ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAF045

PERFORMANCE OPTIMISATION OF TURN-BY-TURN BEAM POSITION MONITOR DATA HARMONIC ANALYSIS

L. Malina[1], J. Coello de Portugal, J. Dilly, P.K. Skowroński, R. Tomás, M. Toplis
CERN, Geneva 23, Switzerland
[1] also at University of Oslo, 0316 Oslo, Norway

Abstract

Nowadays, turn-by-turn beam position monitor data is increasingly utilized in many accelerators, as it allows for fast and simultaneous measurement of various optics parameters. The accurate harmonic analysis of turn-by-turn data costs beam time when needed online. Generally, the electronic noise is avoided by cleaning of the data based on singular value decomposition. In this paper, we exploit the cleaning procedure to compress the data for the harmonic analysis. This way the harmonic analysis is sped up by an order of magnitude. The impact on measurement accuracy is discussed.

INTRODUCTION

Optics measurements in storage rings can be performed by exciting the beam and acquiring turn-by-turn (TbT) beam position monitor (BPM) data of the coherently oscillating beam [1]. In the analysis process, TbT BPM data is first cleaned, which reduces the amount of information. Later the harmonic analysis is performed on cleaned TbT data BPM by BPM. A framework presented here implements new methods to increase the speed of harmonic analysis. This framework replaces SUSSIX [2, 3] in optics measurements analysis software in the LHC. It also implements BPM by BPM harmonic analysis, further referred to as "bpm" method.

SINGULAR VALUE DECOMPOSITION

In order to improve analysis precision and accuracy, TbT data needs to be cleaned of the noise, for example BPM electronic noise. This is done using methods [4–6] based on Singular Value Decomposition (SVD). The SVD of a matrix A is given by

$$A = USV^T, \tag{1}$$

where columns of U and V are normalized eigenvectors of $A^T A$ (left-singular vectors) and AA^T (right-singular vectors), respectively. S is a positively definite diagonal matrix of singular values ordered in decreasing order. The TbT BPM data decomposition contains all temporal and spatial information about physical modes of beam motion. The noise floor removal is performed by keeping only the modes corresponding to the largest singular values, as shown in Figure 1. Table 1 presents the typical TbT matrix dimensions in the LHC, N_{BPMs} and N_{turns}, together with N_{modes}, number of singular values to be kept. This reduces the size of data and information. Matrix A with dimensions (500x6600) is approximated by USV^T matrices with dimensions: (500x12), (12x12) and (12x6600). In the second step, TbT data are recomposed by matrix multiplication of the reduced matrices. The size of the data after the recomposition is the same as the input one (500x6600), which is about a factor 40 larger than the reduced USV^T matrices. However, the amount of information is still reduced.

Table 1: Typical Parameters of TbT Data and its SVD Cleaning

N_{BPMs}	No. BPMs (per plane)	500
N_{turns}	No. turns acquired	6600
N_{modes}	No. singular values	12

HARMONIC ANALYSIS

The actual lattice properties are contained in the frequency information of TbT BPM data. The Discrete Fourier Transform (DFT) is obtained performing Fast Fourier Transformation (FFT) on the cleaned TbT data from a single BPM x_n:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N} \tag{2}$$

The equation has the form of inner product of x_n and $e^{-i2\pi kn/N}$, which means that X_k is a multiplicative complex coefficient of a signal with frequency k/N. In case of FFT k is an integer smaller than N. The refined frequency of the strongest signal is found using Jacobsen frequency interpolation with bias correction [7] based on 3 DFT peaks (the maximal amplitude $|X_{k_p}|$ and two neighbouring samples $X_{k_p \pm 1}$):

$$\delta = \frac{\tan(\pi/N)}{(\pi/N)} Real\left[\frac{\left(X_{k_p-1} - X_{k_p+1}\right)}{\left(2X_{k_p} - X_{k_p-1} - X_{k_p+1}\right)}\right], \tag{3}$$

where $\delta \in [-0.5, 0.5]$ is a correction to the frequency of DFT peak. The refined complex amplitude of the signal is obtained from the inner product of a unit signal with pure frequency $\left(k_p + \delta\right)/N$ with the TbT data:

$$X_{k_p+\delta} = \sum_{n=0}^{N-1} x_n e^{i2\pi n\left(k_p+\delta\right)/N} \tag{4}$$

This signal is subtracted from the TbT data and the whole procedure starting with FFT is repeated [8], in the LHC typically 300 times. As a result the TbT data is approximated by the sum of the 300 strongest harmonics $h \sum_{j=0}^{299} h_j$. The basis forms a linear vector space.

