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Performance of Truncated Type-II Hybrid ARQ Schemes with Noisy Feedback over Block Fading Channels

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Abstract—This paper considers truncated type-II hybrid automatic repeat-request (ARQ) schemes with noisy feedback over block fading channels. With these ARQ techniques, the number of retransmissions is limited, and, similar to forward error correction (FEC), error-free delivery of data packets cannot be guaranteed. Bounds on the average number of transmissions, the average coding rate as well as the reliability of the schemes are derived using random coding techniques, and the performance is compared with FEC. The random coding bounds reveal the achievable performance with block codes and maximum-likelihood soft-decision decoding. Union upper bounds and simulation results show that over block fading channels, these bounds can be closely approached with simple terminated convolutional codes and soft-decision Viterbi decoding. Truncated type-II hybrid ARQ and the corresponding FEC schemes have the same probability of packet erasure; however, the truncated ARQ schemes offer a trade-off between the average coding rate and the probability of undetected error. Truncated ARQ schemes have significantly higher average coding rates than FEC at high and medium signal-to-noise ratio even with noisy feedback. Truncated ARQ can be viewed as adaptive FEC that adapts to the instantaneous channel conditions.

Index Terms—Block fading channel, error control, noisy feedback, random coding bound, truncated ARQ, union bound.

I. INTRODUCTION

A GOOD model for a wireless communication environment with slowly moving terminals is the block fading channel, see, e.g., [1] and [2]. Over a block fading channel, most of the time, the received signal is strong enough and the data packets are received correctly even without forward error correction (FEC). Occasionally, the received signal fades resulting in bursts of errors. Traditionally, interleaving is employed to randomize these errors such that FEC techniques for memoryless channels can be utilized [3], [4]. Due to a delay constraint

imposed by many services, ideal (infinite) interleaving is not possible. Furthermore, powerful low rate coding wastes capacity at times when the channel is good. In communication systems with a feedback channel, the incremental redundancy type-II hybrid automatic repeat-request (ARQ) techniques provide adaptability using error correction only when required [5]–[10].

Occasionally, even with type-II hybrid ARQ, a large number of retransmissions may be required, resulting in an unacceptable maximum delay. The maximum delay of ARQ schemes can be reduced by limiting the maximum number of retransmissions, yielding the truncated ARQ techniques [8], [11]–[15]. In [8], a type-II hybrid ARQ scheme with a finite receiver buffer using rate 1/2 convolutional code was analyzed over a two-state Markov channel. In [11], a constant rate ARQ scheme, which reserves some limited capacity for retransmissions (and therefore limits the number of retransmissions), was considered over an additive white Gaussian noise (AWGN) channel using a type-I hybrid technique with BCH codes. A truncated type-II hybrid ARQ scheme using block codes and only one retransmission was studied in [12] over an ideally interleaved Nakagami fading channel. In [13], a time-diversity ARQ scheme (pure ARQ with packet combining) employing a limited number of transmissions was analyzed. The numerical results were presented for MSK over a block fading channel. In [14], a truncated type-I hybrid ARQ scheme with convolutional codes and code combining was considered over an ideally interleaved Rayleigh fading channel assuming error-and-erasures Viterbi decoding. In [15], a truncated incremental redundancy hybrid ARQ scheme with rate compatible punctured convolutional codes was analyzed over an AWGN channel and over an ideally interleaved Rayleigh fading channel, assuming independent decoding attempts. In all these studies, a noiseless feedback channel was assumed.

In this paper, truncated type-II hybrid ARQ schemes are analyzed over a block fading channel assuming noisy feedback. Since the number of retransmissions is limited, truncated ARQ cannot guarantee error-free delivery of data packets, similar to pure FEC. We compare truncated type-II hybrid ARQ with pure FEC using two different approaches: 1) random coding techniques for block codes [2] and 2) union upper bounds for specific terminated convolutional codes [16]. The first approach

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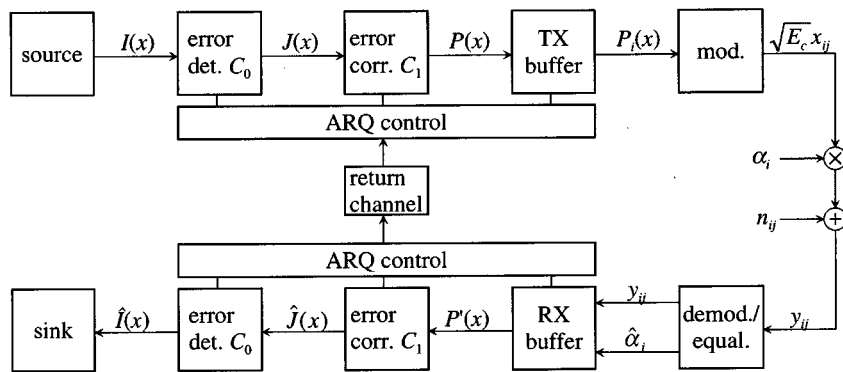


Fig. 1. Generic system model for the ARQ schemes over a block fading channel.

yields the achievable performance with block codes and maximum-likelihood soft-decision decoding. The second approach reveals how close we can approach the random coding performance by the use of a popular coding technique, namely, terminated convolutional codes with soft-decision Viterbi decoding.

II. SYSTEM MODEL

A generic system model for ARQ schemes is presented in Fig. 1. We describe separately the truncated ARQ protocol and the physical layer issues.

A. Truncated Type-II Hybrid ARQ Scheme with Convolutional Codes

We consider a truncated generalized type-II hybrid ARQ scheme with a limited number of retransmissions which is similar to the schemes in [7], [8], [12]. The maximum number of transmissions is denoted by F . We only consider full retransmission schemes here, i.e., the retransmission packet has the same length as the first transmission, extension to partial retransmission schemes is straightforward. We describe the ARQ scheme assuming terminated convolutional codes for error correction; numerical results are also presented for random block codes.

The data packet $I(x)$ of k bits, see Fig. 1, is first encoded with a high rate binary systematic (n, k) error detection code C_0 into an n -bit code word $J(x)$, i.e., $n - k$ parity bits are added. Then m tail bits are added to terminate the code trellis, and this block of $n + m$ bits is encoded with a rate $1/F$ convolutional code C_1 with F generator polynomials, $G_1(x), G_2(x), \dots, G_F(x)$ into F output blocks of $n + m$ bits: $P_1(x) = J(x)G_1(x)$, $P_2(x) = J(x)G_2(x)$, etc. The first transmission is the output of one generator polynomial, $P_i(x)$, and the other outputs are stored in the transmitter for possible retransmissions. The length of each packet is $L_p = n + m$ bits. Each transmission/packet as such employs a $(n + m, k)$ code, i.e., there are $n - k + m$ parity bits when a single packet is detected for errors and $n - k$ parity bits after decoding.

