Performance of Multiuser Diversity Reception in Rayleigh Fading CDMA Channels

Milica Stojanovic and Zoran Zvonar

Abstract—**Error probability of an adaptive multiuser diversity receiver is evaluated in terms of channel fading rate and the number of code-division multiple access users. Fading-induced performance loss, which leads to error probability floor, is established for the proposed coherent combining scheme and compared to that of differentially coherent receiver with equal-gain combining.**

I. INTRODUCTION

MULTIUSER receiver design for fading multiple-access channels, including detection structures and estimation algorithms, and the impact of imperfect channel estimation on the receiver performance are of growing importance in the context of the new generation of wireless systems. Methods for the receiver design based on statistical modeling of a fading process with memory are presented in [1]. Related work on multiuser receiver design has focused on performance comparison between differentially coherent multiuser receiver and adaptive coherent decorrelating receiver which utilizes estimation of fading process based on Kalman filtering [2], [3]. Different approach has been pursued in [4] by developing optimal receiver and consequently addressing suboptimal decision-directed realizations. Estimation part of the optimal receiver is based on Kalman filtering, and by replacing detection part by linear decorrelating receiver in [5] authors have arrived to the same receiver structure presented in [2].

In this paper we extend the work on multiuser receivers by analyzing an adaptive decision-directed coherent receiver with diversity reception and impact of imperfect channel estimates on the diversity improvement in rapidly fading channels. Multiuser diversity receiver employs a low-complexity near–far resistant decorrelating detector [6] in each diversity branch and utilizes the joint MMSE estimates of the fading processes as channel references for carrier recovery and diversity combining. The average error probability of such an adaptive receiver is a function of the channel estimation error covariance matrix. We compare the performance of adaptive coherent receiver to that of a differentially coherent receiver with equal-gain combining, showing regions where each may be preferable depending on the fading dynamics, MAI and available signal-to-noise ratio (SNR).

II. SYSTEM MODEL

A synchronous code-division multiple-access (CDMA) channel is considered, where K active users are subject to independent, frequency-nonselective Rayleigh fading. The equivalent complex baseband received signal for the lth diversity channel of a centralized receiver is given by

$$r_l(t) = \sum_{k=1}^{K} \sum_i b_k(i) c_{k,l}(i) s_k(t - iT) + n_l(t), \qquad l = 1, \dots L \tag{1}$$

where $\{b_k(i)\}$ are independent identically distributed (i.i.d.) binary phase-shift keyed (BPSK) data symbols, and $\{s_k(t)\}$ are unit-energy signature waveforms with support $[0, T]$ equal to one symbol interval. The additive white Gaussian noise (AWGN) processes $\{n_l(t)\}$ are assumed to be independent, of equal power spectral density N_0. Frequency-nonselective fading for each user/receiving element pair is modeled as a complex gain $c_{k,l}(i)$, assumed to be constant within one symbol interval. A nonselective fading model applies when the signal bandwidth is much smaller than the coherence bandwidth of the channel. For Rayleigh fading, the distortions $c_{k,l}(i)$ are modeled as stationary sequences of Gaussian random variables. The fading processes are taken to be independent among the users and diversity branches, with identical normalized correlation functions $E\{c_{k,l}(i+j) c_{k,l}^*(i)\} = E_k R_c(j)$, where E_k is the kth user's received energy.

The received signal at the lth antenna element can be expressed in vector notation as[1]

$$r_l(t) = \sum_i \mathbf{s}^T(t - iT) \mathbf{B}(i) \mathbf{c}_l(i) + n_l(t) \tag{2}$$

where $\mathbf{s}(t) = [s_1(t) \cdots s_K(t)]^T, \mathbf{B}(i) = \mathrm{diag}[b_1(i) \cdots b_K(i)]$, and $\mathbf{c}_l(i) = [c_{1,l}(i) \cdots c_{K,l}(i)]^T$. Assuming the knowledge of signature waveforms and the existence of perfect bit-timing of all the users, the optimal receiver incorporates a bank of matched filters, whose outputs, sampled at the symbol rate, are given by

$$\mathbf{y}_l(i) = \int_{iT}^{(i+1)T} r_l(t) \mathbf{s}^*(t - iT)\, dt = \mathbf{R}\mathbf{B}(i)\mathbf{c}_l(i) + \boldsymbol{\eta}_l(i) \tag{3}$$

Paper approved by B. D. Woerner, the Editor for Wireless Spread Spectrum of the IEEE Communications Society. Manuscript received July 3, 1996; revised March 11, 1998. This paper was presented in part at the Communication Theory Mini-Conference, San Francisco, CA December 1994.

