Performance of Asymmetric Digital Subscriber Lines in an Impulse Noise Environment

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***Abstract*—This letter presents a numerical study of the impact of impulse noise on asymmetric digital subscriber lines (ADSL). Methods for simulating the effect of impulse disturbances on a discrete multitone system are first presented, and actual measured noise bursts are then used for the simulations as if they were deterministic signals, in order to characterize their effects on ADSL systems. It is shown that while a combination of coding, interleaving, and 6-dB margin is adequate in protecting ADSL systems from isolated impulses, an impulse train with long duration will cause a significant number of error bits in the system. In this case, a tradeoff among the number of error seconds, the maximum reach, and the coding delay must be made.**

***Index Terms*—Asymmetric digital subscriber lines (ADSL), discrete multitone (DMT), forward error correction (FEC), impulse noise, Reed–Solomon (RS) codes.**

I. INTRODUCTION

OCCASIONALLY, large nonstationary electromagnetic disturbances may be coupled into telephone wires, resulting in impulse noise. The objective of this paper is to present a study of the impact of impulse noise on asymmetric digital subscriber line (ADSL) systems. The study is carried out in two steps. First, the effect that a deterministic time-domain impulse has on the performance of a multicarrier system is examined, and a method to accurately simulate the performance of a discrete multitone (DMT)-based G.dmt-compliant [1] ADSL system is described. Methods to simulate the effect of impulse noise on a system with Reed–Solomon (RS) codes do not appear to have been presented in the open literature, to the best of the authors' knowledge. Second, realistic impulses as measured by France Télécom are used to simulate both uncoded and RS-coded ADSL systems. The performance is characterized in terms of both the probability of bit error and the number of error bytes for isolated impulses, and in terms of the number of error seconds for impulse trains.

Previous studies in this area [2]–[6] mostly rely on statistical models of the impulse noise. This letter recognizes that the accurate modeling of impulse noise is not an easy task, and instead relies on measured impulses to study the behavior of an actual system in a real environment. As it is expected and will be verified by the simulations, the impact of impulse noise on a practical ADSL system depends strongly on the impulse amplitude, its duration, the interarrival time, and the spectral characteristics of the impulse.

II. ADSL PERFORMANCE IN GAUSSIAN NOISE

This section presents a method to compute the probability of bit error in a multicarrier system with RS-based forward error correction (FEC) in an additive white Gaussian noise (AWGN) environment. Specific values are used to simplify the description of the method.

Assuming an RS codeword of 200 bytes, 16 bytes of which are parity, for an RS codeword to be correctable, at most, eight bytes can be in error. It is well known [7] that the miscorrection probability of RS codes is very small, and it will be ignored in the following calculations. In an AWGN or properly equalized channel, the probability of error $P_{\rm byte}$ for each byte is approximately the same. When $P_{\rm byte}$ is small, as in typical ADSL deployments, the probability of codeword error is closely approximated by the probability that nine erroneous bytes occur, as events with eight or fewer errors are corrected by the code, and events with more than nine errors occur with much lower probability. Hence, the probability of codeword error is approximately

$$\binom{200}{9} P_{\rm byte}^{9}(1-P_{\rm byte})^{200-9}.$$

Due to the small $P_{\rm byte}$, for each byte in error, only a single bit is likely to be wrong. Thus, the fraction of bits in error in the codeword is roughly $9/200/8 = 5.6 \cdot 10^{-3}$. Therefore, in order to achieve a target bit-error rate (BER) of 10^{-7}, $P_{\rm byte}$ must satisfy

$$10^{-7} = \binom{200}{9} P_{\rm byte}^{9}(1-P_{\rm byte})^{191} 5.6 \cdot 10^{-3}$$

or $P_{\rm byte} = 0.0065$. Taking the first term of the binomial expansion $P_{\rm byte} = 8P_b(1-P_{\rm bit})^8 \approx 8P_{\rm bit}$, the probability of bit error should be $P_{\rm bit} = P_{\rm byte}/8 = 0.0065/8 = 8.1 \cdot 10^{-4}$ in order to attain an overall BER 10^{-7} after the decoding of the RS code. The signal-to-noise (SNR) gap for quadrature amplitude modulation (QAM) for $P_{\rm bit} = 8.1 \cdot 10^{-4}$ is found by noticing that $2Q(10.5\ {\rm dB}) = 8.1 \cdot 10^{-4}$. Hence, the coding gain is equal to $14.5\ {\rm dB} - 10.5\ {\rm dB} = 4.0\ {\rm dB}$. Table I summarizes the coding gains of the RS code for small $P_{\rm byte}$ under various system conditions.

