CHAPTER 8

PERIOD BY THE WAVE-GROUP METHOD

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ABSTRACT

The wave-group period, T_g, is obtained by a simple manual procedure from the periodic wave groups and sequences that appear in strip-chart records. A dozen or more values are commonly derived from a 20-minute record. When plotted as a time-series, T_g values form patterns that represent individual wave trains, each generated by a synoptic weather event. Swell trains and wind-wave trains can be distinguished by their characteristic period distributions.

For swell trains, the mean wave-group frequency, $\overline{f}_g$, obtained from the best-fit line to the time-series plot of f_g values (reciprocal T_g values), is equivalent to the frequency of maximum energy density, f_m, obtained from spectral analysis. The distance and time of origin for a swell train can be determined from the time rate of change of $\overline{f}_g$, and with the use of weather maps the generating area can be identified and the deep-water arrival direction of the swell train at the wave gage determined.

The zero-line crossing period, T_z, represents an integration of the waves from all wave trains present and has no synoptic significance.

INTRODUCTION

Period is ordinarily obtained from a wave record by computer analysis of the wave information in digitized form to produce an energy density spectrum, or by manual analysis of an analog trace written on a strip chart to produce one or more of a half dozen kinds of period measures. Computer algorithms have also been developed for picking off crests, troughs, and zero-line crossings from analog data for use in obtaining several of the latter period measures.

Spectral analysis has an important advantage over other methods in that it yields the energy-density distribution over the full range of frequencies considered, although from this writer's observations it is only the period associated with the dominant frequency peak present that is used in most practical applications. In addition, there is adequate demonstration of the fact that prominent maxima in the spectrum represent wave trains from individual generating areas. The latter frequently can be followed in the wave records for several days.

Among the non-spectral procedures for processing analog records, the wave-group method has received little attention in the literature (Harris, 1970; Thompson, 1970). Nevertheless, as will be demonstrated, the

wave-group period is equivalent to the spectral peak period for swell, and in addition it is similarly identified with individual wave trains and is therefore a synoptically significant period parameter. The zero-line crossing method, which may be regarded as the standard non-spectral method of analysis by reason of its common use and frequent reference in the literature, is also examined in relation to the wave-group and spectral methods.

The results presented in this paper are principally drawn from the analysis of swell generated in moving cyclonic storms and propagated over distances of several hundred miles or more to wave gages on the California coast located in the general direction of storm advance. The data were recorded using pressure-type sensors mounted in 25 to 40 feet of water.

Period and frequency are referred to interchangeably throughout the paper in accord with conventional useage and ease of visualization.

WAVE-GROUP METHOD

Those who work with analog strip-chart records are familiar with the sequences of more or less periodic waves that appear at intervals in the record. Examples are illustrated in Figure 1. All wave sequences that can be identified in a record are here termed wave groups, whether they appear as discrete groups, as in Figure 1(a), or combined with other waves to form an irregular profile.

The wave-group period, T_g, can be obtained by identifying the first and last waves in a periodic sequence, and dividing the time interval between by the number of waves in the sequence. The measurement is made between those equivalent parts of the wave form that the analyst judges to give the best measure in each case, e.g., between selected crests, troughs, centers of wave mass, or zero-line crossings. Measurement between centers of mass are most useful when the wave shape is irregular and crest or trough measures are unsatisfactory (Figure 1(b)). Zero-line crossings should in general be avoided because most wave sequences are not symmetrical with respect to the zero line of the record due to super-imposition on other waves that are ordinarily present. Occasionally, more than one wave sequence can be identified from the same time interval (Figure 1(d)). A quick check of the period of each wave in the sequence is recommended to avoid sequences containing phase changes.

The identification of periodic sequences requires judgment and seems at times to be quite subjective; nevertheless, even T_g values derived from waves of highly irregular form ordinarily prove to be associated with a recognized wave train. Rapid improvement in the identification and evaluation of wave sequences comes with a little practice, and the measurement of T_g for a selected sequence is a very quick operation. Best results in both quantity and quality of the data obtained requires the judgment of an analyst having a basic understanding of the fundamentals of wave interference, such as are given by Manley (1945).

A 20-minute record will frequently contain a dozen or more sequences of 3 or more waves. The T_g values obtained should be identified whenever

possible with individual wave trains present, as described in the next
section. Averaging of the values from a given wave record without
segregating them will yield an integrated period measure of uncertain
significance, and a primary advantage of the wave-group method will have
been discarded. The wave-group method is meant here to include both the
mechanics of obtaining the wave-group period from a wave sequence and
the association of the T_g values with wave trains present so as to yield
synoptically significant period information.

SYNOPTIC WAVE TRAINS

Values of T_g plotted as a time series usually form well-defined patterns,
each representing an individual wave train generated by a synoptic weather
event, such as a cyclonic storm, occurring locally or at a distance from
the recording station. These synoptic wave trains can ordinarily be
recognized, from their distinctive period distributions, either as swell
generated at a distance from the wave gage or as wind waves and associ-
ated young swell of local origin. Wave trains may appear singly in the
T_g data, but two or more frequently are present at once. Most of the
time individual trains can be distinguished from one another, but at times
their period distributions merge and cannot be separated, and at other
times during periods of very low waves the T_g values fall in a random
pattern across the spectrum and no wave trains can be recognized. The
identification of wave trains and their recognition as sea or swell from the
period data are usually correctly confirmed by the associated wave-height
data and weather maps.

