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Periodic Cellular Automata of Period-3

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ABSTRACT

Cellular automata are a new approach that has been rapidly used in many area of science. In this paper, we introduce a notion of periodic one-dimensional cellular automata. In addition, we calculate entropy of some periodic cellular automata of period-3 and present the simulation for some fractal type periodic cellular automata.

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INTRODUCTION

Cellular automata were originally introduced by John von Neumann in 1940 as a model of self-reproducing biological system. After that, cellular automata have been developed by the followers including John H. Coway in his 'Game of Life' project and Stephen Wolfram in his book entitled 'A New Kind of Science'. Cellular automaton is a self-organizing behavior system which depends on previous state under the effect of the specific local rule. It is a model of system which contains a grid of cells. Each cell has finite state and neighbourhood.

An elementary cellular automaton is the simplest type of cellular automaton (Wolfram, 1983). It has two possible states (0 or 1) and simplest neighbourhood where the cell itself, left and right neighbourhood. Martinez (2013) mentioned that Wolfram classified it into four classes of behavior which are Class I (uniform), Class II (periodic), Class III (chaotic) and Class IV (complex).

In this paper, we introduce periodic one-dimensional cellular automata of period three. We include the example of entropy of some periodic of period – 3 and present some simulation of that kind cellular automata.

Periodic Cellular Automata of Period-3:

Let $A=\{0,1,2,\cdots,r-1\}$ be an alphabet. Let

$x=\cdots x_{-k}x_{-k+1}\cdots x_{-1}\,.\,x_0x_1\cdots x_k\cdots$

a bi-sequence over A. A full shift space $A^{\mathbb{Z}}=\left\{x=(x_i)_{i\in\mathbb{Z}} : x_i\in A, \forall i\in\mathbb{Z}\right\}$ is a set of all bi-sequences. Let $x_{[-k,k]}=x_{-k}x_{-k+1}\cdots x_{-1}x_0x_1\cdots x_{k-1}x_k$ be a central block and $B_{2k+1}(A)=\left\{b=b_{-k}b_{-k+1}\cdots b_{-1}b_0b_1\cdots b_{k-1}b_k : b_i\in A\right\}$ be a set of all central blocks. The shift space $A^{\mathbb{Z}}$ is a Cantor space with respect to the following metric

$$d(x,y)=\begin{cases}2 & x_0\neq y_0\\ 2^{-k} & x_{[-k,k]}=y_{[-k,k]}\\ 0 & x=y\end{cases}$$

Let $\sigma : A^{\mathbb{Z}}\to A^{\mathbb{Z}}$ be a shift mapping $[\sigma(x)]_i=x_{i+1}$ for any $i\in\mathbb{Z}$.

Definition 1.1:

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A mapping $F : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is called *a cellular automata* if one has that $F \circ \sigma = \sigma \circ F$, i.e., $F(\sigma(x)) = \sigma(F(x))$ for any $x \in A^{\mathbb{Z}}$. (Hedlund, 1969 and Lind and Marcus, 1995)

In this paper, we aim to define periodic cellular automata of period – 3.

Definition 1.2:

A mapping $F : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is called *periodic cellular automata* of periodic – 3 if one has that $F \circ \sigma^{(3)} = \sigma^{(3)} \circ F$ i.e., $F(\sigma(\sigma(\sigma(x)))) = \sigma(\sigma(\sigma(F(x))))$ for any $x \in A^{\mathbb{Z}}$.

Let us define the following mapping $F_{012} : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ by local rules $f_0, f_1, f_2 : B_{2k+1}(A) \to A$

$$\left[F_{012}(x)\right]_i = \begin{cases} f_0\left(x_{-k}x_{-k+1}\cdots x_{-1}x_0x_1\ldots x_{k-1}x_k\right) & i \equiv 0 \, mod(3) \\ f_1\left(x_{-k}x_{-k+1}\cdots x_{-1}x_0x_1\ldots x_{k-1}x_k\right) & i \equiv 1 \, mod(3) \\ f_2\left(x_{-k}x_{-k+1}\cdots x_{-1}x_0x_1\ldots x_{k-1}x_k\right) & i \equiv 2 \, mod(3) \end{cases} \quad (1)$$

Proposition 1.1:

A mapping $F_{012} : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is a continuous periodic cellular automata of period – 3.