The operation of the receiver is similar to that in [7], [8], and [12], but we assume a soft-decision decoder. After receiving a packet, the syndrome of the hard decision version of the packet $P'_i(x)$ is checked in two steps using both C_1 and C_0 , see [8]. If errors are detected, a negative acknowledgment (NAK) is sent to

the transmitter asking for a retransmission and the soft-decision vector $P'_i(x)$ along with the channel state information (CSI) is stored in the receiver buffer. The transmitter now sends the next code word $P_{i+1}(x)$ stored in the buffer. The second transmission is first checked for errors in two steps in the same way as for $P'_i(x)$. If errors are detected, the soft-decision vectors $P'_i(x)$ and $P'_{i+1}(x)$ are interleaved to form a punctured code word in C_1 which is decoded using the Viterbi decoder producing an estimate $\hat{J}_{i, i+1}(x)$ and the syndrome of $\hat{J}_{i, i+1}(x)$ is checked with C_0 . If there are errors, a third transmission $P_{i+2}(x)$ is requested and $P'_{i+1}(x)$ is stored in the receiver buffer. These retransmissions are continued until a correct packet is received or the subsequent Viterbi decoding of the two, three, etc. most recent packets is successful. When all the F transmissions have been detected to be in error and even the combined and Viterbi decoded packet is detected to be in error, the packet is erased and the process starts for a new data packet.

Throughout this paper, an ideal selective repeat protocol is assumed. Due to the noisy feedback, some of the acknowledgment (ACK) messages may be lost. This leads to a situation where the transmitter and the receiver are not synchronized: the receiver is waiting for a different packet than the transmitter is sending [17]. To recover such situations, we assume the following: 1) the packets are numbered, the numbering can be cyclic [4], [17], [18]; 2) the transmitted code vector for a given packet depends on the frame number of the underlying system, i.e., the first transmission of a given packet can be any of the code vectors and the receiver always knows which decoder to use, even if the ACK is lost; 3) for the schemes which combine several packets, we assume that the combining is carried out in steps: first the most recent packet is detected alone, then two most recent packets are combined, etc.; 4) an ACK is assumed to acknowledge not only the current packet but all the previous correctly received packets, too; and 5) an erroneous NAK cannot be interpreted as an ACK, and we assume that all the erroneous or lost ACK's are interpreted as NAK's, and therefore, a lost NAK has no effect and a lost ACK causes an extra retransmission.

B. Truncated Type-II Hybrid ARQ Scheme with Random Block Codes

We will also analyze the truncated type-II hybrid ARQ schemes using random block codes. The error detection code C_0 is assumed to be a high rate binary systematic (n, k) block

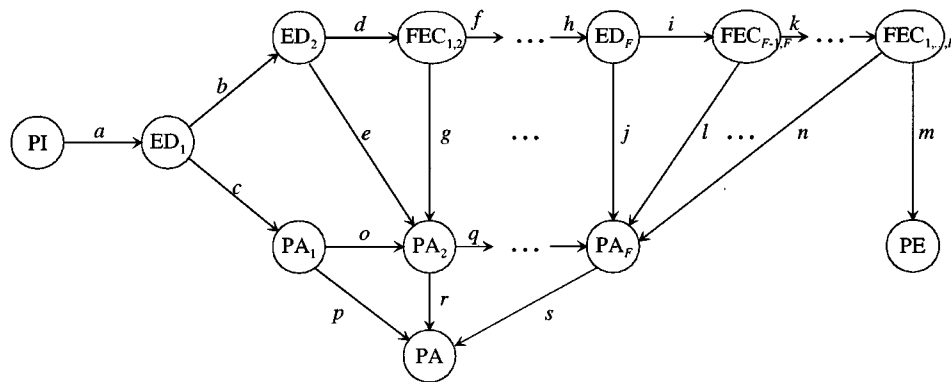


Fig. 2. State diagram of the truncated type-II hybrid ARQ scheme with F transmissions and noisy feedback.

code. The error correction code C_1 is assumed to be a random (n_F, n) block code, where $n_F = F(n + m)$.¹ Each code word is divided into F packets of length $L_p = n + m$, which are transmitted one by one when requested.² Otherwise, the operation of the ARQ scheme is the same as with terminated convolutional codes, i.e., in the receiver, soft-decision vectors are combined and maximum-likelihood decoded using ideal CSI.

C. Block Fading Channel

The block fading channel model used in this paper is shown as part of Fig. 1. The input symbols to the channel are denoted by x_{ij} , where i indicates the channel block and j the sample within the block. We assume binary antipodal modulation, i.e., $x_{ij} = \pm 1$. The modulated signal is passed through a block fading channel with a fading envelope α_i , which is assumed to be constant during a block of N bits. Furthermore, the fading envelopes of different blocks are assumed to be independent of each other and identically Rayleigh distributed with the probability density function

$$f(\alpha_i) = \frac{\alpha_i}{\sigma^2} e^{-\alpha_i^2/2\sigma^2} \quad (1)$$

where $2\sigma^2 = E[\alpha_i^2]$ is the average power of the fading envelope α_i . We assume that the channel is constant during the transmission of a packet, i.e., $N = L_p$. The received signal samples at the output of the single path block fading channel assuming coherent detection can be written as

$$y_{ij} = \sqrt{E_c} \alpha_i x_{ij} + n_{ij}, \quad i = 1, 2, \dots; j = 1, 2, \dots, N \quad (2)$$

where E_c is the energy per transmitted bit and n_{ij} 's are zero mean white Gaussian noise samples with a variance of $N_0/2$. We assume unquantized soft decisions, i.e., samples y_{ij} are stored in the receiver buffer, and the channel estimate is also assumed to be perfect, i.e., $\hat{\alpha}_i = \alpha_i$.

D. Maximum-Likelihood Detection

The performance of ARQ as well as FEC schemes will be analyzed using both random block codes and specific terminated

convolutional codes. For both codes, maximum-likelihood soft-decision decoding with ideal CSI is assumed. For convolutional codes we assume that each transmitted packet is one of the F outputs of the rate $1/F$ encoder, thus after f transmissions we have a rate $1/f$ (punctured) convolutional code. The Viterbi algorithm is used for decoding of this code and the branch metrics are calculated for path r as [15]

$$\lambda_j^{(r)} = \sum_{i=1}^f \alpha_i x_{ij}^{(r)} y_{ij}. \quad (3)$$

III. PERFORMANCE MEASURES FOR TRUNCATED ARQ

In this section, the average number of transmissions, the average coding rate as well as the reliability of the truncated type-II hybrid ARQ schemes are derived. The random coding bounds are given in the next section.

