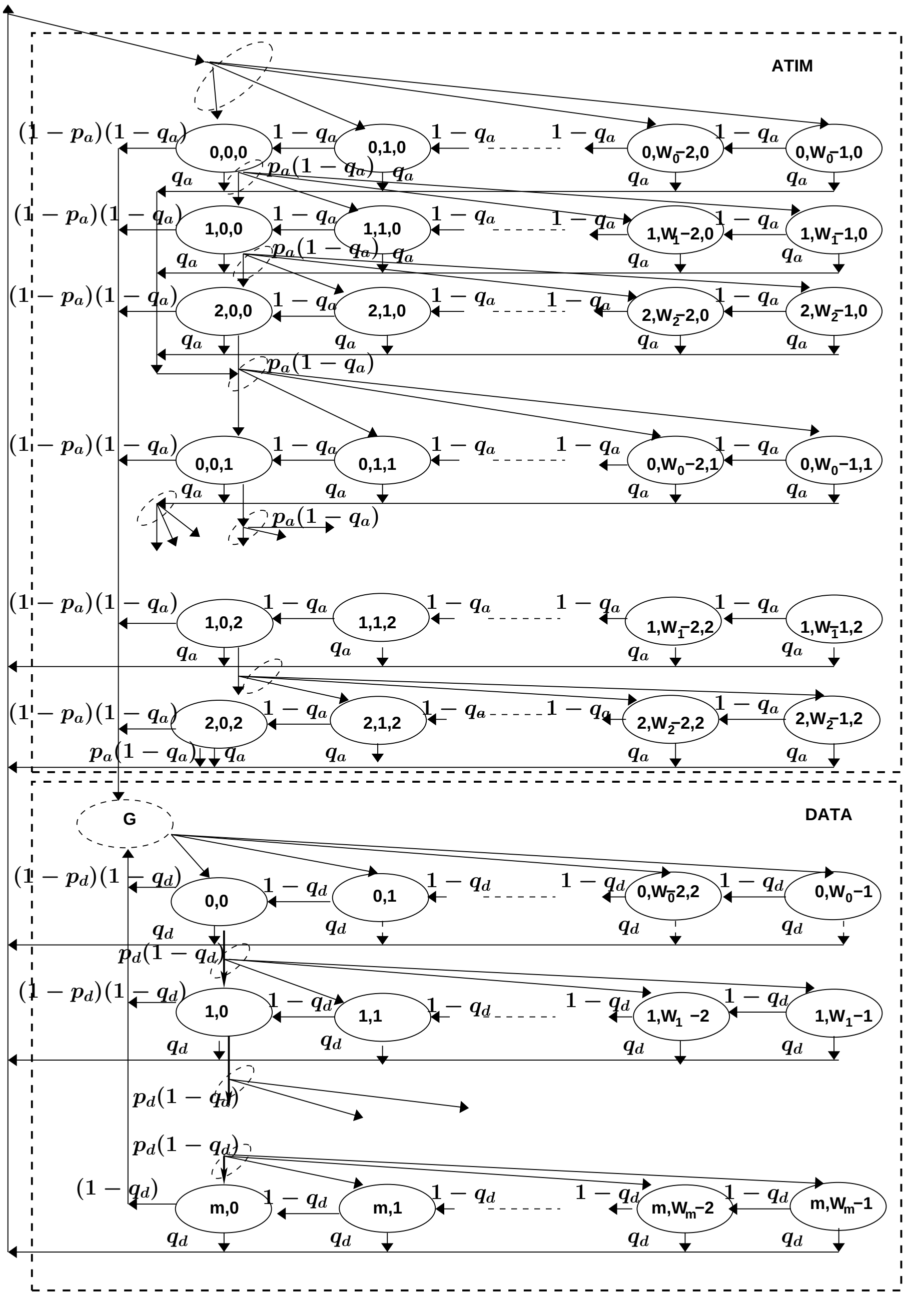

Figure 2: Markov model for data frame transmission in power save mode

- The ninth equation indicates the end of data window has been reached at any backoff stage or any backoff counter, resulting in dropping of the data frame.

$$\left\{\begin{array}{lll}
(1) & P\{(i,j,k)^{'}|(i,j+1,k)^{'}\} = 1-q_a, & i \in [0,2], j \in [0,W_i-1], k \in [0,2]; \\
(2) & P\{(0,j,k+1)^{'}|(i,j',k)^{'}\} = \frac{q_a}{W_0}, & i \in [0,2], j \in [0,W_0-1], k \in [0,1], j' \in [0,W_i-1]; \\
(3) & P\{(0,j,0)^{'}|(i,j',2)^{'}\} = \frac{q_a}{W_0}, & i \in [0,2] j \in [0,W_0-1], j' \in [0,W_i-1]; \\
(4) & P\{G|(i,0,k)^{'}\} = (1-p_a) \times (1-q_a), & i \in [0,2], k \in [0,2], \\
(5) & P\{(0,j,0)^{'}|(2,0,2)^{'}\} = \frac{p_a \times (1-q_a)}{W_0}, & j \in [0,W_0-1]; \\
(6) & P\{(0,j,k+1)^{'}|(2,0,k)^{'}\} = \frac{p_a \times (1-q_a)}{W_0}, & j \in [0,W_0-1], k \in [0,1]; \\
(7) & P\{(i+1,j,k)^{'}|(i,0,k)^{'}\} = \frac{p_a \times (1-q_a)}{W_i}, & i \in [0,1], j \in [0,W_i-1], k \in [0,2]; \\
(8) & P\{(i,j)^{''}|(i,j+1)^{''}\} = 1-q_d, & i \in [0,m], j \in [0,W_i-1]; \\
(9) & P\{(0,j,0)^{'}|(i,j_0)^{''}\} = \frac{q_d}{W_0}, & i \in [0,m], j \in [0,W_0-1], j_0 \in [0,W_i-1]; \\
(10) & P\{(0,j)^{''}|(i,0)^{''}\} = \frac{(1-p_d) \times (1-q_d)}{W_0}, & i \in [0,m], j \in [0,W_0-1]; \\
(11) & P\{(i+1,j)^{''}|(i,0)^{''}\} = \frac{p_d \times (1-q_d)}{W_i}, & i \in [0,m], j \in [0,W_i-1]; \\
(12) & P\{(0,j)^{''}|(m,0)^{''}\} = \frac{(1-q_d)}{W_0}, & j \in [0,W_0-1]
\end{array}\right. \qquad (1)$$

- The tenth equation models the successful transmission of a data frame.
- The eleventh equation indicates that the station increases the backoff stage and chooses the backoff counter uniformly after an unsuccessful transmission of a data frame within the data window.
- The twelfth equation models the unsuccessful transmission of a data frame at the last backoff stage.