Proof. Let us first show that $F_{012} : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is continuous. Let $x \in A^{\mathbb{Z}}$ and $F_{012}(x) = y \in A^{\mathbb{Z}}$. Let $\varepsilon > 0$ and $m_0 \in \mathbb{N}$ such that $\frac{1}{2^{m_0}} < \varepsilon$. We choose $\delta > 0$ such that $\delta < \frac{1}{2^{1+k+m_0}}$ and let $u \in A^{\mathbb{Z}}$ be any element such that $d(u,x) < \delta$. This means that $x_{[-k-m_0,k+m_0]} = u_{[-k-m_0,k+m_0]}$. Let $F_{012}(u) = v \in A^{\mathbb{Z}}$. We then get that $\left[F_{012}(x)\right]_{[-m_0,m_0]} = \left[F_{012}(u)\right]_{[-m_0,m_0]}$, i.e., $d\left(F_{012}(u), F_{012}(x)\right) < \varepsilon$. This means that $F_{012} : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is continuous.

Let us show that $F_{012}\left(\sigma\left(\sigma\left(\sigma(x)\right)\right)\right) = \sigma\left(\sigma\left(\sigma\left(F_{012}(x)\right)\right)\right)$ for any $x \in A^{\mathbb{Z}}$. In fact, we have that

$$\left[F_{012}\left(\sigma\left(\sigma(x)\right)\right)\right]_i = \begin{cases} f_0\left(\left[\sigma^{(3)}(x)\right]_{-k+i}\cdots\left[\sigma^{(3)}(x)\right]_i\cdots\left[\sigma^{(3)}(x)\right]_{k+i}\right) & i \equiv 0 \, mod(3) \\ f_1\left(\left[\sigma^{(3)}(x)\right]_{-k+i}\cdots\left[\sigma^{(3)}(x)\right]_i\cdots\left[\sigma^{(3)}(x)\right]_{k+i}\right) & i \equiv 1 \, mod(3) \\ f_2\left(\left[\sigma^{(3)}(x)\right]_{-k+i}\cdots\left[\sigma^{(3)}(x)\right]_i\cdots\left[\sigma^{(3)}(x)\right]_{k+i}\right) & i \equiv 2 \, mod(3) \end{cases}$$

$$= \begin{cases} f_0\left(x_{-k+i+3}\cdots x_{i+3}\cdots x_{k+i+3}\right) & i \equiv 0 \, mod(3) \\ f_1\left(x_{-k+i+3}\cdots x_{i+3}\cdots x_{k+i+3}\right) & i \equiv 1 \, mod(3) \\ f_2\left(x_{-k+i+3}\cdots x_{i+3}\cdots x_{k+i+3}\right) & i \equiv 2 \, mod(3) \end{cases}$$

$$\left[\sigma^{(3)}\left(F_{012}(x)\right)\right]_i = \left[F_{012}(x)\right]_{i+3} = \begin{cases} f_0\left(x_{-k+i+3}\cdots x_{i+3}\cdots x_{k+i+3}\right) & i \equiv 0 \, mod(3) \\ f_1\left(x_{-k+i+3}\cdots x_{i+3}\cdots x_{k+i+3}\right) & i \equiv 1 \, mod(3) \\ f_2\left(x_{-k+i+3}\cdots x_{i+3}\cdots x_{k+i+3}\right) & i \equiv 2 \, mod(3) \end{cases}$$

Therefore, $F_{012} : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is a continuous periodic cellular automata of period – 3.

