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***Abstract*—Power line communications (PLC) system performance is severely affected by the presence of impulsive noise and channel gain. Many communication systems employ a combination of forward error correction and automatic repeat request (ARQ) to detect transmission errors known as hybrid ARQ (HARQ). Thus, HARQ can be used to improve the reliability of a PLC system. In this paper, we evaluate the performance of a PLC system under the influence of impulsive noise over Rayleigh and Log-Normal channel gains using HARQ with chase combining scheme. Closed-form expressions of the analytical outage probability are derived for these two channel gain scenarios. Using the expression for outage probability, we also compute the average throughput for the PLC system under the given channel gain scenarios. It is observed by the analysis that the outage performance of the considered PLC system is significantly improved with the use of HARQ.**

***Index Terms*—PLC, Hybrid ARQ, fading channels, impulsive noise, performance analysis.**

I. Introduction

The Internet of Things (IoT) is expected to grow significantly to 50 billion connected devices by 2020, which provides valuable opportunities to consumers, manufacturers, and utility providers [1]. While wireless communications is an essential enabling technique for IoT, the adverse effects of wireless propagation and requirement on battery life imply that wireless solution cannot fit all requirements of IoT. To this regard, the industry and academia have explored to connect the IoT devices via the power line thanks to its ubiquitous nature, widespread availability, and power line communications (PLC) has already proven to be a robust and cost-optimized solution in the field of smart home, Industry 4.0, smart grid, etc. [1].

PLC channel is tremendously different from the wireless channel. The channel gain statistics in PLC environments are not well established compared to wireless communications [2]–[10]. A vast number of measurement results show that distributions such as Rayleigh and Log-Normal are applicable for defining the path amplitudes in PLC channels [2]–[8]. In [3], wideband propagation measurements were undertaken in the 30 kHz to 100 MHz band in various indoor channel environments. The peak and notch widths of the measured transfer functions were experimentally verified to follow the Rayleigh distribution. The statistical multipath features of a PLC channel over a bandwidth of 30 MHz have been studied in [4] and the Rayleigh channel gain is justified for the first arrival path. It is also claimed that due to the random signal reflection behaviour through the network, the magnitudes of the other paths also follow a Rayleigh distribution. In [5], a three node PLC network is taken where the nodes are connected to a main NAYY150SE power cable through NAYY50SE house connection cables. The outage probability curve for the source to destination direct transmission at 50 kHz transmission frequency for the PLC channel overlaps with that of the wireless channel assuming Rayleigh fading in [5, Fig. 2]. This also supports that the Rayleigh channel model is applicable to PLC.

Besides the Rayleigh channel model, it has also been proposed that the amplitude gain of the PLC channel follows the Log-Normal distribution [6]–[8]. The measurement campaign conducted in US urban and sub-urban homes over low and medium voltage power lines across frequencies of 1.8 MHz to 30 MHz in [6] suggested that the Log-Normal distribution indeed models PLC channel gain. In [8], a two-conductor transmission line approach for cabling components was used to model the PLC channel transfer function on account of multipath propagation and the path amplitudes were demonstrated to follow the Log-Normal distribution. The additive noise in PLC can be classified into two broad categories: background noise and impulsive noise, which is mostly modeled by the Gaussian-mixture distributions, e.g., Bernoulli-Gaussian or Middleton's class-A distributions [10].

The impulsive noise present in the communication channels can largely degrade the system performance [11]. To improve the performance of communication systems, the hybrid automatic repeat request (HARQ) scheme can be implemented in the link layer [12, pp. 227–236], which combines automatic repeat request with forward error correction. HARQ is usually categorized into chase combining (CC) and incremental redundancy (IR), depending on whether the retransmission is identical to the original transmission or it consists of new redundancy bits from the channel encoder [13].

Many communication services (especially in the industry settings) enabled by the PLC techniques requires robust and reliable transmission [14], [15]. Hence, to cope up with the harsh nature of the PLC channel, it is necessary to utilize data link layer protocols such as HARQ to improve the PLC system performance. For instance, it was shown via simulations in [16] that HARQ with distributed space-time block codes can be used to improve the throughput of PLC systems. Despite the significant importance of HARQ in modern communication systems, almost all of the analysis involving ARQ have been focused on the wireless communication systems [13], [16]–[20] and the performance of HARQ with PLC systems is very limitedly explored in literature to the best of authors' knowledge.