3.3. Model Analysis

Let $b^{'}_{i,j,k}$ and $b^{''}_{i,j}$ be the stationary distributions of the Markov chain for the ATIM and data windows, respectively. Here,

$$\begin{aligned} b^{'}_{i,j,k} &= \lim_{t\to\infty} P\{s(t)=i, b(t)=j, a(t)=k\}, \\ &\qquad i \in [0,2], j \in [0,W_i-1], k \in [0,2] \end{aligned}$$

and

$$\begin{aligned} b^{''}_{i,j} &= \lim_{t\to\infty} P\{s(t)=i, b(t)=j\}, \\ &\qquad i \in [0,m], j \in [0,W_i-1] \end{aligned}$$

To obtain a closed-form solutions for the Markov chain presented in Fig. 2, iterative equation (2) and equation (3) are used:

$$b^{'}_{i,0,k} = \tfrac{p_a(1-q_a)}{W_i} \textstyle\sum_{l=0}^{W_i-1} (1-q_a)^l b^{'}_{i-1,0,k} \quad 0 < i \leq 2 \qquad (2)$$

$$b^{''}_{i,0} = \tfrac{p_d(1-q_d)}{W_i} \textstyle\sum_{l=0}^{W_i-1} (1-q_d)^l b^{''}_{i-1,0} \quad 0 < i \leq m \qquad (3)$$

The Markov chain presented in Fig. 2 is a regular chain. So for each $j \in [0, W_i-1]$ and $k \in [0,2]$, we have equation (4) and equation (5). The values of M and N are

$$M = \frac{1}{W_0}\Big[p_a(1-q_a)b^{'}_{2,0,k-1} + q_a \sum_{i=0}^{2}\sum_{j=0}^{W_i-1} b^{'}_{i,j,k-1}\Big]$$

$$N = \frac{1}{W_0}\Big[(1-p_d)(1-q_d)\sum_{i=0}^{m-1} b^{''}_{i,0} + (1-q_d)b^{''}_{m,0}\Big]$$

Let τ_a be the probability that a station transmits an ATIM frame in a randomly chosen slot. This can be obtained as

$$\begin{aligned} \tau_a &= \sum_{k=0}^{2}\sum_{i=0}^{2} b^{'}_{i,0,k} \qquad (6) \\ &= \sum_{k=0}^{2}\sum_{i=0}^{2} \Big(\frac{C \times p_a}{q_a}\Big)^i \prod_{j=1}^{i} \frac{\{1-C^{W_j}\}}{W_j} b^{'}_{0,0,k}, \end{aligned}$$

here, $C = (1-q_a)$. From the equation (4), value of

$$b^{'}_{i,j,k} = \begin{cases} \frac{1}{W_0}, & i=0, j=W_0-1, k=0; \\ \frac{1}{W_0} \times \sum_{l=0}^{W_0-(j+1)} (1-q_a)^l, & i=0, j\in[0,W_0-2], k=0; \\ M, & i=0, j=W_0-1, k\in[1,2]; \\ M \times \sum_{l=0}^{W_0-(j+1)} (1-q_a)^l, & i=0, j\in[0,W_0-2], k\in[1,2]; \\ \frac{p_a(1-q_a)}{W_i} \times \sum_{l=0}^{W_i-(j+1)} (1-q_a)^l b^{'}_{i-1,0,k}, & i\in[1,2], j\in[0,W_i-1], k\in[0,2] \end{cases} \quad (4)$$

$$b^{''}_{i,j} = \begin{cases} N, & i=0, j=W_0-1; \\ N \times \sum_{l=0}^{W_0-(j+1)} (1-q_d)^l, & i=0, j\in[0,W_0-2]; \\ \frac{p_d(1-q_d)}{W_i} \times \sum_{l=0}^{W_i-(j+1)} (1-q_d)^l b^{''}_{i-1,0}, & i\in[1,m], j\in[0,W_i-1], \end{cases} \quad (5)$$

the $b^{'}_{0,0,k}$ can be written as

$$\begin{aligned} b^{'}_{0,0,k} &= M \times \sum_{l=0}^{W_0-1} (1-q_a)^l \\ &= \frac{1}{W_0}[p_a(1-q_a)b_{2,0,k-1} + q_a \sum_{i=0}^{2} \sum_{j=0}^{W_i-1} b_{i,j,k-1}] \\ &\times \sum_{l=0}^{W_0-1} (1-q_a)^l \end{aligned} \quad (7)$$

The equation (7) shows the relation between $b^{'}_{0,0,k}$ and $b^{'}_{0,0,k-1}$ for $k \in [1,2]$. The value of τ_a can be obtained by solving equation (2), equation (4) and equation (7). The relation between p_a and τ_a is

$$p_a = 1-(1-\tau_a)^{(n-1)}. \quad (8)$$

where n is the number of stations in the network. The value of τ_a and p_a can be solved numerically using fixed point iteration. Let P_{as} denotes the probability that an ATIM frame transmission is successful. P_{as} can be calculated as follows:

$$P_{as} = \frac{n\tau_a(1-\tau_a)^{(n-1)}}{1-(1-\tau_a)^n}. \quad (9)$$

Similarly, to find out the probability of success for a data frame transmission after successfully transmitting an ATIM frame, let τ_d be the probability that a station transmits a data frame in a randomly chosen slot in the data window. So the value of τ_d depends on the value of τ_a. The former can be represented as:

$$\tau_d = \sum_{i=0}^{m} b^{''}_{i,0} \quad (10)$$

From equation (3) the value of $b^{''}_{i,0}$ can be expressed in terms of $b^{''}_{0,0}$ as follows:

$$\sum_{i=0}^{m} b^{''}_{i,0} = \sum_{i=0}^{m} \Big(\frac{p_d(1-q_d)}{q_d}\Big)^i \prod_{j=1}^{i} \frac{\{1-(1-q_d)^{W_j}\}}{W_j} b^{''}_{0,0} \quad (11)$$