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAF045

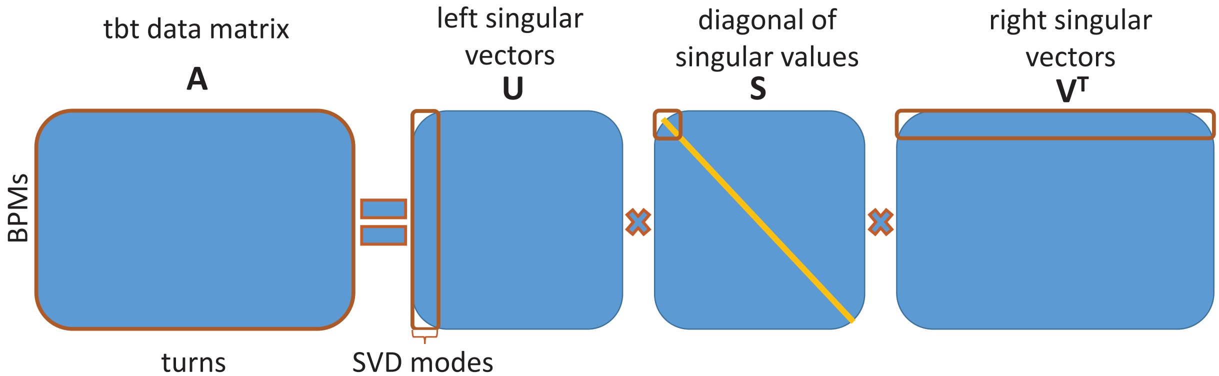

Figure 1: Sketch of the TbT data recomposition for noise floor removal. The decomposition of raw TbT data (in blue) gives full matrices, which are then reduced, as only several largest singular values are kept for the clean TbT data recomposition (in brown edges).

HARMONIC ANALYSIS OF DECOMPOSED DATA

As SVD and refined harmonic analysis are both linear operations, they can be combined. The cleaned TbT data (Equation (2)) can be reconstructed from SVD matrices elements:

$$x_{jn} = \sum_{l=0}^{N_{modes-1}} u_{jl} s_{ll} v_{nl}, \tag{5}$$

where j is the BPM number. The complex coefficients corresponding to an arbitrary frequency a/N are given by:

$$X_{ja} = \sum_{n=0}^{N-1} \sum_{l=0}^{N_{modes-1}} u_{jl} s_{ll} v_{nl} e^{i2\pi na/N} = \tag{6}$$

$$= \sum_{l=0}^{N_{modes-1}} u_{jl} \sum_{n=0}^{N-1} s_{ll} v_{nl} e^{i2\pi na/N}, \tag{7}$$

second summation represents the complex coefficient corresponding to frequency a/N in a l^{th} row of SV^T.

Putting all this together, we obtain complex coefficients corresponding to frequency a/N in cleaned TbT data from all BPMs as a linear combination of complex coefficients corresponding to the same frequency in the rows of SV^T with the multiplication factor being the rows of U. In order to measure the frequencies, two algorithms were developed, performing harmonic analysis on:

- the sum $\sum_{l=0}^{N_{modes}-1} s_{ll} v_{nl}$ of the rows of reduced SV^T giving a single set of frequencies, hereafter referred to as "fast" method
- each of the rows of reduced SV^T, giving a union of frequencies, found for every row, hereafter referred to as "svd" method

The complex coefficients, corresponding to such sets of frequencies, are calculated for each of the rows of reduced SV^T matrix by the inner product in time domain (last sum in Equation (7)). At this point the vectors in frequency domain are no longer orthogonal. The perturbation of the orthogonality of the two vectors (in frequency domain) is influenced by two factors:

- the difference (in the time domain) between the vector under study and the vector the harmonic analysis was performed on
- the spectral response of a windowing function, that can be used to filter the signal in time domain

A rectangular window, which does not change the signal and has the best frequency resolution, is used in the following. On the other hand, the rectangular window has larger spectral response in other frequencies compared to other windowing functions [9], which should be kept in mind.