A. Average Number of Transmissions

The average number of transmissions for the truncated type-II hybrid ARQ scheme will be derived using the state diagram (or flow graph) with directed branches labeled with the transition probabilities and exponents of variable T [18], [19]. The state diagram for the truncated type-II hybrid ARQ scheme with F transmissions is given in Fig. 2. The states are labeled as: packet initialization (PI), error detection after i th transmission (ED_i), combining, decoding, and subsequent error detection of packets i and j ($FEC_{i,j}$), packet erasure (PE) and packet accepted after the i th transmission (PA_i). State PA is for the final packet acceptance. Several intermediate packet acceptance states (PA_i) are needed due to the noisy feedback and the truncation: the lost ACK's can only cause a limited number of extra transmissions for the truncated ARQ schemes. In those states, the packet has already been accepted by the receiver but the transmitter is still waiting for the ACK and has not yet removed the packet from the retransmission buffer. The final packet acceptance (PA) indicates that the ACK has been correctly received or the maximum number of retransmissions is reached. A packet erasure occurs if the packet after F transmissions and all the combining and decoding operations is still detected to be in error. Error detection in the ED_i -states is based on both C_1 and C_0 , in the $FEC_{i,j}$ -state only on C_0 . The labels of the directed branches between states are defined in Table I separately for the calculation

¹Here, the m 'tail' bits are part of C_1 and can take any values, they are added simply to have comparable results with terminated convolutional codes.

²Each packet and combination of packets can be viewed as punctured code words of C_1 .

TABLE I
DEFINITION OF THE BRANCH LABELS IN FIG. 2 FOR THE TRUNCATED ARQ SCHEME WITH F TRANSMISSIONS

label	for T_r	for $P_E = P_{PE} + P_{ue}$
a	T	1
b	$TP(R_d^{(1)})$	$P(R_d^{(1)})$
c	$1 - P(R_d^{(1)})$	$P(R_e^{(1)})$
d	$P(R_d^{(2)})$	$P(R_d^{(2)})$
e	$1 - P(R_d^{(2)})$	$P(R_e^{(2)})$
f	$TP(D_d^{(1,2)} R_d^{(1)}, R_d^{(2)})$	$P(D_d^{(1,2)} R_d^{(1)}, R_d^{(2)})$
g	$1 - P(D_d^{(1,2)} R_d^{(1)}, R_d^{(2)})$	$P(D_e^{(1,2)} R_d^{(1)}, R_d^{(2)})$
h	$TP(D_d^{(1,\dots,F-1)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(2,\dots,F-1)})$	$P(D_d^{(1,\dots,F-1)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(2,\dots,F-1)})$
i	$P(R_d^{(F)})$	$P(R_d^{(F)})$
j	$1 - P(R_d^{(F)})$	$P(R_e^{(F)})$
k	$P(D_d^{(F-1,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(1,\dots,F-1)}, R_d^{(F)})$	$P(D_d^{(F-1,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(1,\dots,F-1)}, R_d^{(F)})$
l	$1 - P(D_d^{(F-1,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, R_d^{(F)})$	$P(D_e^{(F-1,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(1,\dots,F-1)}, R_d^{(F)})$
m	$P(D_d^{(1,\dots,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(2,\dots,F)})$	$P(D_d^{(1,\dots,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(2,\dots,F)})$
n	$1 - P(D_d^{(1,\dots,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(2,\dots,F)})$	$P(D_e^{(1,\dots,F)} R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(2,\dots,F)})$
o, q	$TP(R_d^{(ACK)})$	$P(R_d^{(ACK)})$
p, r	$1 - P(R_d^{(ACK)})$	$1 - P(R_d^{(ACK)})$
s	1	1

of the average number of transmissions T_r and the probability of error P_E . $R_d^{(i)}$, $R_e^{(i)}$, and $R_c^{(i)}$ denote, respectively, the events “ i th received sequence contains detected errors,” “undetected errors,” and “no errors.” $D_d^{(i,j)}$, $D_e^{(i,j)}$, and $D_c^{(i,j)}$ denote the events “combined and decoded sequence formed from packets i and j contains detected errors,” “undetected errors,” and “no errors,” respectively. $R_d^{(ACK)}$ denotes the event “ACK packet contains detected errors.” Each transmission is assumed to be independent of each other.

In order to calculate the average number of transmissions, we first define the transfer function between the initial node PI and the two end nodes PA and PE (both PA and PE have to be included when calculating the average number of transmissions; in [12] and [8], PE was not included). For $F = 2$, the transfer function is given by $T_{PI \rightarrow PA+PE} = acp + acos + ab(es + dgs + dm)$. Substituting the labels defined in Table I, we get the following generator function:

$$G^{(2)}(T) = T \left(1 - P(R_d^{(1)}) \right) \left(1 - P(R_d^{(ACK)}) \right) + T^2 \left[\left(1 - P(R_d^{(1)}) \right) P(R_d^{(ACK)}) + P(R_d^{(1)}) \right]. \quad (4)$$

The average number of transmissions $T_r^{(2)}$ can now be calculated as [18]

$$T_r^{(2)} = \left[\frac{\partial}{\partial T} G^{(2)}(T) \right] \Big|_{T=1} = 1 + P(R_d^{(1)}) \left[1 - P(R_d^{(ACK)}) \right] + P(R_d^{(ACK)}). \quad (5)$$

Notice that the combining and decoding of two packets does not affect the average number of transmissions which is also intuitively correct: if $F = 2$ then two transmissions are always needed if errors are detected in the first transmission independent of the result of the combining and decoding.

Similarly, for the truncated type-II hybrid ARQ scheme with F transmissions, the average number of transmissions $T_r^{(F)}$ is given by

$$\begin{aligned} T_r^{(F)} &= (1 - P_{ACK}) \sum_{i=1}^{F-1} i \sum_{j=1}^i \left(P_{Dd}^{(j-1)} - P_{Dd}^{(j)} \right) P_{ACK}^{i-j} \\ &\quad + F \left[\sum_{j=1}^{F-1} \left(P_{Dd}^{(j-1)} - P_{Dd}^{(j)} \right) P_{ACK}^{F-j} + P_{Dd}^{(F-1)} \right] \\ &= 1 + \sum_{j=1}^{F-1} \left[P_{Dd}^{(j)} \left(1 - P_{ACK}^{F-j} \right) + P_{ACK}^j \right] \\ &= 1 + \sum_{j=1}^{F-1} P_{Dd}^{(j)} + \sum_{j=1}^{F-1} \left[P_{ACK}^j - P_{Dd}^{(j)} P_{ACK}^{F-j} \right] \end{aligned} \quad (6)$$

where $P_{Dd}^{(i)}$ is the joint probability of all the decoding events up to i th packet and is defined as

$$\begin{aligned} P_{Dd}^{(i)} &= P \left(D_d^{(1,\dots,i)}, D_d^{(2,\dots,i)}, \dots, D_d^{(i-1,i)}, D_d^{(1,\dots,i-1)}, \right. \\ &\quad \left. D_d^{(2,\dots,i-1)}, \dots, D_d^{(1,2)}, R_d^{(i)}, R_d^{(i-1)}, \dots, R_d^{(1)} \right) \\ P_{Dd}^{(1)} &= P(R_d^{(1)}) \quad \text{and} \quad P_{Dd}^{(0)} = 1 \end{aligned} \quad (7)$$

and $P_{\text{ACK}} = P(R_d^{\text{(ACK)}})$. (Notice the difference between the exponents and superscripts.) For noiseless feedback, (6) reduces to a truncated version of the average number of transmissions for an untruncated ARQ scheme, see, e.g., [10]. The expressions for the average number of transmissions (5) and (6) clearly show how the noisy feedback increases the average number of transmissions, but also that the number of transmissions is limited even if the feedback channel is useless, i.e., $P(R_d^{\text{(ACK)}}) = 1$. $P_{D_d}^{(i)}$ can be upper bounded by [10]

$$P_{D_d}^{(i)} = P\left(R_d^{(1)}, R_d^{(2)}, D_d^{(1,2)}, \dots, D_d^{(1,\dots,i)}\right) \leq P\left(D_d^{(1,\dots,i)}\right). \quad (8)$$

This upper bound is very tight as will be seen in the simulation results.

