Performance of Turbo Codes in Interleaved Flat Fading Channels with Estimated Channel State Information *

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Abstract- **This paper investigates the performance of a turbo coded system transmitting over correlated flat fading channels with channel interleaving and both perfect and estimated channel state information. We show that channel interleaving is necessary on a correlated fading channel, and that performance degrades as the product of fading bandwidth and symbol duration (BT_s) decreases. We consider both Rayleigh and Rician fading, and compare the performance of Log-Maximum A Posteriori (Log-MAP) and Soft Output Viterbi Algorithm (SOVA) based decoding algorithms. It is shown that for fading channels, the performance of the Log-MAP algorithm is considerably superior to that of the SOVA algorithm. A simple method for estimating the fading amplitudes and noise variance is proposed, and the impact of its use is investigated. It is shown that estimating the fades degrades performance only slightly, and estimating the noise variance does not noticeably affect performance.**

I. Introduction

Several recent studies have demonstrated that turbo codes can achieve remarkable bit error performance over flat Rayleigh fading channels [1]-[6]. However, many of the published simulation studies make two unrealistic assumptions. The first assumption is that the fades are fully interleaved and thus the fading amplitudes are statistically independent realizations of a Rayleigh random variable. In order for this assumption to be valid, a channel interleaver is required and must have a depth greater than the ratio t_c/T_s, where t_c is the channel coherence time and T_s is the symbol duration. There are many instances when this requirement is not met, such as when communication is between a fixed base station and a slowly moving mobile. When the fading is very slow, the interleaver does not satisfactorily separate the fades and performance suffers. The second assumption is that precise estimates of the noise variance and fading amplitudes are available at the decoding algorithm. In practical systems, the channel must be estimated at the receiver. Because of the low signal to noise ratios typical of turbo coded systems, it is difficult to obtain perfect estimates of the fading amplitudes. Thus, the performance of turbo codes operating in fading environments will be degraded when the channel is estimated at the receiver.

The performance of turbo codes in the presence of non fully-interleaved flat Rayleigh fading channels has been addressed by Hall and Wilson [5],[6]. In this study it was shown that the performance of turbo codes degrades quickly as the product of fading bandwidth and symbol time (BT_s) decreases and no additional channel interleaving is used. Some methods for channel interleaving were introduced and simulation results were presented. The issue of channel estimation has been addressed by Jordan and Nichols [7]. This discussion of channel estimation for turbo codes was limited to the effect of noise variance estimation errors in conjunction with binary symmetric (BSC) and additive white Gaussian noise (AWGN) channels. It was shown that turbo codes could tolerate noise variance estimation errors of less than 3 dB in an AWGN environment, but estimation errors greater than 3 dB sharply degrade performance. No discussion was provided regarding how the noise variance estimate is obtained or how fading amplitude estimation errors affect the performance of turbo codes on flat fading channels. An alternative approach to the noise variance estimation problem was presented by Hoeher in [8], which recommends setting the noise variance equal to a "break-point" — that is to simply use the variance that corresponds to the desired operating point.

While there are many studies that investigate the performance of turbo codes over flat Rayleigh fading channels, we are not aware of similar studies concerning Rician fading. Rician fading arises when there is a direct specular component along with the diffuse energy, a situation that occurs when there is a line of sight (LOS) path.

This paper addresses some of the gaps in the literature concerning the performance of turbo codes over flat fading channels. The main objective is to investigate the effects of channel estimation on turbo coded systems operating over correlated flat fading channels with channel interleaving. To this end, we propose a simple channel estimator based on a low pass FIR filter. A general channel model is presented that allows us to consider both Rician

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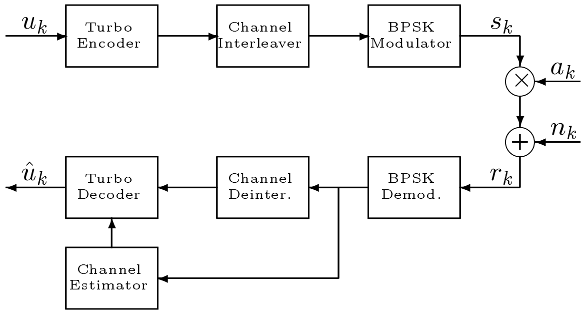

Figure 1: System model

and Rayleigh fading. Additionally, we consider the effect of the type of decoding algorithm on performance. Both the Log-Maximum A Posteriori (Log-MAP) algorithm of [9], and the Soft Output Viterbi Algorithm (SOVA) of [10] are considered.