ACCURACY

The accuracy of the harmonic analysis performed on decomposed TbT data is studied in this section. TbT data matching the LHC lattice injection optics was simulated along with the betatron resonances of known frequency, phase and amplitude. Realistic noise of about 8 % amplitude compared to coherent betatron motion at focusing quadrupoles was added. Results of the afore-mentioned analysis corresponding to a given spectral line consist of its frequency $\in (0, 1)$, initial phase in units of 2π and its amplitude. The accuracy is estimated by the root mean square of the difference to the value defined in a simulation in a set of all BPMs. The two methods ("svd" and "fast") are compared to the original harmonic analysis the "bpm" method. The betatron tune is found in the spectra from all three methods. Accuracies of the frequency and phase of the found betatron tunes as a function of number of turns are shown in Figures 2 and 3.

Both "svd" and "fast" methods have comparable accuracy or slightly better accuracy in frequency and phase compared to the "bpm" method. An exception is the "svd" analysis performed on a low number of turns, where it shows less accurate results. The differences in relative amplitude accuracy are not shown as they are negligible. Generally, "svd" and "fast" methods seem to be better suited for larger number

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAF045

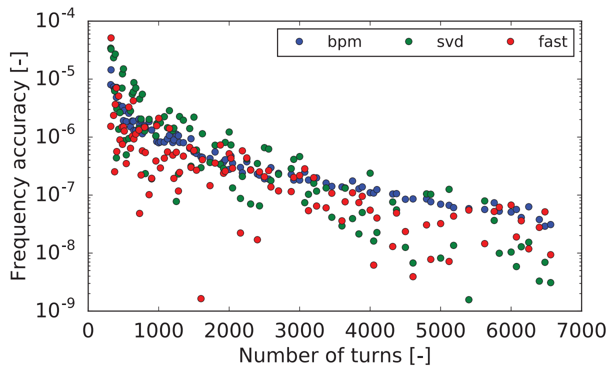

Figure 2: Accuracy of the betatron tune (the strongest spectral line) frequency as a function of number of turns.

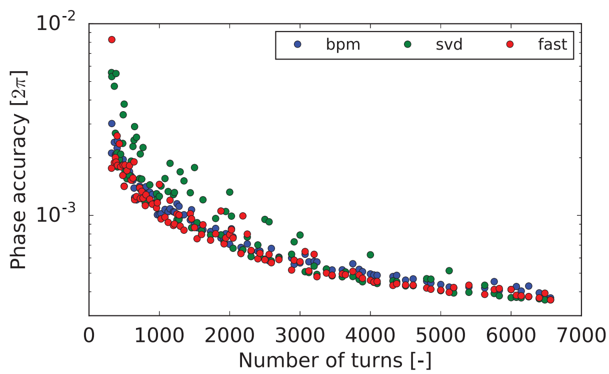

Figure 3: Accuracy of the betatron tune (the strongest spectral line) phase as a function of number of turns.

of turns and larger noise levels. For small number of turns or small noise levels, the situation is opposite. Additionally, a weaker spectral line with about 14 % amplitude at focusing BPMs and 0.01 away in frequency from the betatron tune was investigated. Here, the methods perform all similarly in terms of frequency accuracy, as shown in Figure 4. In terms of its phase accuracy, shown in Figure 5, the "bpm" method is better than the other two. The amplitude accuracy shows similar behaviour as the phase accuracy.