M. Stojanovic is with the Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115 USA (e-mail: milica@cdsp.neu.edu).

Z. Zvonar is with Analog Devices, Communications Division, Wilmington, MA 01887 USA (e-mail: Zoran.Zvonar@analog.com).

Publisher Item Identifier S 0090-6778(99)02184-4.

[1] Symbols T, $*$, and H denote transpose, conjugate, and conjugate transpose, respectively.

where $\mathbf{R} = \int_0^T \mathbf{s}^*(t)\mathbf{s}^T(t)dt$ is the cross-correlation matrix of signature waveforms, and the output AWGN is zero-mean with covariance $\mathrm{Cov}[\boldsymbol{\eta}_l] = N_0\mathbf{R}$.

III. ADAPTIVE COHERENT DIVERSITY COMBINING

Performance of a linear-complexity decorrelating receiver may greatly be improved by the use of diversity. An adaptive receiver with diversity combining is depicted in Fig. 1. The receiver performs joint MMSE estimation of all the users' fading coefficients, and uses these estimates for carrier recovery and coherent combining. The output of the decorrelating filter in the lth diversity branch is given by

$$\mathbf{z}_l(i) = \mathbf{R}^{-1}\mathbf{y}_l(i) = \mathbf{B}(i)\mathbf{c}_l(i) + \boldsymbol{\xi}_l(i) \tag{4}$$

where the output AWGN vector is characterized by $\mathrm{Cov}[\boldsymbol{\xi}_l] = N_0\mathbf{R}^{-1}$. The decorrelator outputs may be combined in different ways. We propose a suboptimal combining strategy in which the decorrelator outputs corresponding to a particular user are combined disregarding those of other users. The combiner for each user, however, is of maximal-ratio type, yielding a decision variable of the form

$$b_k(i) = \mathrm{Re}\left\{\sum_{l=1}^{L} \hat{c}_{k,l}^*(i) z_{k,l}(i)\right\} \tag{5}$$

where $\hat{c}_{k,l}(i)$ denotes the estimate of the fading distortion. Estimation of complex channel coefficient is equivalent to carrier recovery in the system. For independent diversity, it suffices to assign an independent estimator to each of diversity branches, as shown in Fig. 1.

We now turn our attention to performance analysis of diversity receiver. Gaussian statistics of $c_{k,l}(i)$ imply Gaussian statistics of a linear, unbiased estimate $\hat{c}_{k,l}(i)$. Assuming the same statistics of fading processes in all diversity branches, the estimation error covariance, observed in steady state for each diversity, is given by $\mathbf{G} = \mathrm{Cov}[\mathbf{c}_l(i) - \hat{\mathbf{c}}_l(i)]$. For the decision variable given by (5), and neglecting the effect of error propagation through the channel estimator, the bit error probability for the kth user is given by [7]

$$P_k^{(L)} = \left(\frac{1-\mu_k}{2}\right)^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \left(\frac{1+\mu_k}{2}\right)^l = p_L(\mu_k). \tag{6}$$

The parameter μ_k is obtained as

$$\mu_k = \frac{E\{\hat{c}_{k,l}^*(i) z_{k,l}(i)\}}{\sqrt{E\{|\hat{c}_{k,l}(i)|^2\} E\{|z_{k,l}(i)|^2\}}} = \sqrt{\frac{E_k R_c(0) - G_{kk}}{E_k R_c(0) + N_0 D_{kk}}} \tag{7}$$

where $G_{kk} = [\mathbf{G}]_{kk}, D_{kk} = [\mathbf{R}^{-1}]_{kk}$, and the last line follows from the fact that the MMSE channel estimate is orthogonal to the estimation error.

Defining the the kth user's average signal to noise ratio as $\gamma_k = R_c(0)E_k/N_0$, and the corresponding normalized estimation error variance as $\Gamma_k = G_{kk}/E_k R_c(0)$, the parameter μ_k, can be expressed as

$$\mu_k = \sqrt{(1-\Gamma_k)\frac{\gamma_k}{\gamma_k + D_{kk}}}. \tag{8}$$

Hence, besides the SNR, the error probability of the coherent decorrelating detector depends on the normalized estimation error variance Γ_k, and, through the factor D_{kk}, on the cross correlations of normalized signature waveforms. In contrast to the case of perfect estimation, an error probability floor is observed, defined as $P_{\mathrm{BPSK}\infty}^{(L)} = \lim_{\gamma_k \to \infty} P_k^{(L)} = p_L(\sqrt{1-\Gamma})$. In this case, as will become apparent from the error covariance analysis, a given user does not suffer from the estimation errors of other users, i.e. the error probability floor in CDMA scenario is equal to the error probability floor observed for single-user transmission over the same channel. Performance degradation results from channel tracking inaccuracies, and is ultimately determined by the fading rate of the channel.

