

## **Performance of Submerged Active Breakwaters in a Hydraulic Model**

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### ***Abstract***

A submerged active breakwater consists of a large buoyant cylinder that is held horizontal beneath the free surface of the water, by a spring and damper restraint system. The cylinder will be forced to oscillate in a certain mode, in response to an incident wave train. If properly “tuned”, the cylinder can absorb a considerable fraction of the incident wave energy. Utilization of this concept may provide a number of potential benefits including; 1) a no loss to fish habitat; 2) a depth-independent materials cost; 3) a scheme easily adaptable to long term water level changes (such as those which occur naturally in the Great Lakes and those which are anticipated with sea level rise); and 4) the capability of adequately protecting a coastal area while maintaining boat access and water circulation. Knowledge of these devices however, is currently limited to performance in; 1) regular wave trains of narrow frequency bands, 2) zero angle of incidence between the wave crest and the structure, and 3) waves of small amplitude.

Research evaluating the performance of submerged active breakwaters was performed in a two-dimensional wave flume in the Queen’s University Coastal Engineering Research Laboratory (QUCERL). Both single cylinders and multi-cylinders placed in series were evaluated. Transmission coefficients in the range of 0.3 to 0.7 were measured over a broad range of conditions, indicating the possibility of these devices being used in prototype situations to achieve the benefits described above.

### ***INTRODUCTION***

In the field of coastal zone protection, with a general trend of global sea level rise, the design and implementation of coastal defense structures is becoming significantly more complex. In response to the detrimental effects of conventional breakwaters, such as elimination of fish habitat and water stagnation, a variety of new breeds of breakwaters are emerging in an effort to meet the new demands and restrictions of coastal protection.

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A submerged active breakwater is one such variety. Consisting primarily of a large circular cylinder, this type of breakwater is designed to remove a large portion of incident wave energy while maintaining fish habitat and remaining fully submerged.

In order to investigate these devices, a series of experiments was performed at the Centre for the Aquatic Environment (CAE), Queen's University, Canada. The testing program was completed to broaden the existing experimental data base, evaluate the performance of these devices subject to irregular waves, evaluate the influence of the depth of submergence and to examine the performance of a multi-cylinder array of submerged active breakwaters. Specifically, tests were focused on determining the performance of a specific type of submerged active breakwater by examining the effects of wave height, wave period, the degree of randomness of the waves and the depth of submergence. In addition, a significant portion of the testing was aimed at observing the attenuation abilities of a system of two such devices placed in parallel.

### BACKGROUND

The type of submerged active breakwater under consideration has been previously studied by Evans *et al*<sup>2</sup> and Davis<sup>4,5</sup> and was referred to as the "Submerged Cylinder Wave Energy Device" or the "Bristol Cylinder". Figure 1 shows a typical setup for a submerged active breakwater. Essentially, the device consists of a large buoyant cylinder that is

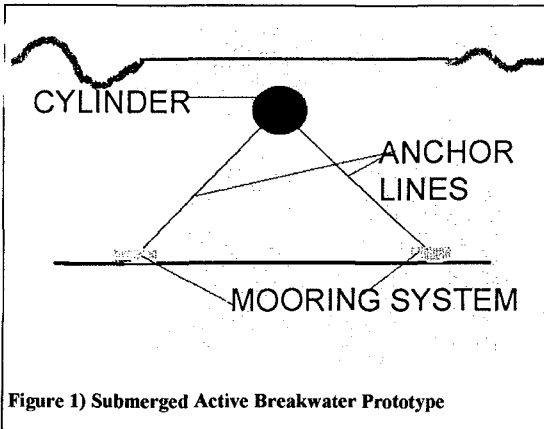


Figure 1) Submerged Active Breakwater Prototype

submerged to a desired depth by a system of cables, springs and some damping mechanism. As the on-coming wave motion interacts with the cylinder, it is forced to oscillate, typically in circular orbits in some phase related to that of the water particle orbits. By adjusting the stiffness of the springs and degree of damping in the restraining system, the device can be "tuned" to the incoming wave frequency and wave energy. When this tuning condition is satisfied, the device is capable of absorbing a

considerable portion of the incident wave energy. As the cylinder oscillates in a circular orbit beneath the free surface, the damping mechanism absorbs the energy of the waves, causing little or no reflection and in certain cases, virtually eliminating the transmitted wave.