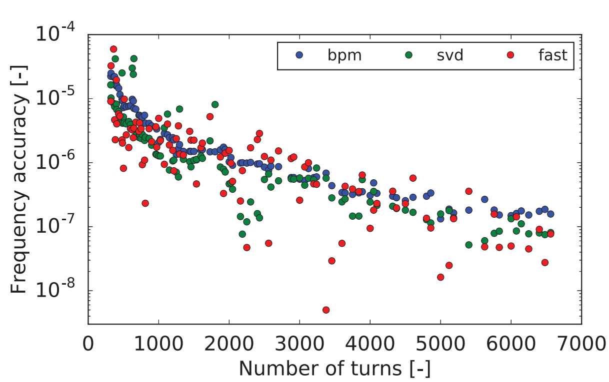

Figure 4: Frequency accuracy of weaker spectral line as a function of number of turns.

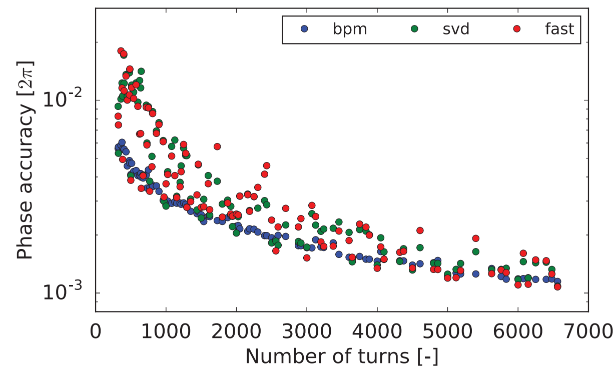

Figure 5: Phase accuracy of weaker spectral line as a function of number of turns.

SPEED UP

Harmonic analysis performed on decomposed TbT BPM data is faster by up to a factor of N_{BPM}/N_{modes} using the "svd" method and up to a factor of N_{BPM} in the "fast" method. Harmonic analysis by "fast" method of one set of LHC TbT BPM data takes about 2 seconds in a single thread compared to about 18 seconds in 32 threads in the "bpm" method.

CONCLUSIONS AND OUTLOOK

New techniques combining data cleaning together with harmonic analysis have been developed. Their usage results in a speed up by factor about 300 in the LHC. This is possible by analysing decomposed data directly instead of the recomposed data. The analysis is comparably or more accurate in terms of frequency. However currently, it is less accurate in terms of phase and amplitude of smaller spectral lines. This can be potentially overcome by the choice of a windowing function, addressing the orthogonality perturbation, which will be studied. It needs to be stressed, that both new algorithms are better suited for noisy data, compared to standard method.

REFERENCES

[1] R. Tomás, M. Aiba, A. Franchi, and U. Iriso, “Review of linear optics measurement and correction for charged particle accelerators”, *Phys. Rev. Accel. Beams*, vol. 20, p. 054801, 2017.

[2] R. Bartolini and F. Schmidt, “A Computer Code for Frequency Analysis of Non-Linear Betatron Motion”, CERN, Geneva, Switzerland, Rep. CERN-SL-NOTE-98-017-AP, 1998.

[3] T. Bach and R. Tomás, “Improvements for Optics Measurements and Corrections Software”, CERN, Geneva, Switzerland, Rep. CERN-ACC-NOTE-2013-0010, 2013.

[4] J. Irwin et al., “Model-Independent Beam Dynamics Analysis”, *Phys. Rev. Letters* Vol. 82, p. 1684, 1999.

[5] R. Calaga and R. Tomás, “Statistical Analysis of RHIC beam position monitors performance”, *Phys. Rev. ST Accel. Beams*, vol. 7, p. 042801, 2004.

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAF045

[6] X. Huang, S. Y. Lee, E. Prebys and R. Tomlin, "Application of independent component analysis to Fermilab Booster", *Phys. Rev. ST Accel. Beams*, vol. 8, p. 064001, 2005.

[7] Ç. Candan, "A method For Fine Resolution Frequency Estimation From Three DFT Samples", *IEEE Signal Procces. Lett.*, vol. 18, p. 351-354, 2011.

[8] J. Laskar, "Frequency analysis for multi-dimensional systems. Global dynamics and diffusion", *Physica D* vol. 67, p. 257-281, 1993.

[9] G. Heinzel, A. Rüdiger and R. Schilling, "Spectrum and spectral density estimation by the Discrete Fourier transform (DFT), including a comprehensive list of window functions and some new flat-top windows", 2002, http://edoc.mpg.de/395068