B. Average Coding Rate

In order to be able to compare the truncated ARQ schemes with pure FEC, we define an average coding rate for ARQ as follows (cf. [15]):

$$R_{\text{av}} = \frac{1}{T_r} \frac{k}{n+m} \quad (9)$$

where T_r is the average number of transmissions, and we assume an (n, k) error detection code and m tail bits for the terminated convolutional encoder. Notice that the expression for R_{av} is the same as the throughput for the untruncated ARQ schemes with a selective repeat protocol.

For the FEC schemes using convolutional codes, we define the average coding rate as

$$\begin{aligned} R_{\text{av, FEC}} &= R_{\text{FEC}} R_{\text{CRC}} \\ &= \left(\frac{k_c}{n_c} \frac{n}{n+m}\right) \frac{k}{n} \\ &= \left(R_c \frac{n}{n+m}\right) \frac{k}{n} \\ &= R_c \frac{k}{n+m} \end{aligned} \quad (10)$$

where $R_c = k_c/n_c$ is the rate of the convolutional code, $R_{\text{FEC}} = R_c n/(n+m)$, i.e., it takes into account the tail bits, too, and $R_{\text{CRC}} = k/n$. If no error detection is used, then $R_{\text{CRC}} = 1$, i.e., $n = k$.

We will see later that the probability of packet erasure will be practically the same for the truncated type-II hybrid ARQ scheme and the FEC scheme, if we select $R_c = 1/F$, where F is the maximum number of transmissions for the truncated ARQ scheme. Then we notice that for low signal-to-noise ratios (SNRs), when $T_r = F$, $R_{\text{av}} = R_{\text{av, FEC}}$. (If $R_{\text{CRC}} = 1$, the FEC scheme will have slightly higher average coding rate.) However, whenever the average number of transmissions is less than the maximum, i.e., $T_r < F$, the truncated ARQ scheme has higher average coding rate: $R_{\text{av}} > R_{\text{av, FEC}}$. At high SNRs, $T_r = 1$ and the truncated ARQ scheme has F times higher average coding rate (\approx throughput) than the FEC scheme: $R_{\text{av}} = F R_{\text{av, FEC}}$.

C. Reliability

For the truncated type-II hybrid ARQ schemes, there are two types of error events: packets are accepted with undetected errors and packets are erased. The total probability of error P_E for the truncated ARQ schemes is defined as the sum of the probability of undetected error P_{ue} and the probability of packet erasure P_{PE} . The error probability of the truncated ARQ schemes is compared with the error probability of the FEC schemes.

The probability of packet erasure P_{PE} for the truncated type-II hybrid ARQ scheme with $F = 2$ can be calculated by substituting the labels from Table I into the transfer function between states PI and PE (see Fig. 2), i.e., $T_{\text{PI} \rightarrow \text{PE}} = abdm$

$$\begin{aligned} P_{\text{PE}}^{(2)} &= P\left(R_d^{(1)}\right) P\left(R_d^{(2)}\right) P\left(D_d^{(1,2)} \mid R_d^{(1)}, R_d^{(2)}\right) \\ &= P\left(D_d^{(1,2)}, R_d^{(1)}, R_d^{(2)}\right) \\ &= P_{D_d}^{(2)} \leq P\left(D_d^{(1,2)}\right). \end{aligned} \quad (11)$$

The probability of undetected error is obtained from the transfer function between states PI and PA: $T_{\text{PI} \rightarrow \text{PA}} = ac(p + os) + abs(e + dg)$. Substituting the labels from Table I yields

$$\begin{aligned} P_{\text{ue}}^{(2)} &= P\left(R_e^{(1)}\right) + P\left(R_d^{(1)}\right) P\left(R_e^{(2)}\right) \\ &\quad + P\left(D_e^{(1,2)}, R_d^{(1)}, R_d^{(2)}\right). \end{aligned} \quad (12)$$

Finally, the total probability of error for the truncated type-II hybrid ARQ scheme with $F = 2$ is

$$\begin{aligned} P_E^{(2)} &= P_{\text{ue}}^{(2)} + P_{\text{PE}}^{(2)} \\ &= P\left(R_e^{(1)}\right) + P\left(R_d^{(1)}\right) P\left(R_e^{(2)}\right) \\ &\quad + P\left(D_e^{(1,2)}, R_d^{(1)}, R_d^{(2)}\right) + P\left(D_d^{(1,2)}, R_d^{(1)}, R_d^{(2)}\right) \\ &\leq P\left(R_e^{(1)}\right) + P\left(R_d^{(1)}\right) P\left(R_e^{(2)}\right) + P\left(D_e^{(1,2)}\right) \\ &\quad + P\left(D_d^{(1,2)}\right) \\ &= P\left(R_e^{(1)}\right) + P\left(R_d^{(1)}\right) P\left(R_e^{(2)}\right) + \left[1 - P\left(D_e^{(1,2)}\right)\right] \end{aligned} \quad (13)$$

where we have used the upper bounding defined in (8). We can clearly see that the reliability of the truncated ARQ schemes does not depend on the quality of the feedback channel: the noisy feedback only increases the average number of transmissions but does not affect the error probabilities.³

Similarly, for F transmissions using the state diagram in Fig. 2 and the labels in Table I, we get (14)–(16), shown at the bottom of the next page.

For the FEC schemes, the reliability is the probability of error, which is often defined as the total probability of errors, including both detected and undetected errors. When an error detection code is used with FEC, we can define both the probability of packet erasure, $P_{\text{PE}} = P(D_d)$ (equals the probability of detected errors), and the probability of undetected errors, $P_{\text{ue}} = P(D_e)$, where D_d , D_e , and D_c are the events that the

³Notice that [12, eq. (21)] yields the same result as (13) above when the term $(1-y)$ is replaced with the term P_d^2 (this small error, which does not affect the numerical results, originates from [7]).

FEC decoded sequence contains detected errors, undetected errors or no errors, respectively. For the FEC schemes, the total probability of error is simply

$$P_E = P_{PE} + P_{ue} = P(D_d) + P(D_e) = 1 - P(D_c). \quad (17)$$

When comparing the probability of packet erasure for the truncated type-II hybrid ARQ scheme (14) with the corresponding FEC scheme, we notice that the upper bound for the truncated ARQ scheme is the same as the exact result for the FEC scheme. Here, the code rate of the convolutional code for the FEC scheme is assumed to be $R_c = 1/F$ and the coded block is assumed to be transmitted over $L = F$ channel blocks, i.e., the same parameters as in the last decoding attempt of the truncated type-II hybrid ARQ scheme. In practice, these error probabilities are the same since the upper bound (8) is very tight. This implies that both schemes have the same diversity order for the probability of packet erasure [2].