Figure 2 presents values of T_g derived from 5-minute records made
hourly for 7 days at Monterey, California, and reveals three successive
wave trains arriving during otherwise quiet sea conditions. The continuous
decrease of period with time from the initial appearance of each train
clearly marks them as swell. The T_g distribution in a typical swell train
also displays a relatively small scatter about the mean, which is consistent
with the narrow spectral bandwidth character of swell. About the time of
appearance of the first T_g values in a newly arriving swell train, the wave
height normally rises above the energy level of the existing waves; however,
on uncommon occasions during intervals of very low wave height (half a
foot or less) the period distribution has unmistakably revealed the presence
of a swell train when its height did not rise above the energy level of the
background. When one swell train replaces another in time, the T_g values
associated with each train during the interval of overlap (when the energy
levels are of the same order of magnitude) may appear either as independent
distributions, as in the case of Trains 1 and 2 of Figure 2, or they may form
a continuous transition zone, as occurs from Train 2 to Train 3. Whether
the period distribution is continuous or discontinuous appears to be deter-
mined by the magnitude of the period separation between the trains. The
swell properties described here are typical of swell generated in approaching
cyclonic storms.

Wind waves and associated local swell are revealed in the T_g data by the
initial appearance of short periods, which increase with time as the sea
grows until the wave height peak is reached. The T_g values display wide
scatter, consistent with the broadband spectral nature of a sea, but tend to
be concentrated near the upper envelope of the distribution. Following

passage of the wave peak, the T_g values for the young swell trailing in after the wind has fallen off diminish with time, and the period distribution takes on more the appearance of a distant swell train. No clearcut example of a wind-wave train is available for illustration.

T_g values obtained from the analysis of a series of wave records can be put to practical use best by fitting a curve to the time-series data associated with each wave train identified. In the case of a swell train, a curve fitted to the T_g values will represent the mean wave-group period for the train, $\overline{T}_g$. If reciprocal f_g values are plotted instead, as shown in Figure 3 for the swell trains of Figure 2, the best-fit curve will be, similarly, the mean wave-group frequency for the train, $\overline{f}_g$; this curve should properly be linear for distant swell trains, as discussed in a later section. The $\overline{f}_g$ curve, as will be shown in the next section, agrees closely with the frequency of maximum energy density, f_m, for the swell train as determined by spectral analysis. In the case of wind-wave trains, no comparative analyses of the same wave records have been made by the wave-group and spectral methods; however, inductive evidence indicates that a curve drawn near the upper envelope of the T_g distribution and through the densest concentration of values will give a reasonable approximation of T_m for the wave train.

In this manner, a time series of T_m curves (or f_m curves) can be produced from which the user can pick off the value or values prevailing for any selected time. When more than one wave train is recognized to be present at a given time, the dominant train can be identified by the fact that its period will be associated with the wave sequences of largest amplitude. The dominant train can often be identified also from examination of the wave-height distribution in relation to the period data. For most practical applications, when more than one wave train is present the ocean surface can be characterized by using the observed wave height along with the period of the dominant train.

EQUIVALENCE OF $\overline{f}_g$ AND f_m FOR SWELL

In the case of swell trains generated in wind areas located several hundred miles or more from the recording station, the mean wave-group frequency, $\overline{f}_g$, and the spectral peak frequency, f_m, derived from the same wave records by the respective methods of analysis have been found to agree closely in all comparisons made.

By way of demonstration, a swell train recorded at Monterey during otherwise quiet wave conditions was selected, and 20-minute strip-chart records made every 6 hours for its 2-day life were analysed by the two methods. The analog traces, written at a chart speed of 2 inches per minute, were converted for spectrum computation to digital data using a digitizer, and analysis was performed using the Blackman-Tukey (1958) method. The energy densities were computed to an upper limit of 0.24 Hertz using a frequency increment of 4.16 milliHertz, with 90 per cent confidence limits estimated at 0.60 and 2.10. The results are presented in the form of spectral density topography in Figure 4.

The prominent ridge in the topography extending across the lower part of the figure represents the swell train of interest. The frequency along

the ridge line, or spectral peak, is f_m, and is seen to increase with time in the characteristic manner of swell. The swell train reached its maximum height of two feet at 0200Z on 29 November. Several hours later, a new swell train with considerably more energy and of lower frequency began to arrive and caused the frequency peak to shift (just off the graph) in the same manner as shown in Figure 3 between Trains 2 and 3.

Values of f_g obtained from periodic sequences in the wave records are shown as dots in Figure 4, and are seen to be concentrated about the ridge line of the swell train in the lower part of the graph and to be scattered about a dying wave train in the upper left corner. This association of the f_g values with the spectral masses of the two trains is also illustrated in Figure 6, which shows the 14 f_g values obtained from Wave Record No. 2.

The f_g values shown in Figure 4 for the swell train of interest were averaged to obtain $\overline{f}_g$, and these values are plotted in Figure 5. Also plotted are the values of maximum energy density frequency, f_m, obtained from the spectral computation for each wave record. The two sets of values are also tabulated in Table 1. f_m exceeded $\overline{f}_g$ in six of the seven records by an average amount, expressed as period, of 0.3 seconds. Agreement within one second is typical of all comparisons we have made on swell trains.

$\overline{f}_g$ for a given wave record can be obtained, as just illustrated, by averaging the f_g or T_g values associated with the swell train of interest. It can also be obtained in the form of the mean or the median of the distribution of frequency or period of the individual waves composing the appropriate wave sequences. These procedures each give results that agree closely. The simplest and most practical means of obtaining $\overline{f}_g$ for an entire swell train, however, is to fit a straight line visually to the time plot of the f_g values for the train, as illustrated in Figure 3.

It is pertinent here to note that Harris (1970) compared six kinds of period measures and found the highest correlation to occur between the most prominent period in a 7-minute wave record and the spectral peak period. He stated that the period correlation might be improved if the data were stratified in some way so as to make the data samples more homogeneous. The present study indicates that stratification of a basic sort can usefully be performed on Pacific Coast wave data by differentiating between swell and wind waves where possible before manipulating the data, and that correlation can be further improved by use of a larger sample of periods such as is derived from the wave sequences appearing in wave records.