1. *Entropy of Periodic Cellular Automata: Example:*

An entropy of a periodic cellular automata $F : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is defined by the following formula

$$h(F) = \lim_{w\to\infty}\lim_{t\to\infty}\frac{\log R(w,t)}{t} \quad (2)$$

where $R(w,t)$ is the number of distinct rectangles of width w and height t occurring in space-time diagram (for more details see [Cattaneo *et al*. (2000), Hurd *et al*. (1992), Blancard *et al*. (1997)]. In this section, we shall provide a few examples for periodic cellular automata of period-3 in which their entropy might or might nor be zeros.

Let $A_3 = \{0,1,2\}$ and $B_3(A_3) = \{b = b_0b_1b_2 : \, b_i \in A_3\}$ be a set of all 3-blocks. Let $f_k : B_3(A_3) \to A_3$ be a local rule $f_k(b_0b_1b_2) = b_k$ for $k \in A_3$.

Let us define the following periodic cellular automata

$$\left[\boldsymbol{F}_{012}(\boldsymbol{x})\right]_i = \begin{cases} f_0\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 0 \,\mathbf{mod}(3) \\ f_1\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 1 \,\mathbf{mod}(3) \\ f_2\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 2 \,\mathbf{mod}(3) \end{cases} = \begin{cases} x_i & i \equiv 0 \,\mathbf{mod}(3) \\ x_{i+1} & i \equiv 1 \,\mathbf{mod}(3) \\ x_{i+2} & i \equiv 2 \,\mathbf{mod}(3) \end{cases} \tag{3}$$

$$\left[\boldsymbol{F}_{120}(\boldsymbol{x})\right]_i = \begin{cases} f_1\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 0 \,\mathbf{mod}(3) \\ f_2\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 1 \,\mathbf{mod}(3) \\ f_0\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 2 \,\mathbf{mod}(3) \end{cases} = \begin{cases} x_{i+1} & i \equiv 0 \,\mathbf{mod}(3) \\ x_{i+2} & i \equiv 1 \,\mathbf{mod}(3) \\ x_i & i \equiv 2 \,\mathbf{mod}(3) \end{cases} \tag{4}$$

$$\left[\boldsymbol{F}_{201}(\boldsymbol{x})\right]_i = \begin{cases} f_2\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 0 \,\mathbf{mod}(3) \\ f_0\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 1 \,\mathbf{mod}(3) \\ f_1\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 2 \,\mathbf{mod}(3) \end{cases} = \begin{cases} x_{i+2} & i \equiv 0 \,\mathbf{mod}(3) \\ x_i & i \equiv 1 \,\mathbf{mod}(3) \\ x_{i+1} & i \equiv 2 \,\mathbf{mod}(3) \end{cases} \tag{5}$$

$$\left[\boldsymbol{F}_{021}(\boldsymbol{x})\right]_i = \begin{cases} f_0\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 0 \,\mathbf{mod}(3) \\ f_2\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 1 \,\mathbf{mod}(3) \\ f_1\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 2 \,\mathbf{mod}(3) \end{cases} = \begin{cases} x_i & i \equiv 0 \,\mathbf{mod}(3) \\ x_{i+2} & i \equiv 1 \,\mathbf{mod}(3) \\ x_{i+1} & i \equiv 2 \,\mathbf{mod}(3) \end{cases} \tag{6}$$