The value of the $b^{''}_{0,0}$ can be obtained from the normalized condition:

$$1 = \sum_{i=0}^{m} \sum_{j=0}^{W_i-1} b^{''}_{i,j} \quad (12)$$

Using the value of $b^{''}_{i,j}$ from equation (5), the equation (12) can be written as

$$1 = \sum_{i=0}^{m} \sum_{j=0}^{W_i-1} \frac{1-(1-q_d)^{W_i-j}}{1-(1-q_d)^{W_i}} b^{''}_{i,0}$$

From equation (3),

$$1 = \sum_{i=0}^{m} \sum_{j=0}^{W_i-1} \frac{p_d(1-q_d)}{W_i} \sum_{l=0}^{W_i-(j+1)} (1-q_d)^l b^{''}_{i-1,0}$$

From the above equation the value of the $b_{0,0}$ can be written as

$$b^{''}_{0,0} = \frac{1}{A} \quad (13)$$

where

$$A = \sum_{i=0}^{m} \Big(\frac{p_d(1-q_d)}{q_d}\Big)^i \times \tag{14}$$

$$\prod_{j=1}^{i} \frac{1-(1-q_d)^{W_j}}{W_j} \Big[\frac{W_i}{1-(1-q_d)^{W_i}} - \frac{(1-q_d)}{q_d}\Big].$$

Now the value of τ_d can be calculated from equation (10), equation (11) and equation (13). The relation between p_d and τ_d is

$$p_d \;=\; 1-(1-\tau_d)^{(n'-1)}. \tag{15}$$

where $n' = \lceil n \times P_{as} \rceil$ and n is the number of stations in the network. The quantity $\lceil n \times P_{as} \rceil$ denotes the expected number of active communication pairs in the data window after the completion of the ATIM window. However, this quantity is not equal to the number of active stations, as a sender may have multiple receivers. For simplicity, it has been assumed that a sender can send data frames to a single receiver in a data window. Thus the number of active stations in a data window is proportional to $n \times P_{as}$. So at the beginning of each data window, the network with $n \times P_{as}$ number of stations are in saturation condition. Let P_{tr} be the probability that there is at least one data frame transmission in the considered slot. Let P_{ds} be the joint probability that a data frame is transmitted successfully after the successful transmission of an ATIM frame. The values of P_{tr} and P_{ds} are given by

$$P_{tr} \;=\; 1-(1-\tau_d)^{n\times P_{as}} \tag{16}$$

$$P_{ds} \;=\; \frac{n \times P_{as}\tau_d(1-\tau_d)^{(n\times P_{as}-1)}}{P_{tr}}. \tag{17}$$

3.4. Analysis and Estimation of q_a and q_d

It has been assumed in the model in Fig. 2 that q_a is the probability of reaching the end of an ATIM window and q_d is the probability of reaching the end of a data window. The probability q_a depends on the network size as well as ATIM window size. We assume a fixed ATIM window size and that all stations satisfy the saturation condition. For a fixed ATIM window size, the value of q_a is constant. In [16], the authors show that the performance of the network is maximum with ATIM window size $20ms$. From analysis of simulation results it has been observed that the probability of success in ATIM window with ATIM window size = $20ms$ is similar with the theoretical model for $q_a = 0.002$. In this paper, ATIM window size is assumed to be fixed of size $20ms$. Accordingly, in theoretical analysis q_a is assumed as 0.002.

However, the size of the data window is considerably larger than the size of the ATIM window, and the network size has a large impact on the probability of reaching the end of the data window. The probability value q_d is proportional to the number of active stations in the data window and the data window size. From the assumption that a station can send data to a single station only in a particular data window, the quantity $n \times P_{as}$ denotes the expected number of active stations in a data window. So,

$$q_d \propto n \times P_{as} \tag{18}$$

This can be written as,

$$q_d = c \times n \times P_{as} \tag{19}$$

where c is the proportionality constant. For a fixed ATIM window size, the value of c depends on the size of the data window. The impact of the parameter c on beacon interval is analyzed using simulation results.

3.5. Saturation Throughput Analysis

The fraction of time the channel is used to successfully transmit payload bits is called the system throughput [2]. Let S denote the normalized system throughput in the data window. The normalized saturation throughput at the data window is given by

$$S_{\text{DATA}} \;=\; \frac{E[\text{payload transmitted in a slot}]}{E[\text{duration of a slot}]}$$

Let, $E[p]$ be the average packet payload size (in terms of time unit, e.g., μs). $P_{ds}P_{tr}$ is the probability that payload information is transmitted successfully in a slot. The average length of a slot in a data window is computed by considering three mutually exclusive and exhaustive cases. $(1-P_{tr})$ is the probability that a slot is empty, $P_{ds}P_{tr}$ is the probability of successful transmission of data and $(1 - P_{ds})P_{tr}$ is the collision probability for a data frame. Therefore,

$$S_{\text{DATA}} = \frac{P_{ds}P_{tr}E[p]}{(1 - P_{tr})\sigma + P_{ds}P_{tr}T_s + (1 - P_{ds})P_{tr}T_c} \tag{20}$$

Here T_s and T_c are the average time the channel is sensed busy because of a successful transmission or a collision respectively, and σ is the empty slot time. T_s and T_c can be calculated as follows,

$$\begin{aligned} T_s &= \text{DIFS} + H + E[P] + 2\delta + \text{SIFS} + \text{ACK} \qquad (21) \\ T_c &= \text{DIFS} + H + E[P] + \text{SIFS} + \text{ACK}_{\text{TO}} \end{aligned}$$

It has been assumed that all packets have the same size, so $E[p] = P$ is the average payload. The ACK timeout (ACK_{TO}) is added in T_c according to the standard [1] specification that a station waits for an EIFS time when the channel is sensed busy because of collision. $\text{EIFS} = \text{SIFS} + \text{ACK}_{\text{TO}} + \text{DIFS}$. Let $H = \text{PHY}_{\text{hdr}} + \text{MAC}_{\text{hdr}}$ be the packet header and δ the propagation delay.