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TABLE I
PARAMETERS OF RS FEC FOR ADSL SYSTEMS

Target data rate		608 kbps	1.216 Mbps	2.048 Mbps
Number of bytes per DMT		23	42	68
Fast mode	DMT symbols per RS	1	1	1
	RS codeword length	23 bytes	42 bytes	68 bytes
	Parity length	4 bytes	4 bytes	6 bytes
	RS coding gain	3.0 dB	2.78 dB	3.2 dB
	Coding overhead	128 kbps	128 kbps	192 kbps
Interleave Mode	DMT symbols per RS	8	4	2
	RS Codeword length	200 bytes	184 bytes	152 bytes
	Parity length	16 bytes	16 bytes	16 bytes
	RS coding gain	4.00 dB	4.03 dB	4.16 dB
	Coding overhead	64 kbps	128 kbps	256 kbps

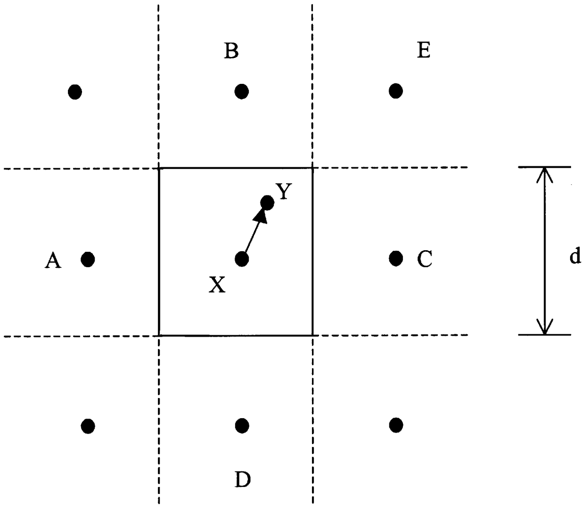

Fig. 1. Probability of error calculation.

III. ADSL PERFORMANCE WITH IMPULSE NOISE

Suppose that a tone of the DMT symbol is hit by a deterministic impulse shown in Fig. 1 as an arrow from the original constellation point *X* to the new location *Y*. If *Y* is outside the decision boundary, an error is almost certain to occur, so $P_e = 1$, and $P_{\mathrm{bit}} = 1/2$. If *Y* is outside the decision boundary, a symbol error is almost certain to occur, and $P_{\mathrm{bit}} = 1/2 \sum_j Q(d_j/2\sigma)$, where d_j's are the distances between *Y* and the boundaries of the decoding regions of its neighbors *A*, *B*, *C*, *D*, and *E*. Strictly speaking, summing the probability is only valid when the dimensions are orthogonal. The above formula is a union bound. The factor 1/2 accounts for the conversion between the probability of symbol error and the probability of bit error. It represents a worst-case scenario, and a practical system with Gray code bit mapping may have lower probability of bit error.

In an uncoded system, the probability of bit error of the entire DMT symbol can be found by averaging the probability of bit error in each tone: $P_{\mathrm{bit,DMT}} = \sum_i b_i P_{\mathrm{bit}}(i) / \sum_i b_i$, where $P_{\mathrm{bit}}(i)$ is the probability of bit error and b_i the number of bits of the ith tone.

In a coded system, in order to evaluate the expected number of error bytes in an RS codeword, the probability of byte error is first computed as $P_{\mathrm{byte}} = 1 - \prod_{i=1}^{8} (1 - P_{\mathrm{bit}}(i))$. Since the byte boundaries do not necessarily coincide with the tone boundaries, the probabilities of bit error $P_{\mathrm{bit}}(i)$ for each bit may be different. In the absence of coding, the expected number of error bytes is just the sum $\sum_k P_{\mathrm{byte}}(k)$, summed over all bytes in the codeword. When an RS code is used, the codewords with fewer than nine errors are corrected. Let $P_{\mathrm{error}}(n)$ denote the probability that there are n error bytes in an RS codeword. Then, the average number of byte errors is computed as follows: $\sum_{n>8} nP_{\mathrm{error}}(n) = \sum_n nP_{\mathrm{error}}(n) - \sum_{n\leq 8} nP_{\mathrm{error}}(n) = \sum_k P_{\mathrm{byte}}(k) - \sum_{n\leq 8} nP_{\mathrm{error}}(n)$. To reduce the computational effort, $P_{\mathrm{error}}(n)$ can be very closely approximated by selecting only the large probability terms which typically correspond to the few bytes with large $P_{\mathrm{byte}}(k)$.