In this paper, we investigate the performance of HARQ with CC for a PLC system in the presence of impulsive noise over Rayleigh and Log-Normal channel gain conditions. Closed-form expressions for the outage probability of the considered PLC system are derived for each of these channel conditions. We also compare the outage performance of the PLC system with and without the use of HARQ with CC. We then proceed to evaluate the average throughput of the PLC system under different channel conditions. We further obtain useful insights into the system performance by varying the parameters of the impulsive noise, transmission rate, and number of HARQ rounds.

II. System and Channel Models

The input/output model of a PLC system with channel gain and additive noise can be expressed as

$$y = h \cdot x + z, \tag{1}$$

where x is the channel input with energy $\mathcal{E}_x$, i.e., $\mathrm{E}[|x|^2] = \mathcal{E}_x$, $\mathrm{E}[\cdot]$ is the expectation operator and the parameter h denotes the Rayleigh or Log-Normally distributed channel gain. The frequency selectivity of a fading channel depends on the coherence bandwidth, which determines the range of frequencies over which a channel can be considered flat. If the maximum bandwidth of the transmitted signal is less than the coherence bandwidth, then all frequency components of the signal experience the same amount of fading and the channel is considered to be flat. The coherence bandwidth for the indoor PLC channel up to 100 MHz and for the cruise ship up to 50 MHz have been measured and tabulated in [3], [21], respectively. It has been concluded that the amount of fading and the coherence bandwidth of the PLC channel depend upon the topology of the power network. If the branching and coupling of the network are well managed, then the coherence bandwidth is large. For the PLC system under consideration, we assume that the bandwidth of the transmitted signal is less than the coherence bandwidth and hence, the PLC channel is assumed to be flat [3], [21]–[24].

The random variable (RV) z represents the background and potentially impulsive noise following the Bernoulli-Gaussian model. The noise sample z in PLC is represented by

$$z = n_1 + \zeta n_2, \tag{2}$$

where n_1 and n_2 denote the background and impulsive noise components of the Bernoulli Gaussian PLC noise, respectively. In (2), ζ is a Bernoulli RV with parameter p which signifies the probability of occurrence of the impulsive component of the noise. It is known that n_1, n_2, and ζ are independent of each other. Consequently, the probability density function (PDF) $f_z(z)$ of the Bernoulli-Gaussian RV z is a Gaussian-mixture distribution and can be expressed as [2]

$$f_z(z) = (1-p)\cdot\mathcal{N}(0,\sigma_g^2) + p\cdot\mathcal{N}(0,(K+1)\sigma_g^2), \tag{3}$$

where $\mathcal{N}(0,\sigma^2)$ denotes the Gaussian PDF with zero mean and σ^2 variance, σ_g^2 and $\sigma_b^2 = (1+K)\sigma_g^2$ are the variance of the background noise and impulsive components of the Bernoulli-Gaussian noise, respectively; and K is the ratio of the impulsive noise power σ_b^2 to the background noise power σ_g^2.

Given the channel gain statistics, h, the instantaneous signal-to-noise ratio (SNR), γ, of the PLC link is given as

$$\gamma = \begin{cases} \frac{|h|^2\cdot\mathcal{E}_x}{\sigma_g^2} = |h|^2\cdot\overline{\gamma}_g = \gamma_g, & \text{only background noise} \\ \frac{|h|^2\cdot\mathcal{E}_x}{(1+K)\cdot\sigma_g^2} = |h|^2\cdot\overline{\gamma}_b = \gamma_b, & \text{both background and} \\ & \text{impulsive noises} \end{cases} \tag{4}$$

where the subscripts g and b in the SNRs represent good channel condition with only background noise and bad channel condition with both background and impulsive noise, respectively.

For HARQ with CC scheme, the mutual information is obtained by combining received SNR over the k rounds of retransmission. An outage after k HARQ rounds means that the accumulated total mutual information I_k at the destination node is still less than the transmission rate R. The outage probability $P_{out}(k)$ after k HARQ transmission rounds can be expressed as [17], [18]

$$\begin{aligned} P_{out}(k) &= \Pr(I_k < R) \\ &= \Pr\Big(\log_2\Big(1+\sum_{i=1}^{k}\gamma_i\Big) < R\Big) \\ &= \Pr\Big(\gamma_{(k)} < 2^R - 1\Big), \end{aligned} \tag{5}$$

where γ_i is the SNR at the i-th round, where the channel gain varies independently from one HARQ round to another; $\gamma_{(k)} = \sum_{i=1}^{k}\gamma_i$ is the accumulated SNR after k HARQ rounds.