S_{DATA} provides the channel throughput at the data window only. The overall channel throughput has the ATIM overhead included with the data window channel throughput. The overall normalized throughput (S) can be calculated as,

$$S = S_{\text{DATA}} \times \frac{\text{Data Window size}}{\text{Duration of Beacon Interval}} \tag{22}$$

This section provided an analytical model to calculate the network throughput in IEEE 802.11 DCF power save mode. The next section provides the theoretical model for delay analysis in IEEE 802.11 DCF PSM using the discrete time Markov Model presented in Fig. 2.

4. Analytical Model for Delay Analysis

The delay for a data frame transmission in MAC layer in PSM depends on the delay in ATIM window to transmit an ATIM frame successfully. Let $\overline{D}$ be the average delay to transmit a data frame successfully, following a successful transmission of an ATIM frame. Then,

$$\overline{D} = \overline{D^{(a)}_{\text{succ}}} + \overline{D^{(d)}_{\text{succ}}}. \tag{23}$$

Here $\overline{D^{(a)}_{\text{succ}}}$ is the average delay in transmitting an ATIM frame successfully and $\overline{D^{(d)}_{\text{succ}}}$ is the average delay in transmitting a data frame successfully. Let $P^{(a)}_{\text{succ}}(i,k)$ be the probability that an ATIM frame is transmitted successfully at the i^{th} backoff stage of the k^{th} ATIM window, and is given by

$$P^{(a)}_{\text{succ}}(i,k) = X^i_k(1 - p_a)(1 - q_a). \tag{24}$$

Here X^i_k is the probability that a station will try to send an ATIM frame in the i^{th} backoff stage of the k^{th} ATIM window. We have

$$X^i_k = \begin{cases} L^i, & k = 0; \\ L^{(3+i)} + q_a \times L^i, & k = 1; \\ L^{(3*2+i)} + 2 \times q_a \times L^{(3+i)} + q_a^2 \times L^i, & k = 2; \end{cases} \tag{25}$$

Here, $L = p_a(1 - q_a)$. It can be observed from equation (24) that the total backoff value up to the i^{th} backoff stage has not been considered in the delay calculation for successful transmission of an ATIM frame, because a station has to wait for the complete ATIM window to transmit a data frame. The probability $P^{(a)}_{\text{drop}}$ that a packet is dropped because of the retry limit being exceeded in the last ATIM window is given by

$$P^{(a)}_{\text{drop}} = 1 - \sum_{k=0}^{2}\sum_{i=0}^{2} P^{(a)}_{\text{succ}}(i,k). \tag{26}$$

The expression $P^{(a)'}_{\text{succ}}(i,k)$ denotes the conditional probability that the backoff process of a packet transmission ends at the i^{th} stage of the k^{th} ATIM window, given that the data frame is transmitted successfully. So,

$$P^{(a)'}_{\text{succ}}(i,k) = \frac{P^{(a)}_{\text{succ}}(i,k)}{1 - P^{(a)}_{\text{drop}}}. \tag{27}$$

According to the IEEE 802.11 [1] standard for IBSS in PSM, all stations stay in active mode for the whole period of the ATIM window. Assume that $D^{(a)}(k)$ is the delay experienced up to the k^{th} ATIM window to transmit an ATIM frame successfully. $\text{ATIM}_{\text{size}}$ represents the ATIM window size, which is assumed fixed. We have

$$D^{(a)}(k) = k \times \text{BeaconInterval} + \text{ATIM}_{\text{size}}. \tag{28}$$

Here k $\in \{0, 2\}$. From the equation (27) and equation (28), the value of $\overline{D^{(a)}_{\text{succ}}}$ can be written as

$$\overline{D^{(a)}_{\text{succ}}} = \sum_{k=0}^{2}\sum_{i=0}^{2} P^{(a)'}_{\text{succ}}(i,k) \times D^{(a)}(k). \tag{29}$$

Let $P^{(d)}_{\text{succ}}(i)$ denote the probability of transmitting a data frame successfully in the i^{th} backoff stage of the data window. Fig. 2 shows that $p_d(1-q_d)$ is the probability that there is a collision within the data window. Therefore,

$$P^{(d)}_{\text{succ}}(i) = \{p_d(1-q_d)\}^i\{(1-p_d)(1-q_d)\}. \tag{30}$$

The value $P^{(d)'}_{\text{succ}}(i,b)$ denotes the probability that the backoff process of a packet transmission ends at the i^{th} stage of the data window, with total backoff value b up to the i^{th} backoff stage, given that the data frame is transmitted successfully.

$$P^{(d)'}_{\text{succ}}(i,b) = \frac{P^{(d)}_{\text{succ}}(i,b)}{1-P^{(d)}_{\text{drop}}} \tag{31}$$

Here $P^{(d)}_{\text{drop}}$ is the probability that a data frame is dropped because of exceeding the retry limit in the data window, $P^{(d)}_{\text{succ}}(i,b)$ is the probability that a data frame is transmitted at the i^{th} stage and b is the the sum of backoff values up to the i^{th} backoff stage. Therefore,

$$P^{d}_{\text{succ}}(i,b) = P^{(d)}_{\text{succ}}(i)Pr(B(i)=b)$$
$$P^{(d)}_{\text{drop}} = 1 - \sum_{i=0}^{m}\{p_d(1-q_d)\}^i\{(1-p_d)(1-q_d)$$

Here m is the maximum retry limit to transmit a data frame in the data window and $B(i)$ is the total backoff value up to i^{th} backoff stage.

Let $P^{(d)}_{\text{idle}}$, $P^{(d)}_{\text{col}}$ and $P^{(d)}_{\text{succ}}$ be the probability that a randomly chosen slot in the data window is idle, leads to a collision and results in successful transmission, respectively.

$$\begin{aligned}
P^{(d)}_{\text{idle}} &= (1-\tau_d) \\
P^{(d)}_{\text{succ}} &= n \times P_{as}\tau_d(1-\tau_d)^{(n\times P_{as}-1)} \\
P^{(d)}_{\text{col}} &= 1-(1-\tau_d) - n \times P_{as}\tau_d(1-\tau_d)^{(n\times P_{as}-1)}.
\end{aligned}$$