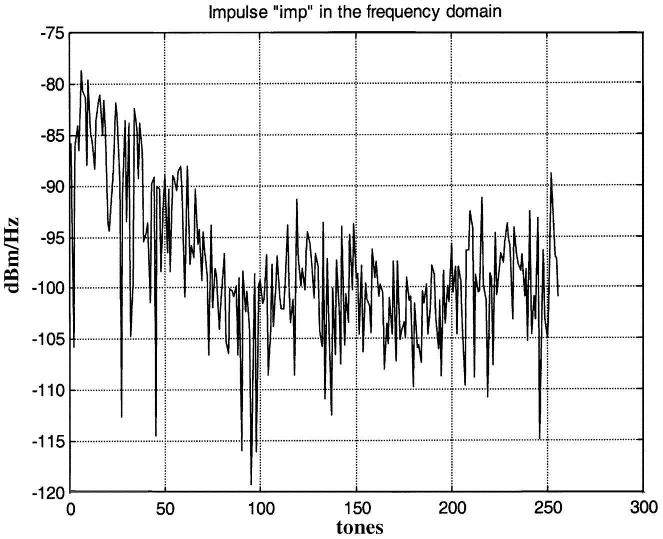

Fig. 2. Frequency-domain plot of the impulse "imp."

IV. SIMULATION RESULTS

The performance of both coded and uncoded systems is first evaluated for three representative impulse samples (named "imp," "ex," and "raf"). The "imp" impulse lasts for 500 μs (i.e., is two DMT symbols long) and has the highest peak voltage. Its frequency domain plot is given in Fig. 2 as an example. "ex" has slightly smaller peak voltage, and only lasts for one DMT symbol, whereas "raf" has the smallest peak voltage, and is about 2.5 DMT symbols long. The average BER for the uncoded system over a DMT symbol at the maximum reaches for each margin value and for three different noise models is shown in Table II. SC1 and SC2 are proprietary crosstalk models for the network of France Télécom. As expected, the BER largely depends on the duration of the impulse. The BER also depends on the noise model. Impulse noise will have a more severe effect on systems designed for AWGN channels compared to channels with crosstalk. This is due to the fact that a system designed for a crosstalk environment has to be more robust, since the power of crosstalk is higher compared to AGWN. In general, the service range of a system designed to cope with crosstalk is smaller, and the distance between the constellation points larger. Consequently, it is more immune to impulse noise as well. Since the impulse simulation is

TABLE II
PROBABILITY OF BIT ERROR FOR EACH OF THE THREE IMPULSES (UNCODED)

	Rate	608kbps			1.216Mbps			2.048Mbps		
Impulse	Noise	AWGN	SC1	SC2	AWGN	SC1	SC2	AWGN	SC1	SC2
“imp”	0dB	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0
high peak	6dB	0.5	0.5	0.3	0.5	0.4	0.25	0.5	0.3	0.25
2 DMT	12dB	0.5	0.2	0.7	0.5	0.2	0.7	0.5	0.1	0.07
“ex”	0dB	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
smaller peak	6dB	0.5	0.4	0.3	0.5	0.35	0.3	0.5	0.35	0.2
1 DMT	12dB	0.5	0.35	0.25	0.5	0.3	0.2	0.5	0.3	0.2
“raf”	0dB	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
smallest peak	6dB	0.4	0.08	0.05	0.3	0.8	0.05	0.3	0.1	0.05
2.5 DMT	12dB	0.3	0.019	0	0.25	0.038	0.005	0.22	0.13	0.007

TABLE III
EXPECTED NUMBER OF ERROR BYTES AT MAXIMUM REACHES AS A FUNCTION OF INTERLEAVER DEPTH, IMPULSE, MARGIN, CROSSTALK MODEL, AND TARGET RATE