To quantify the effects of channel estimation errors, we analyze a Rayleigh fading process $\mathbf{c}_l(i)$ represented by a Gauss–Markov model, whose parameters can be obtained with sufficient accuracy through extensive measurements [1]. When the model parameters are known, the MMSE estimator can be realized by the Kalman filter and its error covariance matrix is obtained from the corresponding discrete-time Ricatti equation. For the case of joint estimation of all the coefficients of the channel vector $\mathbf{c}_l(i)$, the fact that the measurement equation (4) is data dependent may be overcome by solving a quasistationary Ricatti equation using the procedure described in [8].

If explicit channel estimation is too complex, or if the received signal phase is varying too rapidly to allow planar tracking, one may resort to differentially coherent detection. In this case, the decision variable is given by (5) with $\hat{c}_{k,l}(i) = z_{k,l}(i-1)$. The bit error probability remains in the form (6) with the parameter μ_k now given by

$$\mu_k = \varrho \frac{\gamma_k}{\gamma_k + D_{kk}}$$

where $\varrho = |R_c(1)/R_c(0)|$ describes the fading dynamics. Similarly as in the BPSK case, an error probability floor is observed, given by $P_{\mathrm{DPSK}\infty}^{(L)} = p_L(\varrho)$. Again, this value is independent of the presence of interfering users and determined solely by the fading dynamics.

IV. DISCUSSION OF RESULTS

Several numerical examples are presented to illustrate the performance of the proposed multiuser receiver. In the first set of examples a first-order Gaus-Markov model is used for each of the independent fading process:

$$c_{k,l}(i+1) = ac_{k,l}(i) + \nu_{k,l}(i), \quad k = 1 \cdots K, \quad l = 1 \cdots L \tag{9}$$

The model parameter a is related to the Doppler spread $B_d = \omega_d/\pi$ of the channel through $a = \exp(-\omega_d T)$. Fig. 2 shows the error probability of coherent decorrelating detector for BPSK signaling as a function of the average SNR per bit, chosen equal for all the users, $\mathrm{SNR} = L\gamma_k =$