The concept for this type of submerged active breakwater was first studied on a theoretical basis by Evans<sup>1</sup> who built on the early work on submerged cylinders by Dean<sup>6</sup>,

Ursell<sup>7</sup> and Ogilvie<sup>8</sup>. From all of the work presented by Evans and Davis, the primary goal of the Bristol Cylinder was the conversion of readily available wave power to usable electrical energy.

Several benefits and drawbacks are immediately apparent; others become more apparent when one considers the previous studies and experiment of Evans<sup>1</sup>, Evans *et al*<sup>2</sup> and Davis<sup>3,4</sup>. Benefits for such devices at the prototype scale include:

- No loss to fish habitat.
- A material cost that is virtually independent of the depth of water.
- A scheme that is potentially adaptable to long-term water level changes, such as those apparent in the Great Lakes and also those anticipated with global sea level rise.
- The adequate protection of a coastal region while maintaining more than sufficient water circulation to prohibit stagnation.
- The suitability for use in conjunction with newer bio-engineered shore protection methods which are capable of withstanding some degree of wave energy.
- The ability to maintain boat access across the breakwater.

Some of the current fundamental shortcomings include the following:

- Previous testing and theory has shown adequate performance of submerged active breakwaters in wave attenuation only for regular waves of small amplitude over narrow frequency bands.
- Progressive wave crests are limited in angle to a direct wave attack, an incident angle of zero degrees.
- Devising a robust spring and damping unit that can be used in a prototype scenario may prove to be a challenging task.

## RESEARCH GOALS

In an effort to broaden the experimental background of submerged active breakwaters, a few key shortcomings were to be addressed. Initially, since most, if not all published experimental data is based on device performance with regular sinusoidal wave trains, the experiments were to be conducted using random wave signals as well. Developing seas, indicative of Great Lakes conditions, were to be modelled by using Jonswap spectra with a specified significant wave height,  $H_s$ , and a peak period,  $T_p$ .

Secondly, a single device in operation produces a typical performance curve that is narrow in bandwidth over which it provides adequate wave attenuation. In an effort to improve the frequency range, two devices, tuned to different conditions, were to be tested in parallel. This was meant to partially validate the “n” cylinder theory presented by Evans *et al*<sup>3</sup>. This dual-cylinder system was also to be examined under random wave conditions.

The devices would then be reversed in order to observe the potential benefits of having a smaller period-tuned device in front versus behind.

Thirdly, the performance of individual and dual systems is to be examined in four different depths of submergence using irregular wave conditions.

### **NUMERICAL MODEL**

A numerical model to be used to provide a comparison between the theoretical efficiency (or ability to reduce the height of the transmitted waves) with experimental data was developed based primarily on the underlying assumptions of irrotational flow around the cylinder and first-order small amplitude wave theory. From an external force balance equating the oscillatory wave forces to the resistive forces of the cylinder, including added mass and hydrodynamic damping terms related to the cylinder motion, and the spring and damper effects, the efficiency can be calculated as,

$$E = \frac{4\omega^2 db}{\{k - (m + a)\omega^2\}^2 + \omega^2(b + d)^2} \quad [1]$$

where  $\omega$  = angular wave frequency ( $2\pi/T$ ), rads/s,  $d$  = damper constant, Ns/m,  
 $b$  = hydrodynamic damping, Ns/m,  $k$  = spring constant, N/m,  
 $m$  = cylinder mass per unit length, N/m,  $a$  = added mass, N/m.