The probabilities of undetected error, on the other hand, are very different: for the FEC scheme, there is only one probability term, whereas for the truncated type-II hybrid ARQ scheme, there are several terms involved in (15). The term with the lowest diversity order dominates the results. For the truncated type-II hybrid ARQ scheme with any number of transmissions, it is the probability of having undetected errors in a single transmission, i.e., $P(R_e^{(i)})$. Over the block fading channel, this implies that the truncated type-II hybrid ARQ scheme has a probability of undetected error asymptotically with a diversity order of $\lambda = 1$. On the other hand, the probability of undetected error for the corresponding FEC scheme will have a diversity order dictated by the code rate R_c and the number of channel blocks L over which the coded block is interleaved [2]. For $R_c = 1/F$ and $L = F$, we have a diversity order of $\lambda = F$.

IV. RANDOM CODING BOUNDS

In this section, we will consider the average performance of truncated type-II hybrid ARQ schemes over the ensemble of all block codes, often referred to as the random coding bounds. Then we know that there exists at least one code which gives performance not worse than the bound.

Consider first the average number of transmissions, which is given by (6) and can be upper bounded using (8). Assume for the moment noiseless feedback. Then for a given code C_1 we have

$$T_r^{(F)} \leq 1 + \sum_{j=1}^{F-1} P(D_d^{(1, \dots, j)}). \quad (18)$$

Each term $P(D_d^{(1, \dots, j)})$ in (18) is the probability of detecting errors when decoding a rate $R^{(j)} = n/[j(n+m)]$ code which is a punctured version of C_1 . The average number of transmissions for the ensemble of all block codes is given by

$$\overline{T_r^{(F)}} \leq 1 + \overline{\sum_{j=1}^{F-1} P(D_d^{(1, \dots, j)})} = 1 + \sum_{j=1}^{F-1} \overline{P(D_d^{(1, \dots, j)})} \quad (19)$$

where the bar represents code ensemble average, and $\overline{P(D_d^{(1, \dots, j)})}$ is the ensemble average error probability of the rate $R^{(j)}$ block codes of length $n_j = j(n+m)$.⁴ The result in (19) indicates that the average number of transmissions for the ensemble of block codes can be upper bounded by summing the ensemble average error probabilities. In other words, we can calculate a random coding upper bound for each probability term separately and get an upper bound on the ensemble average of $T_r^{(F)}$. The random coding upper bound on the error probability over the block fading channels was derived in [2]

$$\begin{aligned} \overline{P(D_d^{(1, \dots, j)})} &\leq \int_{\alpha_1} \dots \int_{\alpha_L} \min_{0 \leq \rho(\alpha) \leq 1} \\ &\cdot \left[2^{R^{(j)}NL\rho(\alpha)} \left(\prod_{i=1}^L 2^{-NE_0(\rho(\alpha), \mathbf{q}|\alpha_i)} \right) \right] \\ &\cdot \prod_{l=1}^L f(\alpha_l) d\alpha_1 \dots d\alpha_L \end{aligned} \quad (20)$$

⁴The ensemble average in (19) is over $2^{Mn_{F-1}}$ different codes where $n_{F-1} = (F-1)(n+m)$ is the length of the punctured code after $F-1$ transmissions, and $M = 2^n$ is the number of code words. For higher rate punctured codes this means that each punctured code is taken several times.

$$P_{PE} = P_{D_d}^{(F)} \leq P(D_d^{(1, \dots, F)}) \leq 1 - P(D_c^{(1, \dots, F)}) \quad (14)$$

$$\begin{aligned} P_{ue} &= P(R_e^{(1)}) + P(R_d^{(1)})P(R_e^{(2)}) + P(D_e^{(1,2)}, R_d^{(1)}, R_d^{(2)}) + \dots + P(D_e^{(2, \dots, F)}, D_d^{(3, \dots, F)}, \dots, D_d^{(F-1, F)}, D_d^{(1, \dots, F-1)}, \\ &\quad D_d^{(2, \dots, F-1)}, \dots, D_d^{(1,2)}, R_d^{(F)}, \dots, R_d^{(1)}) + P(D_e^{(1, \dots, F)}, D_d^{(2, \dots, F)}, \dots, D_d^{(F-1, F)}, \\ &\quad D_d^{(1, \dots, F-1)}, D_d^{(2, \dots, F-1)}, \dots, D_d^{(1,2)}, R_d^{(F)}, \dots, R_d^{(1)}) \\ &\leq \sum_{j=0}^{F-1} \sum_{i=1}^{F-j} P(D_d^{(1, \dots, i-1)}) P(D_e^{(i, \dots, i+j)}) \end{aligned} \quad (15)$$

$$\begin{aligned} P_E &= P_{PE} + P_{ue} \\ &\leq P(D_d^{(1, \dots, F)}) + \sum_{j=0}^{F-1} \sum_{i=1}^{F-j} P(D_d^{(1, \dots, i-1)}) P(D_e^{(i, \dots, i+j)}). \end{aligned} \quad (16)$$

where $E_0(\rho(\alpha), \mathbf{q}|\alpha_i)$ is a conditional Gallager function, $\rho(\alpha)$ is a parameter to be optimized [a function of the fading vector $\alpha = (\alpha_1, \dots, \alpha_L)$], \mathbf{q} is the input distribution, the channel block length $N = L_p$, and the number of channel blocks involved in the decoding process $L = j$. Similarly, we can calculate random coding bounds for the average number of transmissions with noisy feedback (P_{ACK} is independent of the code used in forward channel and can be considered as a constant when averaging over the code ensemble).

For the average coding rate, the random coding bound is a lower bound

$$\begin{aligned} \bar{R}_{\text{av}} &= \frac{1}{T_r^{(F)}} \frac{k}{n+m} \\ &\geq \frac{1}{T_r^{(F)}} \frac{k}{n+m} \\ &\geq \frac{1}{1 + \sum_{j=1}^{F-1} P(D_d^{(1, \dots, j)})} \frac{k}{n+m} \end{aligned} \quad (21)$$

where we have used the Jensen inequality for convex functions [20] and the upper bound (19). And finally for the reliability, the random coding upper bound is given by

$$\begin{aligned} \bar{P}_E &= \bar{P}_{\text{PE}} + \bar{P}_{\text{ue}} \\ &\leq P(D_d^{(1, \dots, F)}) + \sum_{j=0}^{F-1} \sum_{i=1}^{F-j} \\ &\quad \cdot \overline{P(D_d^{(1, \dots, i-1)}) P(D_e^{(i, \dots, i+j)})} \\ &= \overline{P(D_d^{(1, \dots, F)})} \\ &\quad + \sum_{j=0}^{F-1} \sum_{i=1}^{F-j} \overline{P(D_d^{(1, \dots, i-1)}) P(D_e^{(i, \dots, i+j)})}. \end{aligned} \quad (22)$$

The bar within the sum can be cut since the two probability terms are independent (the decoding operations involve disjoint parts of the code words).