Here τ_d as calculated in equation (10), represents the probability that a station transmits a data frame in the randomly chosen slot in the data window and $n \times P_{as}$ is the number of station in active mode in the data window. Then,

$$T_{avg} = P^{(d)}_{\text{idle}}\sigma + P^{(d)}_{\text{succ}}T_s + P^{(d)}_{\text{col}}T_c. \tag{32}$$

where T_s and T_c are the average time the channel is sensed busy because of a successful transmission or a collision, respectively, of the data frame in the data window and σ is the empty slot time. Let $D^{(d)}_{\text{succ}}(i,b)$ be the delay to transmit a data frame successfully in the i^{th} backoff stage of the data window when the sum of the backoff value up to stage i is b. For simplicity, lets assume that b is the average value of the contention window. Since $(W_i - 1)$ is the maximum contention window size at the i^{th} backoff stage, the value of b is $\frac{Wi}{2}$. The value of $D^{(d)}_{\text{succ}}(i,b)$ can be calculated from equation (21) and equation (32).

$$D^{(d)}_{\text{succ}}(i,b) = b \times T_{\text{avg}} + i \times T_c + T_s. \tag{33}$$

From the equation (31) and equation (33), the value of $\overline{D^{(d)}_{\text{succ}}}$ can be written as

$$\overline{D^{(d)}_{\text{succ}}} = \sum_{i=0}^{m}\sum_{b=0}^{B(j)_{\max}} P^{(d)'}_{\text{succ}}(i,b) \times D^{(d)}_{\text{succ}}(i,b). \tag{34}$$

Here, $B(j)_{\max} = \sum_{j=0}^{i}(W_j - 1)$. Using equation (29) and equation (34), the average delay $\overline{D}$ to transmit a data frame can be calculated. Therefore, the total average delay, calculated using equation (23) is,

$$\begin{aligned}\overline{D} =&\overline{D^{(a)}_{\text{succ}}} + \overline{D^{(d)}_{\text{succ}}}\\ =&\sum_{k=0}^{2}\sum_{i=0}^{2} P^{(a)'}_{\text{succ}}(i,k) \times D^{(a)}(k) +\\ &\sum_{i=0}^{m}\sum_{b=0}^{B(j)_{max}} P^{(d)'}_{\text{succ}}(i,b) \times D^{(d)}_{\text{succ}}(i,b).\end{aligned}$$

The next section provides a theoretical model for average power consumption in IEEE 802.11 DCF power save mode.

5. Analytical model for Power Consumption

Let $\overline{PW}$ be the average power consumed by a station per unit time. The station may be in one of the radio modes viz. transmit, receive, idle and sleep. It is assumed that the same power is consumed in transmit and receive mode. Let PW_{idle}, $\text{PW}_{\text{tx/rx}}$ and PW_{sleep} be the power consumed per unit time in idle, transmit/ receive and sleep modes respectively. $\overline{PW}$ can be written as

$$\overline{PW} = \frac{P}{T}. \tag{35}$$

Here P is the total power consumed in an interval T time units which includes transmit/receive time, idle time and sleep time. P can be represented as

$$\begin{aligned}P =&\overline{T_{\text{tx/rx}}} \times PW_{\text{tx/rx}} + \overline{T_{\text{idle}}} \times PW_{\text{idle}} +\\ &\overline{T_{\text{sleep}}} \times PW_{\text{sleep}}\end{aligned} \tag{36}$$

Similarly T can be represented as

$$T = \overline{T_{\text{tx/rx}}} + \overline{T_{\text{idle}}} + \overline{T_{\text{sleep}}} \tag{37}$$

Here $\overline{T_{\text{tx/rx}}}$ is the expected time spent in transmit mode by a station (a station is in transmit mode when it transmits ATIM frame or data frame). $\overline{T_{\text{idle}}}$ and $\overline{T_{\text{sleep}}}$ are the expected time spent in idle and sleep mode respectively.

$\overline{T_{\text{tx/rx}}}$ has following components:

- Average time spent to transmit an ATIM frame in the ATIM window ($\overline{T^{a}_{\text{tx/rx}}}$), and
- Average time spent to transmit data frame in the data window ($\overline{T^{d}_{\text{tx/rx}}}$), after successful transmission of ATIM frame in the ATIM window.

Therefore,

$$\begin{aligned}\overline{T_{\text{tx/rx}}} =&\overline{T^{a}_{\text{tx/rx}}} + \overline{T^{d}_{\text{tx/rx}}}\\ =&\sum_{k=0}^{2}\sum_{i=0}^{2} P^{(a)}_{\text{succ}}(i,k) \times (i \times T_{\text{acol}} + T_{\text{asucc}}) +\\ &\sum_{i=0}^{m} P^{(d)}_{\text{succ}}(i) \times (i \times T_c + T_s)\end{aligned} \tag{38}$$

The values of $P^{(a)}_{\text{succ}}(i,k)$ and $P^{(d)}_{\text{succ}}(i)$ are presented in equations (24) and equation (30) respectively. The value of T_c and T_S are presented in equation (21). T_{asucc} and T_{acol} are the average time for which the channel is sensed busy because of a successful transmission and a collision of an ATIM frame respectively. Therefore,

$$\begin{aligned}T_{\text{asucc}} &= \text{ATIM}_{\text{framesize}} + \delta + \text{SIFS} + \text{ATIMACK}_{\text{TO}} + \delta.\\ T_{\text{acol}} &= \text{ATIM}_{\text{framesize}} + \text{SIFS} + \text{ATIMACK}_{\text{TO}}.\end{aligned}$$

Here $\text{ATIM}_{\text{framesize}}$ is the size of an ATIM frame and $\text{ATIMACK}_{\text{TO}}$ is the timeout interval for ATIMACK .