From this relationship, it can be seen that to maximize the efficiency of the system, the spring rate and damper constant must be adjustable such that the following conditions can be met,

$$d = b \quad \text{and} \quad k = (m + a)\omega^2. \quad [2]$$

The added mass term,  $a$ , is representative of the additional mass of water that is moved in conjunction with the motion of the cylinder. The hydrodynamic damping term,  $b$ , can be thought of as the wave-making ability of the cylinder. Curves of the variation of these two parameters with wave number can be found in McIver<sup>9</sup>. By using the efficiency equation, we can essentially choose a desirable tuned wave period and then examine the predicted performance of a cylinder over a wide range of frequencies subjected to small amplitude waves. This results in the typical theoretical performance curves shown in Figures 2 and 3 in terms of wave height reduction (transmission coefficient) and energy removal (power absorption efficiency). Since the added mass and hydrodynamic damping terms vary to a large extent with wave frequency and depth of submergence, so do the predicted efficiency curves. Since this paper deals primarily with breakwater transmission, only transmission curves will be discussed.

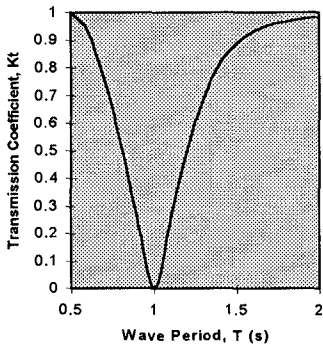


Figure 1. A typical cylinder response curve - wave transmission versus period

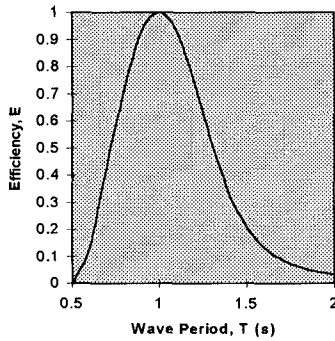
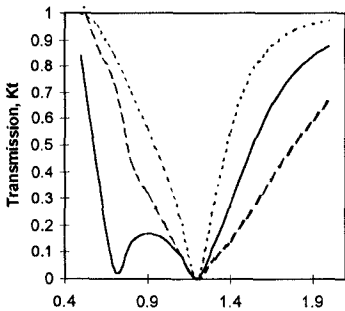


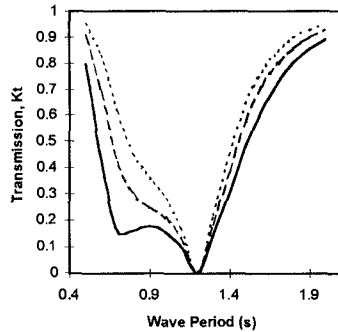
Figure 2. A typical cylinder response curve - power absorption efficiency versus period

Additional parameters that affect these curves are the cylinder's specific gravity and the spring and damper rates. Figures 4, 5, 6 and 7 show the variation in cylinder performance as a function of these parameters.



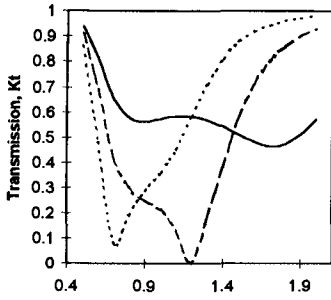
--- S/A=1.05    — S/A=1.25    ··· S/A=2.0

Figure 4. Variation in transmission with cylinder specific gravity

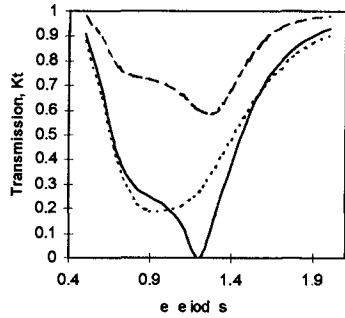


— SG=0.1    --- SG=0.5    ··· SG=0.9

Figure 5. Variation in transmission with depth of submergence



— K=Loose    - - - K=Medium    ····· K=Stiff



- - - =Sm    — =Medium    ····· =L

Figure 6. Variation in transmission with spring stiffness

Figure 7. Variation in transmission with damping rate

From the experimental work of Davis<sup>4,5</sup>, it was shown that the numerical model can predict the performance of a single cylinder quite accurately provided the modeller has a good estimate of the spring and damper rates being used.