V. EVALUATING THE PROBABILITIES FOR BLOCK FADING CHANNEL

For the block fading channel with a channel block per packet, the probability of receiving a packet in error $P(R_d^{(i)})$ is calculated as follows. Assuming that the probability of undetected error is negligible, i.e., $P(R_e^{(i)}) \ll P(R_d^{(i)})$, then the conditional probability $P(R_d^{(i)}|\alpha_i)$ can be approximated

$$P(R_d^{(i)}|\alpha_i) \approx 1 - P(R_e^{(i)}|\alpha_i) = 1 - (1 - P_b(\alpha_i))^{L_p} \quad (23)$$

where $P_b(\alpha_i) = Q(\alpha_i \sqrt{2E_c/N_0})$ is the conditional bit-error probability for antipodal modulation. Averaging over fading yields

$$\begin{aligned} P(R_d^{(i)}) &\approx 1 - P(R_e^{(i)}) \\ &= 1 - \int_0^\infty (1 - P_b(\alpha_i))^{L_p} f(\alpha_i) d\alpha_i \end{aligned} \quad (24)$$

where we have used the fact that the fading envelope is constant during one packet and which, due to the (L_p) th power of the function $Q(x)$, will be evaluated numerically.

When calculating the performance of the ARQ schemes using specific terminated convolutional codes, the probability of detecting errors in a combined/decoded sequence formed from packets i, \dots, j , $P(D_d^{(i, \dots, j)})$, will be upper bounded using the union upper bounds. Assuming that the probability of undetected error is negligible, the conditional probability $P(D_d^{(i, \dots, j)}|\alpha)$ can be upper bounded as

$$\begin{aligned} P(D_d^{(i, \dots, j)}|\alpha) &\leq 1 - P(D_e^{(i, \dots, j)}|\alpha) \\ &\leq 1 - \left(1 - P_E^{(i, \dots, j)}(\alpha)\right)^n \end{aligned} \quad (25)$$

where $P_E^{(i, \dots, j)}(\alpha)$ is the conditional error event probability which can be upper bounded using the component distance properties of the convolutional code (see [16] or [22] for details). The average probability of detected errors after decoding in a frequency-nonsselective block fading channel is

$$\begin{aligned} P(D_d^{(i, \dots, j)}) &\leq 1 - \int_0^\infty \dots \int_0^\infty \left(1 - P_E^{(i, \dots, j)}(\alpha_1, \dots, \alpha_L)\right)^n \\ &\quad \cdot \prod_{l=1}^L f(\alpha_l) d\alpha_1 \dots d\alpha_L. \end{aligned} \quad (26)$$

This L -fold integral has to be evaluated numerically.

The probability of undetected error for a packet transmitted over a single channel block can be upper bounded as [4]

$$\begin{aligned} P(R_e^{(i)}|\alpha_i) &\leq 2^{-(n+m-k)} \left[1 - (1 - P_b(\alpha_i))^{n+m}\right] \\ &= 2^{-(n+m-k)} \left[1 - P(R_e^{(i)}|\alpha_i)\right] \end{aligned} \quad (27)$$

where we have assumed $n + m - k$ parity bits. This is an upper bound on the average probability of an undetected error for an ensemble of linear systematic $(n + m, k)$ codes over a BSC and it tells that there exist linear codes with the undetected error probability lower than the bound. Strictly speaking, the bound may not always be true for our case since the m tail bits are part of the error correction code which is not optimized for error detection. A looser bound using $n - k$ instead of $n + m - k$, however, is always true. Averaging (27) over the fading yields

$$P(R_e^{(i)}) \leq 2^{-(n+m-k)} \left[1 - P(R_e^{(i)})\right] \quad (28)$$

and $[1 - P(R_e^{(i)})]$ is given by (24).

The error detection after decoding of C_1 is based on the (n, k) code C_0 , i.e., there are $n - k$ parity bits. If we assume proper stochastic channel transforms (see [21] for details) between the two codes C_0 and C_1 , the conditional probability of undetected error is given by [21]

$$\begin{aligned} P(D_e^{(i, \dots, j)}|\alpha) &= \frac{2^k - 1}{2^n - 1} \left[1 - P(D_e^{(i, \dots, j)}|\alpha)\right] \\ &\leq 2^{-(n-k)} \left[1 - P(D_e^{(i, \dots, j)}|\alpha)\right]. \end{aligned} \quad (29)$$

TABLE II
CODE PARAMETERS FOR THE DIFFERENT TRUNCATED TYPE II HYBRID ARQ SCHEMES USED FOR NUMERICAL RESULTS

	F	L_p	$C_0(n, k)$	$C_1(n_F, n)$	R	R_c	m	gen. polynomials
2 transmissions	2	500	(494,470)	(1000,494)	0.494	1/2	6	(133,171)
3 transmissions	3	500	(494,470)	(1500,494)	0.3293	1/3	6	(135,161,147)
4 transmissions	4	500	(494,470)	(2000,494)	0.247	1/4	6	(135,161,147,163)

The average probability of undetected error for the block fading channel is obtained by averaging (29) over the fading vector α

$$P(D_c^{(i, \dots, j)}) = \frac{2^k - 1}{2^n - 1} [1 - P(D_c^{(i, \dots, j)})] \quad (30)$$

and $[1 - P(D_c^{(i, \dots, j)})]$ is upper bounded by (20) or (26).

We assume that the ACK message is transmitted in a channel block over the block fading channel, i.e., without interblock interleaving. The ACK message can be part of a data packet or a short separate packet. In this work, we approximate the probability of detecting an ACK in error as

$$P(R_d^{(\text{ACK})}) = \beta P(R_d^{(i)}) \quad (31)$$

where β is a parameter which depends on the ACK packet length and the amount of error correction used for ACK's. The value of β varies between 0 and 1. Error-free feedback is modeled by $\beta = 0$, and $\beta = 1$ approximates the case where no additional error correction is used. For instance, a rate 1/2 coded 20 bits ACK message can be approximated with $\beta \approx 0.4$.

VI. NUMERICAL RESULTS

In this section, numerical results for the performance of the truncated type-II hybrid ARQ schemes over a block fading Rayleigh channel are given using the random coding bounds. Then, the results are compared with those obtained with the union upper bounds and the simulations for specific terminated convolutional codes. For further numerical results, see [22].

The parameters assumed for the numerical results are given in Table II. F is the maximum number of transmissions, $L_p = n + m$ is the packet length, C_0 is the error detection code with $n - k = 24$ parity check bits, R is the rate of the equivalent block code C_1 used for the random coding upper bounds, m is number of tail bits for the convolutional codes of rate R_c . The channel block length $N = L_p = 500$. The generator polynomials of the convolutional codes given in octal form are used for the union bounds for specific convolutional codes as well as for the simulations.