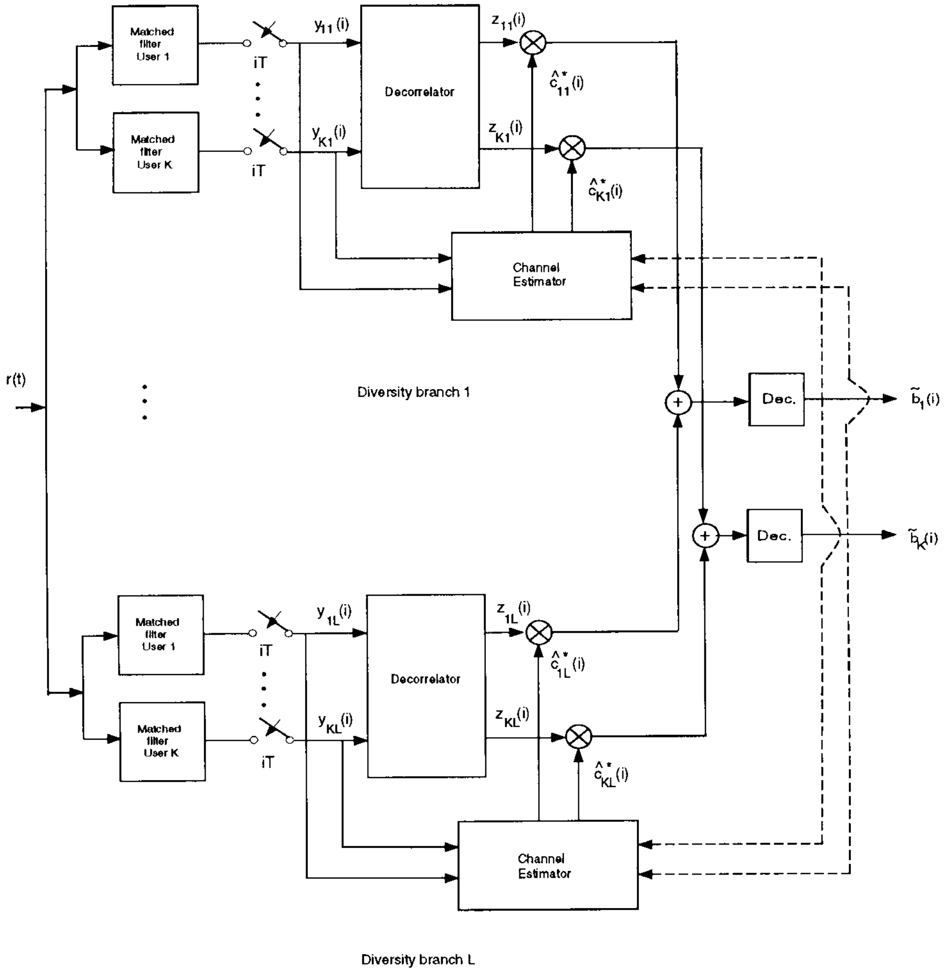

Fig. 1. Adaptive multiuser receiver with diversity combining.

$\sum_{k=1}^{K} E\{|c_{k,l}(i)|^2\}/N_0$. The performance of single and dual diversity receivers is compared for the given normalized fading rate $\omega_d T = 0.001$, in cases of single-user and $K = 20$ users. Flip-coin signature sequences of length 127 are used to determine the values of cross-correlations in $\mathbf{R}$. Reference curves present the single-user bound with perfect knowledge of the fading distortion. Total performance degradation due to the presence of other active users is less than 1 dB at the given value of fading rate. For the case of a simple first-order model it is interesting to add that the probability of error saturates at the same level for coherent and differentially coherent reception ($\Gamma = 1 - |a|^2$). The error floor is induced by fading dynamics, and is not affected by MAI. However, the degradation of differentially coherent reception becomes significant for more realistic fading models, as will be discussed in the second example.

The effects of estimation errors on coherent diversity detection at varying fading rates is analyzed in Fig. 3. Shown in this figure is the penalty ratio, $P^{(L)}(B_d T, K)/P^{(L)}(0, K)$, of the error probabilities with and without estimation errors, for coherent decorrelator with $K = 20$ users. The estimation error penalty is seen to increase with the order of diversity while largely depending on the fading dynamics. Despite the fact that its effectiveness diminishes with an increase in fading rate, diversity remains an effective way of improving performance in the presence of fading and MAI, for all practical values of fading rate.

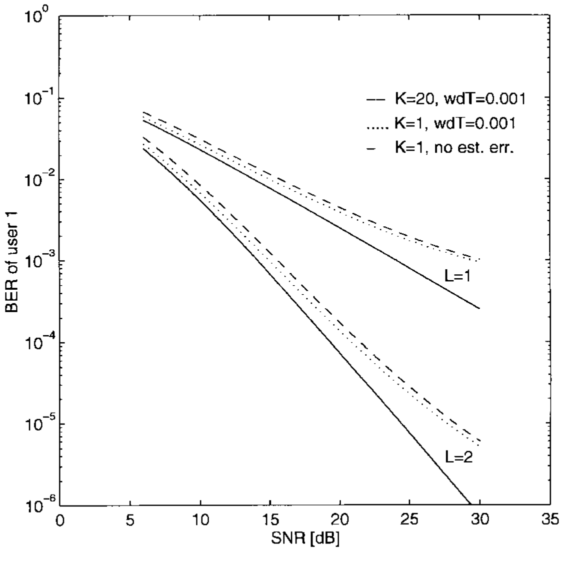

Fig. 2. Performance of single and dual diversity coherent detection in a nonselective Rayleigh fading CDMA channel (first-order model).

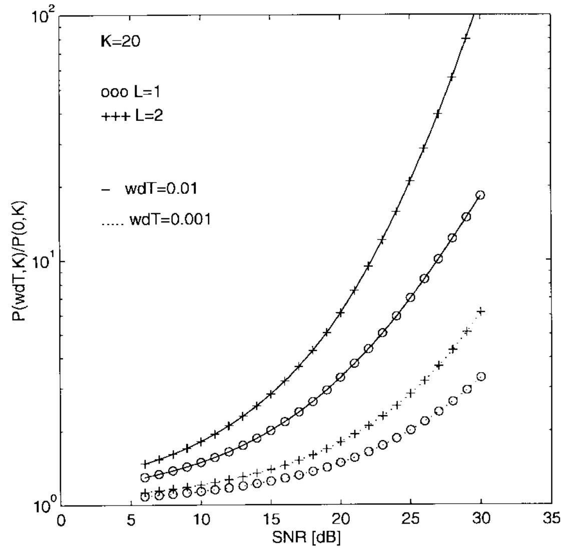

Fig. 3. Estimation error penalty of adaptive coherent decorrelating receivers in a nonselective Rayleigh fading CDMA channel (first-order model).