Clearly, to obtain the outage performance of PLC with HARQ, we need to derive the statistics of the RV $\gamma_{(k)}$.

III. Outage Performance of HARQ over PLC

In this section, we derive the outage performance statistics of HARQ for PLC systems under Rayleigh and Log-Normal channel gain conditions.

A. Rayleigh Channel Gain

With Rayleigh distributed channel gain, the PDF of the channel gain on the i-th HARQ round is

$$f_{h_i}(h) = \frac{h}{\sigma_i^2}\exp\Big(\frac{-h^2}{2\sigma_i^2}\Big), \tag{6}$$

where σ_i is the scale parameter of the distribution.

Hence, the distribution $f_{\gamma_i}(\gamma)$ of the instantaneous SNR in the i-th HARQ round can be given by using (4) as

$$f_{\gamma_i}(\gamma) = \frac{(1-p)}{2\sigma_i^2 \cdot \overline{\gamma}_{g,i}} \cdot e^{\left(-\frac{\gamma}{2\sigma_i^2\overline{\gamma}_{g,i}}\right)} + \frac{p}{2\sigma_i^2 \cdot \overline{\gamma}_{b,i}} \cdot e^{\left(-\frac{\gamma}{2\sigma_i^2\overline{\gamma}_{b,i}}\right)}. \tag{7}$$

Since $\gamma_{(k)}$ is the sum of k independent but not necessarily identical (i.n.i.d.) distributed exponential RVs, the PDF $f_{\gamma_{(k)}}(\gamma)$ of the RV $\gamma_{(k)}$ is expressed as follows [25]:

$$\begin{aligned} f_{\gamma_{(k)}}(\gamma) =&(1-p) \cdot \sum_{i=1}^{k} \prod_{\substack{j=1 \\ j\neq i}}^{k} \frac{\left(\frac{1}{2\sigma_i^2\overline{\gamma}_{g,i}}\right)\exp\left(-\frac{\gamma}{2\sigma_i^2\overline{\gamma}_{g,i}}\right)}{2\overline{\gamma}_{g,j}\cdot\sigma_j^2\cdot\left(\frac{1}{2\overline{\gamma}_{g,j}\cdot\sigma_j^2}-\frac{1}{2\overline{\gamma}_{g,i}\cdot\sigma_i^2}\right)} \\ &+ p \cdot \sum_{i=1}^{k} \prod_{\substack{j=1 \\ j\neq i}}^{k} \frac{\left(\frac{1}{2\sigma_i^2\overline{\gamma}_{b,i}}\right)\exp\left(-\frac{\gamma}{2\sigma_i^2\overline{\gamma}_{b,i}}\right)}{2\overline{\gamma}_{b,j}\cdot\sigma_j^2\cdot\left(\frac{1}{2\overline{\gamma}_{b,j}\cdot\sigma_j^2}-\frac{1}{2\overline{\gamma}_{b,i}\cdot\sigma_i^2}\right)}. \end{aligned} \tag{8}$$

Remark 1: For the special case when the k RVs, γ_i, are independent and identically distributed (i.i.d.) exponential RVs, it is well known that $\gamma_{(k)}$ follows the Erlang distribution.

From the PDF of $\gamma_{(k)}$, the cumulative distribution function (CDF) is obtained by integrating (8) with respect to (w.r.t.) γ as

$$\begin{aligned} F_{\gamma_{(k)}}(\gamma) =&(1-p) \cdot \sum_{i=1}^{k} \prod_{\substack{j=1 \\ j\neq i}}^{k} \frac{1-\exp\left(-\frac{\gamma}{2\sigma_i^2\overline{\gamma}_{g,i}}\right)}{2\overline{\gamma}_{g,j}\cdot\sigma_j^2\cdot\left(\frac{1}{2\overline{\gamma}_{g,j}\cdot\sigma_j^2}-\frac{1}{2\overline{\gamma}_{g,i}\cdot\sigma_i^2}\right)} \\ &+ p \cdot \sum_{i=1}^{k} \prod_{\substack{j=1 \\ j\neq i}}^{k} \frac{1-\exp\left(-\frac{\gamma}{2\sigma_i^2\overline{\gamma}_{b,i}}\right)}{2\overline{\gamma}_{b,j}\cdot\sigma_j^2\cdot\left(\frac{1}{2\overline{\gamma}_{b,j}\cdot\sigma_j^2}-\frac{1}{2\overline{\gamma}_{b,i}\cdot\sigma_i^2}\right)}. \end{aligned} \tag{9}$$