The average coding rate R_{av} for the truncated type-II hybrid ARQ scheme with $F = 2, 3,$ and 4 is shown in Fig. 3 with noiseless ($\beta = 0$), and with noisy ($\beta = 0.5$ and 1.0) feedback. The (average) coding rates of the corresponding FEC schemes are also shown. The truncated ARQ scheme and the corresponding FEC scheme have the same error probability (strictly speaking the same probability of packet erasure) for a given \bar{E}_c/N_0 , see the curves in Fig. 4. It is clearly seen, that the truncated ARQ scheme gives much higher average coding rate than the FEC scheme, especially at the high SNR. The FEC scheme is slightly

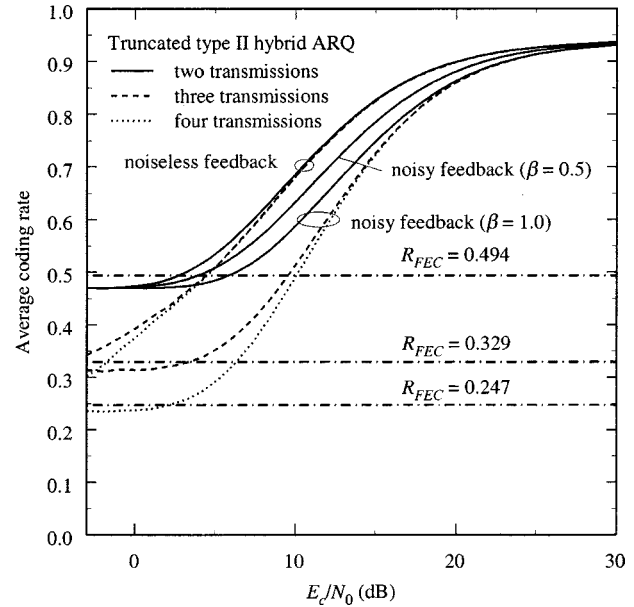


Fig. 3. Average coding rate R_{av} for the truncated type-II hybrid ARQ schemes with two, three, and four transmissions, noiseless ($\beta = 0$) and noisy feedback ($\beta = 1.0$ and 0.5), compared with the coding rate of FEC schemes with the same probabilities of error.

better only at the very low average SNR ($\bar{E}_c/N_0 < 5$ dB). Notice that for the FEC scheme, we have selected $R_{CRC} = 1$, i.e., no error detection, in order to have the highest possible average coding rate. Due to the truncation, the average coding rate of the truncated ARQ scheme never drops very low. The noisy feedback reduces the average coding rate of the truncated ARQ scheme, but the error probability is not affected. The difference between the noiseless and the noisy feedback with $\beta = 1.0$ is about 3–5 dB for a given average coding rate. The schemes with a larger number of transmissions are more sensitive to the ACK errors. Even with the noisy feedback, the truncated ARQ schemes have a clear advantage over the FEC schemes.

In Fig. 4, the probability of error P_E , given by (13) and (16), is shown for the truncated type-II hybrid ARQ schemes with $F = 2, 3,$ and 4 transmissions. In the same figure, the error probability of the corresponding FEC schemes is given. The FEC schemes are chosen such that they have the same probability of packet erasure P_{PE} as the corresponding truncated ARQ schemes. We can clearly see that the error probability for the truncated ARQ schemes is dominated by the probability of packet erasures; only at very low error probabilities ($< 10^{-9}$), the undetected errors for the first transmission start to increase the total error probability. Fig. 4 shows also the case where the fixed code rate of the FEC scheme has been increased to the same value as the average coding rate of the truncated ARQ

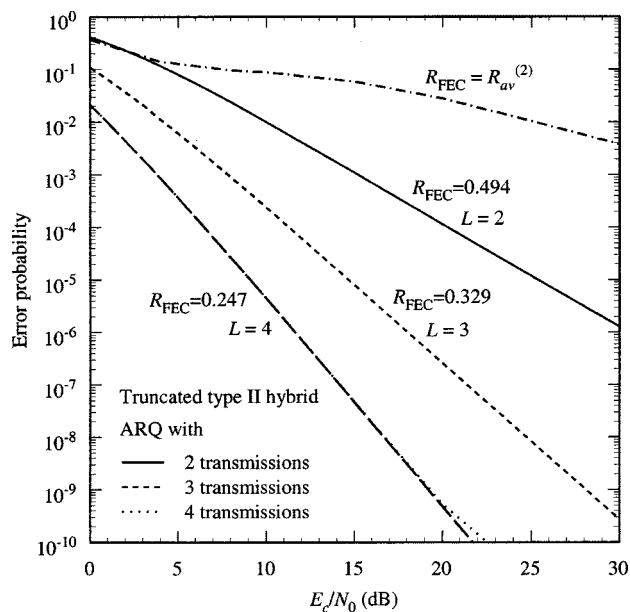


Fig. 4. Probability of error for the truncated type-II hybrid ARQ schemes with $F = 2, 3,$ or 4 and for the FEC schemes with code rates $R = 0.494$ for $L = 2,$ $R = 0.329$ for $L = 3,$ and $R = 0.247$ for $L = 4$ (for $L = 2$ and $3,$ the curves for FEC and ARQ are the same), as well as for an FEC scheme with the same average coding rate for each \bar{E}_c/N_0 as the truncated ARQ with $F = 2$ (noiseless feedback), over the block fading Rayleigh channel.

scheme with $F = 2,$ i.e., for a given \bar{E}_c/N_0 both the truncated ARQ scheme and the FEC scheme have the same average coding rate and the probability of error is compared. This comparison is especially unfavorable for the FEC schemes since the increase in the code rate implies that the FEC scheme loses in the diversity gain (when the code rate is increased over $R = 0.5$ in the case of $L = 2$ channel blocks, the diversity order is reduced to one, see [2, Theorem 1]). This is clearly seen in Fig. 4 where the error probability curves for the truncated ARQ scheme and the FEC scheme have different slopes.

The probability of undetected error P_{ue} for the truncated type-II hybrid ARQ schemes with two, three, and four transmissions as well as for the corresponding FEC schemes is depicted in Fig. 5. All the truncated ARQ schemes have asymptotically at the high SNR's the same probability of undetected error which is due to the fact that the first transmission dominates the results [$P(R_e^{(1)})$ has diversity order of $\lambda = 1$]. The FEC schemes, on the other hand, achieve the maximum possible diversity order ($\lambda = L$) [2] since the error detection is performed only once after the decoding of C_1 . Although the probability of undetected error for the truncated ARQ schemes decreases very slowly with increasing SNR, in many applications this is not a problem: by specifying enough parity bits for the error detection code $C_0,$ the probability of undetected error can be made low enough and the increased probability of undetected error can be traded off for higher average coding rate, see Fig. 3.