Further theoretical work carried out by Evans *et al*<sup>3</sup> demonstrated mathematically that any number of devices could be used in parallel with no destructive interference. The performance of a system of “n” cylinders could thus be predicted using superposition of the individual performance curves for each cylinder. Since the addition of any device could only cause additional reduction of the transmitted wave, an “n” cylinder system could only be an improvement over an “n-1” cylinder system. The outcome of this work showed that any degree of wave attenuation could be achieved over a wider frequency band by adding additional devices in parallel.

This effect can be seen in Figures 8 and 9 in terms of wave transmission and power absorption efficiency. In addition to this broadened frequency band, the system would be capable of performing satisfactorily in progressively larger waves due to the subsequent attenuation by successive cylinders.

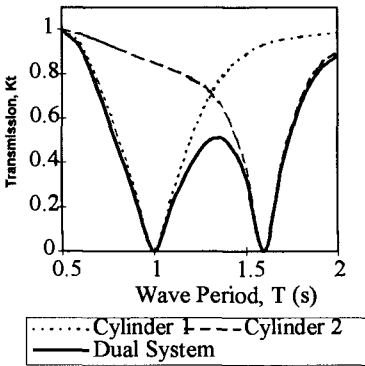


Figure 8. Wave transmission past a system of two cylinders in parallel, tuned to 1.0 s and 1.5 s

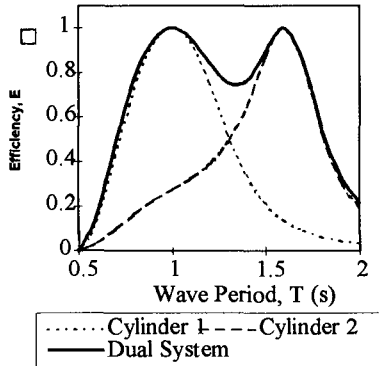


Figure 9. Power absorption efficiency of a two cylinder system, tuned to 1.0 s and 1.5 s

For the dual-cylinder system, Evans *et al*<sup>3</sup> expresses the power absorption efficiency as a function of the individual cylinder efficiencies,

$$E(T) = 1 - [1 - E_1(T)] \cdot [1 - E_2(T)] \quad [3]$$

where E is shown as a direct function of the incident wave period, T. In addition, assuming the reflection is truly zero, the transmission coefficient of the dual-cylinder system can be expressed as,

$$K_T = K_{T_1} \cdot K_{T_2} \quad [4]$$

The creation of this “n” cylinder model is restricted by the linear simplification that the transmitted wave is restricted to the fundamental harmonic.

**EXPERIMENTAL SETUP**

Tests were performed in a two-dimensional wave flume in the Queen’s University Coastal Engineering Research Laboratory. The flume is 1.2 m deep and is approximately 50 m long. The water depth was maintained at 86.0 cm for all tests described in this paper. Both regular and irregular waves were utilized in the testing program. In order to determine the performance of a device for all conditions tested, a sweep of wave periods with a constant wave height was performed. Waves of 2.0, 3.0 and 5.0 cm were used with periods of 0.7 s to 2.0 s at 0.1 s intervals. Although this setup allows a conversion to any scale, the anticipated scale of 1:60 would project performance of a 12 m diameter cylinder in 1.2 m, 1.8 m and 3 m waves with periods ranging from 5.4 s to 15.5 s.