ZERO-LINE CROSSING METHOD

The zero-line crossing method (also called zero-upcrossing and zero-crossing method) involves determining the mean water level or zero line, usually visually, then counting the number of intervals between upcrossings (or downcrossings) of the zero line and dividing this number into the duration of the wave record to obtain T_z. The zero-line crossing method and the wave-group method are similar in that both basically involve the measurement of wave intervals relative to a datum surface, but differ in

that the datum used in the first method is of constant elevation and repre-
sents the still water level for the entire wave record (after the tides are
filtered out) and the datum in the second method is the ordinarily irregular
water surface on which a wave sequence is superimposed. Because of the
difference in datums, T_z can be obtained by use of a computer algorithm
that picks off zero crossings (Wilson, Wu, and Baird, 1972), but the task
of discriminating individual wave sequences and measuring T_g remains
beyond the accomplishment of computer programming.

The relationship of the zero-line crossing period, in the form of its
reciprocal, f_z, to $\bar{f}_g$ and f_m is illustrated for the swell train of interest
in Figures 5 through 7. In Figure 5, and in Table 1, f_z is seen to differ
considerably from the other two values and to display large variability
from record to record. In the first four wave records f_z migrates from
near the spectral peak of the dying wave train in Record No. 1 toward the
spectral peak of the growing swell train. Subsequently, f_z roughly approxi-
mates $\bar{f}_g$ and f_m for the dominant swell train. The relationship of f_z to the
spectra of the first four records is best shown in Figure 7.

This example serves to illustrate the fact that T_z is not equivalent to $\bar{T}_g$
or T_m in the case of a swell train (wind waves have not been examined
sufficiently), and that it has no synoptic significance, i.e., it gives no
information about the number of wave trains present, their dominant
periods, or their identification as sea or swell, and unlike $\bar{T}_g$ and T_m it
yields no information about the wind field that produced the swell (discussed
in next section). Since a single period value is derived from a given record,
T_z is clearly an integration of the waves associated with all wave trains
present. It is evident that the zero-line crossing period should be used
with these characteristics and limitations in mind.

In other evaluations, Harris (1970) found a poor correlation between
the zero-line crossing period (T_{ZUC}) and the period corresponding to the
frequency of maximum energy density per unit frequency (T_{FM}) from wave
records recorded at Atlantic City, and presented a short series of compara-
tive data showing T_z to be smaller than T_m in every record analysed.
Wilson, Wu, and Baird (1972) compared T_z and T_m from recordings made
at Western Head, Nova Scotia and Ocean Station Papa and stated they found
no strong correlations.

SOME APPLICATIONS

Swell Source and Direction Determination

Using spectrally analysed wave data, Munk, et al., (1963), showed that
swell trains arriving from generating areas located at distances greater
than a few hundred miles from the wave gage are revealed on a frequency-
time graph by ridges in the spectral energy density topography, and noted
that the ridge axes are linear or very nearly so. A straight ridge line
implies a point source for the swell train in space and time. In view of the
fact that a storm system produces swell for the duration of its passage over
the ocean, the point source computed from wave records appears to repre-
sent that location and moment in the storm history at which the maximum
energy density in the sea directed toward the wave gage was generated.

The distance from the wave gage to the effective swell source, D_O, can be computed from the slope of the ridge line, df_m/dt, using the relationship

$$D_o = \frac{g}{4\pi \, df_m/dt}$$

or

$$D_o = \frac{1.515}{df_m/dt}$$

where D_O is in nautical miles, f_m is in Hertz, and t is in hours. The effective origin time, t_o, is given by the intercept of the ridge line with zero frequency.

In view of the equivalence of wave-group frequency and spectral peak frequency in swell, it is evident that the effective origin of a swell train can also be computed using frequencies obtained manually through application of the wave-group method. In regard to the swell train presented in Figure 4, for example, a straight line fitted to the f_g data has a slope of 0.4220 milliHertz per hour, giving a source distance of 3590 nautical miles from the wave gage at Monterey and an origin time of 0918Z/23 November, or about 5-1/2 days before the swells were first detected. A straight-line fit to the spectral peak frequencies derived from the same wave records (Figure 5) gives $df_m/dt = 0.4175$ milliHertz per hour, $D_O = 3622$ nautical miles, and $t_o = 0654Z/23$ November. The swell source in space and time computed from the two data sets is seen in this example to agree within approximately $1/2°$ latitude and 2-1/2 hours.

With regard to application of this procedure, it should be pointed out that the peak frequency theoretically can shift as a wave train passes through shoal water to the wave gage due principally to differential amplification of the spectral components by refraction. A significant shift could produce an erroneous computed swell source; however, it can be demonstrated that because of the narrow-banded character of swell spectra no shift effectively occurs, even when the rate of change of the refraction factor with frequency for a given shoal-water site is large. It may be concluded that nearly everywhere in shoal water the peak frequency of swell is conserved for purposes of swell origin computation.

Records from a single wave gage provide no directional information that can be used to locate the wind area in which the swell was generated. The storm responsible can be identified, however, using the computed origin distance and time derived from the wave records combined with weather charts. An arc of great-circle radius D_O drawn on a weather chart of or near time t_o will be found to intersect the storm system. The effective swell source point within the storm can be taken as the point of intersection of the arc of radius D_O and the axis of the maximum winds directed toward the recording station, as illustrated in the example in Figure 8.

When the location of the source point is established, a great-circle trajectory drawn from the point to the wave gage may be considered for practical engineering purposes to give the deep-water swell direction Ψ_o, for the wave-gage site (and for any other location on the adjacent coast).