The results for the truncated type-II hybrid ARQ schemes so far have been calculated using the random block codes. These results show what can be achieved with the optimal codes in a block fading Rayleigh channel. In order to see, how close we can get with the terminated convolutional codes, the performance of the truncated ARQ scheme was calculated using the union upper

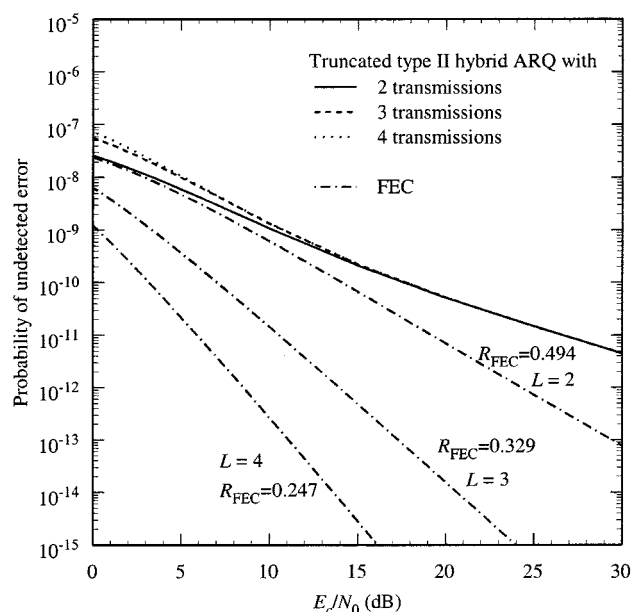


Fig. 5. Probability of undetected error for the truncated type-II hybrid ARQ schemes with two, three, or four transmissions as well as for the FEC schemes with code rates $R = 0.494$ for $L = 2,$ $R = 0.329$ for $L = 3,$ and $R = 0.247$ for $L = 4,$ over the block fading Rayleigh channel.

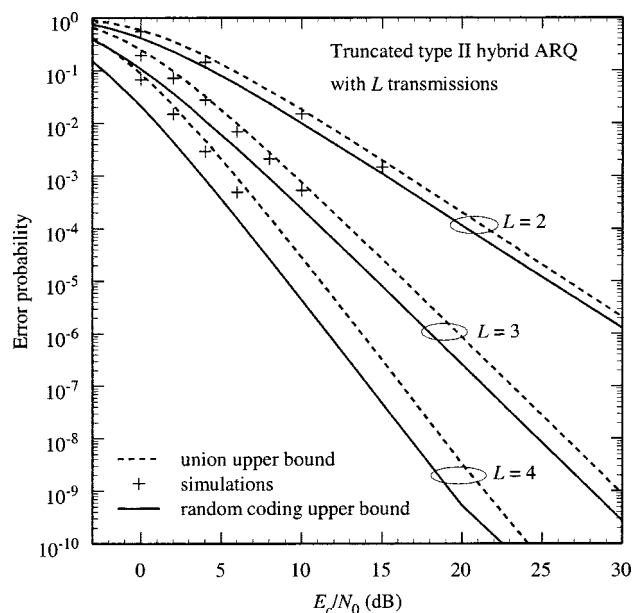


Fig. 6. Error probability of the truncated type-II hybrid ARQ schemes with L transmissions calculated using the random coding upper bounds for block codes, and the union upper bounds and simulations for specific terminated convolutional codes.

bounds on specific convolutional codes as well as using simulations. See Table II for the code parameters. The error probability for the truncated ARQ schemes using the random coding upper bounds on block codes, the union upper bounds on specific convolutional codes as well as the simulations for convolutional codes are depicted in Fig. 6. The difference between the union upper bounds and the random coding upper bounds is 1–2 dB, the difference is larger for the larger number of transmissions. The simulations show that the union upper bounds are quite tight (within 1 dB) and also, that the selected terminated

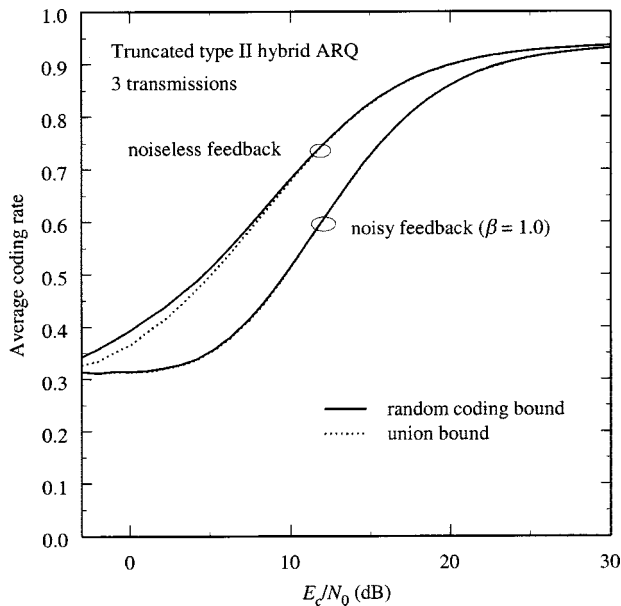


Fig. 7. Lower bounds on the average coding rate for the truncated type-II hybrid ARQ schemes with three transmissions based on the random coding bounds for block codes and the union bounds for specific convolutional codes, noiseless, and noisy feedback.

convolutional codes are quite good as block codes. It is natural that with such large block lengths ($FL_p = 1000\text{--}2000$), that we are using here, better block codes exist but the differences are surprisingly small.

The lower bounds on the average coding rates of the truncated type-II hybrid ARQ schemes with three transmissions using the random coding bounds on block codes and the union upper bounds on specific convolutional codes are compared in Fig. 7. At the low \bar{E}_c/N_0 with noiseless feedback, the random coding bounds give slightly better results; the simulated performance with the terminated convolutional codes is between the two bounds, see Fig. 6. With the noisy feedback, however, the difference disappears, as can be seen in Fig. 7. The results are similar with four transmissions. For two transmissions, the average coding rate can be calculated exactly and both bounding methods yield the same results.

VII. CONCLUSIONS

Type-II hybrid ARQ schemes with a limited number of retransmissions were considered, and a comparison with the FEC schemes was provided using the average coding rate and the reliability as performance measures. New expressions for the average number of transmissions with noisy feedback, as well as for the probability of error, were derived. Since neither truncated ARQ nor pure FEC can guarantee error-free delivery of data packets, the probability of error is due to both erasures and undetected errors.

The truncated type-II hybrid ARQ schemes and the corresponding FEC scheme were shown to yield the same probability of packet erasure; however, truncated ARQ provides a trade-off between the average coding rate and the probability of undetected error. It is shown that even with noisy feedback, the truncated ARQ schemes have a significantly higher average

coding rates than FEC at high and medium SNR. At low SNR, the schemes are comparable. With FEC, a lower probability of undetected error can be achieved, which in many cases is not needed. Truncated ARQ can be viewed as adaptive FEC that adapts to the instantaneous channel conditions. Even with very noisy feedback, the average coding rate does not drop to zero since the number of retransmissions is limited. Therefore, truncated ARQ techniques are attractive alternatives to FEC for services which require limited delay but can tolerate some errors, such as voice or video services.

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