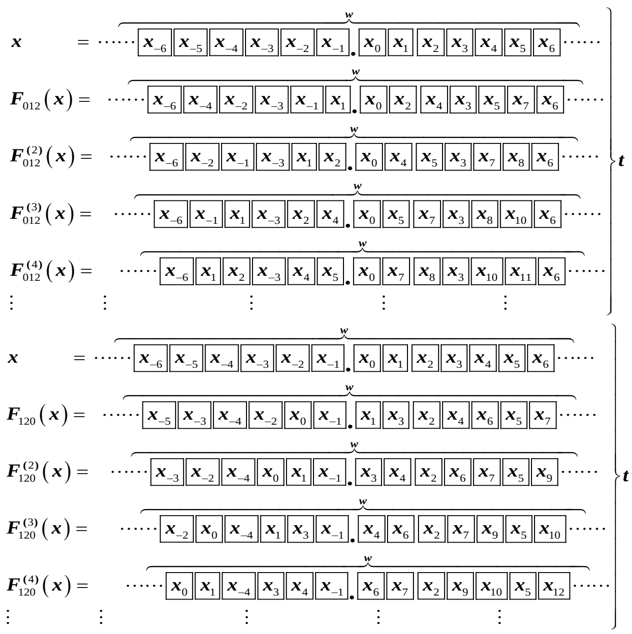
$$\left[\boldsymbol{F}_{102}(\boldsymbol{x})\right]_i = \begin{cases} f_1\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 0 \,\mathbf{mod}(3) \\ f_0\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 1 \,\mathbf{mod}(3) \\ f_2\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 2 \,\mathbf{mod}(3) \end{cases} = \begin{cases} x_{i+1} & i \equiv 0 \,\mathbf{mod}(3) \\ x_i & i \equiv 1 \,\mathbf{mod}(3) \\ x_{i+2} & i \equiv 2 \,\mathbf{mod}(3) \end{cases} \tag{7}$$

$$\left[\boldsymbol{F}_{210}(\boldsymbol{x})\right]_i = \begin{cases} f_2\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 0 \,\mathbf{mod}(3) \\ f_1\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 1 \,\mathbf{mod}(3) \\ f_0\left(x_i x_{i+1} x_{i+2}\right) & i \equiv 2 \,\mathbf{mod}(3) \end{cases} = \begin{cases} x_{i+2} & i \equiv 0 \,\mathbf{mod}(3) \\ x_{i+1} & i \equiv 1 \,\mathbf{mod}(3) \\ x_i & i \equiv 2 \,\mathbf{mod}(3) \end{cases} \tag{8}$$

Theorem 2.1:

The entropies of periodic cellular automatas given by even permutation (3) – (5) are log 3.

Proof. In order to calculate the entropy, we have to calculate the number of distinct rectangles $R(w,t)$ of width w and height t occurring in space-time diagram. Let us show how to calculate $R(w,t)$.



Journal of Applied Science and Agriculture, 10(5) Special 2015, Pages: 89-96

$$\left.\begin{array}{ll}
x & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-5}}\boxed{x_{-4}}\boxed{x_{-3}}\boxed{x_{-2}}\boxed{x_{-1}}_{\bullet}\boxed{x_{0}}\boxed{x_{1}}\boxed{x_{2}}\boxed{x_{3}}\boxed{x_{4}}\boxed{x_{5}}\boxed{x_{6}}}^{w}\cdots \\
F_{201}(x) & = \cdots \overbrace{\boxed{x_{-4}}\boxed{x_{-5}}\boxed{x_{-3}}\boxed{x_{-1}}\boxed{x_{-2}}\boxed{x_{0}}_{\bullet}\boxed{x_{2}}\boxed{x_{1}}\boxed{x_{3}}\boxed{x_{5}}\boxed{x_{4}}\boxed{x_{6}}\boxed{x_{8}}}^{w}\cdots \\
F_{201}^{(2)}(x) & = \cdots \overbrace{\boxed{x_{-3}}\boxed{x_{-5}}\boxed{x_{-1}}\boxed{x_{0}}\boxed{x_{-2}}\boxed{x_{2}}_{\bullet}\boxed{x_{3}}\boxed{x_{1}}\boxed{x_{5}}\boxed{x_{6}}\boxed{x_{4}}\boxed{x_{8}}\boxed{x_{9}}}^{w}\cdots \\
F_{201}^{(3)}(x) & = \cdots \overbrace{\boxed{x_{-1}}\boxed{x_{-5}}\boxed{x_{0}}\boxed{x_{2}}\boxed{x_{-2}}\boxed{x_{3}}_{\bullet}\boxed{x_{5}}\boxed{x_{1}}\boxed{x_{6}}\boxed{x_{8}}\boxed{x_{4}}\boxed{x_{9}}\boxed{x_{11}}}^{w}\cdots \\
F_{201}^{(4)}(x) & = \cdots \overbrace{\boxed{x_{0}}\boxed{x_{-5}}\boxed{x_{2}}\boxed{x_{3}}\boxed{x_{-2}}\boxed{x_{5}}_{\bullet}\boxed{x_{6}}\boxed{x_{1}}\boxed{x_{8}}\boxed{x_{9}}\boxed{x_{4}}\boxed{x_{11}}\boxed{x_{12}}}^{w}\cdots \\
\vdots & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{array}\right\} t$$