Remark 2:For the special case when γ_is are i.i.d. exponential RVs, the CDF of $\gamma_{(k)}$ is given by

$$F_{\gamma_{(k)}}(\gamma) = \frac{(1-p)}{\Gamma(k)}\gamma\Big(k, \frac{\gamma}{\sigma^2\overline{\gamma}_g}\Big) + \frac{p}{\Gamma(k)}\gamma\Big(k, \frac{\gamma}{\sigma^2\overline{\gamma}_b}\Big), \tag{10}$$

where $\gamma(m,x) \triangleq \int_0^x t^{m-1}\exp(-t)dt$ is the lower incomplete gamma function.

The outage probability $P_{out}(k)$ of the PLC system after k HARQ rounds is simply related to the CDF as

$$P_{out}(k) = F_{\gamma_{(k)}}(2^R - 1). \tag{11}$$

On substituting (10) in (11), the outage probability of the PLC system under Rayleigh channel gain is given by (12) on the top of the next page.

B. Log-Normal Channel Gain

When the PLC channel varies according to the Log-Normal distribution, the PDF of the channel gain on the i-th HARQ round is given as follows:

$$f_{h_i}(h) = \frac{1}{\sqrt{2\pi\sigma_i^2}\cdot h} \cdot \exp\Big(-\frac{(\ln(h)-\mu_i)^2}{2\sigma_i^2}\Big), h \geq 0, \tag{13}$$

where μ_i and σ_i are the mean and standard deviation of a new RV defined as the natural logarithm of the RV h_i. Following (4), the PDF $f_{\gamma_i}(\gamma)$ of the instantaneous SNR on the i-th HARQ round under Log-Normal channel gain is expressed as

$$\begin{aligned} f_{\gamma_i}(\gamma) = &\quad \frac{(1-p)}{\sqrt{2\pi\sigma_i'^2}\cdot\gamma} \cdot \exp\Big(-\frac{(\ln(\gamma)-\mu_i')^2}{2\sigma_i'^2}\Big) \\ &+ \frac{p}{\sqrt{2\pi\sigma_i'^2}\cdot\gamma} \cdot \exp\Big(-\frac{(\ln(\gamma)-\mu_i'')^2}{2\sigma_i'^2}\Big), \end{aligned} \tag{14}$$

where $\mu_i' = \ln(\overline{\gamma}_{g,i}) + 2\mu_i$, $\mu_i'' = \ln(\overline{\gamma}_{b,i}) + 2\mu_i$, and $\sigma_i'^2 = 4\sigma_i^2$.

Now, $\gamma_{(k)}$ is the sum of k i.n.i.d. Log-Normally distributed RVs. It is well known in literature [26]–[28] that the PDF of such Log-Normal RVs can be approximated by another Log-Normal RV. We utilize a simple yet accurate approximation mentioned in [28] for the Log-Normal sum with parameters $\mu_{\gamma_{(k)}}$ and $\sigma^2_{\gamma_{(k)}}$ given by

$$\mu_{\gamma_{(k)},g} = 0.5\ln\mathrm{E}[\gamma_{(k),g}^{-2}] - 2\ln\mathrm{E}[\gamma_{(k),g}^{-1}], \tag{15}$$

$$\sigma^2_{\gamma_{(k)},g} = \ln\mathrm{E}[\gamma_{(k),g}^{-2}] - 2\ln\mathrm{E}[\gamma_{(k),g}^{-1}], \tag{16}$$

$$\mu_{\gamma_{(k)},b} = 0.5\ln\mathrm{E}[\gamma_{(k),b}^{-2}] - 2\ln\mathrm{E}[\gamma_{(k),b}^{-1}], \tag{17}$$

$$\sigma^2_{\gamma_{(k)},b} = \ln\mathrm{E}[\gamma_{(k),b}^{-2}] - 2\ln\mathrm{E}[\gamma_{(k),b}^{-1}], \tag{18}$$

where $\mathrm{E}[\gamma_{(k),b}^{-n}]$ and $\mathrm{E}[\gamma_{(k),g}^{-n}]$, $n = 1, 2$, can be evaluated numerically or through simulation using [28, Eq. (9)].