The remainder of this paper is organized as follows. In section II we present the system model. Here the type of encoding and decoding is specified along with the channel model. In section III the effect of channel interleaving on the correlated Rayleigh fading channel is presented to motivate the use of a channel interleaver. In section IV the proposed channel estimator is presented. Simulation results showing the effect of the proposed channel estimator for several different channels and both decoding algorithms are given in section V.

II. System Model

The system model is as shown in Figure 1, and is based on the model used in [4]. Information bits u_k are grouped into frames of 1,024 bits and passed through a rate 1/2, constraint length three turbo encoder with code generator $(5/7)_8$ and random coding interleaver. The code bits are interleaved using a 32 by 64 block channel interleaver, modulated using a Binary Phase Shift Keying (BPSK) modulator, and sent over a flat fading channel. The flat fading channel multiplies each symbol s_k at the output of the modulator by a fading amplitude a_k. A sample n_k from a white Gaussian noise process with double-sided power spectral density (PSD) of $N_o/2 = \sigma_n^2$ is added to the faded symbol and passed to a BPSK demodulator. The soft output from the BPSK demodulator is deinterleaved and passed to the turbo decoding algorithm which produces estimates $\hat{u}_k$ of the data. The turbo decoding algorithm uses eight iterations of either the Log-MAP algorithm [9], or the SOVA algorithm [10] with the normalization technique of [11]. In addition, the BPSK demodulator outputs are sent to a channel estimator, which provides the turbo decoding algorithm with estimates of the fading amplitudes and noise variance.

The process that generates the fading amplitudes is a critical factor in the system model. In [2] it is suggested that if the system has sufficient interleaving, then the fading amplitudes are statistically independent. In our system model we make no such assumption and model fading using a correlated fading process. The fading process can be modeled by either a Rayleigh or Rician distribution, depending on the presence or absence of a specular component. Fading is Rayleigh if the multiple reflective paths are large in number and there is no dominant line-of-sight (LOS) propagation path [12]. If there is a dominant non-fading signal component present, such as a LOS propagation path, then fading is described by a Rician distribution.

The fading amplitude a_k is the magnitude of a complex channel gain and may be represented as [13]:

$$a_k = |(\alpha + x_k) + j(y_k)| \quad (1)$$

where α represents the amplitude of the specular component, and x_k and y_k are samples of zero mean stationary Gaussian random processes each with variance σ_f^2. The ratio of specular to diffuse energy is $\gamma = \alpha^2/2\sigma_f^2$, and the ratio of energy per symbol to one-sided noise spectral density is [13]:

$$\frac{E_s}{N_o} = \frac{(\alpha^2/2 + \sigma_f^2)T_s}{N_o} \quad (2)$$

when $\gamma = 0$ the fading is Rayleigh and when $\gamma > 0$ the fading is Rician.

Since the two Gaussian processes are (jointly) independent, $E[x_k y_k] = 0$. However, each process is self-correlated, and the particular autocorrelation function used is an important consideration. In [6] and [13] an exponential autocorrelation is used. While exponential correlation has the benefit of mathematical tractability, it is not the most realistic model. We use the following autocorrelation, which is based on Clarke's model [14]:

$$R(k) = \frac{1}{\pi B T_s} J_o\left(2\pi B T_s k\right) \quad (3)$$

where $J_o(\cdot)$ is the zero-order Bessel function of the first kind.

III. Channel Interleaving for Correlated Rayleigh Fading

The need for channel interleaving when fading is correlated is best illustrated by an example. Consider a situation where the turbo code described in the preceeding section is used over a flat Rayleigh fading channel with SOVA decoding. The bit error performance of such a scenario is shown in Figure 2 for three common values of BT_s both with and without channel interleaving. When no channel interleaving is used, the bit error performance is very poor for all three values of BT_s. By using the channel interleaver, which has a depth of 32 symbols, performance is greatly improved. With channel interleaving, the performance improves as BT_s increases, which corresponds to a faster moving mobile. The lowest bit error curve in Figure 2 corresponds to the fully interleaved case and serves as a lower bound on performance over correlated fading channels. As can be seen from this figure, the performance loss