		Impulse “imp”			Impulse “ex”			Impulse “raf”		
Coding		FEC, no interleaving ($D = 0$)								
Margin		0 dB	6 dB	12 dB	0 dB	6 dB	12 dB	0 dB	6 dB	12 dB
608 kbps	AWGN	74	73	71	48	47	47	84	81	72
	SC1	72	59	32	46	39	32	73	34	15
	SC2	44	35	21	34	33	29	39	25	2
1.2 Mbps	AWGN	133	131	129	86	85	86	149	138	118
	SC1	125	92	55	81	66	54	109	57	33
	SC2	80	64	31	63	62	48	74	41	9
2.0 Mbps	AWGN	216	213	205	139	138	139	243	225	190
	SC1	187	128	60	124	110	78	144	99	20
	SC2	134	100	46	107	97	71	130	61	16
Coding		FEC with Interleaving $D = 8$								
Margin		0 dB	6 dB	12 dB	0 dB	6 dB	12 dB	0 dB	6 dB	12 dB
608 kbps	AWGN	75	60	56	28	0	0	90	77	64
	SC1	74	21	0	18	0	0	79	1	0
	SC2	7	0	0	2	0	0	2	0	0
1.2 Mbps	AWGN	138	126	124	81	72	73	155	135	119
	SC1	132	89	23	77	56	31	118	29	2
	SC2	83	37	0	60	39	4	74	2	0
2.0 Mbps	AWGN	225	216	202	148	140	137	250	216	176
	SC1	199	121	38	135	100	62	157	93	0
	SC2	140	97	6	113	87	57	134	48	0
Coding		FEC with Interleaving $D = 32$								
Margin		0 dB	6 dB	12 dB	0 dB	6 dB	12 dB	0 dB	6 dB	12 dB
608 kbps	AWGN	9	0	0	8	0	0	13	0	0
	SC1	9	0	0	8	0	0	13	0	0
	SC2	7	0	0	6	0	0	9	0	0
1.2 Mbps	AWGN	8	0	0	8	0	0	16	0	0
	SC1	8	0	0	8	0	0	15	0	0
	SC2	8	0	0	6	0	0	12	0	0
2.0 Mbps	AWGN	209	62	27	6	16	16	14	0	0
	SC1	81	0	0	7	0	0	14	0	0
	SC2	7	0	0	13	0	0	16	0	0
Coding		FEC with Interleaving $D = 64$								
Margin		0 dB	6 dB	12 dB	0 dB	6 dB	12 dB	0 dB	6 dB	12 dB
608 kbps	AWGN	18	0	0	13	0	0	18	0	0
	SC1	17	0	0	13	0	0	17	0	0
	SC2	11	0	0	9	0	0	10	0	0
1.2 Mbps	AWGN	16	0	0	16	0	0	16	0	0
	SC1	16	0	0	15	0	0	16	0	0
	SC2	14	0	0	12	0	0	15	0	0
2.0 Mbps	AWGN	14	0	0	14	0	0	14	0	0
	SC1	14	0	0	14	0	0	14	0	0
	SC2	16	0	0	16	0	0	16	0	0