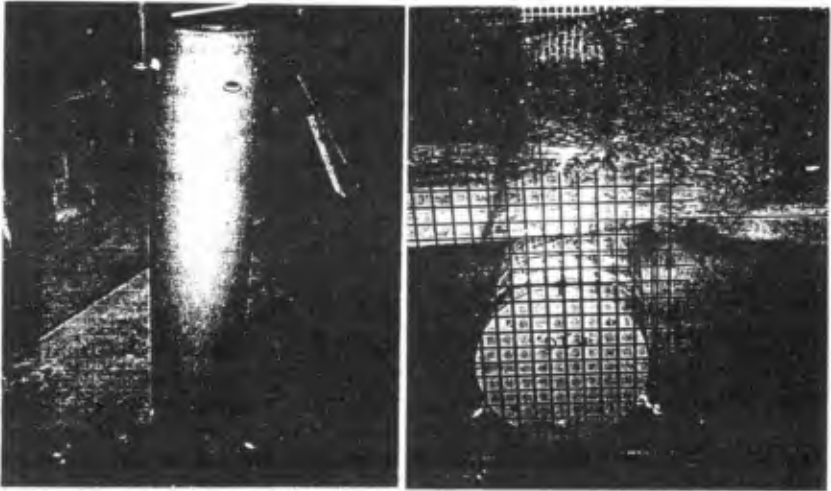


Figure 10. Test Cylinder

Figure 10 shows one of the test cylinders in the dry and in the wave flume. Both cylinders, 1.08 m long with a 10.6 cm (4") radius, were constructed using ABS pipe and symmetrically weighted to give a specific gravity of 0.3. Two 8.0 m long parallel tracks were installed on the flume bottom and equipped with stainless steel, double sealed bearings at the desired anchor points. Two cables at each side, attached at 90° to each other, passed down underneath the bearings and vertically up towards the springs that were located above the water surface.

The springs were automotive leaf springs attached to provide an upward force on the cables. Damping was not added because preliminary tests tended to indicate that there was too much damping already inherent with this cable system.

The wave flume was equipped with ten capacitance wave probes of which eight were sampled during each run. The signals were generated by the GEDAP wave generation and analysis package, developed by the NRC Canadian Hydraulics Center. Sampling was done at 20 Hz. The samples were then analysed by the GEDAP package by zero-crossing analysis and variance spectral density for a number of parameters including incident and transmitted significant wave height, incident and transmitted periods, incident and transmitted wave power and reflection from the device(s). The zero-moment wave height was not used due to the irregular nature of the transmitted spectra.

After the analysis of the wave signal, the efficiency,  $E$ , could be expressed as the fraction of the incident wave power absorbed by the cylinder,



$$E = \frac{P - P_r - P_T}{P} \tag{6}$$

where P is the wave power of the incident (i), reflected (r) and the transmitted (t) components. The power is calculated as per small amplitude wave theory as,

$$P = \frac{\rho g^2 TH^2}{32\pi} \cdot \tanh(kd) \cdot \left\{ 1 + \frac{2kd}{\sinh(2kd)} \right\} \tag{7}$$

where P = wave power, W/m                      ρ = density of water, Kg/m<sup>3</sup>  
 g = acceleration by gravity, m/s<sup>2</sup>            T = wave period, s  
 H = wave height, m                                d = water depth, m  
 k = wave number (2π/L), dimensionless.

Note that with this equation, the efficiency could also be determined easily by the measured wave heights thus knowing the values of the reflection coefficient, K<sub>R</sub>, and the transmission coefficient, K<sub>T</sub>,

$$E = 1 - K^2 - K_T^2 \tag{8}$$

where

$$K = \frac{H}{H} \quad \text{and} \quad K_T = \frac{HT}{H} \tag{9}$$

since the wave power is a function of H squared. This efficiency equation could only be used if the transmitted wave was truly locked to the fundamental, since the power is a function of the wave period, T. As will be discussed later, wave scattering to higher harmonics is common and thus expressing E in terms of K<sub>R</sub> and K<sub>T</sub> alone is invalid.

**RESULTS**

The notation for the devices and systems of devices tested is shown in Table 1. To tune the devices to the desired frequency, the spring and damper rates require adjustment. The intent was to determine a suitable spring rate and then add damping to maximize the efficiency of the devices by the addition of a rubbing strip or "brake" on the cables. This was tested but any addition of frictional damping caused a decrease in the performance.

Device Number	Desired Tuned Period	Location
Device 1	1.0s	Cylinder by itself
Device 2	1.5s	Cylinder by itself
System 3	1.0s, 1.5s	Device 1 in Front, Device 2 in Back
System 6	1.5s, 1.0s	Device 2 in Front, Device 1 in Back

**Table 1            Device descriptions**

Reflection from the cylinders rarely exceeded 5%. Even reflection of up to 10% would cause no more than 1% error in the determination of the efficiency, which is certainly less than the accuracy of the probe sampling and analysis routines combined. The reflection from the cylinders could thus be neglected as well. Measurements of significant wave height and wave power in front and behind the cylinder could therefore directly represent incident and transmitted components.