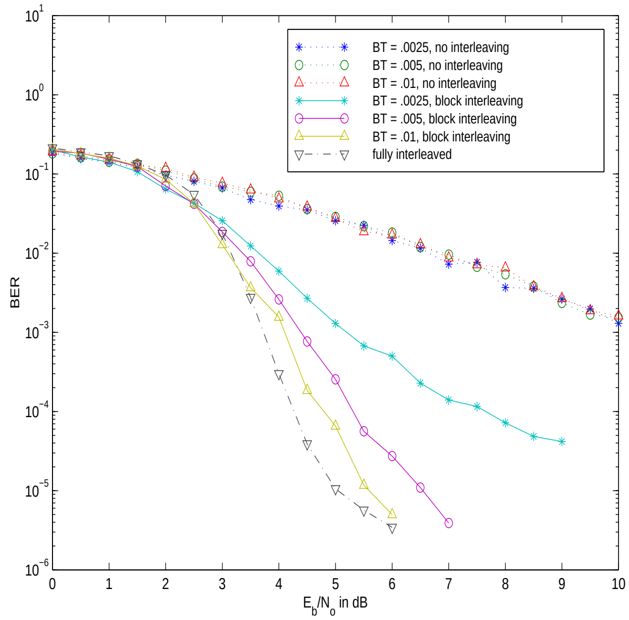

Figure 2: Bit error performance of a turbo code operating over a flat Rayleigh fading channel as parameterized by type of channel interleaving and BT_s. Turbo code is rate 1/2, constraint length 3, and uses a 1,024 bit random coding interleaver with 8 iterations of SOVA decoding.

due to correlation in the fading channel can be significant even when channel interleaving is used. Thus we recommend that the effect of channel correlation and interleaving be taken into account when investigating the performance of turbo codes operating over flat fading channels.

IV. Proposed Channel Estimator

Because of the correlation in the channel, a reasonable estimate of the fading amplitudes can be obtained by passing the absolute value of the received signal r_k through an FIR filter. The estimate of the fading amplitude $\hat{a}_k$ is thus:

$$\hat{a}_k = \sum_{n=0}^{N} h_n |r_{k-n}| \tag{4}$$

where $h_n, n = 0, 1, ..., N$ is the impulse response of the FIR filter, which has order N.

If the exact nature of the fading process is known, then an optimization procedure will yield the best filter design. An adaptive filter, such as the Kalman filter, could automatically optimize itself and track changes in the channel. Instead of seeking an optimal solution to the estimation problem, we chose to use a low pass filter with cutoff at f_d, the Doppler frequency. We found that such a filter is simple and robust, and provides good fading estimates. For the fading estimator, an FIR filter of order $N = 32$ was implemented using the Hamming window method.

While knowledge of the fading amplitudes is critical to the performance of the turbo code, the noise variance estimate is of lesser importance. In [8] it is suggested that the noise variance can simply be set to the desired operating point. For long block lengths and an additive white Gaussian noise (AWGN) channel, the bit error curve is very steep and a break-point is easily identified. However, for fading channels and a modest 1,024 bit block size, the curves are not as steep as for AWGN channels and a break-point is not readily found. A good noise variance estimate can be obtained by computing the sample variance of the quantity $e_k = |r_k| - \hat{a}_k$. The absolute value operation causes this estimate to be biased, but the bias can be removed by multiplying the sample variance by a constant c:

$$\hat{\sigma}_n^2 = \frac{c}{L-1} \sum_{k=1}^{L} ((|r_k| - \hat{a}_k) - \mu)^2 \tag{5}$$

where L is the number of code bits per block and μ is the sample mean of e_k:

$$\mu = \frac{1}{L} \sum_{k=1}^{L} (|r_k| - \hat{a}_k) \tag{6}$$

The optimum value of c depends on the signal to noise ratio and satisfies $1 < c < 2$. For our results we used a factor $c = 1.5$ for all values of E_b/N_o.