A station can be in idle mode either in the ATIM window or in the data window. So $\overline{T_{\text{idle}}}$ can be calculated as,

$$\overline{T_{\text{idle}}} = \overline{T^{a}_{\text{idle}}} + \overline{T^{d}_{\text{idle}}} \tag{39}$$

In the i^{th} backoff stage the station chooses a uniformly distributed backoff counter W, where $W \in \{0, W_i - 1\}$. The station waits for W time units before sending a frame and decrements the backoff counter after each slot time (assuming that each slot is an empty slot). The station transmits the frame when the backoff counter reaches zero. When a station chooses W as the backoff counter in the i^{th} backoff stage then the idle period in that stage is W. As W is uniformly distributed in $\{0, W_i - 1\}$ the station spends $\frac{W_i}{2}$ slots in idle mode in the i^{th} backoff stage on an average. After a successful transmission of ATIM frame the station will remain in the idle mode for the rest of the ATIM window. Therefore,

$\overline{T^a_{\text{idle}}}$ can be written as

$$\overline{T^a_{\text{idle}}} = \sum_{k=0}^{2}\sum_{i=0}^{2} P^{(a)}_{\text{succ}}(i,k) \times \left(\frac{W_i}{2} \times \sigma\right) + \sum_{k=0}^{2}\sum_{i=0}^{2} P^{(a)}_{\text{succ}}(i,k) \times (\text{ATIM}_{\text{size}} - (i \times T_{\text{acol}} + T_{\text{asucc}})) \quad (40)$$

Here $\text{ATIM}_{\text{size}}$ is the time duration of an ATIM window and σ is the duration of one slot time.

For the data window, the idle time can be calculated as,

$$\overline{T^d_{\text{idle}}} = \sum_{i=0}^{2} P^{(d)}_{\text{succ}}(i) \times \left(\frac{W_i}{2} \times \sigma\right) \quad (41)$$

According to the PSM algorithm given in the IEEE 802.11 standard [1], a station can send multiple data frames in the data window. So after one successful data transmission the station may be in transmit mode to transmit the remaining data frames to the same receiver in that data window.

If a station fails to transmit the ATIM frame in the ATIM window then it goes to the sleep mode in the following data window. Then

$$\overline{T_{\text{sleep}}} = \sum_{k=0}^{2}\sum_{i=0}^{2} k \times \text{DATA}_{\text{size}} \times (1 - P^{(a)}_{succ}(i,k)) \quad (42)$$

Here, $\text{DATA}_{\text{size}}$ represents the time duration of the data window. From equation (35), equation (38), equation (39) and equation (42) the average power consumed by a station per unit time can be calculated.

6. Model Validation and Performance Evaluation

The proposed theoretical model is validated using simulation results obtained from the Qualnet 5.0.1 network simulator [27]. For simulation in IBSS power save mode, the size of the ATIM window is taken to be $20ms$. The throughput for the basic access in power save mode is calculated under the Direct Sequence Spread Spectrum (DSSS) physical layer [1]. The power consumption in different states are taken from the data-sheet of CISCO Aironet 350 Series Client Adapters [28]. The system parameters used in the calculation are listed in Table 1. CW_{min} is the minimum contention window, CW^a_{max} and CW^d_{max} are maximum contention window for ATIM and data window respectively. Qualnet has inbuilt IEEE 802.11 PSM module which is used for the simulation perpose. The simulation is done for 10 different cases with randomly generated seed values and the average is plotted. The simulation uses similar parameter settings as given in Table 1.

Table 1: Parameters used in the simulation

Parameter	**value**
Payload size	1024 bytes
ATIM	28 bytes + PHY header
ACK	14 bytes + PHY header
PHY header	$192\mu s$
MAC header	28 bytes
Basic rate	1Mbps
Data rate	2Mbps
Slot time	$20\mu s$
SIFS	$10\mu s$
DIFS	$50\mu s$
CW_{min}	32
CW^a_{max}	128
CW^d_{max}	1024
$PW_{\text{tx/rx}}$	2.25W (Watt)
PW_{idle}	1.35W (Watt)
PW_{sleep}	0.07W (Watt)

6.1. Analysis of the Proportional Constant c

It has been discussed in subsection 3.4 that for a fixed ATIM window size, the value of the parameter c depends on the size of the data window. Fig. 3 shows the overall normalized throughput calculated for different c values using the theoretical analysis presented in this paper. The figure also shows the overall normalized throughput obtained from simulation for different beacon interval sizes with ATIM window fixed at $20ms$. It can be observed from the

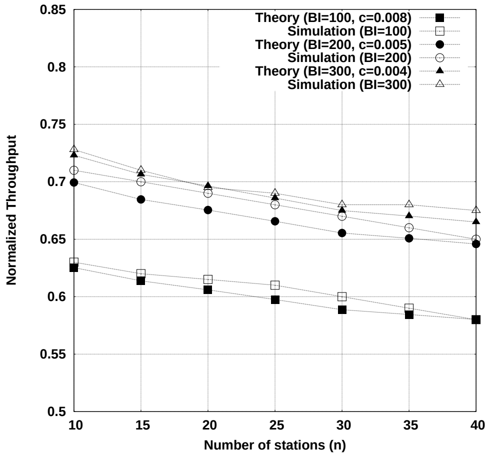

Figure 3: Overall Normalized Throughput (DATA window + ATIM Overhead)

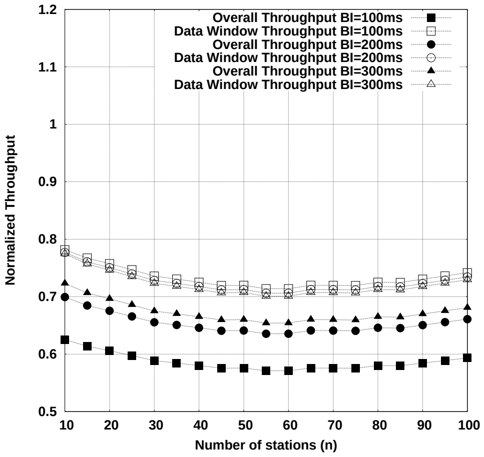

Figure 4: Normalized Throughput in the Data Window vs Overall Throughput (According to Theory)

figure that for a particular beacon interval size in simulation and c value in theory, the theoretical results are at par the simulation results. For $c = 0.008$, the theoretical result matches with the simulation result for beacon interval size $100ms$. Similarly, for $c = 0.005$, the theoretical result matches with the simulation result for beacon interval size $200ms$, and for $c = 0.004$, the theoretical result matches with the simulation result for beacon interval size $300ms$.

Recall that q_d is the probability that the data window ends while transmitting a data frame. For a fixed number of stations and for a fixed ATIM window size, q_d should decrease with the increase of data window size. Furthermore, for a fixed ATIM window size, the number of nodes compete in the data window for data frame transmission, i.e. $n \times P_{as}$, is independent of data window size. So the value of c should decrease with the increase of data window size. This is also reflected in Fig. 3. For a fixed ATIM window size of $20ms$, the values of c are 0.008, 0.005 and 0.004 for beacon interval sizes $100ms$, $200ms$ and $300ms$, respectively. This is consistent with equation (19).