In all cases, $R(w,t) = 3^{w+t-1}$. Therefore, the entropies of periodic cellular automatas given by even permutation (3) – (5) are log 3.

Theorem 2.2:

The entropies of periodic cellular automatas given by odd permutation (6)-(8) are zero.

Proof. In order to calculate the entropy, we have to calculate the number of distinct rectangles $R(w,t)$ of width w and height t occurring in space-time diagram. Let us show how to calculate $R(w,t)$.

$$\left.\begin{array}{ll}
x & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-5}}\boxed{x_{-4}}\boxed{x_{-3}}\boxed{x_{-2}}\boxed{x_{-1}}_{\bullet}\boxed{x_{0}}\boxed{x_{1}}\boxed{x_{2}}\boxed{x_{3}}\boxed{x_{4}}\boxed{x_{5}}\boxed{x_{6}}}^{w}\cdots \\
F_{021}(x) & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{0}}\boxed{x_{0}}_{\bullet}\boxed{x_{0}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{6}}\boxed{x_{6}}\boxed{x_{6}}}^{w}\cdots \\
F_{021}^{(2)}(x) & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{0}}\boxed{x_{0}}_{\bullet}\boxed{x_{0}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{6}}\boxed{x_{6}}\boxed{x_{6}}}^{w}\cdots \\
F_{021}^{(3)}(x) & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{0}}\boxed{x_{0}}_{\bullet}\boxed{x_{0}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{6}}\boxed{x_{6}}\boxed{x_{6}}}^{w}\cdots \\
F_{021}^{(4)}(x) & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{-3}}\boxed{x_{0}}\boxed{x_{0}}_{\bullet}\boxed{x_{0}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{3}}\boxed{x_{6}}\boxed{x_{6}}\boxed{x_{6}}}^{w}\cdots \\
\vdots & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{array}\right\} t$$

$$\left.\begin{array}{ll}
x & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-5}}\boxed{x_{-4}}\boxed{x_{-3}}\boxed{x_{-2}}\boxed{x_{-1}}_{\bullet}\boxed{x_{0}}\boxed{x_{1}}\boxed{x_{2}}\boxed{x_{3}}\boxed{x_{4}}\boxed{x_{5}}\boxed{x_{6}}}^{w}\cdots \\
F_{102}(x) & = \cdots \overbrace{\boxed{x_{-5}}\boxed{x_{-5}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{1}}_{\bullet}\boxed{x_{1}}\boxed{x_{1}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{7}}\boxed{x_{7}}}^{w}\cdots \\
F_{102}^{(2)}(x) & = \cdots \overbrace{\boxed{x_{-5}}\boxed{x_{-5}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{1}}_{\bullet}\boxed{x_{1}}\boxed{x_{1}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{7}}\boxed{x_{7}}}^{w}\cdots \\
F_{102}^{(3)}(x) & = \cdots \overbrace{\boxed{x_{-5}}\boxed{x_{-5}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{1}}_{\bullet}\boxed{x_{1}}\boxed{x_{1}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{7}}\boxed{x_{7}}}^{w}\cdots \\
F_{102}^{(4)}(x) & = \cdots \overbrace{\boxed{x_{-5}}\boxed{x_{-5}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{-2}}\boxed{x_{1}}_{\bullet}\boxed{x_{1}}\boxed{x_{1}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{4}}\boxed{x_{7}}\boxed{x_{7}}}^{w}\cdots \\
\vdots & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{array}\right\} t$$