V. Simulation Results

The performance of the turbo coded system was simulated for three channels: (a) a Rayleigh fading channel (Figure 3), (b) a Rician fading channel with $\gamma = 1$ (Figure 4), and (c) a Rician fading channel with $\gamma = 2$ (Figure 5). In each case the product of fading bandwidth and symbol duration was $BT_s = .005$ and a 32 by 64 block channel interleaver was used. The turbo code used a 1,024 bit random coding interleaver, was rate 1/2 and had constraint length $K = 3$. Decoding was performed using both the Log-MAP algorithm and the normalized SOVA algorithm. In each case eight decoder iterations were used. For each type of decoder and channel model, three cases were investigated: (a) both the fading amplitudes and the noise variance are known by the decoder; (b) the fading amplitudes are estimated using the order 32 low pass FIR filter but the noise variance is known perfectly; and (c) the fading amplitudes are estimated using the FIR filter and the noise variance is estimated using 5.

By looking at Figures 3-5 several observations can be made. Recalling that for Rayleigh fading $\gamma = 0$, we see that as γ increases the performance improves. This is to be expected as higher values of γ correspond to more benign channels. For all three channels, the performance of the Log-MAP algorithm is significantly better than that of the SOVA algorithm. The performance gain of the Log-MAP over the SOVA is slightly better for lower values of γ, with the Rayleigh fading channel showing the greatest gain by using Log-MAP instead of SOVA. This can be explained by noting that Log-MAP is superior to SOVA when the signal-to-noise ratio is low; as γ gets smaller, the nulls

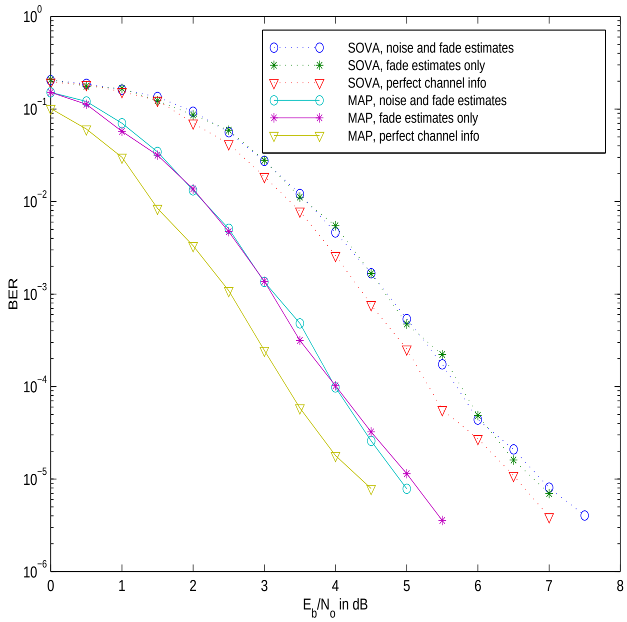

Figure 3: Bit error performance of a turbo code operating over a flat Rayleigh fading channel with $BT_s = .005$ and block channel interleaving as parameterized by type of decoding algorithm and channel estimation. Turbo code is rate 1/2, constraint length 3, and uses a 1,024 bit random coding interleaver with 8 iterations of decoding.

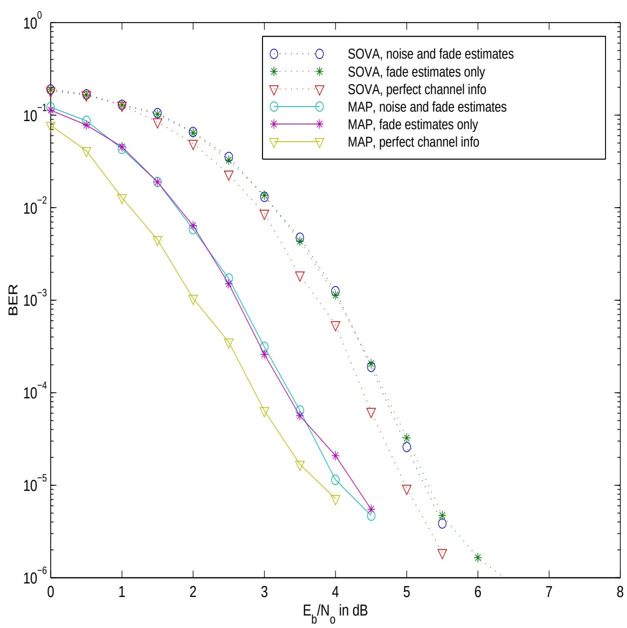

Figure 4: Bit error performance of a turbo code operating over a flat Rician fading channel with $\gamma = 1$, $BT_s = .005$, and block channel interleaving as parameterized by type of decoding algorithm and channel estimation. Turbo code is rate 1/2, constraint length 3, and uses a 1,024 bit random coding interleaver with 8 iterations of decoding.