Because the wave-group method yields information equivalent to that from spectral analysis for determining swell origin and deep-water swell direction, this method has particular attraction as a practical tool in economic situations and geographic regions where computer facilities are unavailable or technical expertise is limited. In this regard, it may be noted that a wave gage is not a necessity, since the mean wave-group period of the dominant swell train present can be obtained from the larger wave sequences by an observer with a stopwatch.

Additional discussion of the applications described here is given by Thompson (1970) and Austin (1972).

Surf Forecasting

Height and period measured during the early stages of arrival of a swell train can be used to make a forecast for the remainder of the train. Measurement of the dominant period, $\overline{T}_g$ or T_m, from wave records or visually over an interval of 6 to 12 hours when converted to frequency will give the rate of change of frequency with time, from which can be obtained an accurate forecast of frequency until the train has passed. The swell height can be predicted to increase continuously until the peak is observed to pass, then to decrease continuously until the train is gone or masked by a new train. Swell trains generated in distant wind-areas are generally observed to last 2 to 5 days. This procedure has proven useful to the Marine Corps, Camp Pendleton, California for forecasting surf.

AREAS FOR FURTHER INVESTIGATION

The distinctive period signature of oceanic swell trains described herein is typical of swell recorded on the west coast of North America. These waves originate in generally approaching cyclonic storms which cross the Pacific in the latitudes of the prevailing westerlies in both hemispheres. Similar swell properties can be expected on any coast where an approaching storm situation occurs, i.e., on most coasts of the oceans. This investigator has not examined wave records from coasts which receive swell trains generated in retreating storms, as occurs on the east coasts of the continents in middle and higher latitudes. There is reason to believe that the period signature of such swell trains may be different, and this question merits investigation.

It is evident from the limited attention given in this paper to wind-wave trains that the relationship between T_g values and wind-wave spectra obtained from analysis of the same wave records needs to be probed further.

The height of the larger waves composing a wave sequence is observed to be related to the height of the wave train with which they are associated in the case of swell. When more than one swell train is present, the dominant train is readily identified by its larger wave groups. This relationship should be examined quantitatively. Possible relationship between the height of a wave sequence and the energy density in the frequency band represented by the wave sequence also appears to merit investigation, and may prove of particular significance in regard to boardband spectra.

There is an important area of engineering application in which the wave-group method may provide information that cannot be provided by computation of the spectrum. Wilson, Wu, and Baird (1972) pointed out that "some marine structures react to individual or groups of waves rather than spectra and there is a real requirement for parameters other than r. m. s. water surface deviation, the moments of the spectrum and some sort of characteristic period". It would appear that such parameters should be developed from knowledge of the wave sequences appearing in wave records. The properties of wave groups and their relationship to the wave train have not been examined to the writer's knowledge beyond what is presented in this paper.

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Table 1: SWELL TRAIN PERIOD (FREQUENCY) BY DIFFERENT METHODS

Wave Record No.	Arrival Time/Date (GMT/1971)	Period in seconds (Frequency in 10^{-5} Hertz)		
		Spectral T_m (f_m)	Wave-Group $\overline{T}_g$ ($\overline{f}_g$)	Zero-Line T_z (f_z)
1	1204/27 Nov	21.83 (4580)	22.55 (4434)	12.24 (8167)
2	1808/27 Nov	20.70 (4830)	20.95 (4773)	12.85 (7784)
3	2355/27 Nov	19.69 (5080)	19.98 (5005)	13.82 (7236)
4	0600/28 Nov	19.05 (5250)	18.90 (5290)	17.40 (5749)
5	1207/28 Nov	17.79 (5620)	18.11 (5522)	15.28 (6546)
6	1810/28 Nov	17.54 (5700)	17.56 (5695)	16.45 (6079)
7	2356/28 Nov	16.21 (6170)	16.54 (6046)	14.30 (6991)