**Regular Wave Results**

The results of extensive testing using regular waves are shown in Figures 12, 13, 14, and 15. Device 1 shows trends in good agreement with the theoretical optimum although a certain degree of difference is expected as the optimum is based on infinitely small waves. The shape of the observed curve is quite consistent with the theory, indicating that the inherent damping is close to the required damping. The theoretical drop in efficiency on the other side of the peak cannot be examined due the limitations in wave generation at periods less than 0.7 s. During the higher frequency waves ( $T < 1.1$  s), transmission was typically 40% to 70% but increased to 80% to 100% at lower frequencies. The data supports the trend that an increase in incident wave height will generally cause a decrease in efficiency. Device 1 transmission peaked with the attenuation of 2 cm waves at 0.7 s to a  $K_T$  of 28%.

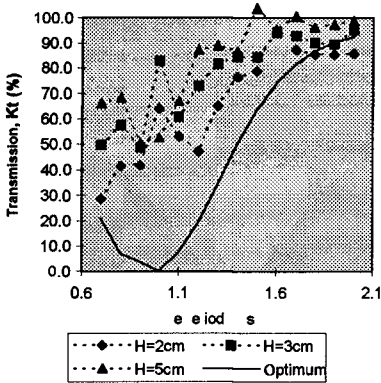


Figure 12. Device 1 transmission in regular waves

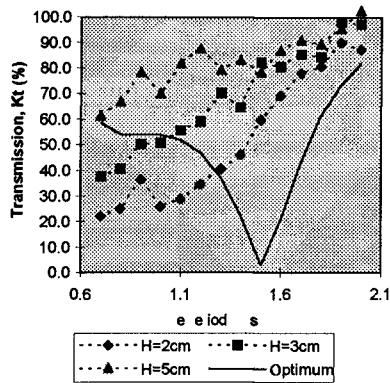


Figure 13. Device 2 transmission in regular waves

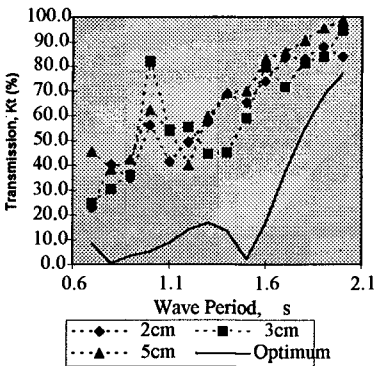


Figure 14. System 3 transmission in regular waves

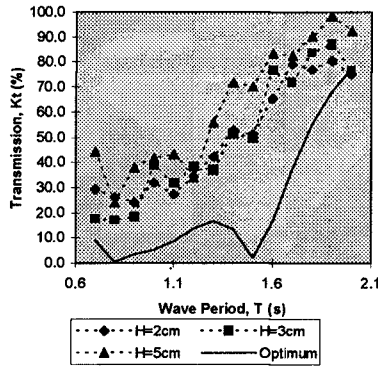


Figure 15. System 6 transmission in regular waves

Experiments with Device 2 were significantly different than the desired optimum. Since the same bearing/cable mechanism was used as with Device 1, inherent damping of was significantly greater than required. This caused skew of the peak to significantly lower wave periods than the desired 1.5 s peak. Despite this problem, Device 2 provided better wave attenuation for 2 cm and 3 cm waves than Device 1 with  $K_T$  values typically 10% less. Wave attenuation for Device 2 peaked again at 0.7 s for 2 cm waves with a  $K_T$  of 22%. It should be noted that the performance curves show a rather broad frequency band over which they perform satisfactorily. The peaks are broad for two reasons, the first being the low specific gravity of the cylinders tested (see Figure 4) and secondly, the excessive damping inherent in the test setup (see Figure 7).