$$\left.\begin{array}{ll}
x & = \cdots \overbrace{\boxed{x_{-6}}\boxed{x_{-5}}\boxed{x_{-4}}\boxed{x_{-3}}\boxed{x_{-2}}\boxed{x_{-1}}_{\bullet}\boxed{x_{0}}\boxed{x_{1}}\boxed{x_{2}}\boxed{x_{3}}\boxed{x_{4}}\boxed{x_{5}}\boxed{x_{6}}}^{w}\cdots \\
F_{210}(x) & = \cdots \overbrace{\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-1}}\boxed{x_{-1}}\boxed{x_{-1}}_{\bullet}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{8}}}^{w}\cdots \\
F_{210}^{(2)}(x) & = \cdots \overbrace{\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-1}}\boxed{x_{-1}}\boxed{x_{-1}}_{\bullet}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{8}}}^{w}\cdots \\
F_{210}^{(3)}(x) & = \cdots \overbrace{\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-1}}\boxed{x_{-1}}\boxed{x_{-1}}_{\bullet}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{8}}}^{w}\cdots \\
F_{210}^{(4)}(x) & = \cdots \overbrace{\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-4}}\boxed{x_{-1}}\boxed{x_{-1}}\boxed{x_{-1}}_{\bullet}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{2}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{5}}\boxed{x_{8}}}^{w}\cdots \\
\vdots & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{array}\right\} t$$

In all cases, $R(w,t)$ is free of t and may take a value in the set $\{3^{w}, 3^{w+1}\}$. Therefore, the entropies of periodic cellular automatas given by odd permutation (6) – (8) are zero.

Journal of Applied Science and Agriculture, 10(5) Special 2015, Pages: 89-96

2. Fractal Structure Periodic Cellular Automata:

Example 3.1:

Let us consider the following local rules that have same fractal structures.

$f_{18}(1,1,1) = f_{18}(1,1,0) = f_{18}(1,0,1) = f_{18}(0,1,1) = f_{18}(0,1,0) = f_{18}(0,0,0) = 0$ and $f_{18}(1,0,0) = f_{18}(0,0,1) = 1$

$f_{26}(1,1,1) = f_{26}(1,1,0) = f_{26}(1,0,1) = f_{26}(0,1,0) = f_{26}(0,0,0) = 0$ and $f_{26}(0,1,1) = f_{26}(1,0,0) = f_{26}(0,0,1) = 1$

$f_{90}(1,1,1) = f_{90}(1,0,1) = f_{90}(0,1,0) = f_{90}(0,0,0) = 0$ and $f_{90}(1,1,0) = f_{90}(1,0,0) = f_{90}(0,1,1) = f_{90}(0,0,1) = 1$

We define periodic cellular automata of period-3

$$[\boldsymbol{F}_{012}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{18}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{26}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{90}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{021}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{18}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{26}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{90}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{102}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{18}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{26}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{90}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{120}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{18}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{26}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{90}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{201}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{18}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{26}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{90}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{210}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{18}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{26}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{90}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \end{cases}$$

Fig. 1: Simulation of periodic cellular automata $\boldsymbol{F}_{012}(\boldsymbol{x})$.

This figure implies for $\boldsymbol{F}_{021}(\boldsymbol{x}), \boldsymbol{F}_{102}(\boldsymbol{x})$, $\boldsymbol{F}_{120}(\boldsymbol{x}), \boldsymbol{F}_{201}(\boldsymbol{x})$ and $\boldsymbol{F}_{210}(\boldsymbol{x})$. They have exactly the same fractal structures.

Example 3.2:

Let us consider the following local rules that have two different fractal structures.