Note: The zero-line method integrates all wave trains present.
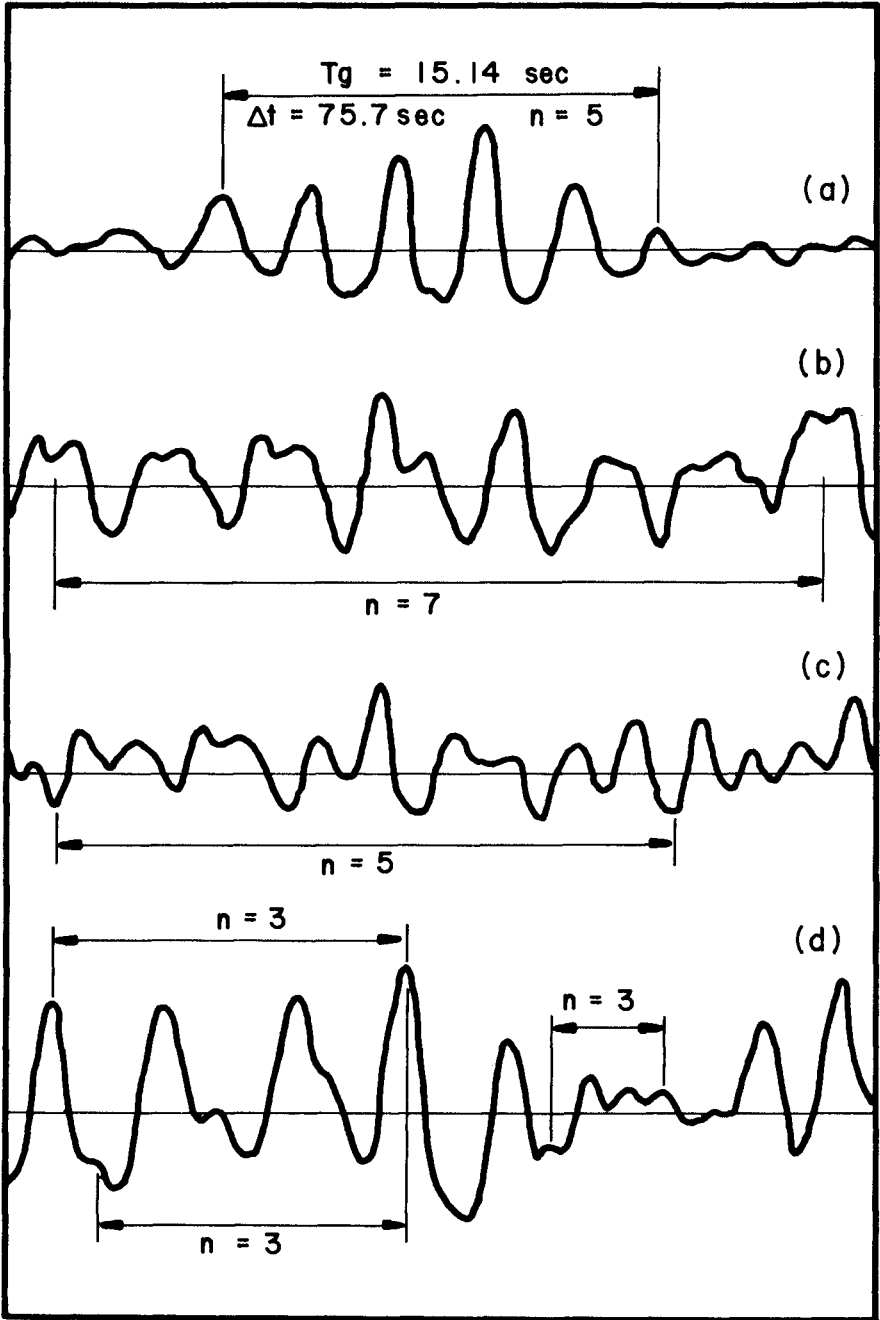
Figure I: PERIODIC WAVE SEQUENCES
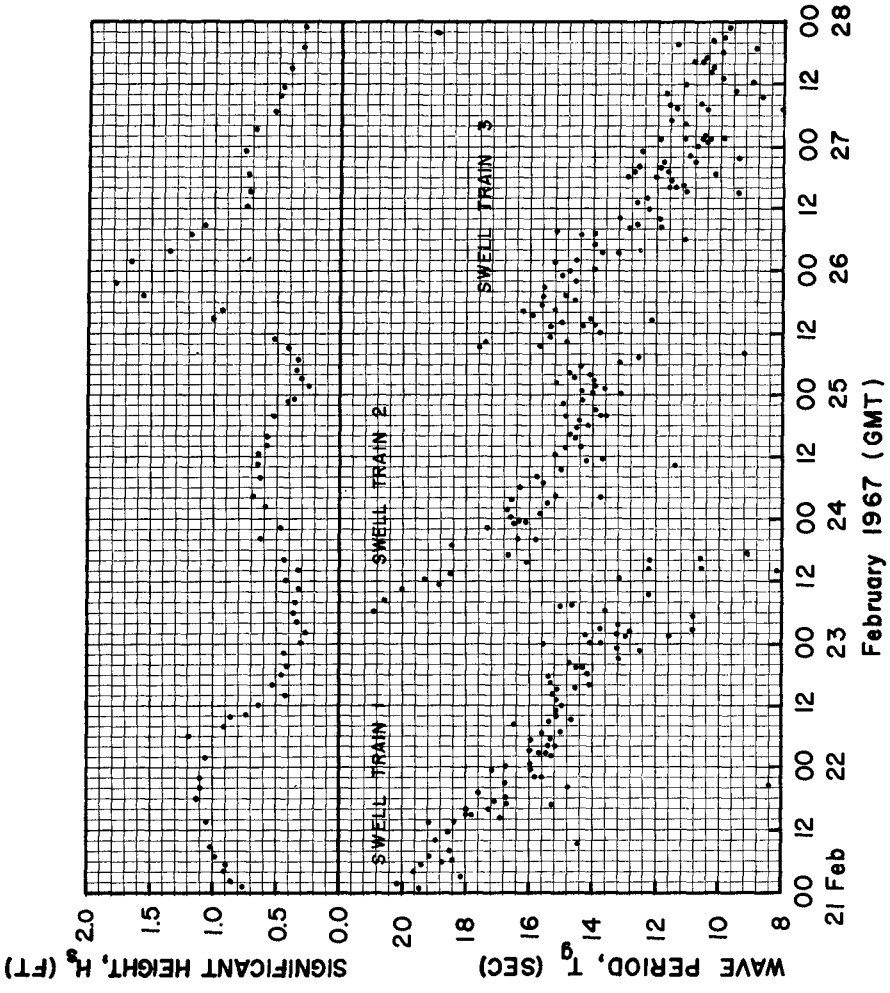
Figure 2: WAVE DATA FOR THREE SWELL TRAINS
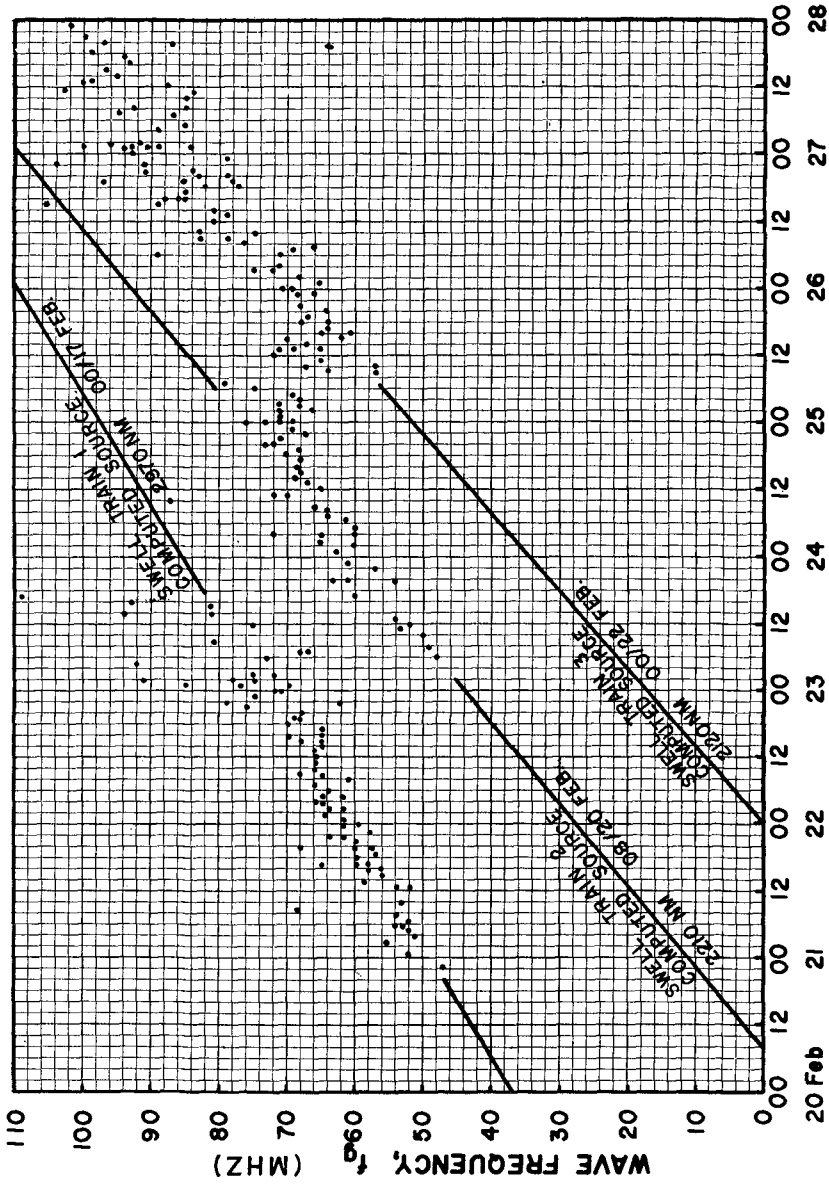
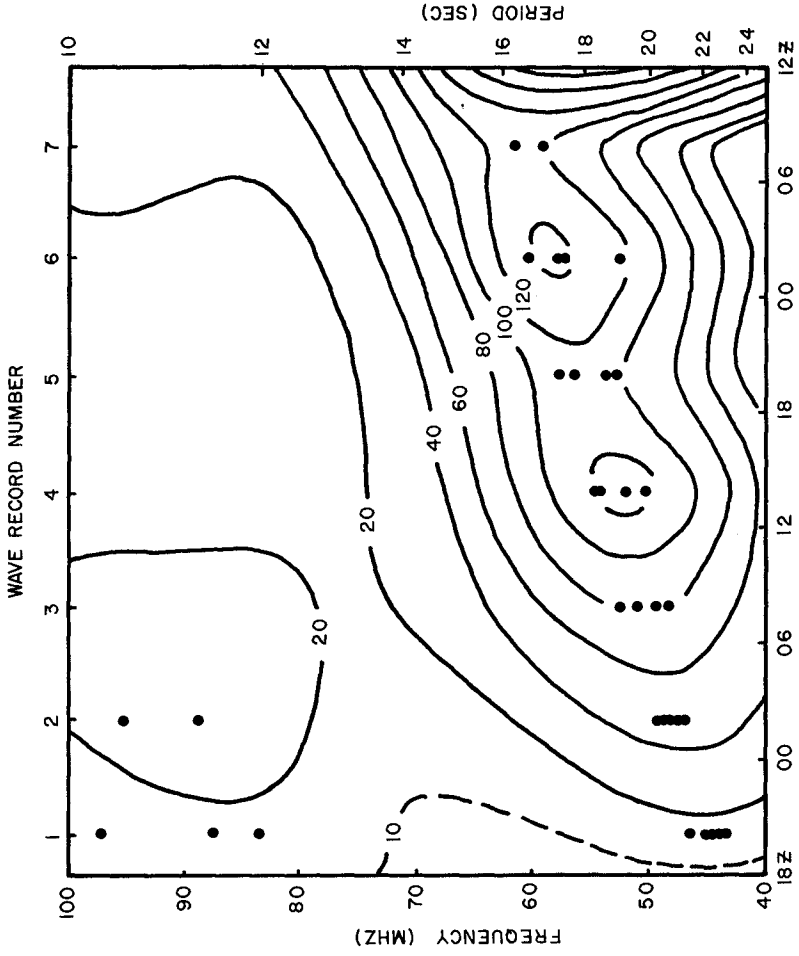FIGURE 3: FREQUENCY-TIME DISTRIBUTION FOR THREE SWELL TRAINS