Hence, the PDF $f_{\gamma_{(k)}}(\gamma)$ of the RV $\gamma_{(k)}$ is expressed as follows:

$$\begin{aligned} f_{\gamma_{(k)}}(\gamma) =&\frac{(1-p)}{\sqrt{2\pi\sigma^2_{\gamma_{(k)},g}}\cdot\gamma} \cdot \exp\Big(-\frac{(\ln(\gamma)-\mu_{\gamma_{(k)},g})^2}{2\sigma^2_{\gamma_{(k)},g}}\Big) \\ &+ \frac{p}{\sqrt{2\pi\sigma^2_{\gamma_{(k)},b}}\cdot\gamma} \cdot \exp\Big(-\frac{(\ln(\gamma)-\mu_{\gamma_{(k)},b})^2}{2\sigma^2_{\gamma_{(k)},b}}\Big). \end{aligned} \tag{19}$$

The CDF $F_{\gamma_{(k)}}(\gamma)$ of the RV $\gamma_{(k)}$ follows immediately by integrating the PDF in (19) w.r.t. γ, which leads to the following expression:

$$F_{\gamma_{(k)}}(\gamma) =(1-p) \cdot Q\Big(\frac{\mu_{\gamma_{(k)},g} - \ln\gamma}{\sigma_{\gamma_{(k)},g}}\Big) \tag{20}$$

$$+ p \cdot Q\Big(\frac{\mu_{\gamma_{(k)},b} - \ln\gamma}{\sigma_{\gamma_{(k)},b}}\Big), \tag{21}$$

where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(\frac{-u^2}{2}\right)du$ is the Gaussian q-function.

The outage probability $P_{out}(k)$ of the PLC system under Log-normal channel gain after k HARQ rounds can, thus, be obtained as

$$\begin{aligned} P_{out}(k) =&F_{\gamma_{(k)}}(2^R - 1) \\ =&(1-p) \cdot Q\Big(\frac{\mu_{\gamma_{(k)},g} - \ln(2^R-1)}{\sigma_{\gamma_{(k)},g}}\Big) \end{aligned} \tag{22}$$

$$+ p \cdot Q\Big(\frac{\mu_{\gamma_{(k)},b} - \ln(2^R-1)}{\sigma_{\gamma_{(k)},b}}\Big). \tag{23}$$

$$P_{out}(k) = (1-p)\cdot\sum_{i=1}^{k}\prod_{\substack{j=1\\ j\neq i}}^{k}\frac{1-\exp\left(-\frac{2^R-1}{2\sigma_i^2\overline{\gamma}_{g,i}}\right)}{2\overline{\gamma}_{g,j}\cdot\sigma_j^2\cdot\left(\frac{1}{2\overline{\gamma}_{g,j}\cdot\sigma_j^2}-\frac{1}{2\overline{\gamma}_{g,i}\cdot\sigma_i^2}\right)} + p\cdot\sum_{i=1}^{k}\prod_{\substack{j=1\\ j\neq i}}^{k}\frac{1-\exp\left(-\frac{2^R-1}{2\sigma_i^2\overline{\gamma}_{b,i}}\right)}{2\overline{\gamma}_{b,j}\cdot\sigma_j^2\cdot\left(\frac{1}{2\overline{\gamma}_{b,j}\cdot\sigma_j^2}-\frac{1}{2\overline{\gamma}_{b,i}\cdot\sigma_i^2}\right)} \quad (12)$$

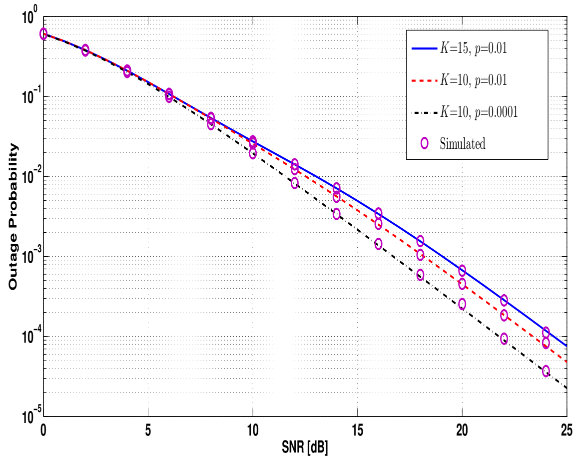

Fig. 1: Comparison of outage probability under Rayleigh channel gain for varying values of K and p, $\sigma_1^2 = 0.5$, $\sigma_2^2 = 1$, and $R = 2$ bps/Hz.