$f_{82}(1,1,1) = f_{82}(1,0,1) = f_{82}(0,1,1) = f_{82}(0,1,0) = f_{82}(0,0,0) = 0$ and $f_{82}(1,1,0) = f_{82}(1,0,0) = f_{82}(0,0,1) = 1$

Journal of Applied Science and Agriculture, 10(5) Special 2015, Pages: 89-96

$f_{22}(1,1,1) = f_{22}(1,1,0) = f_{22}(1,0,1) = f_{22}(0,1,1) = f_{22}(0,0,0) = 0$ and $f_{22}(1,0,0) = f_{22}(0,1,0) = f_{22}(0,0,1) = 1$

We define periodic cellular automata of period-3

$$[\boldsymbol{F}_{012}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{22}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{021}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{22}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{102}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{22}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{120}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{22}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{201}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{22}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \\ \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \end{cases}$$

$$[\boldsymbol{F}_{210}(\boldsymbol{x})]_i = \begin{cases} \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 2\,\mathbf{mod}(3) \\ \boldsymbol{f}_{22}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 1\,\mathbf{mod}(3) \\ \boldsymbol{f}_{82}(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}) & \boldsymbol{i} \equiv 0\,\mathbf{mod}(3) \end{cases}$$

Fig. 2: Simulation of periodic cellular automata $\boldsymbol{F}_{012}(\boldsymbol{x})$ and $\boldsymbol{F}_{210}(\boldsymbol{x})$.

Fig. 3: Simulation of periodic cellular automata $\boldsymbol{F}_{021}(\boldsymbol{x})$ and $\boldsymbol{F}_{120}(\boldsymbol{x})$.

Fig. 4: Simulation of periodic cellular automata $\boldsymbol{F}_{102}(\boldsymbol{x})$ and $\boldsymbol{F}_{201}(\boldsymbol{x})$.

Journal of Applied Science and Agriculture, 10(5) Special 2015, Pages: 89-96

In these three figures show different structures and none of them are fractal.

Example 3.3:

Let us consider the following local rules that have three different fractal structures.

$f_{126}(1,1,1)=f_{126}(0,0,0)=0$ and $f_{126}(1,1,0)=f_{126}(1,0,1)=f_{126}(1,0,0)=f_{126}(0,1,1)=f_{126}(0,1,0)=f_{126}(0,0,1)=1$

$f_{146}(1,1,0)=f_{146}(1,0,1)=f_{146}(0,1,1)=f_{146}(0,1,0)=f_{146}(0,0,0)=0$ and $f_{146}(1,1,1)=f_{146}(1,0,0)=f_{146}(0,0,1)=1$

$f_{161}(1,1,0)=f_{161}(1,0,0)=f_{161}(0,1,1)=f_{161}(0,1,0)=f_{161}(0,0,1)=0$ and $f_{161}(1,1,1)=f_{161}(1,0,1)=f_{161}(0,0,0)=1$

We define periodic cellular automata of period-3

$$\left[\boldsymbol{F}_{012}(\boldsymbol{x})\right]_i=\begin{cases} \boldsymbol{f}_{126}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 0\,\mathbf{mod}(3)\\ \boldsymbol{f}_{146}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 1\,\mathbf{mod}(3)\\ \boldsymbol{f}_{161}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 2\,\mathbf{mod}(3)\end{cases}$$

$$\left[\boldsymbol{F}_{021}(\boldsymbol{x})\right]_i=\begin{cases} \boldsymbol{f}_{126}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 0\,\mathbf{mod}(3)\\ \boldsymbol{f}_{146}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 2\,\mathbf{mod}(3)\\ \boldsymbol{f}_{161}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 1\,\mathbf{mod}(3)\end{cases}$$

$$\left[\boldsymbol{F}_{102}(\boldsymbol{x})\right]_i=\begin{cases} \boldsymbol{f}_{126}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 1\,\mathbf{mod}(3)\\ \boldsymbol{f}_{146}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 0\,\mathbf{mod}(3)\\ \boldsymbol{f}_{161}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 2\,\mathbf{mod}(3)\end{cases}$$