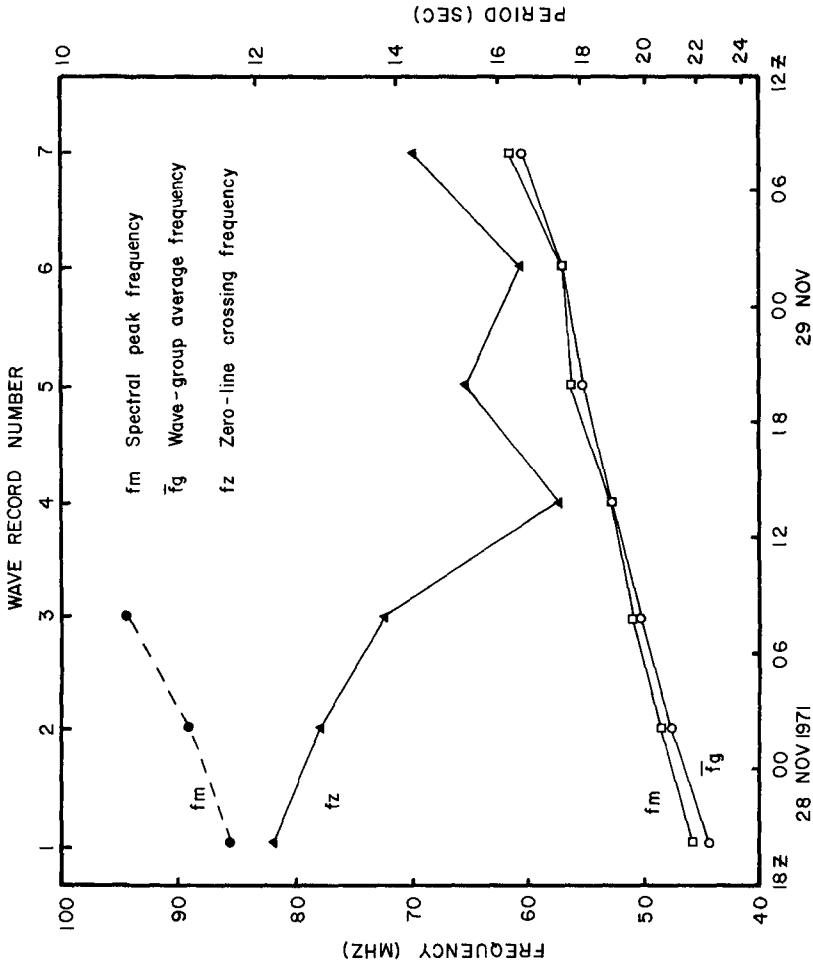Figure 4 : VALUES OF fg SUPERIMPOSED ON SPECTRAL DENSITY TOPOGRAPHY