Unless stated otherwise, the value of c is taken to be 0.005 for rest of the analysis in this paper. This c value is equivalent to ATIM window size of $20ms$ and data window size of $180ms$.

6.2. Saturation Throughput

Fig. 4 shows the normalized data window throughput and overall throughput for different beacon interval sizes, calculated using different c values as stated in the previous subsection. The ATIM window size is kept fixed at $20ms$. Two important observations can be drawn out from this figure. First, the overall throughput is less than the data window throughput because of ATIM overhead. For BI=$200ms$ and ATIM window size=$20ms$, there is an ATIM overhead that reduces overall throughput by 10%. The second observation is that the data window throughput is almost the same for different beacon interval sizes. If the ATIM window size is fixed, the number of stations competing for channel access in the data window, given by $n \times P_{as}$, is almost constant, and so the data window throughput remain close to each other for all the beacon interval sizes. However, as the beacon interval size increases, the overall throughput tends to increase. With the increase of beacon interval size, the ATIM overhead decreases (since the ATIM window size is kept fixed). The overall throughput reduction for BI=$100ms$ is 20%, whereas with BI=$300ms$, the throughput reduction due to ATIM overhead is 6.67%.

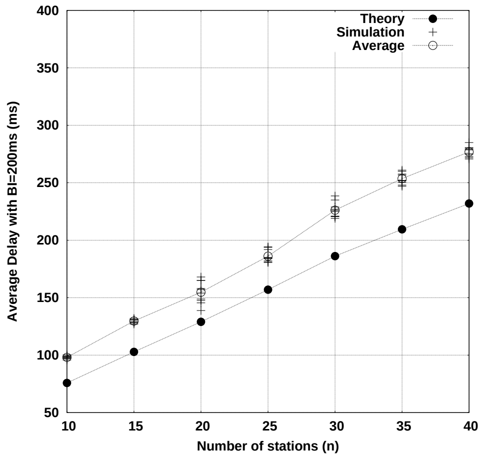

Figure 5: Average Delay for Data Frame Transmission

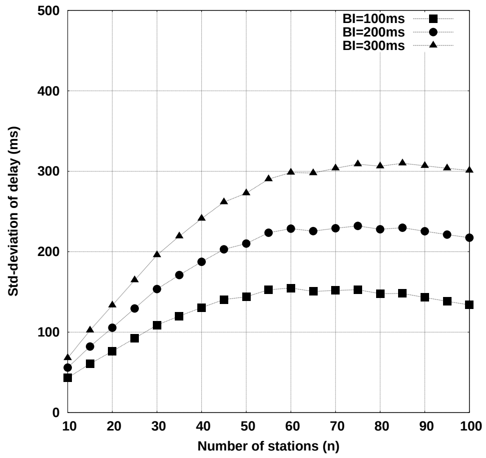

Figure 7: Standard Deviation for Delay

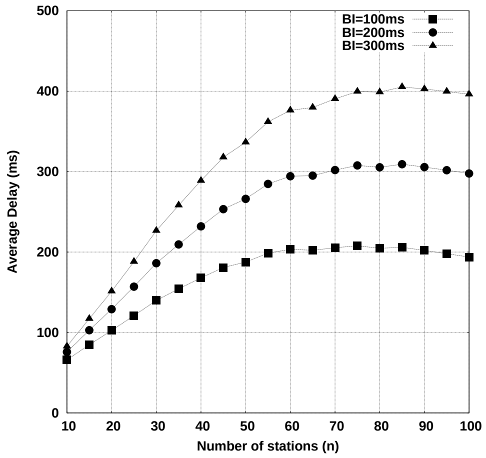

Figure 6: Average Delay for Different Beacon Intervals

6.3. Delay Analysis

Fig. 5 presents average delay against different network sizes. The figure shows that the theoretical and simulation results are close. Fig. 6 shows the impact of the beacon interval size on the average delay for different network sizes. As the beacon interval size increases, the average delay increases. The MAC layer delay is the average delay between the time a packet is en-queued at the MAC layer interface queue, and the time the packet gets transmitted successfully. Thus the average delay has two components - the average waiting time at the MAC layer interface queue when the station tries to get access to the channel, plus the average transmission time. The stations that fail to transmit an ATIM frame successfully in the ATIM window, have to wait for the rest of the beacon interval to get the next chance to transmit another ATIM frame in the next ATIM interval. This increases the average waiting time to get a chance to transmit data frames. As the beacon interval size increases, the average waiting time also increases. For this reason, the average delay increases with the increase of beacon interval size.

Fig. 7 presents the standard deviation of the delay to successfully transmit a data frame as calculated from the analytical model for delay analysis. From Fig. 7, it can be observed that the standard deviation is very high. For example for a network size of 30 and BI=200ms, the average delay is 186ms and standard deviation is 153ms. To verify the above result, the simulation result in Fig. 8 presents the observed delay to transmit a data frame for (BI=200ms) with 30 wireless stations. From Fig. 8, it can be concluded that there is unfairness in data frame transmission and the delay distribution is not a uniform distribution. This results in a very high value for the standard deviation of the delay.

It is worthwhile to mention that the average delay and jitter (i.e. standard deviation of delay) increases

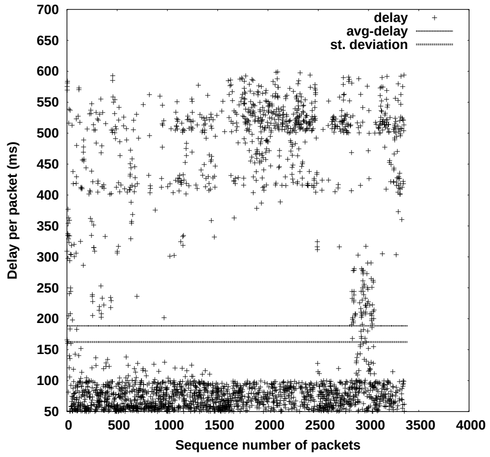

Figure 8: Delay per Packet (ms), Simulation Results for Number of Wireless Stations = 30