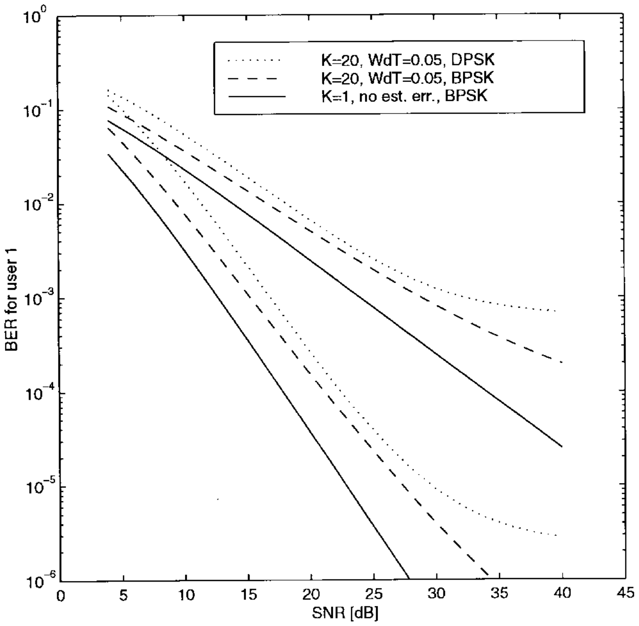

Fig. 4. Performance comparison of single and dual diversity, coherent and differentially coherent detection (second-order fading model).

In Fig. 4 comparison is made between coherent and differentially coherent receivers for a second-order critically damped fading model [1], whose Doppler power spectrum most closely describes that of a realistic mobile radio channel. The normalized 3-dB Doppler spread is set to $\omega_d T = 0.05$. Fading dynamics is again shown to be the major factor limiting the performance of both coherent and differentially coherent receiver. Although both suffer from imperfect channel estimation, the performance of coherent decorrelating receiver remains superior to that of a differentially coherent one, with the exact gain depending on the available SNR. While for low to medium SNR the difference in performance stays within a few dB, for higher SNR coherent receiver significantly outperforms the differentially coherent one as it tends to a lower error floor.

It should be pointed out that diversity combiners analyzed in here are based on heuristic modifications of their optimal counterparts, obtained by substituting the true channel responses by their estimates. If, in addition, the knowledge of the estimation error statistics were incorporated into the combiner parameters, better performance would result. The analysis presented allows extension to different receiver structures, higher level modulation techniques, as well as multipath fading channels. Mathematical treatment of such receivers is a subject of ongoing research.

REFERENCES

[1] R. Haeb and H. Meyr, “A systematic approach to carrier recovery and detection of digitally phase modulated signals on fading channels,” *IEEE Trans. Commun.*, vol. 37, pp. 748–754, July 1989.
[2] Z. Zvonar and D. Brady, “A comparison of differentially coherent and coherent multiuser detection with imperfect phase estimates in a Rayleigh fading channel,” in *Proc. IEEE Int. Symp. Information Theory*, San Antonio, TX, Jan. 1993, p. 48.
[3] Z. Zvonar and M. Stojanovic, “Performance of antenna diversity multiuser receivers in CDMA channels with imperfect fading estimation,” *Wireless Personal Commun.*, vol. 3, pp. 91–110, 1996.
[4] S. Vasudevan and M. Varanasi, “Achieving near-optimum asymptotic efficiency and fading resistance over the time-varying Rayleigh-faded CDMA channels,” *IEEE Trans. Commun*, vol. 44, pp. 1130–1143, Sept. 1996.
[5] S. Vasudevan, “Multiuser detectors for CDMA fading channels,” Ph.D. dissertation, Dept. Electr. & Comp. Eng., Univ. Colorado, Boulder, Nov. 1993.
[6] R. Lupas and S. Verdu, “Near-far resistance of multiuser detectors in asynchronous channels,” *IEEE Trans. Commun.*, vol. 38, pp. 496–508, Apr. 1990.
[7] J. Proakis, *Digital Communications*. New York: McGraw Hill, 1989.
[8] M. Stojanovic, J. Proakis, and J. Catipovic, “Analysis of the impact of channel estimation errors on the performance of a decision-feedback equalizer in fading multipath channels,” *IEEE Trans. Commun.*, vol. 43, pp. 877–886, Feb./Mar./Apr. 1995.