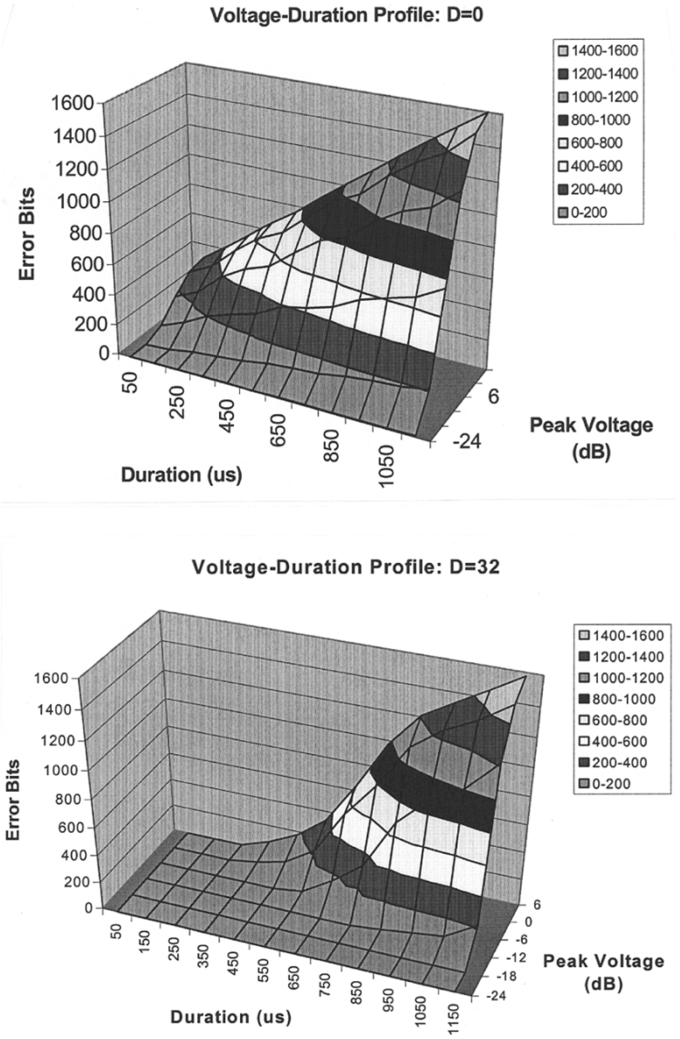

Fig. 3. Voltage-duration profile for interleaver depth $D = 0$ and $D = 32$.

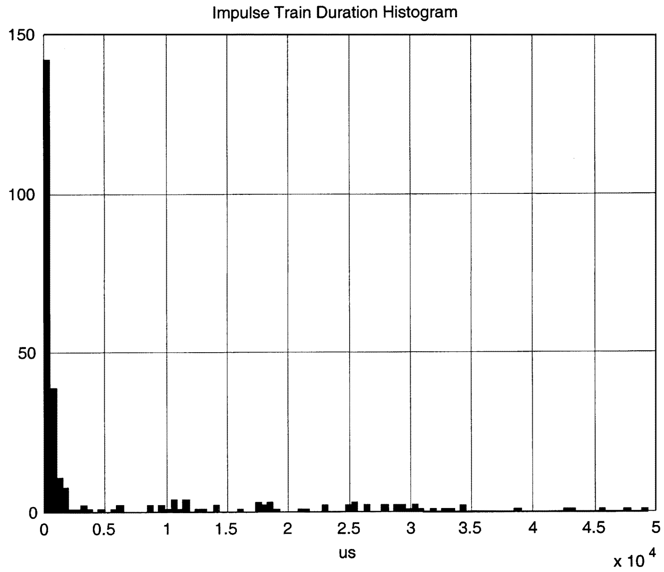

Fig. 4. Duration histogram of the impulses used for the simulations.

performed at the maximum reaches, systems designed for a crosstalk environment appear to be more robust against impulse noise. Table III presents the coded system performance with and without interleaving for the three representative impulses. It is assumed that impulses occur infrequently so that no RS codeword can ever contain bytes corrupted by two different impulses. In all three cases, complete protection again impulse attacks is obtained with a combination of 6-dB noise margin, and an RS code with interleaver depth of 64. Neither noise margin, nor coding, alone is adequate.

Next, the impulse "raf" is used to illustrate how the performance of ADSL is affected by impulse characteristics. The number of error bits is plotted against the impulse peak voltage and the impulse duration. The original impulse is 600 μs long. Shorter impulses are created by truncating, and longer impulses by concatenating replicas of "raf." Impulses with different peak voltages are created by scaling. The peak voltage axis is in decibels, and 0 dB corresponds to the peak voltage of the original impulse. The results are plotted in Fig. 3. AWGN noise is assumed, and a 6-dB margin is included. The maximum ranges for the 2.048 Mb/s system are used. It is interesting to observe that in the fast mode, the number of error bits increases linearly with duration. In the interleaved mode, the number of error bits is suppressed for short duration impulses, but eventually grows linearly again as the interleaver breaks down for longer impulses.

Finally, the effect of impulse trains, i.e, bursts of subimpulses close to each other, is characterized using the notion of error seconds. In this study, all impulse trains are less than 1/16 s long. So, each impulse train can cause, at most, one error second. Therefore, determining the number of error seconds is equivalent to determining the number of impulse trains that cannot be corrected by FEC.

In the simulations, 269 impulse trains occurring over a two-day period were used. The duration histogram of the impulse trains is plotted in Fig. 4. Most impulse trains are less than 3 ms long (or 12 DMT symbols), but the longest ones can last up to 50 ms (or 256 DMT symbols). The number of error seconds is summarized in Table IV. No interleaving is used in the fast mode. In the medium and long delay modes, 16 and 64 DMT symbols, respectively, are interleaved.