System 3 performed worse than anticipated. Firstly, with the first cylinder performing satisfactorily, scattering of the 0.8 s to 1.2 s to the second harmonic proved to be of too high a frequency for the second device to provide any attenuation of its own. It actually had a detrimental effect, especially with the smaller waves. Possible reasons for include the fact that the inherent damping and friction in the system did not permit the rear cylinder to oscillate in the typical circular orbit and for the most part, the cylinder did not move. The small wave forces generated by the reduced wave were insufficient to cause any motion. Additionally, the smaller period scattered waves of 0.4 s to 0.6 s are believed to experience additional shoaling and their energy tends to pass primarily over the top of the cylinder. In one case, a transmission of 31% occurred across the first cylinder and 133% across the second which resulted in an overall system transmission of 42%. It's unfortunate that the wave generator used could not properly generate waves of such high frequencies such that the performance of single devices under very small period waves could be tested. Wave attenuation peaked at 0.7 s with a  $K_T$  of 23% for 2 cm waves and tapered off almost linearly at higher periods.

When the position of the cylinders was reversed, the performance of the system improved. For System 6, the greatest attenuation resulted in  $K_T$  values of 17% to 18% for wave periods of 0.7 s, 0.8 s and 0.9 s for 3 cm waves. The data tends to approximate the theoretical optimum better with a typical improvement greater than 5% across the entire spectrum. Overall, transmissions of 50% and less were achieved over 0.7 s to 1.2 s for all wave heights tested, providing better results than system 3 and both cylinders individually. Again, the wave attenuation for the larger waves improved significantly while the 2 cm wave attenuation improved to a lesser degree. The main beneficial effect of a dual-cylinder system is that the performance curves of the larger 3 cm and 5 cm waves are significantly improved. No significant widening of the performance curves was achieved primarily due to the lack of difference between the individual devices' peak periods.

### *Irregular Wave Results*

The results with irregular waves are shown in Figures 17, 18, 19 and 20. The response curves are significantly smoother as no standing waves develop. In comparing these results with those of regular tests, observation shows that larger wave heights cause less drop in efficiency than with regular waves, for all devices tested. Wave attenuation of 5 cm waves always improved, for 3 cm waves it usually improved mildly while for 2 cm

performance was rarely improved. All devices tend to perform significantly better at longer periods with irregular waves.