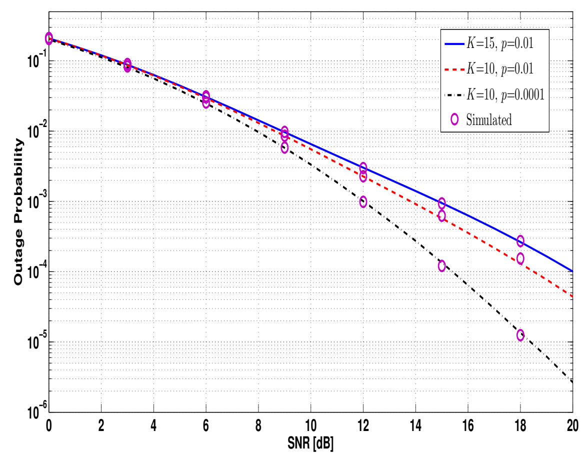

Fig. 2: Comparison of outage probability under Log-Normal channel gain for different values of K and p, $\sigma_1^2 = 0.5$, $\sigma_2^2 = 1$, $\mu_1 = \mu_2 = 0.5$, and $R = 2$ bps/Hz.

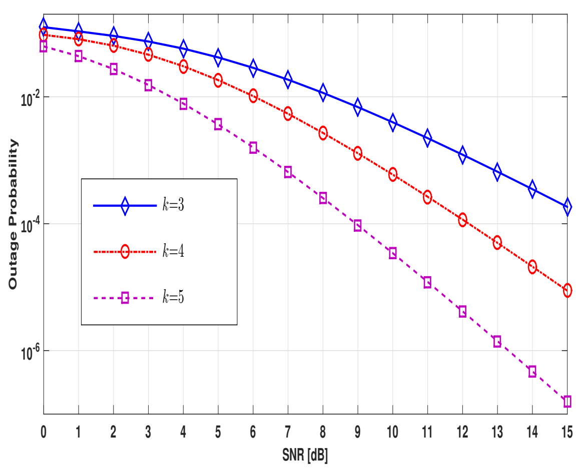

Fig. 3: Comparison of outage probability under Rayleigh channel gain for different number of HARQ rounds, $p = 0.1$ and $K = 10$.

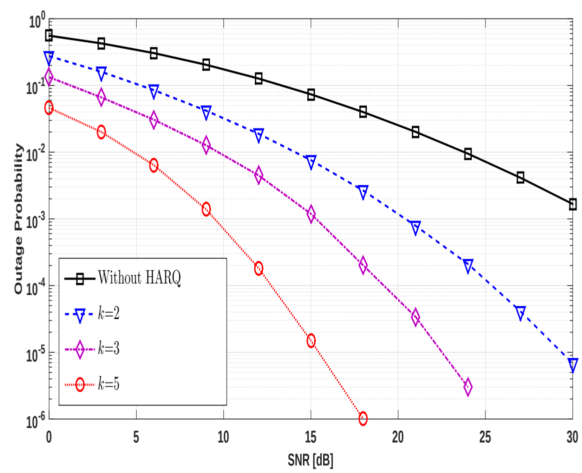

Fig. 4: Comparison of outage probability under Log-Normal channel gain for varying HARQ rounds, $p = 0.1$, $K = 10$, $R = 2$ bps/Hz.

IV. Average Throughput Analysis

The average throughput is an important performance metric for ARQ-based systems. Two definitions of throughput are usually considered in literature. The first measure of throughput is the long term (LT) average throughput which takes into account the steady state behaviour of several message transmissions. This LT throughput measure is given by [19, Eq. (4)]

$$\mathcal{T}_{LT} = \frac{R}{\sum_{n=0}^{k-1} P_{out}(n)}, \quad (24)$$

where $P_{out}(n)$ denotes the outage probability after n HARQ transmission rounds and k is the maximum number of the HARQ rounds. Contrary to (24), in order to track the short term variations in the channel, another throughput measure known as delay limited (DL) throughput is defined as follows [10, Eq. (5)]:

$$\mathcal{T}_{DL} = \sum_{n=1}^{k}\frac{R}{n}[P_{out}(n-1) - P_{out}(n)], \quad (25)$$

with $P_{out}(0) = 1$ for $n = 1$.