Figure 5: COMPARISON OF FREQUENCIES OBTAINED FROM MANUAL AND SPECTRAL ANALYSES
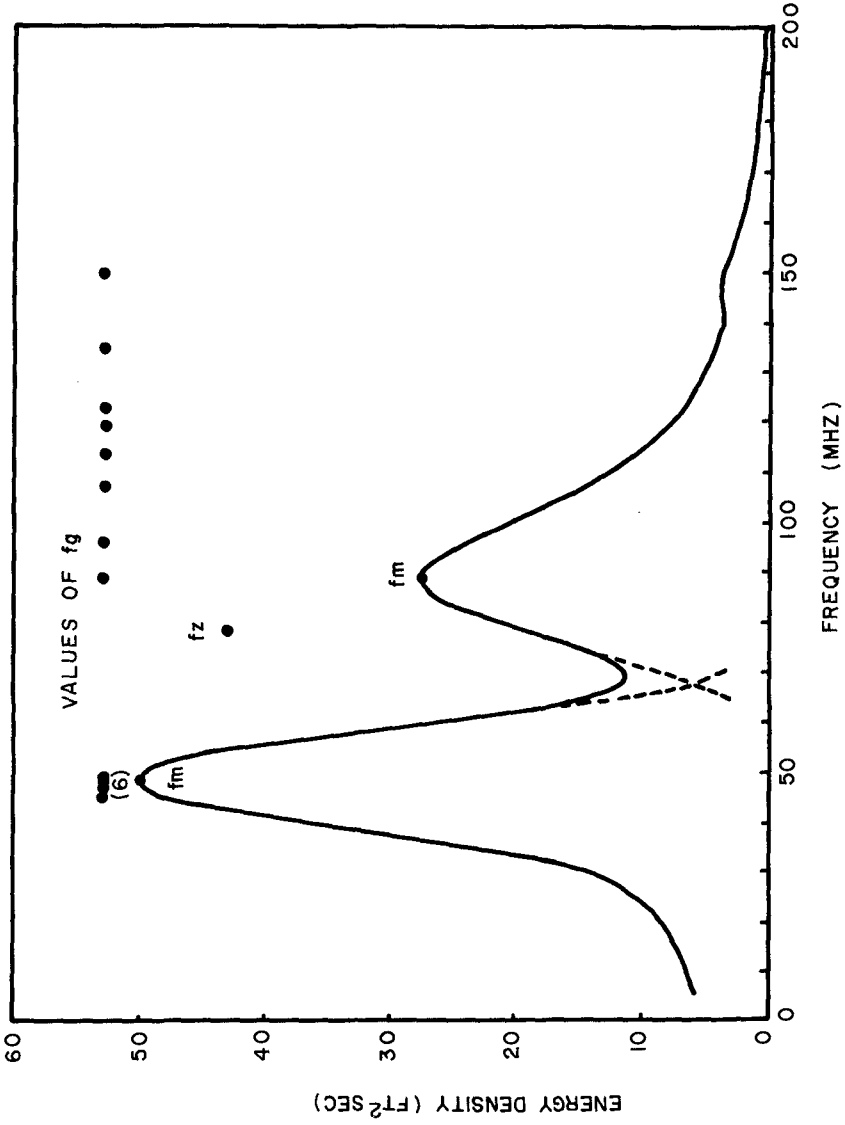
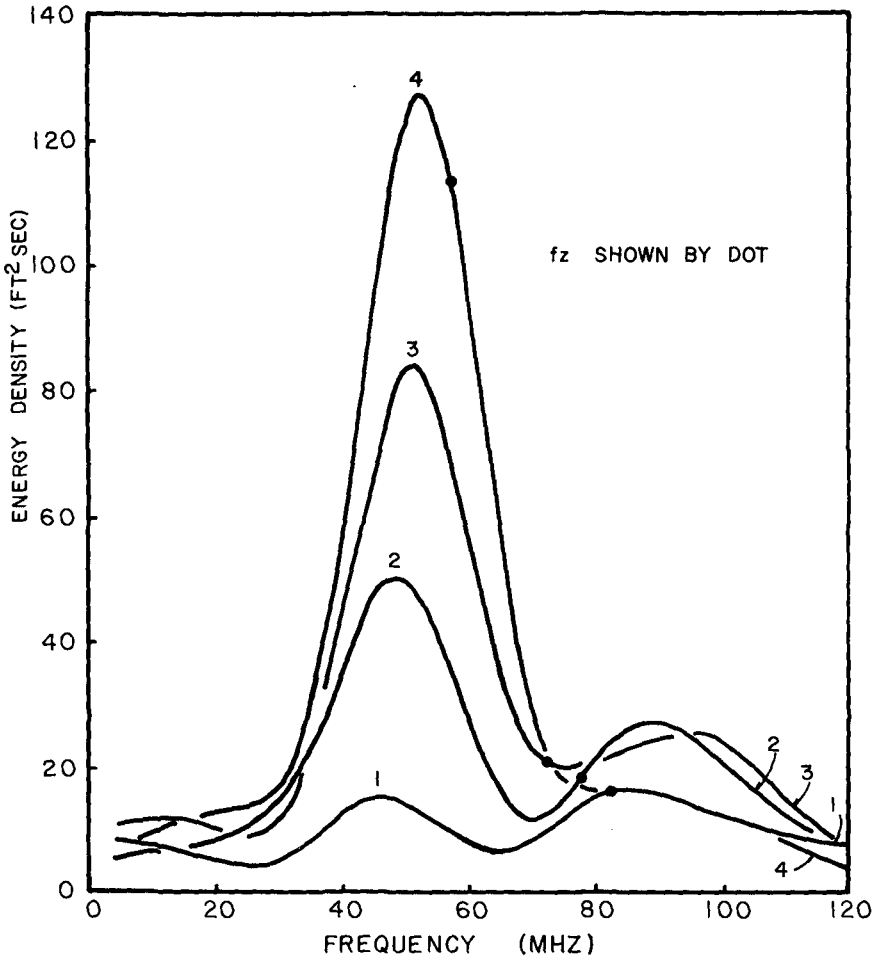Figure 6: FREQUENCY SPECTRUM OF WAVE RECORD NO. 2