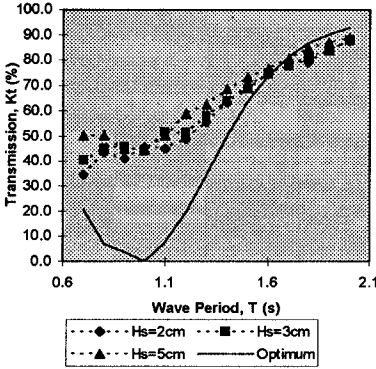


Figure 17. Device 1 transmission in irregular waves

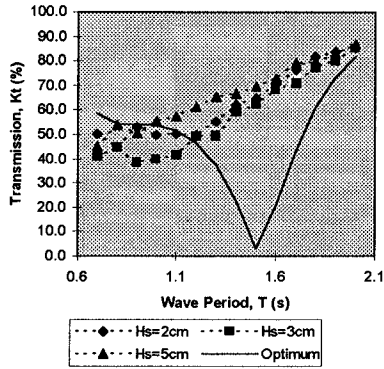


Figure 18. Device 2 transmission in irregular waves

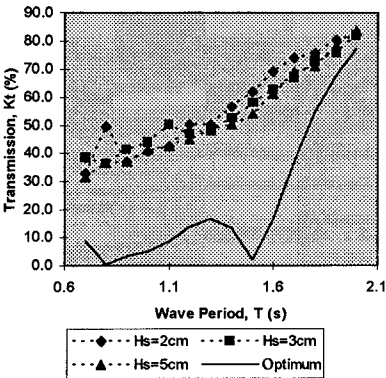


Figure 19. System 3 transmission in irregular waves

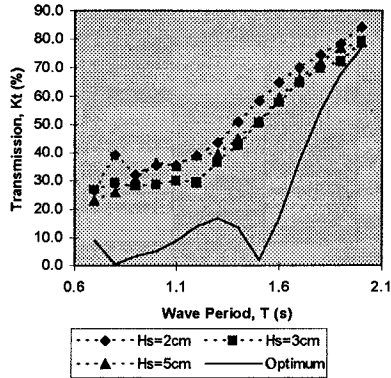


Figure 20. System 6 transmission in irregular waves

These two latter points are likely due to the nature of a Jonswap spectrum. First, the x-coordinate on the graphs indicates the peak period of the spectrum. With regular waves, the peak period is also equal to the average period. However, with irregular wave spectra, the average period is less than the peak period, such that the overall device performance may reflect response to the average period as opposed to the peak period. Second, we are using the significant wave height as the governing wave height parameter. For regular waves, it is more representative of the average wave height, as opposed to irregular wave signals, where the average wave height is significantly less than the significant wave height. The typical drop in efficiency in 2 cm significant waves can be attributed to this as

well, since the average height of the 2 cm irregular waves was likely insufficient to generate the required oscillatory forces to create ideal movement of the cylinder.

The peak attenuation of Devices 1 and 2 in random seas is quite consistent with the regular wave tests. It can be seen more clearly with this data, that the peak period of Device 2 is approximately 1.0 s whereas the peak of Device 1 may be 0.7 s or less. The only significant difference in performance is for waves of larger periods, where Device 2 causes 5% to 10% more attenuation than Device 1. Device 1 peaked at 0.7 s with 2 cm waves with a  $K_T$  of 34%. Device 2 peaked with a  $K_T$  of 39% at 0.9 s with 3 cm waves.

The difference in performance based on which device is in front can also be observed easier with the irregular waves. The wave attenuation is up to 15% more for System 6 than System 3. System 3 peaked at 0.7 s with 5 cm waves with a  $K_T$  of 32%, while system 6 peaked at 0.7 s with 5 cm waves with a  $K_T$  of 23%.

Essentially, the efficiency of these breakwaters was only very slightly worse in random developing seas than in regular sinusoidal waves. The linear theory is also clearly capable of predicting trends in their performance under random waves. This is very beneficial since the numerical model could therefore be used for prototype design with relative confidence that it will perform as intended in random wave conditions.

## ***SUMMARY***

A variety of very general, conservative conclusions can be drawn from these results.

1. Wave transmission of less than 50% can be obtained by a single breakwater over a relatively large frequency range for waves less than  $0.3A$ , where  $A$  represents the radius of the cylinder. This broad frequency envelope can be attributed primarily to the use of cylinders with a low specific gravity.
2. Wave transmission of less than 50% can be obtained by a dual-breakwater system over a slightly larger frequency range for waves less than  $0.5A$ .
3. A general trend exists whereby the efficiency of the breakwater drops when subjected to waves of increasing size.
4. The spring rate and degree of damping are crucial in tuning a cylinder to the intended design conditions. Over-damping tends to broaden the performance curve and drop the theoretical peak efficiency slightly.
5. The numerical model, which predicted this test data satisfactorily, is sufficient to predict trends in the performance of submerged active breakwaters. It also shows that a wider performance envelope can be achieved when placing two devices in parallel, tuned to different wave periods. The test data cannot confirm the widened envelop, but it shows that even with devices tuned to approximately the same peak period, an overall improvement in efficiency can be obtained.
6. The order in which the individual tuned cylinders are placed appears to have some effect on the dual-breakwater system performance curves. Positioning the cylinder tuned to the longer periods in front and shorter periods behind provided better results

when examining the combined systems. Designing and positioning cylinders with the second device tuned to the second harmonic would likely have some practical advantages. Unfortunately, the numerical model and experimental data cannot fully support this hypothesis at this time.

7. A two-cylinder system can cause wave attenuation of larger waves to a level that may be acceptable relative to the attenuation caused by a single cylinder.
8. Current testing is being performed to observe the effects of varying the depth of submergence on a breakwater with pre-set tuned conditions. A comparison will be made to the numerical model.

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