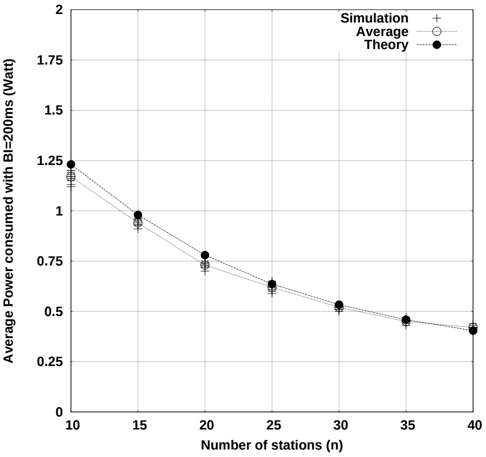

Figure 9: Average power consumed for data frame transmission

with increase in duration of Beacon Interval. For example, with BI=300ms and network size of 30 nodes, delay and jitters are 226.6ms and 200ms respectively. Obviously, it will be significantly very high for larger BI duration, which is impractical from application point-of-view. Considering this, the simulation and theoretical results upto BI=300ms are reported in this paper.

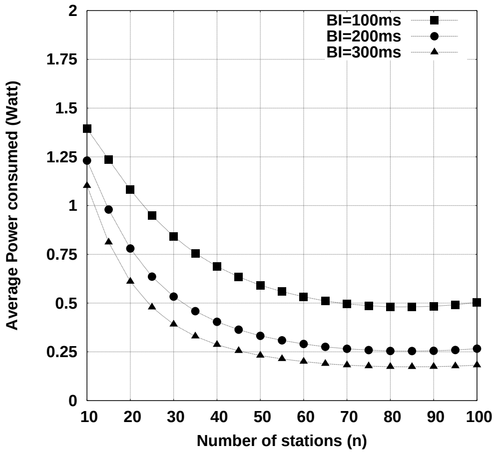

Figure 10: Average power consumed for different beacon intervals

6.4. Power Consumption

Fig. 9 shows the average power consumption with respect to the number of stations in IEEE 802.11 IBSS power save mode. The result obtained from the analytical model for power consumption in section 5 has been verified through simulation. Fig. 10 gives a comparison of the average power consumption for packet transmission for different beacon intervals as obtained from the analytical model. It can be seen from the figure that as the size of beacon interval increases, average power consumption decreases. This is because, with a large beacon interval the stations remain in sleep mode for large amounts of time, and thus save power.

Furthermore, the figure shows that the average power consumption for a network of size 10 is more than the average power consumption for a network of size 20. This is because for small network sizes, contention is less, and so most of the stations are in transmit or receive state, leading to higher power consumption. However, as the network size increases, contention becomes high, and on average $n - n \times P_{as}$ stations go to sleep mode in the data window. This reduces the average power consumption.

Table 2: Comparison between WPSM and PSM with 30 Nodes

Data Window Throughput		
	WPSM	**PSM**
BI=100ms	0.712	0.73583
BI=200ms	0.712	0.72822
BI=300ms	0.712	0.72315

Overall Normalized Throughput		
	WPSM	**PSM**
BI=100ms	0.712	0.58867
BI=200ms	0.712	0.65540
BI=300ms	0.712	0.67494

Average Delay		
	WPSM	**PSM**
BI=100ms	140 ms	139.845 ms
BI=200ms	140 ms	186.165 ms
BI=300ms	140 ms	226.612 ms

Average Power Consumption		
	WPSM	**PSM**
BI=100ms	1.4 Watt	0.84139 Watt
BI=200ms	1.4 Watt	0.53326 Watt
BI=300ms	1.4 Watt	0.39072 Watt

6.5. Comparison between DCF with and without Power Save Mode

A comparison between IEEE 802.11 DCF with Power Save Mode (PSM) and without Power Save Mode (WPSM) is given in Table 2. The performance for IEEE 802.11 DCF without PSM is obtained from the models given in [3], [15], [19] which are generalizations of Bianchi's model [2]. For WPSM, the values do not depend on BI. However, for the ease of presentation these values are given in the table.

It can be noted from the table that the overall throughput obtained for the proposed model for power save mode (PSM) is marginally less than the one obtained for without power save mode (WPSM), whereas in PSM the data window throughput is very high. The loss in overall throughput is because of the ATIM window overhead. On the other hand, the contention is less in the data window as some of the nodes that fail to transmit ATIM frame successfully in the ATIM window, go to sleep mode in the data window. This makes the data window throughput marginally higher. However, the ATIM transmission imposes an extra control message overhead that reduces overall channel throughput.

From Table 2, it can also be observed that the average delay is more in case of PSM compared to WPSM. Delay is more because of the extra time introduced due to ATIM frame transmission. If a station fails to transmit an ATIM frame in the ATIM window, then it has to wait for rest of the beacon interval to get next chance to transmit another ATIM frame. That is why, as the size of the beacon interval increases (with fixed ATIM window size), the average delay increases, as shown in Table 2 and Fig. 6.

Table 2 shows that the average power consumption is less in PSM compared to WPSM. For PSM, the average power consumption is calculated for different beacon interval, with fixed ATIM window size of $20ms$. From the table, it can be observed, that with BI=$100ms$ at PSM, average power consumption is 60.71% of average power consumption at WPSM. With BI=$300ms$ at PSM, average power consumption is 27.14% of average power consumption at WPSM.

The above analysis shows that in IEEE 802.11 PSM, power saving comes at the cost of network throughput and delay. The network throughput for IEEE 802.11 DCF in PSM is marginally less than the throughput obtained in IEEE 802.11 DCF. For a fixed ATIM window size with saturation condition, as the size of the BI increases, then network throughput increases and power consumption decreases. However, the network delay increases considerably. Therefore, it can be said that by tuning the value of q_d based on the number of active stations in the data window, the network lifetime can be increased while keeping the network performance at par without power save mode.

7. Conclusion

In this paper a discrete time Markov chain model is presented for the transmission of ATIM and data frames in IEEE 802.11 DCF power save mode. The theoretical results for probability of success and normalized throughput are close to the simulation results. This paper introduces analytical models for delay analysis and power consumption of the IEEE 802.11 DCF in PSM. From these results, the network life time can also be analyzed. The analysis of the effect of the duration of beacon interval on the network performance justifies the need for an adaptive beacon window based power saving mechanism to maximize network lifetime without degrading network performance.

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