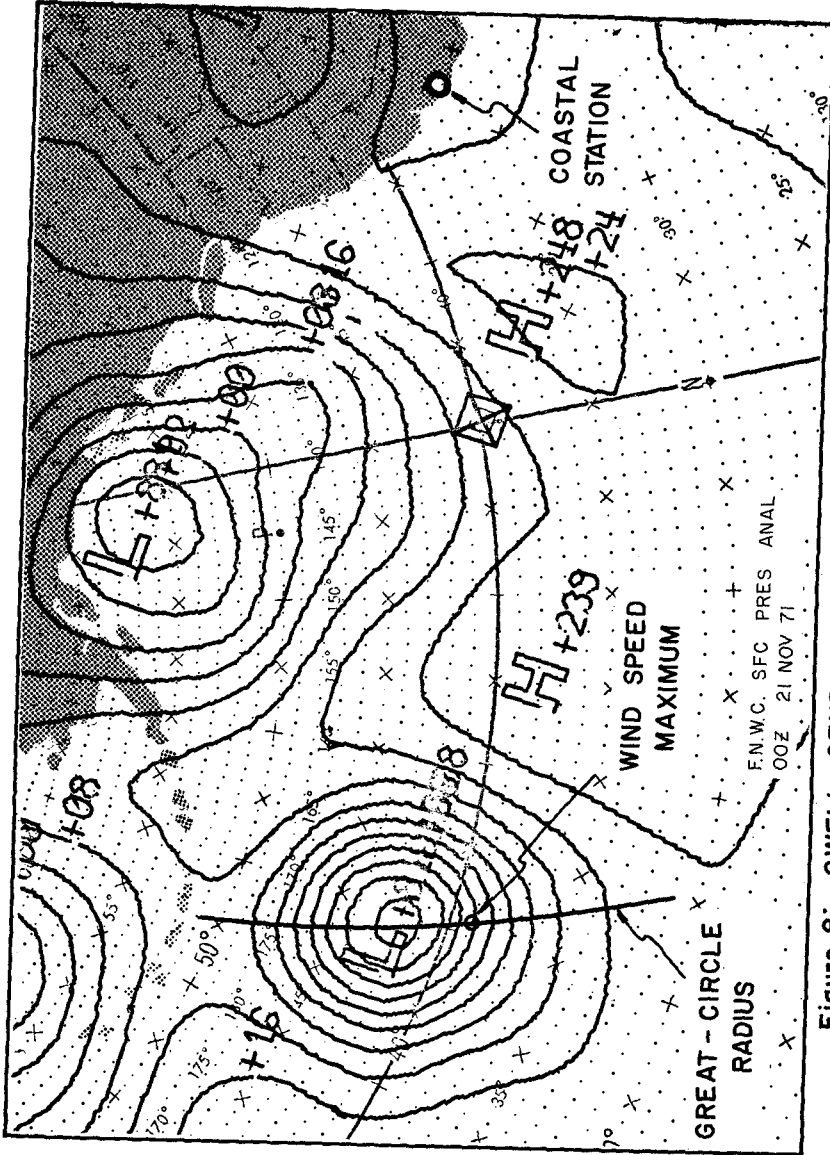Figure 7: FREQUENCY fz IN RELATION TO SPECTRA
FOR WAVE RECORDS I THROUGH 4

Figure 8: SWELL GENERATION POINT WITH RESPECT TO A COASTAL STATION