In this paper, we evaluate the DL throughput given by (25) for the considered PLC system subjected to Rayleigh and Log-Normal channel gains under the impact of impulsive noise. The average throughput $\mathcal{T}_{DL}$ for Rayleigh distributed channel gain is obtained by substituting (12) in (25). On the other hand, when the PLC channel is Log-Normal, $\mathcal{T}_{DL}$ is computed by substituting (23) into (25).

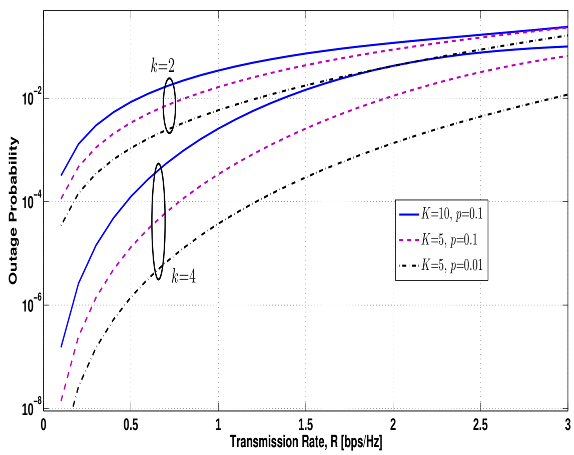

Fig. 5: Comparison of outage probability under Rayleigh channel gain for different values of R for SNR=10dB.

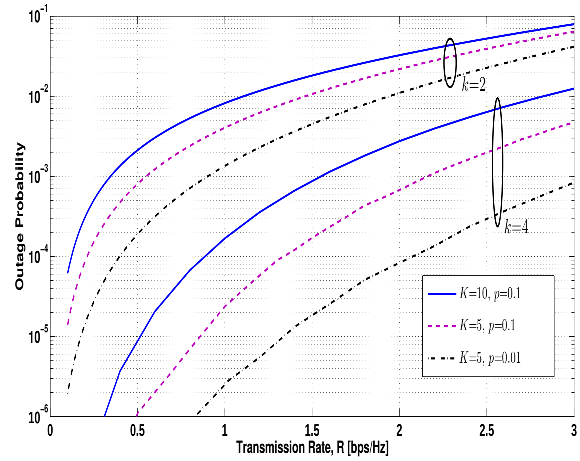

Fig. 6: Comparison of outage probability under Log-Normal channel gain for different values of R for SNR=10dB.

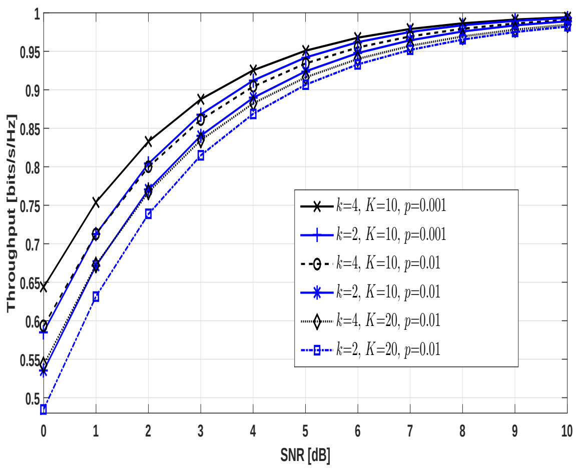

Fig. 7: Comparison of throughput under Rayleigh channel gain for different values of p and K for various HARQ rounds.

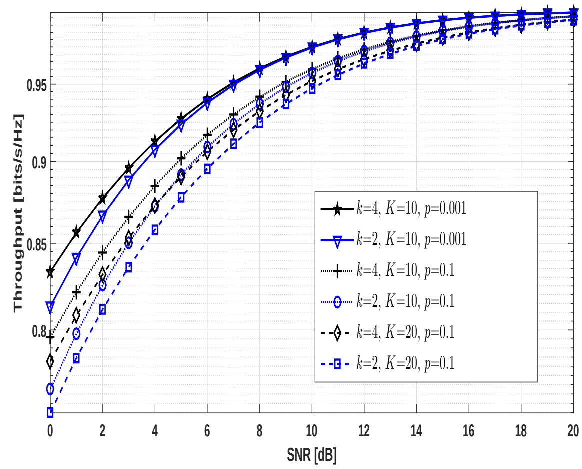

Fig. 8: Comparison of throughput under Log-normal channel gain for different values of p and K for various HARQ rounds.