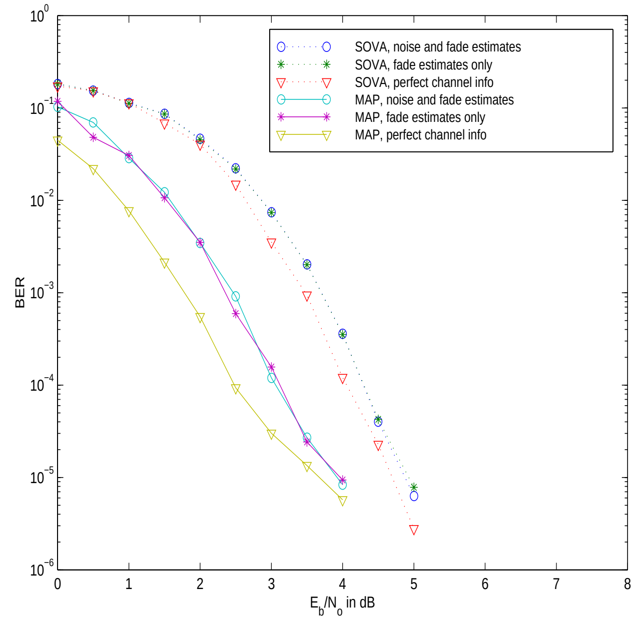

Figure 5: Bit error performance of a turbo code operating over a flat Rician fading channel with $\gamma = 2$, $BT_s = .005$, and block channel interleaving as parameterized by type of decoding algorithm and channel estimation. Turbo code is rate 1/2, constraint length 3, and uses a 1,024 bit random coding interleaver with 8 iterations of decoding.

get deeper and thus the SNR can be very poor at times. For BER of 10^{-4} and perfect channel estimates the Log-MAP outperforms the SOVA by between 1.5 dB (Rician, $\gamma = 2$) to 2.0 dB (Rayleigh). For this reason we reiterate the suggestion in [8] and recommend using the Log-MAP algorithm to decode turbo codes, especially if the channel is flat faded. Compared to the SOVA algorithm, the increase in decoder complexity required by the Log-MAP algorithm is more than compensated for by its superior performance.

For all the cases considered, there is a slight performance penalty when estimating the fading amplitudes using the proposed order 32 low pass FIR filter. This penalty is about 0.25 dB for the SOVA algorithm at a BER of 10^{-4}. The penalty for using the fading estimator is more severe for the Log-MAP algorithm; at a BER of 10^{-4} we observe a loss of 0.5 dB in the Rician fading channels and 0.75 dB for the Rayleigh fading channel. Apparently the sensitivity of the Log-MAP algorithm that allows it to perform well in low SNR environments also makes it more vulnerable to fading estimation errors. Despite this shortcoming, the Log-MAP algorithm still considerably outperforms the SOVA even when the fading amplitudes are estimated. We also considered a reduced complexity estimator consisting of an order 8 low pass FIR filter, but found the penalty for using it to be about double (in dB) that for using the order 32 filter; the performance of this estimator is not shown here. In all cases we see that estimating the noise variance does not noticeably effect the performance.

VI. Conclusion

In this paper, we investigated the performance of a typical turbo code transmitted over various channels using both the Log-MAP and SOVA decoding algorithms. The channel models incorporate the concepts of correlated fading, channel interleaving, and the presence of a Line-of-Sight (LOS) component. We showed that channel interleaving can be used to mitigate the detrimental effects of correlation in the channel. Even when channel interleaving is used performance degrades as the product of fading bandwidth and symbol duration BT_s decreases, or in turn when the relative velocity between transmitter and receiver gets small. The performance of the Log-MAP algorithm is superior to that of the SOVA algorithm, especially when there is no LOS component.

Additionally, the impact of channel estimation was studied. A simple fading amplitude estimator based on a low pass FIR filter was proposed and the effect of its use was investigated. By using this estimator a slight 0.25 dB penalty was observed for the SOVA and a more severe 0.5-0.75 dB penalty was observed for the Log-MAP. No attempt was made to optimize the estimator and we welcome others to improve its design. In particular, the impact of performing channel estimation using a Kalman filter should be explored. The fading estimator was modified to produce a noise variance estimate, and use of such an estimate did not noticeably affect performance.

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