$$\left[\boldsymbol{F}_{120}(\boldsymbol{x})\right]_i=\begin{cases} \boldsymbol{f}_{126}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 1\,\mathbf{mod}(3)\\ \boldsymbol{f}_{146}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 2\,\mathbf{mod}(3)\\ \boldsymbol{f}_{161}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 0\,\mathbf{mod}(3)\end{cases}$$

$$\left[\boldsymbol{F}_{201}(\boldsymbol{x})\right]_i=\begin{cases} \boldsymbol{f}_{126}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 2\,\mathbf{mod}(3)\\ \boldsymbol{f}_{146}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 0\,\mathbf{mod}(3)\\ \boldsymbol{f}_{161}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 1\,\mathbf{mod}(3)\end{cases}$$

$$\left[\boldsymbol{F}_{210}(\boldsymbol{x})\right]_i=\begin{cases} \boldsymbol{f}_{126}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 2\,\mathbf{mod}(3)\\ \boldsymbol{f}_{146}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 1\,\mathbf{mod}(3)\\ \boldsymbol{f}_{161}\left(\boldsymbol{x}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_{i+1}\right) & \boldsymbol{i}\equiv 0\,\mathbf{mod}(3)\end{cases}$$

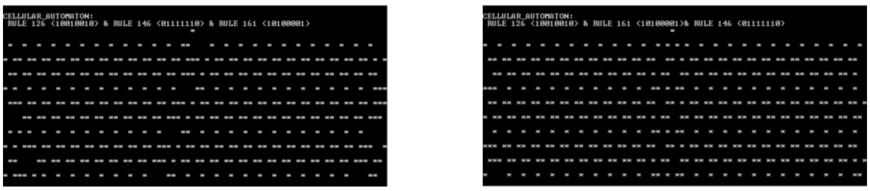


Fig. 5: Simulation of periodic cellular automata $\boldsymbol{F}_{012}(\boldsymbol{x})$ and $\boldsymbol{F}_{021}(\boldsymbol{x})$.

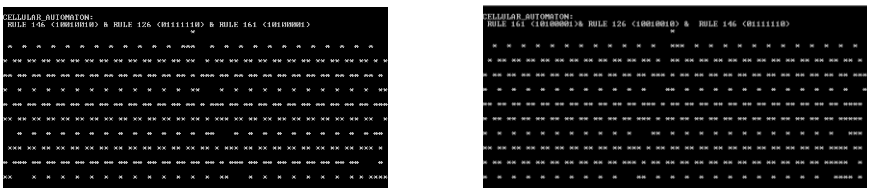
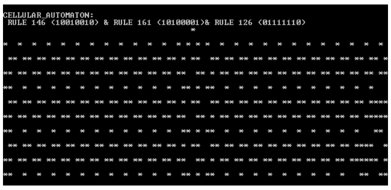


Fig. 6: Simulation of periodic cellular automata $\boldsymbol{F}_{102}(\boldsymbol{x})$ and $\boldsymbol{F}_{120}(\boldsymbol{x})$.

Journal of Applied Science and Agriculture, 10(5) Special 2015, Pages: 89-96



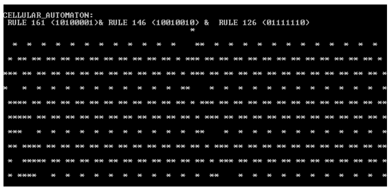


Fig. 7: Simulation of periodic cellular automata $\boldsymbol{F}_{201}(\boldsymbol{x})$ and $\boldsymbol{F}_{210}(\boldsymbol{x})$.

It is clear that all figures do not have fractal structure.

3. Conclusion:

It is shown that the behavior of periodic cellular automata essentially depends on order of components. It is presented examples where for even permutations corresponding entropy is positive and for odd permutations the corresponding entropy is zero.

Since this paper had investigated periodic cellular automata of period – 3 with two states, it is expected that further studies will be carried out on using more states and higher period to see whether there are more interesting cases in the findings.

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