V. Simulation Results And Discussions

In this section, we will discuss the analytical results presented in previous sections to gain a better understanding on the impact of PLC channel and ARQ scheme on the system performance. The SNR (dB) along the x-axis in Figs. 1-4 is $\mathcal{E}_x/\sigma_g^2$. The rate R is set to 1 bps/Hz unless stated otherwise.

A comparison of the analytical outage probability of the PLC system under the combined influence of background noise and impulsive noise in the presence of Rayleigh channel gain is shown in Fig. 1 for two HARQ rounds for different values of K and p. The computational curves are obtained using (12). It is seen from the figure that as the value of the impulsive noise index increases, the outage probability becomes worse. Further, the outage performance deteriorates as the parameter p is increased from 0.0001 to 0.01 due to the detrimental nature of the impulsive noise. We also plot the outage probability for a PLC system subjected to Log-Normal channel gain for different values of K and p in Fig. 2 using (23). It is again observed that the outage performance becomes poor as the impulsive noise parameters K and p are increased. Moreover, we observe from Figs. 1 and 2 that the outage probability is 10^{-4} at 22 dB SNR for $K = 10$ and $p = 0.0001$ under Rayleigh distributed channel gain while this outage probability is achieved at around 15 dB SNR only under Log-Normal channel gain. This indicates that Rayleigh channel deteriorates PLC performance more severely compared to Log-Normal channel. The validity of our analysis is established by close matching with the simulation results. From Figs. 1 and 2, it is also observed that the while the SNR is small, the impulsiveness of the noise does not have a strong impact on the outage probability as in the high SNR region. This trend is in accordance with the observation in terms of BER in [29] where in the low SNR region the performance is mainly impacted by the background Gaussian component of the Bernoulli Gaussian noise.

The analytical outage probability of the considered PLC system for different number of HARQ rounds (k) is plotted for Rayleigh and Log-Normal channel gains in Fig. 3 and Fig. 4, respectively. We observe that for both cases, the outage performance improves as the number of HARQ rounds increases. In Fig. 4, we also compare the outage probability of the PLC system under Log-Normal channel condition with and without the use of HARQ. It is evident from Fig. 4 that an outage probability of 10^{-2} is achieved at 24 dB SNR when we do not use HARQ, while the same outage probability is achieved at around 14 dB SNR even for two HARQ rounds. Thus, significant SNR gain is achieved using HARQ.

In Figs. 5 and 6, we investigate the behaviour of the analytical outage probability as a function of the transmission rate, R, for different values of impulsive noise parameters, K and p, under Rayleigh and Log-Normal channel gains, respectively. The computational curves in Fig. 5 are obtained using (12) with $\sigma^2 = 1$, while those in Fig. 6 are obtained using (23) with $\mu_i = 0.5$ and $\sigma_i^2 = 1$. It is seen that the outage performance deteriorates as the value of R increases. Further, we note that outage probability is less for more number of HARQ retransmission rounds. Thus, HARQ improves the PLC performance even in the presence of impulsive noise which is evident from these results.

Figure 7 shows the DL throughput of the PLC system subjected to Rayleigh channel gain for different number of HARQ rounds and R=1 bps/Hz. the computational curves are obtained using (25). It is seen that the value of throughput decreases as the impulsive noise index K and the probability of arrival of impulsive noise p increase. We also observe that for a given value of K and p, as the the number of HARQ rounds increases from 2 to 4, the DL throughput also increases. Similar trends are observed in Fig. 8 where we plot the DL throughput of the PLC system under Log-Normal channel gain for various number of HARQ rounds and R=1 bps/Hz.

VI. Conclusions

In this paper, we analysed the outage performance of PLC system using HARQ with CC under the combined influence of impulsive noise and Rayleigh and Log-Normally distributed channel gains. Using the closed-form expression for the outage probability, we also evaluate the DL throughput of the considered PLC system. It is inferred from the numerical plots that the performance of a PLC system can be significantly improved using HARQ with CC. Further performance improvement can be achieved by increasing the number of HARQ retransmission rounds. The detrimental effect of the impulsive noise is also observed through the numerical results.

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