The above results clearly illustrate a number of tradeoffs. First of all, better impulse protection requires longer interleaving delays. Secondly, for a given delay, a higher margin system is able to withstand a larger number of impulses. However, the noise margin comes in the expense of maximum reach. Hence, there is a tradeoff among interleaving delay and maximum reach. As an example, Fig. 5 summarizes those tradeoffs for a 2.048 Mb/s system under a moderate amount of crosstalk (SC1).

V. CONCLUSIONS

Typical impulses occurring on ADSL lines are 20–40 dB larger than either AWGN or near-end crosstalk, and they can be several DMT symbols long. Such significant disturbance can destroy the ADSL performance completely when no FEC is used. Noise margin of 6 or 12 dB alone is not sufficient to protect ADSL from impulse noise. With FEC, a size-64 interleaver and 6 dB of noise margin, almost complete protection against an isolated impulse (of duration up to 500 μs) can be obtained regardless of its peak voltage. However, real ADSL

TABLE IV
NUMBER OF ERROR SECONDS IN TWO DAYS

Margin		0 dB			6 dB			12 dB		
Buffer		FAST	MED delay	LONG delay	FAST	MED delay	LONG delay	FAST	MED delay	LONG delay
608 kbps	AWGN	263	265	258	227	220	102	199	178	71
	SC1	233	236	232	198	144	40	163	76	8
	SC2	208	209	207	145	74	15	69	16	6
1.216 Mbps	AWGN	266	266	260	229	216	99	202	175	70
	SC1	238	234	231	201	144	36	147	67	8
	SC2	208	209	207	147	69	15	75	12	6
2.048 Mbps	AWGN	265	263	260	230	212	92	200	172	61
	SC1	240	233	232	199	135	30	99	39	6
	SC2	216	218	217	150	54	13	72	11	5

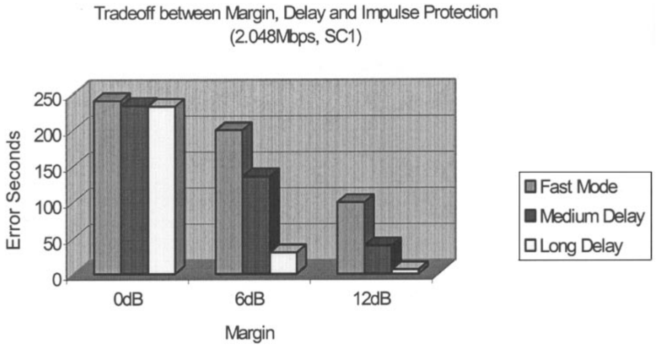

Fig. 5. Tradeoff between margin, delay, and impulse protection.

deployment often experiences extended impulse trains that will occasionally break the FEC code, even when a 6-dB margin and an interleaving depth of 64 are used. The tradeoff among impulse protection, margin, and delay is characterized. These findings are useful for ADSL deployment planning.

REFERENCES

[1] "Draft New Recommendation G.992.1: Asymmetrical Digital Subscriber Line (ADSL) Transceivers (ex G.dmt)," International Telecommunication Union (ITU), Geneva, Switzerland, 1999.

[2] I. Mann, S. McLaughlin, W. Henkel, R. Kirkby, and T. Kessler, "Impulse generation with appropriate amplitude, length, interarrival, and spectral characteristics," *IEEE J. Select. Areas Commun.*, vol. 20, pp. 901–912, June 2002.

[3] W. Henkel, T. Kessler, and H. Y. Chung, "Coded 64-CAP ADSL in an impulse-noise environment—Modeling of impulse noise and first simulation results," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1611–1621, Dec. 1995.

[4] D. B. Levey and S. McLaughlin, "Calculating error-free seconds in xDSL systems corrupted by impulse noise," *IEEE Commun. Lett.*, vol. 5, pp. 319–321, July 2001.

[5] J. W. Modestino and D. H. Sargrad, "Use of coding to combat impulse noise on digital subscriber loops," *IEEE Trans. Commun.*, vol. 36, pp. 529–537, May 1988.

[6] N. Nedev, S. McLaughlin, D. Laurenson, and R. Daley, "ATM cell error performance of xDSL under impulse noise," in *Proc. Int. Conf. Communications*, vol. IV, Helsinki, Finland, June 2001, pp. 1254–1258.

[7] S. B. Wicker, *Error Control Systems for Digital Communication and Storage*. Englewood Cliffs, NJ: Prentice-Hall, 1995.